Production-Based Term Structure of Equity Returns

Hengjie Ai, Mariano M. Croce, Anthony M. Diercks and Kai Li*

Abstract

We study the link between timing of cash flows and expected returns in general equilibrium production economies. Our model incorporates (i) heterogenous exposure to aggregate productivity shocks across capital vintages, and (ii) an endogenous stock of growth options. Our economy features a V-shaped term structure of aggregate dividends in which dividend yields decrease with maturity up to ten years, consistent with the empirical findings of Binsbergen et al. (2012a). Our model also reproduces the empirical negative relationship between cash-flow duration and expected returns in the cross section of book-to-market sorted stocks.

Keywords: Term Structure, Duration, Value Premium

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*Hengjie Ai is an assistant professor at the Carlson School of Management, University of Minnesota (hengjie.ai@gmail.com). Mariano Massimiliano Croce is an assistant professor at the Kenan-Flagler Business School, UNC–Chapel Hill (mmc287@gmail.com). Anthony M. Diercks is a graduate student in the Economics Department of UNC (diercks@live.unc.edu). Kai Li is a graduate student in the Economics Department of Duke University (kai.li@duke.edu). We thank Andrew Abel, Ravi Bansal, Joao Gomes, Amir Yaron, Nick Roussanov, and Lukas Schmid for their helpful comments on our article. We thank our discussant John Heaton. We also thank seminar participants at the 2013 AEA Meetings, the Wharton School, the Kenan-Flagler Business School (UNC), London Business School, the 2013 Carlson School of Management (UMN) Macro-Finance Conference , and the Fuqua School of Business.

1 Introduction

Recent empirical evidence suggests that the expected return on aggregate dividends is decreasing in maturity over a horizon of up to ten years (Binsbergen et al. 2012a). It is also well documented that value stocks feature both higher expected returns and shorter cash-flow duration than growth stocks (see, among others, Dechow et al. 2004 and Da 2009). We present a general equilibrium model with physical capital vintages and intangible capital to account for both.

Consistent with prior work (Ai et al. 2012), we consider a production economy with recursive preferences as in Epstein and Zin (1989) and heterogenous risk exposure of capital vintages. We show that this model generates a V-shaped term structure of equity returns. Specifically, risk premia of zero-coupon equities tend to decline over a maturity horizon of about 10 years, as documented in Binsbergen et al. (2012b) and Boguth et al. (2012). On the other hand, expected zero-coupon equity returns increase over longer maturity horizons, as in standard long-run risk models (Bansal and Yaron 2004).

In addition, we model intangible capital as an endogenous stock of growth options. The market value of a firm reflects the value of both assets in place and growth options, whereas the book value of a firm refers only to tangible assets. For this reason, the market-to-book ratio of a firm is often interpreted as a measure of growth options-intensity. In our model, high book-to-market ratio (value) stocks are tangible capital-intensive and have both a higher long-run risk exposure (Bansal et al. 2005, Bansal et al. 2007, Hansen et al. 2008) and a shorter cash flow duration (Dechow et al. 2004 and Da 2009) than low book-to-market ratio (growth) stocks. This result reconciles the long-run risk and the duration literature.

The reconciliation of the empirical evidence on cash flow duration and expected returns is particularly relevant because it has been a challenge for several leading asset pricing models, such as the habit model of Campbell and Cochrane (1999), the long-run risk framework of Bansal and Yaron (2004), and the rare disaster model of Reitz (1988), Barro (2006) and Gabaix (2009). Focusing on aggregate dividends, Binsbergen et al. (2012a) show that the above mentioned endowment economies imply either an upward sloping or a flat term structure of equity returns.

From the cross-sectional perspective, Santos and Veronesi (2010) and Croce et al. (2007) point out the difficulty in explaining simultaneously the empirical evidence on cash flow duration and expected return on value and growth stocks in standard asset pricing models. In contrast to our model, all the aforementioned endowment-based economies assume a stylized and exogenous stochastic process for dividends. Whether the observed empirical relationship between duration and expected return is consistent with any production economy that generates endogenous dividend processes is an open question. We provide a positive answer.

