Leverage and the Limits of Arbitrage Pricing: Implications for Dividend Strips and the Term Structure of Equity Risk Premia

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ABSTRACT

Negligible pricing frictions in underlying asset markets can become greatly magnified when using no-arbitrage arguments to price derivative claims. Amplification occurs when a replicating portfolio contains partially offsetting positions that lever up exposures to primary market frictions, and can cause arbitrarily large biases in synthetic return moments. We show theoretically and empirically how synthetic dividend strips, which shed light on the pricing of risks at different horizons, are impacted by this phenomenon. Dividend strips are claims to dividends paid over future time intervals, and can be replicated by highly levered long-short positions in futures contracts written on the same underlying index, but with different maturities. We show that tiny pricing frictions can help to reproduce a downward-sloping term structure of equity risk premia, excess volatility, return predictability, and a market beta substantially below one, consistent with empirical evidence. Using more robust return measures we find smaller point estimates of the returns to short-term dividend claims, and little support for a statistical or economic difference between the returns to short- versus long-term dividend claims.

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Abstract

Negligible pricing frictions in underlying asset markets can become greatly magnified when using no-arbitrage arguments to price derivative claims. Amplification occurs when a replicating portfolio contains partially offsetting positions that lever up exposures to primary market frictions, and can cause arbitrarily large biases in synthetic return moments. We show theoretically and empirically how synthetic dividend strips, which shed light on the pricing of risks at different horizons, are impacted by this phenomenon. Dividend strips are claims to dividends paid over future time intervals, and can be replicated by highly levered long-short positions in futures contracts written on the same underlying index, but with different maturities. We show that tiny pricing frictions can help to reproduce a downward-sloping term structure of equity risk premia, excess volatility, return predictability, and a market beta substantially below one, consistent with empirical evidence. Using more robust return measures we find smaller point estimates of the returns to short-term dividend claims, and little support for a statistical or economic difference between the returns to short- versus long-term dividend claims.

I. Introduction

Following Mehra and Prescott (1985), numerous authors have attempted to explain the high average returns on equity relative to historical aggregate consumption risk. Two explanations that have received considerable attention are the habit formation model of Campbell and Cochrane (1999) and the long-run risks model of Bansal and Yaron (2004). Both imply under standard calibrations that long-horizon equity cash flows are riskier and receive higher returns than short-horizon equity cash flows. In other words, these models generate term structures of equity risk premia that are upward-sloping.[1] Lettau and Wachter (2007) point out the importance of the term structure of equity risk premia for empirically evaluating these explanations, and note tension with another regularity, the value premium.[2]

Binsbergen, Brandt, and Koijen ("BBK", 2012) propose to measure the term structure of equity risk premia by calculating the returns on dividend strips, which are claims to dividends paid over future time intervals. Until recently, dividend strips have traded in over-the-counter markets where data are not readily available. BBK provide the insight that dividend strips can be replicated by long-short positions in liquid futures and spot markets, relying on futures-spot parity, or by appropriate positions in puts and calls additionally requiring that put-call parity holds. Using this approach, BBK find higher returns on short-term versus long-term dividend claims, which poses a challenge to traditional models of the equity premium.[3]

We show that the return moments of synthetic dividend strips can be significantly biased because of the impact of small pricing frictions. Our claim may seem surprising.

[1] Rare disaster models (Gabaix, 2008, Barro, 2006, Rietz, 1988) imply a flat term structure of equity risk premia. See Binsbergen, Brandt, and Koijen (2012) for a full discussion of the implications of leading asset pricing models for the term structure of equity risk premia, including prior literature and calibrations.

[2] Chen (2012) offers a reconciliation of an upward-sloping equity premium with the value premium by providing alternative measures of cash flow growth and cash flow duration. He finds that value stocks have duration similar to growth stocks in buy-and-hold portfolios, and greater than growth stocks in rebalanced portfolios.

[3] Croce, Lettau, and Ludvigson (2011) develop a consumption-based model with a downward-sloping equity premium under imperfect information. Wang (2011) shows that imperfect information can help to match a downward-sloping equity premium as well as other real and financial market moments in a production economy.

Dividend strips can be replicated with highly liquid futures contracts, and BBK use careful empirical methods to help alleviate concerns about microstructure frictions. The central insight we offer is that, even when pricing frictions have tiny impacts on any leg of a compound trade involving long and short positions, their cumulative effect relative to the value of the net position can be hundreds of times larger. The amplification of primary-market pricing frictions in a long-short trade occurs because of the effects of implicit leverage.

As an example, suppose one buys an index claim and takes a short position in an offsetting one-year futures contract. The net claim is a synthetic dividend strip that entitles one to all dividends paid on the index over the next year. Due to leverage, the net value of the dividend strip is only a small fraction of the gross value of either the long or the short side of the trade. To see this quantitatively, assume an annualized price-dividend ratio of 50, roughly consistent with recent experience.[4] Normalizing the long value to \$100, the offsetting futures position would have a notional amount in the neighborhood of \$98, and the strip value would be about \$2. Suppose that microstructure frictions have tiny impacts of a few basis points (cents) on either the long side or the short side of the trade, or on the synchronicity of the two prices. These few cents of mispricing may be irrelevant compared to the \$100 gross value of the long side, but are nonetheless substantial in comparison with the \$2 net value of the dividend strip. The magnification of small pricing frictions can cause large biases in synthetic return moments.

One symptom of pricing frictions is the large negative first-order autocorrelation, about -30% at a monthly frequency, in synthetic dividend strip returns. Our analysis shows that two distinct channels explain these extreme reversals. First, negative autocorrelations occur under bid-ask bounce or iid measurement error (Niederhoffer and Osborne, 1966, Blume and Stambaugh, 1983). High implicit leverage in synthetic divi-

[4]During the 1996-2009 sample period considered by BBK, the U.S. annualized price-dividend ratio averaged approximately 60, ranging from a low of 28 to a high of 90, where the monthly stock market price-dividend ratio is calculated following the method of Shiller (2005) with data downloaded from http://www.econ.yale.edu/ shiller/data.htm.

dend strips inflates small measurement errors from the primary markets, causing large negative autocorrelations in the return series.

A second type of microstructure friction, asynchronous price adjustment, also contributes to large negative autocorrelations in dividend strip returns. This result may be surprising since asynchronous price adjustment is typically associated with positive autocorrelation in portfolio returns (e.g., Lo and MacKinlay (1990)). Positive autocorrelation occurs, for example, in a portfolio with positive portfolio weights where all assets have positive exposure to the same underlying fundamental risk. Since each asset in the portfolio responds to the same shock at different times, positive autocorrelations occur. For the long-short portfolios we consider the effects are entirely different. If one futures contract responds more quickly to fundamental news in a long-short calendar spread, the initial return does not fully incorporate the hedging effect of the opposing position, and in a subsequent period the observed return will tend to be reversed. This argument does not require one side of the trade to always be more informationally efficient than the other. Large negative autocorrelations are generated simply by having the long and the short side react to a shock at not precisely the same time.

The negative autocorrelation induced in portfolio returns by microstructure frictions creates an upward bias in average one-month simple returns, following from Blume and Stambaugh (1983). Boguth, Carlson, Fisher, and Simutin ("BCFS", 2012) provide a formula to approximate the bias in simple returns, based on the effects of Jensen's inequality. A substantial portion of the high short-horizon average returns of dividend strips is explained by their extreme negative autocorrelations, in turn associated with microstructure frictions.

We provide a theoretical analysis showing that all of the anomalous findings about dividend strips can be explained by the magnification of small primary-market frictions. In a calibrated model where the true term structure of equity risk premia is flat, we generate in synthetic dividend strips a strongly downward-sloping term structure, excess volatility, large negative return autocorrelations, return predictability from the price-

dividend ratio, and a market model beta substantially less than one.

We further show that alternative measures of returns that are not as sensitive to microstructure frictions lead to different conclusions about the difference between short- and long-term asset returns. In particular, the apparent large returns of dividend strips are substantially diminished in annual rather than monthly return intervals, and in average logarithmic returns.

Our paper relates to the literature on limits to arbitrage, which emphasizes that arbitrageurs may fail to bring prices fully toward their fundamental values for a variety of reasons.[5] An important part of the literature considers constraints on short sales or the leverage of arbitrageurs, as well as fire-sale externalities, as important limitations.[6] The point we make is different, relating to the leverage required to execute an individual arbitrage trade, as opposed to the leverage or exposure of an individual arbitrageur or group of arbitrageurs. In particular, taking partially offsetting positions in underlying markets, as is common in arbitrage strategies, can dramatically lever up small pricing frictions from the primary markets. Because of this amplification, errors in synthetic prices and returns can be orders of magnitude larger than in the underlying markets, creating a significant obstacle for both those who might wish to carry out arbitrage, as well as empiricists interested in estimation and inference.

Section II provides a general framework within which to understand our general idea about the magnification of primary-market frictions when using arbitrage pricing techniques. Section III introduces dividend strips, and provides a back-of-the envelope estimate of the magnitude of the bias in dividend strip mean returns. Section IV gives a theoretical analysis of bias in dividend strip moments. Section V develops our calibrated model, and Section VI provides additional empirical results. Section VII concludes.