The time series properties of asset returns are as informative as those of macroeconomic quantities when assessing the economic mechanisms of DSGE models (Borovička et al. 2011 and Borovička and Hansen 2011). It is standard practice in the macroeconomics literature to confront the model with empirical evidence on the pattern along which fundamental shocks affect macroeconomic quantities. In contrast, research that investigates the implications of general equilibrium models on the time-series properties of asset returns is rare. Our paper fills this gap.

We start our analysis with a production economy with both short- and long-run productivity shocks (Croce 2008) and convex investment adjustment costs (Jermann 1998). We show that (i) the term structure of equity returns in this model is upward sloping, and (ii) the risk premium on the very short term dividend strip is strongly negative. The empirical evidence of Binsbergen et al. (2012a, b), therefore, is an anomaly for classical production economy models.

To understand the above results, note that dividend payout equals capital income minus total cost of investment. If the model is calibrated to match the high volatility of investment and the relatively constant labor share in the data, short-maturity dividends become a powerful hedge against productivity shocks. This is because investment in the model is highly pro-cyclical with respect to both short-run and long-run productivity shocks. In contrast to the consumption cash flow, short-term dividends are not pro-cyclical and hence require a negative risk premium.

Turning our attention to the long-end of the term structure of equities, we document that long-term dividends are as risky as long-term consumption. This is because along the balanced growth path, the long-run properties of investment, capital income, and aggregate consumption are similar. Since the risk premium on the claim to the entire stream of aggregate dividends is a value-weighted average of the zero-coupon equity premia, the dividends risk premium is lower than that associated to aggregate consumption. This result originates the equity premium puzzle in production economy documented by

Rouwenhorst (1995).

Most importantly, we show that the long-term level of the term structure of both dividends and consumption is fully determined by the long-term risk properties of productivity. This implies that the long-end of all term structures is exogenously determined by the assumed characteristics of the productivity process. To resolve both the equity premium puzzle and the term structure anomaly in production economies, it is essential to produce risky short-term dividend strips.

Incorporating heterogenous exposure to aggregate productivity risks across capital vintages dramatically changes the shape of the term structure and allows short-term dividends to be at least as risky as their long term counterparts. Specifically, consistent with the empirical evidence in Ai et al. (2012), our model features zero adjustment cost and a lower productivity risk exposure of new investments relative to capital of older vintages. This setup has two important implications. First, short-run productivity shocks increase output immediately. In the absence of adjustment cost, investment responds strongly to these shocks to smooth consumption. As a result, investment in our model is volatile and pro-cyclical as in the data. In fact, our model largely inherits the success of standard real business cycle models in terms of the dynamics of macroeconomic quantities, which are primarily driven by short-run productivity shocks.

Second, because young capital vintages are less exposed to aggregate productivity shocks than older vintages, good news about future aggregate productivity does not apply to new investments. On one hand, this feature of the model implies that the substitution effect associated with long-run productivity shocks is small, i.e., the agent has no incentive to immediately invest more. On the other hand, the income effect implies that the agent has an incentive to immediately increase consumption, as old capital is expected to be more productive in the future. At the equilibrium, upon the realization of good news for the long-run, our agent finds it optimal to delay investment. Specifically, we observe an immediate short-run drop in investment and a simultaneous positive spike in aggregate dividends.

From a macroeconomic perspective, this response is consistent with recent empirical evidence in the macroeconomic news literature (Barsky and Sims 2011). From an asset pricing perspective, note that the aforementioned spike in aggregate dividends comes in states of the world in which the agent has low marginal utility. Therefore, short-term dividends become extremely risky and carry a substantial risk premium.

In our setting, the long-run behavior of the response of investment to long-run shocks is similar to the response of a frictionless production economy. This is because over time new investments age and become as exposed to long-run shocks as older vintages. The same holds for the response of output and dividends. For this reason, over longer horizons the term structure of equity returns slopes up, as it does in standard long-run risk models. This explains the V-shape of our term structure of equity.