[5] These reasons include uncertainty over how long mispricing can persist and solvency constraints, issues related to the separation of capital from knowledge of mispricing, and the absence of close substitutes with which to arbitrage (see, e.g., Shleifer and Summers (1990), De Long, Shleifer, Summers, and Waldmann (1990), Shleifer and Vishny (1997), Shleifer (1986), Pontiff (2006)).

[6] See, e.g., Miller (1977), Kyle and Xiong (2001), Gromb and Vayanos (2002), Morris and Shin (2004), Allen and Gale (2005), Brunnermeier and Pedersen (2009), Stein (2009), Frazzini and Pedersen (2011).

II. Microstructure Bias in Levered Portfolios

The key idea of our paper is that the leverage implicit in any long-short partially hedged strategy can significantly amplify the importance of even tiny microstructure frictions present in any leg of the strategy. We first develop this idea in a general setting.

A. Amplification of Microstructure Frictions

We start by assuming a trading strategy where the fundamental value V of a synthetic claim can be decomposed into the true values of long (L) and short (S) positions:

$$V = L - S > 0. \tag{1}$$

In practice, pricing frictions make the fundamental values of L, S, and hence V imperfectly observable. Relevant pricing frictions could be bid-ask spreads, trading costs, or delays in price discovery or trade execution.

We show that even when such pricing frictions are negligible in the primary markets where L and S are traded, they can have large impacts on synthetic pricing of V. Let the observed value of the synthetic claim be given by

$$V^o = L^o - S^o, \tag{2}$$

where

$$L^o = L(1 + \varepsilon_L), \tag{3}$$

$$S^o = S(1 + \varepsilon_S). \tag{4}$$

We assume that observed prices are unbiased, $\mathbb{E}[\varepsilon_L] = \mathbb{E}[\varepsilon_S] = 0$, and for sake of generality in this section, no further assumptions on ε_L or ε_S are imposed.

The elasticities of the synthetic price V^o with respect to changes in the microstructure noises in the primary markets are given by

$$\frac{\partial V^o/V^o}{\partial \varepsilon_L/(1+\varepsilon_L)} = \frac{L^o}{L^o - S^o} \tag{5}$$

$$\frac{\partial V^o/V^o}{\partial \varepsilon_S/(1+\varepsilon_S)} = -\frac{S^o}{L^o - S^o}. \tag{6}$$

Importantly, the absolute values of these elasticities increase without bound in the observed leverage as $S^o \uparrow L^o$. Hence, even when a claim can be replicated using instruments that are highly liquid and whose prices are observed with negligible pricing errors, the pricing errors on the synthetic claim itself may be arbitrarily large. In the rest of this paper we show how these magnified pricing errors impact all return moments of the synthetic claim, including means, variances, covariances, and autocorrelations.

B. Measurement Error and Bias in Return Moments

The effect of microstructure frictions on moments of observed returns is a fundamental topic in finance. Blume and Stambaugh (1983) and other authors show how measurement errors create positive bias and negative autocorrelation in the returns of portfolios, such as equal-weighted portfolios, that are frequently rebalanced to fixed weights.[7]

In non-rebalanced portfolios such as stock market indices, asynchronous price adjustment of portfolio components causes positive autocorrelations and impacts volatilities.[8] Scholes and Williams (1977) and Lo and MacKinlay (1990) show that such microstructure frictions do not alter the mean returns of buy-and-hold portfolios; however, both studies consider only logarithmic returns. In more recent work, BCFS show that microstructure frictions also impact the average simple returns and alphas of buy-and-hold portfolios. Most empirical studies focus on average simple monthly returns as indicators of performance, but longer horizon returns are more robust to microstructure noise.

Following from our results in this section, negligible microstructure frictions in primary markets can generate large mispricing in synthetically-valued claims. From prior literature, these pricing errors can effect a variety of return moments. We now show these effects in the context of dividend strips.

[7] Roll (1983) investigates the effects of daily rebalancing over multiple periods. Recently, Asparouhova, Bessembinder, and Kalcheva (2012) show that even lower-frequency monthly rebalancing can result in strong biases in returns, especially for portfolios of illiquid stocks whose average returns can be overstated by more than 0.4% monthly.

[8] See for example Niederhoffer and Osborne (1966), Scholes and Williams (1977), Lo and MacKinlay (1990), Boudoukh, Richardson, and Whitelaw (1994), Ahn, Boudoukh, Richardson, and Whitelaw (2002)

III. Dividend Strip Returns

Following BBK, we introduce dividend strips and describe their empirical properties. We also provide a first approximation of the magnitude of the bias in the mean simple returns of synthetic dividend strips.

A dividend strip entitles the holder to all dividends paid between dates $t+T_1$ and $t+T_2$, where t denotes the current date. The value of a dividend strip at date t is:

$$\mathcal{P}_{t,T_1,T_2} \equiv \sum_{\tau=T_1+1}^{T_2} E_t\left(\frac{M_{t+\tau}}{M_t} D_{t+\tau}\right), \tag{7}$$

where M_t and D_t respectively denote the stochastic discount factor and dividend payments at date t. When $T_1 = 0$, the dividend strip is the "short-term asset." Under absence of arbitrage the short-term asset value is:

$$\mathcal{P}_{t,T} \equiv \mathcal{P}_{t,0,T} = S_t - e^{-r_{t,T}T} F_{t,T}, \tag{8}$$

where S_t denotes the spot value of the equity claim, $F_{t,T}$ is the forward price, and $r_{t,T}$ is the risk-free rate of interest for a bond maturing at date $t+T$. In the general case where $T_1 \geq 0$ the price of a dividend strip is:

$$\mathcal{P}_{t,T_1,T_2} = \mathcal{P}_{t,T_2} - \mathcal{P}_{t,T_1} = e^{-r_{t,T_1}T_1} F_{t,T_1} - e^{-r_{t,T_2}T_2} F_{t,T_2}. \tag{9}$$

One can use put-call parity to rewrite (8) and (9), substituting portfolios of puts, calls, and bonds for futures prices:

$$\mathcal{P}_{t,T} = S_t + p_{t,T} - c_{t,T} - X e^{-r_{t,T}T}, \tag{10}$$

$$\mathcal{P}_{t,T_1,T_2} = p_{t,T_2} - p_{t,T_1} - c_{t,T_2} + c_{t,T_1} - X(e^{-r_{t,T_2}T_2} - e^{-r_{t,T_1}T_1}), \tag{11}$$

where $p_{t,T}$ and $c_{t,T}$ are respectively puts and calls maturing at $t+T$ with common strike X. BBK consider two specific return series. The first dividend strip return equals the return on a short-term asset:

$$R_{1,t} = \frac{\mathcal{P}_{t,T} + D_t}{\mathcal{P}_{t-1,T+1}}, \tag{12}$$

where the maturity T varies between approximately 1.3 and 1.9 years.[9] The second dividend strip return is

$$R_{2,t} = \frac{\mathcal{P}_{t,T_1,T_2}}{\mathcal{P}_{t-1,T_1+1,T_2+1}}, \tag{13}$$

where $T_1 \approx T_2 - 1$ year. The second strategy does not require replicating the index or collecting dividends, and involves trades only in futures contracts.

A. Properties of the Return Series

Using a February 1996 to October 2009 sample period, BBK report the following key facts about the return series $R_{1,t}$ and $R_{2,t}$:

- The one-month average returns of both short-term dividend strips are larger than the one-month average returns on the S&P 500 (annualized 11.6% and 11.2% versus 5.6%). The return differences persist after controlling for standard risk factors.
- The volatilities of the short-term dividend strips are substantially higher than the S&P 500 index (standard deviations of monthly returns are 7.8% and 9.7% versus 4.7%). The volatilities of the dividend strips are substantially larger than the volatilities of subsequent dividend realizations.
- The dividend strip returns have estimated betas of about 0.5 in market model regressions.
- The return series $R_{1,t}$ is highly predictable by lagged values of the price-dividend ratio of a 1.5 year short-term dividend strip.

Both series show very large negative autocorrelation, about -0.30. If the strips were tradeable at the observed prices, an average return of 30% per month per dollar invested would be available by following a return reversal strategy. Hence, measurement error must be present in the observed prices.

[9] Specifically, in January of any given year, the maturity T is chosen according to the available contract expiring in the fall of the following year. This contract will have a maturity of about $T = 1.85$ at purchase. This contract is held for six months, during which period T decreases by 1/12 each month. On July 1, this contract is sold and a new contract with maturity of approximately $T = 1.85$ years is purchased.

B. Approximating the Impact on Dividend Strip Average Returns

We use a formula provided in BCFS to estimate the return a buy-and-hold investor can expect to obtain from trading in the short-term asset. Let $r_{it} = \ln(R_{it}) \sim \mathcal{N}(\mu_i, \sigma_i^2)$ denote monthly log returns. We assume that log returns aggregated over n months have a normal distribution with variance σ_{in}^2. These assumptions are exactly satisfied if r_{it} is a stationary ARMA(p, q) process with Gaussian innovations, and approximately hold in more general cases. The buy-and-hold mean is $\bar{R}_{in}^{BH} \equiv \mathbb{E}[R_{i,t+1} \cdots R_{i,t+n}]$ and the rescaled monthly mean is $\bar{R}_{in}^{RS} \equiv \mathbb{E}[R_{it}]^n$. BCFS show that

$$\frac{\bar{R}_{in}^{RS}}{\bar{R}_{in}^{BH}} = e^{n\sigma_i^2(1-VR_{in})/2}, \tag{14}$$

where $VR_{in} \equiv \sigma_{in}^2/(n\sigma_i^2)$ is the variance ratio. When the variance ratio is one, for example if returns are iid, rescaled monthly returns are an unbiased estimate of the investment return of the buy-and-hold investor. In contrast, a negative autocorrelation implies high short- relative to long-horizon returns. Although (14) is derived from an assumption of lognormal distributions, BCFS show that it is empirically extremely accurate for a wide range of portfolios.

In Table 1, we show the ratio (14) for dividend strip returns and the S&P 500. Panel A lists the primary moments used in the formula. Panel B presents the average one-month returns for each series rescaled to an annual horizon, the estimated twelve-month buy-and-hold return average, and an estimate of the variance ratio for each investment.[10] The average monthly returns of the dividend strip series when rescaled to an annual horizon appear to suggest annual returns of 14.8% and 14.3% for R_1 and R_2 respectively. However, both series have variance ratios far below one, and their estimated annual buy-and-hold returns of 12.8% and 10.1% are substantially lower than

[10] Since no variance ratios are reported in BBK, we use the 1-lag autocorrelation to approximate the variance ratio using

$$VR(q) = 1 + 2\sum_{k=1}^{q-1}(1 - k/q)\,\rho_k, \tag{15}$$

as in Campbell, Lo, and MacKinlay (1997, Eq. 2.4.19). For the higher order autocorrelations, we assume $\rho_k = 0$ for $k \geq 2$. We can alternatively impose the restrictions implied by an AR process, specifying $\rho_k = \rho_1^k$ for $k \geq 2$. The estimated horizon effects are affected little by this assumption.

the average monthly returns would suggest.

These first calculations show that the large negative autocorrelations of dividend strip returns and high short-horizon monthly returns relative to longer-horizon annual returns are related symptoms. The root cause of these symptoms can be traced to small pricing frictions, as we now show.

IV. Microstructure Bias in Synthetic Dividend Strips

We now extend the general idea of Section II to show how microstructure frictions impact synthetic dividend strip returns. Assume that the observed index level S_t^o relates to fundamental value S_t by

$$S_t^o = S_t + \rho(S_{t-1} - S_t) + S_t(e^{\eta_t} - 1), \tag{16}$$

where $\rho(S_{t-1} - S_t)$, $0 \leq \rho \leq 1$, accounts for slow index adjustment, and $S_t(e^{\eta_t} - 1)$ captures iid measurement error. The futures price is for now taken to be unaffected by microstructure frictions (i.e., $F_{t,T}^o = F_{t,T}$). If one uses the futures-spot parity relation (8) to impute the short-term asset value, the observed price is

$$\begin{aligned} \mathcal{P}_{t,T}^o &\equiv S_t^o - e^{-r_{t,T}T} F_{t,T}^o & (17) \\ &= \mathcal{P}_{t,T} + (S_t^o - S_t). & (18) \end{aligned}$$

The microstructure friction $S_t^o - S_t$ is proportionally much more important relative to the short-term asset price, since $\mathcal{P}_{t,T}$ is considerably smaller than S_t.

Using a procedure similar to Campbell and Shiller (1988), we provide log-linear approximations for the observed prices of the index and short-term asset. Let lower case letters denote the logarithm of their uppercase counterpart, and define the capital gain return $R_t^x = S_t/S_{t-1}$. The observed index level is

$$S_t^o = S_t \left[\rho \left(\frac{1}{R_t^x} - 1 \right) + e^{\eta_t} \right]. \tag{19}$$

Let $\ell_{t,T} \equiv \ln(S_t/\mathcal{P}_{t,T})$ be the logarithm of the implicit leverage of the dividend strip. Also define $\bar{\ell}_T = \mathbb{E}(\ell_{t,T})$ and $\bar{L}_T = exp(\bar{\ell}_T)$. In the special case where $\ell_{t,T}$ is a constant, $\bar{L}_T$ is the implicit leverage of the dividend strip with maturity T. We then show

Proposition 1 *Logarithmic index levels and short-term asset prices are:*

$$s_t^o \approx s_t - \rho r_t^x + \eta_t, \tag{20}$$

$$p_{t,T}^o \approx p_{t,T} + \bar{L}_T(-\rho r_t^x + \eta_t). \tag{21}$$

The amplification of microstructure frictions is summarized by $\bar{L}_T$, reflecting leverage. For dividend strips with a one-year maturity, $\bar{L}_T^{-1} \approx 0.02$ approximates the dividend yield of the index, and $\bar{L}_T \approx 50$ approximates the price-dividend ratio of the index, implying substantial magnification of primary market frictions.

The closed-form expressions for prices in Proposition 1 allow direct analysis of the impact on returns. Let $\Delta_t = \ln(D_t/S_t)$, $\Delta = E(\Delta_t)$, $\delta_t = \ln(D_t/\mathcal{P}_{t,T})$, and $\delta = E(\delta_t)$. We then show

Proposition 2 *Observed logarithmic returns on the index and the short-term asset are respectively*

$$r_t^o \equiv \ln\left(\frac{S_t^o + D_t}{S_{t-1}^o}\right) \approx r_t + \rho(r_{t-1}^x - r_t^x) + \eta_t - \eta_{t-1}, \tag{22}$$

$$r_{1t}^o \equiv \ln\left(\frac{\mathcal{P}_{t,T}^o + D_t}{\mathcal{P}_{t-1,T+1}^o}\right) \approx r_{1t} + \bar{L}_T[\rho(r_{t-1}^x - r_t^x) + \eta_t - \eta_{t-1}] \tag{23}$$

where

$$r_t \equiv \ln\left(\frac{S_t + D_t}{S_{t-1}}\right) \approx \ln(1+e^{\Delta}) + \frac{e^{\Delta}}{1+e^{\Delta}}(\Delta_t - \Delta) + s_t - s_{t-1}, \tag{24}$$

$$r_{1t} \equiv \ln\left(\frac{\mathcal{P}_{t,T} + D_t}{\mathcal{P}_{t-1,T+1}}\right) \approx \ln(1+e^{\delta}) + \frac{e^{\delta}}{1+e^{\delta}}(\delta_t - \delta) + p_{t,T} - p_{t-1,T+1}. \tag{25}$$

First, pricing frictions are magnified in the logarithm of the short-term asset return, where the size of the amplification effect is driven by the implicit leverage $\bar{L}_T$. Second, the means of the observed log returns are not biased, since the means of the observed and true log return series are identical.

While mean log returns are not biased, average simple returns are greatly affected. Following from the standard Jensen's inequality approximation:

$$\mathbb{E}(R_{it}) \approx e^{E(r_{it})+0.5\mathrm{Var}(r_{it})}, \tag{26}$$

any microstructure-induced bias in volatity will impact simple returns.

To see the impact on measured variances, consider the case where dividend yields on the index and dividend strip are constants, and the true index return r_t is iid. The variance of the short-term asset return is then

$$\mathrm{Var}(r^o_{1t}) = \mathrm{Var}(r_t)\left[1 - 2\rho\bar{L}_T + 2\rho^2\bar{L}_T^2 + 2\bar{L}_T^2\frac{\mathrm{Var}(\eta_t)}{\mathrm{Var}(r_t)}\right]. \tag{27}$$

Lead-lag effects inflate variance if $\rho > \bar{L}_T^{-1}$, and measurement error unambiguously increases volatility. Both effects are multiplied by factors on the order of $2\bar{L}_T^2$. If we again consider $\bar{L}_T = 50$, implicit leverage magnifies microstructure-induced variance by 5000 times. Even small primary-market pricing frictions cause apparent excess volatility, which biases the average simple return following (26).

Consistent with standard implications of excess volatility (Shiller, 1981), the observed short-term asset return is predictable with first-order autocovariance

$$\mathrm{Cov}(r^o_{1t}, r^o_{1t-1}) = \mathrm{Var}(r_t)\left[\rho\bar{L}_T - \rho^2\bar{L}_T^2 - \bar{L}_T^2\frac{\mathrm{Var}(\eta_t)}{\mathrm{Var}(r_t)}\right]. \tag{28}$$

Lead-lag effects can cause positive or negative autocovariance depending on whether $\bar{L}_t^{-1} - \rho$ is positive or negative, while measurement error unambiguously biases the measured autocovariance downwards. Leverage again inflates the importance of both frictions by factors on the order of $\bar{L}_T^2$.

The microstructure frictions also impact measured beta. The covariance of the short-term asset return with the index return is

$$\mathrm{Cov}(r^o_t, r^o_{1t}) = \mathrm{Var}(r_t)\left[1 - \rho(1 + \bar{L}_T) + 2\rho^2\bar{L}_T + 2\bar{L}_T\frac{\mathrm{Var}(\eta_t)}{\mathrm{Var}(r_t)}\right]. \tag{29}$$

Lead-lag effects reduce the measured covariance when $1 + \bar{L}^{-1} > 2\rho > 0$. Monthly S&P 500 returns are positively autocorrelated with $\rho \approx 0.1 < (1 + \bar{L}_T^{-1})/2$, and should therefore contribute to downward bias in covariance. The last term in (29) shows that measurement error increases covariance. The magnitude depends on the variance of the measurement error relative to the variance of index return, again magnified by the implicit leverage $\bar{L}_T$ of the short-term asset. The observed beta

$$\beta^o_{1t} \equiv \mathrm{Cov}(r^o_t, r^o_{1t})/\mathrm{Var}(r^o_t) \tag{30}$$

is determined by dividing (29) by the measured index variance

$$\mathrm{Var}(r_t^o) = \mathrm{Var}(r_t)\left[1 - 2\rho + 2\rho^2 + 2\frac{\mathrm{Var}(\eta_t)}{\mathrm{Var}(r_t)}\right]. \tag{31}$$

Since leverage does not affect the index variance, the absolute bias in (31) is expected to be tiny relative to the absolute bias in the covariance (29). The effect of microstructure frictions on the observed dividend strip beta (30) should therefore be dominated by bias in the covariance (29).

Table 2 uses the equations derived in this section to illustrate the impact of implicit leverage and microstructure frictions on various moments of the short-term asset return. The selection of parameters is consistent with the ratio $\mathcal{P}/S$ varying from 1/10 to 1/90. Variation in this parameter can either reflect different assumptions for the annual dividend yield on the S&P 500 index or consideration of a variety of dividend strip strategies with $T - t$ ranging from a few months to over a year. Lead-lag effects in the index are captured by ρ, roughly equal to the first-order autocorrelation of index returns. Finally, the parameter $\sigma(\eta)$ reflects the magnitude of measurement errors.

The table shows effects of microstructure frictions on the monthly mean return, standard deviation, autocorrelations, and market beta. Measurement error biases volatility upwards, in extreme cases more than tripling the unobserved true level of 4%. Average simple returns are also overstated, in some cases more than doubling their true value. Even modest measurement error creates substantial negative autocorrelation. Observed market betas are only modestly inflated. Asynchronous price adjustment also has large impacts, and combining the effects of measurement error and asynchronous price adjustment, key facts associated with dividend strips can be produced.

V. A Structural Model, Calibration, and Implications

We now show that in a simple calibrated model, tiny microstructure effects capture the primary empirical features of dividend strips. The model we use does not endeavor to be consistent with consumption moments, and we take an agnostic stance about preferred models from the literature on consumption-based asset pricing. Rather, we use the

simplest Gordon growth formula model of a flat term structure of equity returns. It is straightforward to see how similar microstructure effects could be appended to any consumption-based asset pricing model that one might advocate, and similar biases in the observed versus fundamental return moments of dividend strips would occur.

Let dividends X_t be given by

$$dX_t = gX_t + \sigma X_t dW_t, \tag{32}$$

where dW_t is the increment of a Wiener process, g is the mean growth rate, and σ^2 the variance. The fair price of the equity claim is given by the Gordon growth formula:

$$S_t = \frac{X_t}{\mu - g}, \tag{33}$$

where μ is the constant return on equity.

A. Delayed Price Adjustment in the Index

We first consider the case where only the observed level of the index at time t, denoted S_t^o, depends on lagged fundamentals:

$$S_t^o = (1 - \rho_1 - \rho_2)S_t + \rho_1 S_{t-1} + \rho_2 S_{t-2}. \tag{34}$$

True and measured returns are given by:

$$R_{Mt} = S_t/S_{t-1} - 1, \tag{35}$$

$$R_{Mt}^o = S_t^o/S_{t-1}^o - 1. \tag{36}$$

The true value of a short-term asset with a claim on the first T years of dividends starting from date t is:

$$\mathcal{P}_{t,T} = S_t(1 - e^{-(\mu-g)T}). \tag{37}$$

We initially assume that the observed futures price is equal to fundamental value:

$$F_{t,T}^o = F_{t,T} = (S_t - \mathcal{P}_{t,T})\, e^{rT}. \tag{38}$$

The observed price of the short-term asset, derived from the observed futures price and index value, is

$$\mathcal{P}^o_{t,T} = \mathcal{P}_{t,T} + \rho_1(S_{t-1} - S_t) + \rho_2(S_{t-2} - S_t). \tag{39}$$

In the special case where $\rho_2 = 0$, the measured return on the short-term asset is

$$\frac{\mathcal{P}^o_{t+1,T-1}}{\mathcal{P}^o_{t,T}} = \frac{\mathcal{P}_{t+1,T-1} - \rho_1(S_{t+1} - S_t)}{\mathcal{P}_{t,T} - \rho_1(S_t - S_{t-1})}. \tag{40}$$

To calibrate the model, we set $r_f = 0.0029$ per month and $\mu = 0.0056$ equal to the sample averages of the monthly risk-free rate and market return reported by BBK. We choose $\sigma = 0.047$ to approximately match monthly market volatility. We consider two values for the growth rate g of dividends. First, we set $g = 0.0042$, which is high from a historical perspective but necessary to approximately match the price-dividend ratio of about 60 over the sample period.[11] Second, we choose a more conservative value $g = 0.0025$ for the growth rate of dividends, which produces a much smaller value for the aggregate price-dividend ratio of about 27.

Table 3, Panel A, shows each calibration and the effect on moments of observed dividend strip returns. In column (i) where the average price-dividend ratio equals 60, return moments are greatly impacted by microstructure frictions. We conservatively set $\rho_1 = 0.03$ and $\rho_2 = 0.01$, which gives a first-order autocorrelation of the market index of 0.031, less than half of what is observed empirically during this sample. The observed mean return of the short-term asset is then 1.10% per month, approximately equal to the average return of the strategy R_1 reported by BBK. The first-order autocorrelation of observed returns is -0.258, also very close to the empirical value of -0.268. The simulated standard deviation of short-term asset returns is 0.116, substantially larger than the volatility of the market return, consistent with the empirically observed excess volatility. The market-model beta is downward-biased relative to its true value of one. In the calibration this downward bias is very strong, and the market model beta of -1.44 is substantially less than the empirical value of 0.449.

[11] Alternative calibrations that use a lower dividend growth rate as well as a lower risk-free rate or market risk premium to approximately match the observed price-dividend ratio produce results similar to those reported for this calibration.

In untabulated results, a regression of the observed short-term asset return on the short-term asset price-dividend ratio produces significantly negative coefficients. For example when the return from time t to $t+1$ is regressed on the period t price-dividend ratio, the simulation produces a regression coefficient of -0.58 versus -0.17 empirically. BBK also lag the price-dividend ratio by one quarter, as a check for the effects of measurement error, and find a coefficient of -0.09 when using the lagged price-dividend ratio. Our simulation produces similar effects, because we have included two lags of asynchronous price adjustment in the observed index level (34). In predictability regressions using the period $t-1$ price-dividend ratio we obtain a coefficient of -0.11. The model thus generates predictability from price-dividend ratios, consistent with empirical evidence.

Overall, these results show that a simple model of asynchronous price adjustment in the market index can quantitatively match the mean return and autocorrelation of market and short-term asset returns, while also qualitatively capturing excess volatility in the short-term asset, downward bias in the market model beta, and negative predictability of short-term asset returns from the short-term asset price-dividend ratio.

In column (ii) of Panel A we consider the second calibration with a more modest growth rate of dividends. To compensate for the smaller price-dividend ratio, we set $\rho_1 = 0.075$ and $\rho_2 = 0.015$. These values produce a first-order autocorrelation of market returns of 0.0822, almost exactly matching the empirically observed statistic of 0.0898. The simulated moments of the dividend strip returns are very similar in this calibration to those in column (i). The average returns of the short-term asset are slightly higher (0.0122) and the autocorrelations somewhat more negative (-0.3277). Both calibrations show the ability to match important quantitative and qualitative aspects of short-term asset returns.

Replicating the short-term asset return R_1 considered in this subsection requires trading in the spot market to replicate the market index. The costs of such a trading strategy may not be small. For this reason, BBK suggest that a more appropriate

strategy to empirically evaluate is the dividend steepener return given in (13), which we now consider.

B. Measurement Error in Prices and the Dividend Steepener

In principle, the dividend steepener return in (13) can be obtained by trading only in futures markets, with a long position in maturity T_1 and a short position in maturity $T_2 > T_1$. We first consider the impact of small amounts of measurement error in futures prices:

$$F^o_{t,T} = F_{t,T} e^{\eta_{t,T}}. \tag{41}$$

We assume that the errors $\eta_{t,T}$ are independently drawn from a normal distribution with mean zero and standard deviation σ_η. Hence, the observed prices are unbiased estimates of the true prices.

Table 3, Panel B, column (i) corresponds to a calibration where the growth rate of dividends $g = 0.0042$ gives a market price-dividend ratio of approximately 60. We set $\sigma_\eta = 0.0009$, which reflects a fairly small measurement error. For comparison, the average bid-ask spread in liquid futures contracts during this period was about 3 basis points. Additionally, the prices used by BBK in most of their analysis are not actual futures prices, but are imputed from put-call parity using S&P 500 index options and the put-call parity relations (10) and (11). Consistent with limits to arbitrage, traded futures prices and those inferred from put-call parity are not exactly identical, consistent with our assumption of small measurement errors in primary-market prices. As a consequence of these small frictions, Table 3 shows that the average observed return of the dividend steepener is 1.17% per month, more than double the true mean of 0.056% per month. Other symptoms of frictions are the large negative autocorrelation of returns of the measured dividend steepener returns, equal to -0.4165 and substantial excess volatility.

Similar effects can be seen in column (ii) of Panel B, where $g = 0.0025$ and the price-dividend ratio is about 27. In this case, we increase the measurement error standard deviation to $\sigma_\eta = 0.0015$. The effect on average returns and variances is smaller than

column (i), but still economically meaningful, inflating mean returns of the dividend steepener by about 50% relative to their true value (0.0086 versus 0.0056).

Hence, either small amounts of price asynchronicity (Panel A of Table 3) or iid measurement error (Panel B) can generate high returns, strong negative autocorrelation, and excess volatility of short-term dividend strips as observed in the data. The primary difference in the consequences of these two microstructure frictions can be seen in the last row of Table 3. The empirical values of the market model regression betas are 0.449 for R_1 and 0.486 for R_2, in between the model values in Panels A and B. A combination of price asynchronicity and measurement error should therefore match all of the main features of dividend strip returns, as we now show.

C. Full Model

Panel C of Table 3 shows two ways of combining price asynchronicity with measurement error. In the first case, we set

$$F^o_{t,T} = [(1-\rho_{1,T})F_{t,T} + \rho_{1,T}F_{t-1,T}]e^{\eta_{t,T}}.$$

Column (i) shows simulated moments for a calibration with a tiny amount of slow price adjustment on the long-side of the steepener ($\rho_{1,T_1} = 0.009$) and no price delays on the short side. Combined with measurement error, this specification matches the mean returns, autocorrelations, excess volatility, and market model regression beta of the empirical data.

Alternatively, it seems unlikely that any one futures contract is always more informationally efficient than another, in the sense of always incorporating current pricing information sooner and more fully. We therefore allow that an iid random variable determines which contract receives a news shock at each date t. Specifically, let

$$F^o_{t,T} = \begin{cases} F_{t,T}e^{\eta_{t,T}} & \text{if } \mathbf{1}_{t,T,lagger} = 0 \\ \left[(1-\rho_{1,lagger})F_{t,T} + \rho_{1,T}F_{t-1,T}\right]e^{\eta_{t,T}} & \text{if } \mathbf{1}_{t,T,lagger} = 1, \end{cases}$$

where $\mathbf{1}_{t,T,lagger}$ equals zero when the contract with maturity T incorporates information efficiently at time t, and equals one when the observed price updates slowly. We assume

that at each date t, one of the contracts T_1 and T_2 incorporates information efficiently, and the other is a lagger. We set $P\left([\mathbf{1}_{t,T_1,lagger} = 1] = 2/3\right)$, so that on average the 18 month contract has more current information. This specification, shown in column (ii) of Panel C also allows us to match the mean returns, autocorrelations, excess volatility, and market model regression beta of the empirical data on observed dividend strip returns.

D. Implicit Leverage and Average Returns

In Figure 1 we vary the maturity T of the dividend strip $\mathcal{P}^o_{t,T}$ from 6 to 60 months, and show the implicit leverage of the claim as well as its mean return, volatility, and autocorrelation. As the maturity of the dividend strip shortens, the implicit leverage in the long-short position increases dramatically. Assuming an annualized price-dividend ratio of approximately 60, a six-month dividend strip has implicit leverage exceeding 100. For longer maturities the implicit leverage falls and the importance of the microstructure effects in the synthetic return series lessens.

In the third panel, the autocorrelation function shows a distinctive pattern of first being negative, reaching a minimum at a short maturity less than one year, and then increasing to eventually become positive. The changing autocorrelation reflects a changing balance between two effects. On the one hand, the value of the short-term asset is a fraction of the value of the aggregate market, and the observed returns of the aggregate market have positive persistence. When the dividend strip has a long maturity this effect dominates. On the other hand, the appearance in the numerator and denominator of (40) of the current and lagged true price change causes negative autocorrelation in returns. This effect is more pronounced for more levered dividend strips with shorter maturities T.

Figure 2 shows similar plots in the case where returns are impacted by iid measurement errors rather than asynchronous price changes. Again, the implicit leverage of dividend strips magnifies the importance of microstructure effects, and shorter maturities show higher apparent average returns and excess volatility. The autocorrelations in

the third panel are always negative consistent with our results in Section 4, and become closer to zero for longer maturities.

In both figures, synthetically replicating a short-term dividend strip requires high leverage in nearly offsetting long and short positions. The greater the leverage, the more the amplification of small pricing errors in the fundamental securities used to create the replicating portfolio, and the stronger is the impact on observed return moments.

E. Cointegration and Price versus Return Correlations

Figure 3 shows a simulation of 180 months of fundamental values and observed prices from the model of Table 3, Panel C, column (ii). The correlation between fundamental and observed prices in this example is 93.67%. For comparison, the final pre-publication version of BBK showed in their Section 5.2 that the correlations between dividend strip prices inferred from futures prices versus put-call parity are of a similar magnitude, 94% and 91% for 6-month and 1-year dividend strips respectively. The measurement errors of dividend strips implied by our model are thus of a magnitude comparable to those seen in the data.

BBK infer that because futures markets are highly liquid and the short-term asset prices obtained from futures and options markets are similar, microstructure frictions are an unlikely explanation for their findings. Our analysis shows that it is important to distinguish between high correlation of prices and high correlation of returns. In all of the models we have developed in this paper, the correlation between fair value and observed short-term asset prices is high. Nonetheless, we find large differences in average returns. Further, despite the high correlation of the two price series in Figure 3, the correlation of the measured and true monthly returns is only 19.82%.

The large difference in the autocorrelations of prices and returns is possible because the true and actual return series in the model are cointegrated. Attempts by arbitrageurs to take advantage of profit opportunities imply that the difference between true and measured prices in financial markets must be small and temporary. However, returns may be different and are predictable in the short-run by differences between

true and measured prices. A similar cointegrating relationship holds for futures prices measured directly or via put-call parity from the options markets. In either case, the similarity of prices does not imply a high degree of closeness in returns, either realized or on average. In particular, despite the high price correlation between true and measured prices of the short-term asset shown in Figure 3, the average monthly measured return is twice as high as the average monthly true return (1.12% versus 0.56%). The higher average of the measured return is offset by its strong negative autocorrelation, which substantially dampens the positive effects of compounding over longer horizons. In general, high correlation of two price series, or apparent cointegration between two price series, is not informative about the closeness of their average returns. When one series is subject to strong microstructure bias, generating significant return autocorrelations, the short-horizon average must be biased to compensate. These biases are greatly mitigated in returns measured over longer horizons.

VI. New Estimates of the Dividend Term Premium

In this Section, we reexamine the performance of dividend strip strategies using return measures that are more robust to microstructure frictions. Following our theoretical arguments, average monthly returns of dividend strips are expected to be biased upwards. To confirm this, Table 4 compares averages of the return series (R_1 and R_2) compounded at different horizons.[12] Whereas the average monthly return of the dividend steepener strategy (R_2) reaches 1.12% (14.30% per year), the average annual return scaled to a monthly frequency amounts to just 0.71% (8.86% per year). By contrast, the average returns of the S&P 500 index are relatively similar across horizons, equalling 0.56% and 0.51% respectively for monthly returns and annual returns rescaled to a monthly horizon. Consequently, using longer-horizon annual returns where the impact of microstructure frictions are proportionately smaller, the difference in performance between the dividend steepener strategy and the index reaches only 0.20% monthly.

[12]We obtain returns from Ralph Koijen's website, http://faculty.chicagobooth.edu/ralph.koijen/data.html

Most of the difference in average returns of the short-term asset and the index is attributable to the first half of the sample. For example, Table 4 shows that the difference in average returns R_1 of the short-term asset and the S&P 500 index amounts to 0.94% monthly in the first half but reaches just 0.27% in the second half. Autocorrelation of the short-term asset returns is more negative and volatility is higher during the 1996-2002 period. Moreover, liquidity of the assets used to compute returns R_1 and R_2 was likely lower earlier in the sample. It is thus not surprising that the horizon effects are particularly striking in the first half of the sample. When the annual compounding horizon is used, the difference in average returns of the short-term asset R_1 and the index amounts to 0.53% monthly in the first half and 0.33% in the second half of the sample. More striking, for R_2 the difference in returns based on an annual compounding horizon is *negative* in the first half of the sample, measuring -0.04% per month, remarkably lower than the monthly return mean difference of 0.73%.

Figure 4 graphically confirms these results. An investment in the dividend steepener strategy in 1996 produced a lower cumulative return than an investment in the S&P 500 index through 2002. A one dollar investment in either strategy grew to the same amount (around $2) by the end of June 2004. Moreover, during the last four years of the sample (2006-2009), cumulative returns of the two investments were largely similar. Thus the difference in average returns of the dividend steepener (1.12%) and the index (0.56%) are not representative of what a long-term investor would realize. To illustrate this point further, Figure 4 plots cumulative returns of two hypothetical strategies that every month earn 1.12% and 0.56%. The differences in these plots are entirely consistent with the effects of small pricing errors in primary markets, which become amplified when using no-arbitrage relations to impute fair values of dividend strips.

VII. Conclusion

We show that small pricing frictions in primary markets become greatly magnified in importance when one uses highly levered long-short positions to replicate and synthet-

ically price financial claims. Such amplification implies large biases in the moments of synthetic return series.

We apply these ideas to dividend strips, and show that the magnification of pricing errors implies upward-biased short-horizon returns, negative autocorrelation, excess volatility, market beta below one, and predictability of returns from the price-dividend ratio, consistent with empirical evidence. The gearing up of primary-market pricing frictions in synthetic dividend strips occurs because of high implicit leverage. These instruments are created from long-short positions in futures or option markets, and have a net value only a fraction of the long and short positions used in their construction.

These ideas are generally important for our ability to understand the effectiveness of arbitrage activity in financial markets. Many standard arbitrage trades, such as the replication of calls or other options, and a variety of calendar- and other types of spreads, involve high leverage. Seemingly negligible frictions of a few basis points in primary markets can imply substantial pricing errors of several percentage points or more in synthetically priced derivative assets. These arguments are consistent with the fact that when dividend strips first started trading on exchanges, their bid-ask spreads were on the order of hundreds of basis points, despite the fact that synthetic replication could be carried out using highly liquid underlying instruments.

Understanding of microstructure frictions is essential for those interested in evidence on the returns of short-term dividend strips. In monthly data, autocorrelations of -30% and R^2 in predictive regressions in excess of 10% simply cannot be sensibly interpreted if one assumes accurately measured prices, and our arguments help to explain these seemingly anomalous features of dividend strip returns. Expected returns are difficult to measure accurately because of the short time series of historical data that is available, but our analysis shows that point estimates of the average monthly return have substantial upward bias. Our research also suggests that alternative approaches to measuring the means or other properties of dividend strip returns should explicitly account for microstructure frictions.

Appendix

Proof of Proposition 1

Equation (20) follows from a Taylor series expansion of the two variable equation (19) around $r_t^x = \ln(R_t^x) = 0$ and $\eta_t = 0$. Equation (21) follows from a Taylor series expansion of the expression

$$\begin{aligned} \mathcal{P}_{t,T}^o &= \mathcal{P}_{t,T} + (S_t^o - S_t) && (42) \\ &= S_t(e^{-\ell_{t,T}} + \rho e^{-r_t^x} - (1+\rho) + e^{\eta_t}) && (43) \end{aligned}$$

around the three points $\ell_{t,T} = \bar{\ell}_T$, $r_t^x = 0$, and $\eta_t = 0$. ∎

Proof of Proposition 2

Equations (24) and (25) are the standard Campbell and Shiller (1988) log-linearizations of the index and dividend strip returns. Equations (22) and (23) then follow from first-differencing the relevant expressions for log ex-dividend index and dividend strip prices in equations (20) and (21). Note that to simplify the expression, the expansion for $\mathcal{P}_{t-1,T+1}^o$ is around $\bar{\ell}_T$, which represents the average dividend-price ratio for the short-term asset $\mathcal{P}_{t,T}^o$. ∎

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Table 1. Back of the Envelope Calculation

	A. Estimates from BBK One-month Returns			B. 12-month Returns		
Portfolio	$\bar{R}$	σ	ρ_1	$\bar{R}_{12}^{RS}$	VR_{12}	$\bar{R}_{12}^{BH}$
$S\&P$	0.0056	0.0469	0.0898	0.0693	1.1646	0.0716
R_1	0.0116	0.0780	-0.2682	0.1484	0.5083	0.1280
R_2	0.0112	0.0965	-0.3668	0.1430	0.3275	0.1008
Difference						
$R_1 - S\&P$	0.060	-0.3580	0.0791	0.0791		0.0564
$R_2 - S\&P$	0.056	-0.4570	0.0737	0.0737		0.0292

Notes: This table reports in Panel A estimated moments for the $S\&P$ index and the two short-term assets from Binsbergen, Brandt, and Koijen (2012). Panel B compares the rescaled simple monthly return $\bar{R}_{in}^{RS} \equiv \mathbb{E}\left[R_{it}\right]^n$ with an estimated buy-and-hold return $\bar{R}_{in}^{BH} \equiv \mathbb{E}\left[R_{i,t+1} \cdots R_{i,t+n}\right]$, $n = 12$ obtained from the approximation

$$\frac{\bar{R}_{in}^{RS}}{\bar{R}_{in}^{BH}} = e^{n\sigma_i^2(1-VR_{in})/2},$$

provided in Boguth, Carlson, Fisher, and Simutin (2012). The key parameter in the approximation, the variance ratio $VR_{in} \equiv \sigma_{in}^2 / \left(n\sigma_i^2\right)$ is computed under the assumption that all autocorrelations of order greater than one are zero.

Table 2. Microstructure Bias in Leveraged Portfolios

	$\rho = 0$			$\rho = 0.05$			$\rho = 0.1$		
$\bar{L}_T$	$\sigma(\eta) = 0$	0.05%	0.1%	$\sigma(\eta) = 0$	0.05%	0.1%	$\sigma(\eta) = 0$	0.05%	0.1%
A. Return Standard Deviation (%)									
10	4.00	4.06	4.24	2.83	2.92	3.16	4.00	4.06	4.24
30	4.00	4.53	5.83	6.32	6.67	7.62	14.42	14.58	15.03
50	4.00	5.34	8.12	11.66	12.19	13.64	25.61	25.86	26.57
70	4.00	6.36	10.68	17.20	17.90	19.85	36.88	37.21	38.18
90	4.00	7.52	13.34	22.80	23.67	26.12	48.17	48.58	49.82
B. Expected Simple Return (%)									
10	0.30	0.30	0.31	0.26	0.26	0.27	0.30	0.30	0.31
30	0.30	0.32	0.39	0.42	0.44	0.51	1.27	1.29	1.36
50	0.30	0.36	0.55	0.90	0.97	1.16	3.56	3.63	3.82
70	0.30	0.42	0.79	1.71	1.84	2.21	7.27	7.40	7.80
90	0.30	0.50	1.12	2.86	3.07	3.70	12.55	12.78	13.46
C. Return Autocorrelation									
10	0.00	-0.02	-0.06	0.50	0.44	0.30	0.00	-0.02	-0.06
30	0.00	-0.11	-0.26	-0.30	-0.32	-0.36	-0.46	-0.46	-0.46
50	0.00	-0.22	-0.38	-0.44	-0.45	-0.46	-0.49	-0.49	-0.49
70	0.00	-0.30	-0.43	-0.47	-0.48	-0.48	-0.49	-0.49	-0.49
90	0.00	-0.36	-0.46	-0.48	-0.49	-0.49	-0.50	-0.50	-0.50
D. Market Beta									
10	1.00	1.00	1.01	0.55	0.56	0.57	0.12	0.13	0.14
30	1.00	1.01	1.04	-0.44	-0.43	-0.40	-1.83	-1.82	-1.78
50	1.00	1.02	1.06	-1.44	-1.42	-1.37	-3.78	-3.76	-3.70
70	1.00	1.02	1.09	-2.43	-2.41	-2.33	-5.73	-5.70	-5.62
90	1.00	1.03	1.11	-3.43	-3.39	-3.30	-7.68	-7.65	-7.53

Notes: This table reports moments of the ex-dividend return of the short-term asset for varying microstructure parametrization ($\rho \in \{0, 0.05, 0.1\}$, $\sigma(\eta) \in \{0, 0.0005, 0.001\}$) and for different price-dividend ratios ($\bar{L}_T \in \{10, 30, 50, 70, 90\}$) following the approximations from Proposition 2. Panel A shows return standard deviations, Panel B expected simple ex-dividend returns, Panel C return autocorrelations, and Panel D observed market betas. The log ex-dividend return has a mean of 0.22% and a standard deviation of 4%, so that simple return in the absence of microstructure frictions average 0.3%.

Table 3. Comparison of Model versus Empirical Moments

	A. Asynchronous Prices		B. Measurement Error		C. Full Model		Empirical Value	
Parameter	(i)	(ii)	(i)	(ii)	(i)	(ii)	R_1	R_2
T_1	0	0	6	6	6	6	0	6
T_2	12	12	18	18	18	18	12	18
g_d	0.0042	0.0025	0.0042	0.0025	0.0042	0.0042		
ρ_{1,T_1}	0.03	0.075	0	0	0.009	0		
ρ_{2,T_1}	0.01	0.015	0	0	0	0		
σ_η	0	0	0.0009	0.0015	0.0009	0.0003		
$\rho_{1,lagger}$	-	-	-	-	-	0.0025		
$P(lagger = T_1)$	-	-	-	-	-	2/3		
Moment								
$\bar{R}_M$	0.0056	0.0056	0.0056	0.0056	0.0056	0.0056	-	-
$\rho_1(R_M)$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	-
$\bar{R}_\mathcal{P}$	0.0056	0.0056	0.0056	0.0056	0.0056	0.0056	-	-
$\rho_1(R_\mathcal{P})$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	-
$\bar{R}^o_M$	0.0054	0.0054	-	-	-	-	0.0056	-
$\bar{R}^o_M - \bar{r}_f$	0.0025	0.0025	-	-	-	-	0.0027	-
$\rho_1(R^o_M)$	0.0313	0.0822	-	-	-	-	0.0898	-
$std(R^o_M)$	0.0454	0.0431	-	-	-	-	0.0469	-
$PD(R^o_M)$	60	27	60	27	60	60	60	-
$\bar{R}^o_\mathcal{P}$	0.0110	0.0122	0.0117	0.0086	0.0110	0.0112	0.0116	0.0112
$\rho_1(R^o_\mathcal{P})$	-0.2582	-0.3277	-0.4165	-0.3703	-0.4088	-0.4080	-0.2682	-0.3668
$std(R^o_\mathcal{P})$	0.1161	0.1249	0.1201	0.0947	0.1156	0.1157	0.0780	0.0965
$\beta^o_\mathcal{P}$	-1.4401	-1.4635	1.0010	1.0030	0.4731	0.4767	0.4488	0.4863

Notes: This table shows moments of observed market and dividend strip returns under calibrations of the models described in Section 3. All models set the risk-free rate to $r_f = 0.0029$ per month, the market return drift to $\mu = 0.0056$, and the volatility of dividends to $\sigma = 0.047$ per month. In each column, the model is simulated for 200,000 months to obtain the moments reported in the table.

Table 4. Horizon Effects in Short-Term Asset Returns

	Rescaled Average Returns					p-values			
				$R_i - SP$		$P(R_i \leq SP)$		$P(R_i = SP)$	
Horizon	R_1	R_2	SP	R_1	R_2	R_1	R_2	R_1	R_2
A. Full Sample									
1 month	1.16	1.12	0.56	0.60	0.57	0.13	0.18	0.27	0.36
3 months	1.04	0.91	0.57	0.46	0.34	0.18	0.23	0.36	0.47
6 months	1.03	0.88	0.56	0.48	0.32	0.19	0.24	0.38	0.47
12 months	0.94	0.71	0.51	0.43	0.20	0.23	0.34	0.46	0.67
165 months	0.85	0.65	0.44	0.41	0.21	-	-	-	-
1 month log	0.85	0.65	0.44	0.41	0.21	0.22	0.37	0.44	0.73
B. First Half									
1 month	1.59	1.39	0.65	0.94	0.73	0.17	0.26	0.34	0.51
3 months	1.32	0.87	0.67	0.65	0.20	0.25	0.41	0.49	0.82
6 months	1.29	0.79	0.61	0.67	0.18	0.26	0.42	0.52	0.83
12 months	1.20	0.63	0.67	0.53	-0.04	0.32	0.52	0.63	0.97
1 month log	1.10	0.64	0.52	0.58	0.12	0.27	0.46	0.54	0.91
C. Second Half									
1 month	0.72	0.86	0.46	0.27	0.40	0.29	0.22	0.57	0.45
3 months	0.75	0.96	0.47	0.28	0.48	0.22	0.10	0.45	0.20
6 months	0.77	0.96	0.50	0.28	0.46	0.17	0.05	0.33	0.10
12 months	0.67	0.79	0.34	0.33	0.45	0.11	0.04	0.22	0.09
1 month log	0.60	0.66	0.36	0.24	0.29	0.30	0.28	0.61	0.56

Notes: This table reports average returns (in percent) of the two short-term assets (R_1 and R_2) and the S&P 500 index (SP), calculated as $(\mathbb{E}[R_{i,t+1} \cdots R_{i,t+n}])^{1/n}$, where $R_{i,t}$ is the asset gross return in month t and n is the compounding horizon. Overlapping windows are used. Also reported are the differences in average returns of the short-term assets and the index and the associated p-values (based on a one-tailed test) computed using Newey-West (1987) methodology with n lags. Full sample covers 1996:2-2009:10, first half is 1996:2-2002:12, and second half is 2003:1-2009:10.

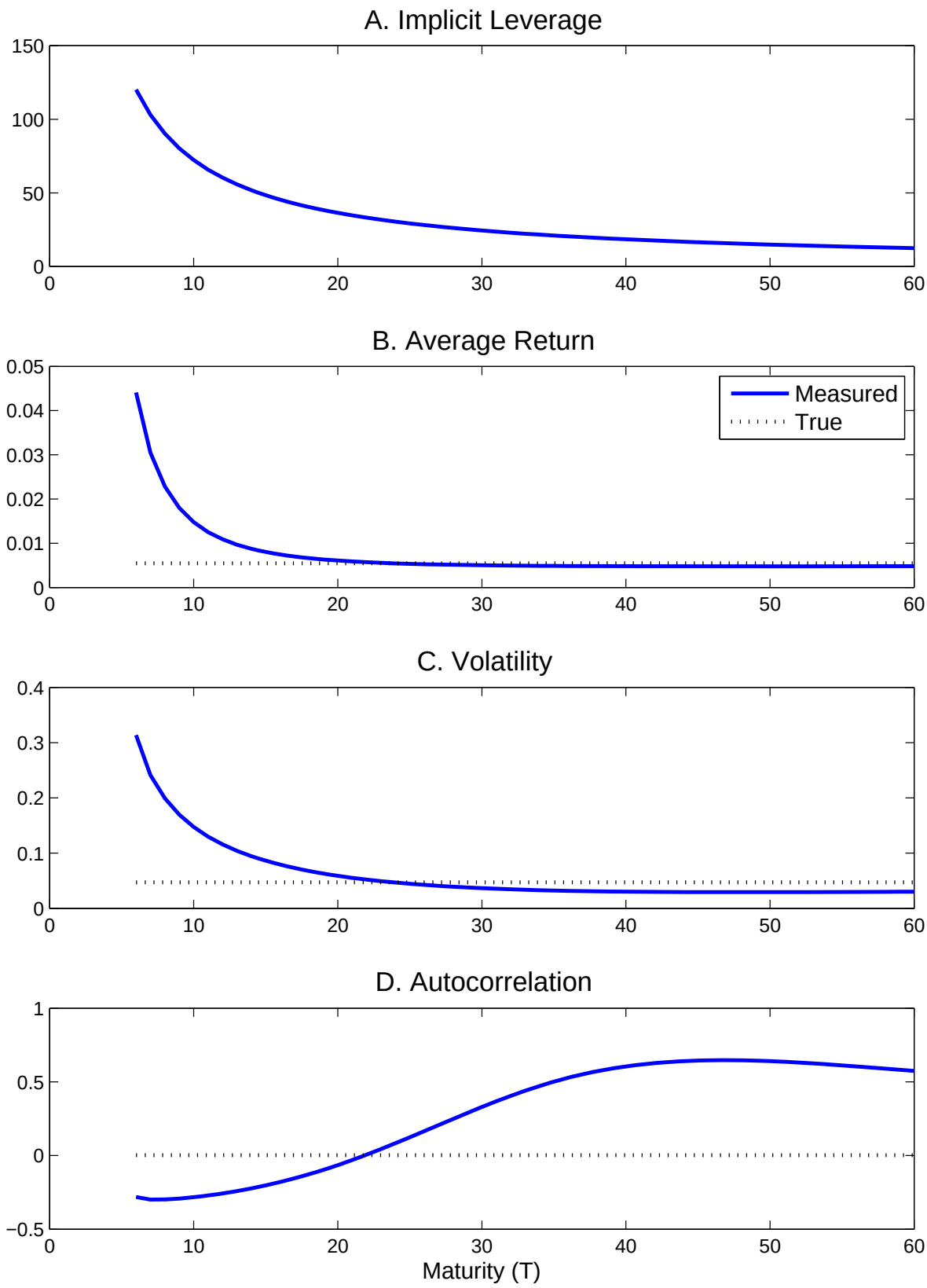

Figure 1. The Effects of Implicit Leverage on Dividend Strip Returns: Case I, Asynchronous Price Adjustment. This figure plots the average returns, volatility, and autocorrelation of the short-term asset return for different maturities T of the short-term asset. We use the base calibration of Table 3, Panel A, column (i), in which asynchronous prices cause small persistence in index returns.

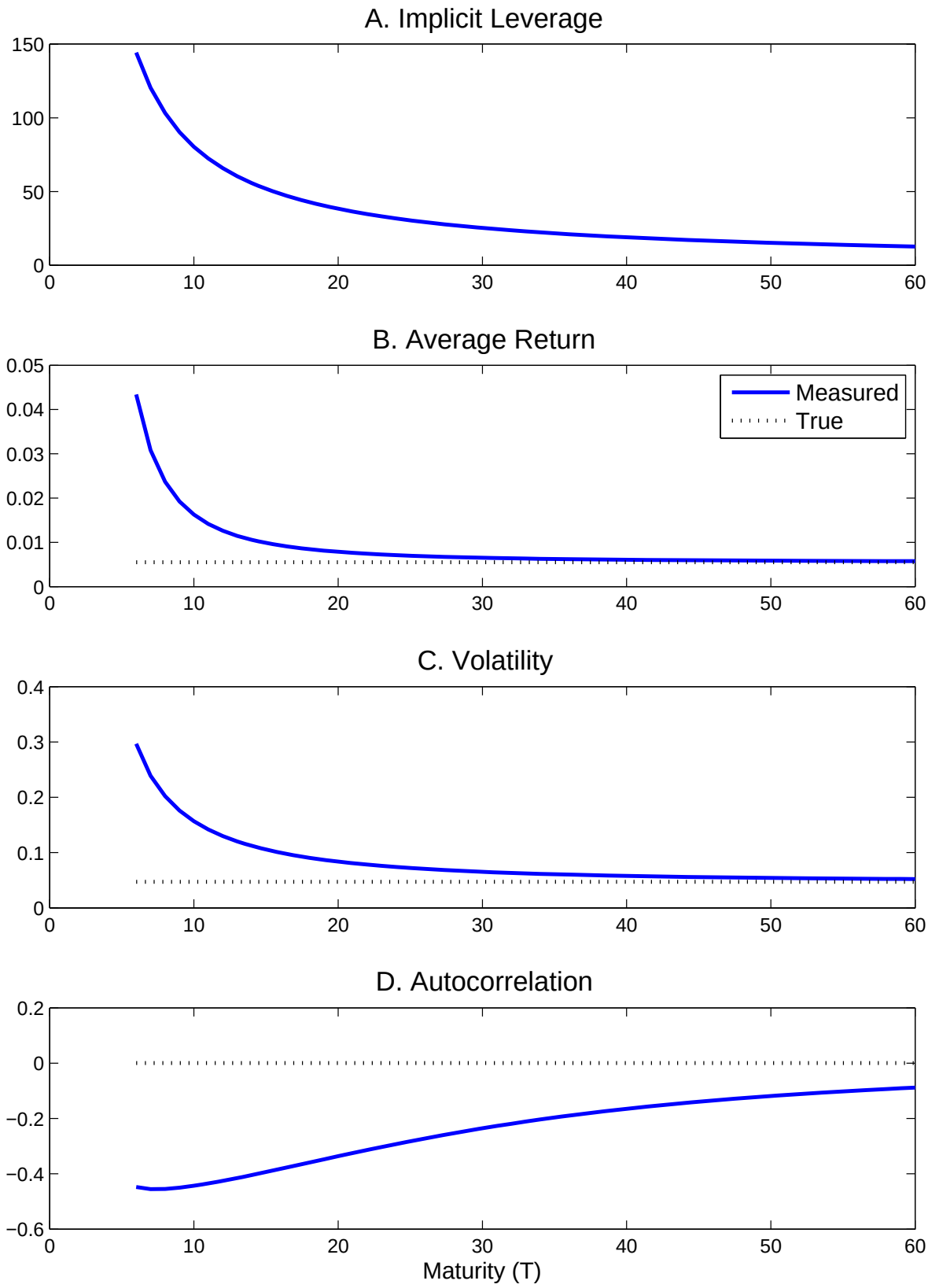

Figure 2. The Effects of Implicit Leverage on Dividend Strip Returns: Case II, Measurement Error. This figure plots the average returns, volatility, and autocorrelation of the short-term asset return for different maturities T of the short-term asset. We use the base calibration of Table 3, Panel B, column (i), in which measurement error impacts futures prices. We set the maturity T_1 of the long position to one month and the maturity T_2 of the short position varies along the horizontal axis of the figure.

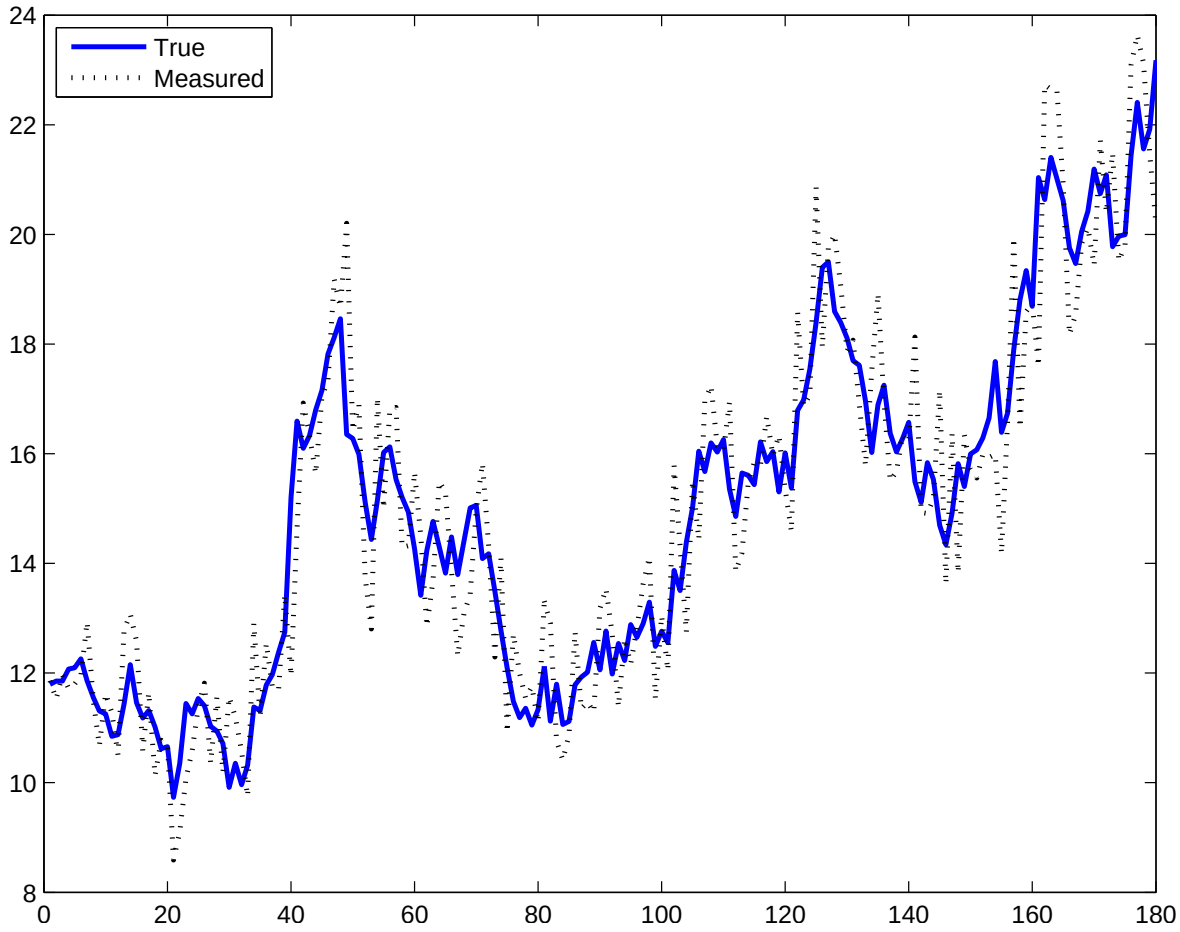

Figure 3. Cointegration between True and Measured Dividend Strip Prices. This figure plots simulated true and measured dividend strip prices using 180 months of data drawn from the calibration given in Table 3, Panel C, column (ii). The correlation of the two price series is high (approximately 94%), but the correlation of returns is much lower (20%). The unconditional average of the measured return series is approximately twice as high as the unconditional average of the true return series. (See Table 3.)

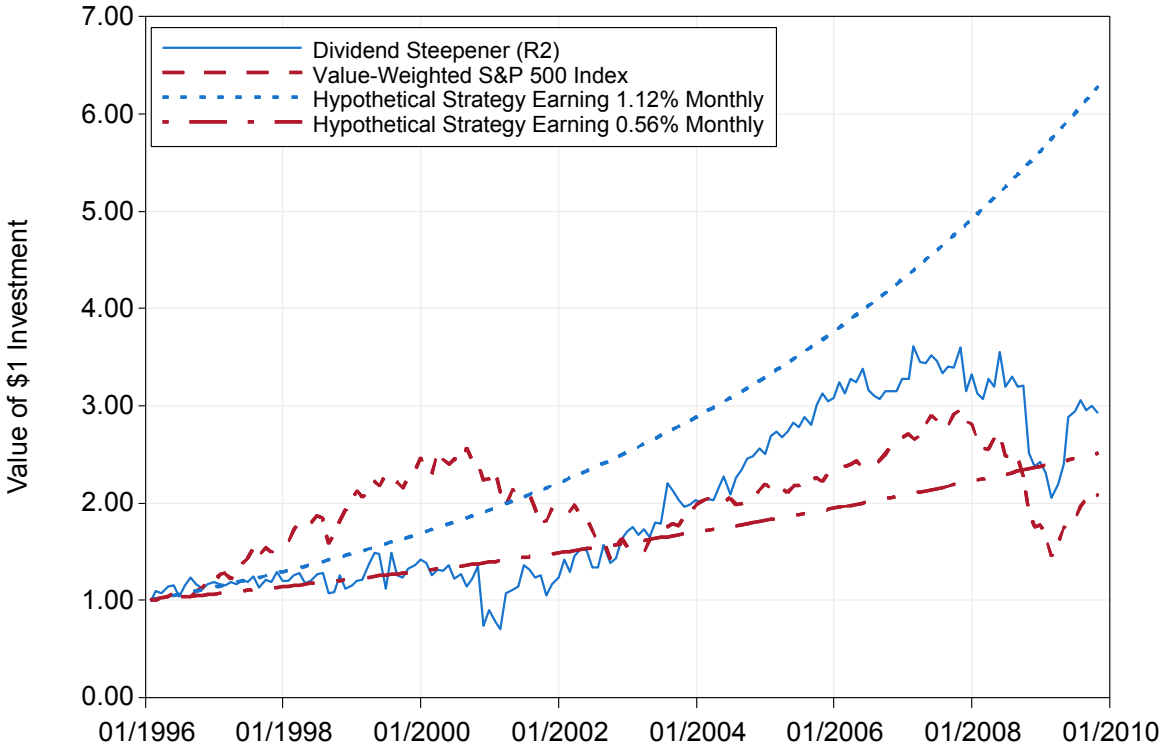

Figure 4. Comparison of Investment Performance. This figure plots the value of a \$1 investment in the dividend steepener strategy (R_2), the value-weighted S&P 500 index, and two hypothetical strategies whose returns each month equal to the average returns of the dividend steepener strategy (1.12%) and the S&P 500 index (0.56%).