In the final step of our analysis, we introduce intangible capital and study the joint implications of our model on the term structure of aggregate dividend returns and on the link between cash flow duration and expected returns in the cross section. We model intangible capital as growth options and physical capital as assets in place (Ai 2007). This implies that our cross-sectional risk heterogeneity can be tied back to risk differences across tangible and intangible capital, in the spirit of Hansen et al. (2005) and Li (2009).

In our model, growth options produce no cash-flows until they are exercised, whereas assets in place always produce cash-flows. This implies that our growth options have longer duration than assets in place. As in Ai et al. (2012), growth options have a negative exposure to long-run productivity shocks and require a lower compensation for risk than assets in place. Since in our economy growth stocks are option-intensive whereas value stocks are assets in place-intensive, our model reproduces the cross-sectional negative relationship between cash flow duration and expected returns (Dechow et al. 2004 and Da 2009). In addition, we replicate the failure of the one-factor CAPM in explaining the cross-section of returns, as growth options and assets in place have different exposures to short- and long-run shocks.

The rest of the paper is organized as follows. In the next section, we discuss further the existing literature. In section 2, we detail our model. Section 3 reports no-arbitrage conditions required to price zero-coupon equities. In section 4, we study the term structure of equity returns in a standard production economy with convex adjustment costs. In section 5, we introduce capital vintages. In section 6, we consider intangible capital. Section 7 concludes.

2 Discussion of the Literature

Using an affine quadratic discount rate, Lettau and Wachter (2007, 2011) are the first ones to explore correlation structures consistent with the negative relationship between cash flow duration and expected returns in the cross section. Santos and Veronesi (2010) study the impact of cash flow risk on the aggregate term structure of equity returns in a habit-based model. Croce et al. (2007) study the term structure of equities in a learning model with long-run risks. Belo et al. (2012) emphasize the role of financial leverage as a device to reallocate risk from the long-end to the short-end of the term structure of net equity payout.

All the aforementioned studies produce a downward sloping term structure of equity returns. We differ from these papers in at least two respects. First, we study dividends that are a direct outcome of investment and production decisions, whereas previous studies focus on exogenously specified cash-flows. This allows us to tie the asset pricing implications of our model to the dynamics of several macroeconomic quantities. More broadly, our production economy framework enables us to impose joint structural restrictions on cash-flows and the pricing kernel using data about both macroeconomic aggregates and cross-sectional returns. Second, our results suggest that the term structure of equity may be V-shaped, as opposed to being monotonic and decreasing over maturities.

Similarly to us, Gomes et al. (2003) propose a general equilibrium model with both tangible assets and growth options. We differ from Gomes et al. (2003) in several dimensions. First, in our setting, the ability to produce and store growth options makes them an insurance factor, as options are less risky than assets in place. In the Gomes et al. (2003) economy, growth options are not storable and represent an additional risk source, as in Berk et al. (1999) and Carlson et al. (2004).

In Gomes et al. (2003), options are a device to shift the whole term structure of equity returns upward and obtain higher equity premium. In our setting, adding options to the model with tangible capital shifts the term structure of dividends downward. Since heterogenous exposure to producibility shocks across capital vintages makes short-term dividends very risky, the average level of our term structure of dividends is consistent with the equity premium observed in the data.

Most importantly, in Gomes et al. (2003) the term structure of equity is upward sloping, implying that value stocks have longer duration than growth stocks, in contrast to the duration evidence. This is because in Gomes et al. (2003) the term structure of equity returns is mainly a reflection of their upward sloping term structure of real bonds. In our model, the term structure of real bonds is downward sloping and the equity term structure is V-shaped.

Papanikolaou (2011), and Kogan and Papanikolaou (2009, 2010, 2012) focus on investment specific shocks to explain cross-sectional returns in production-based general equilibrium models. A broader review of the production based asset-pricing literature is provided by Kogan and Papanikolaou (2011). This class of models can produce a downward sloping term structure, consistent with the Binsbergen et al. (2012a) findings. We complement Kogan and Papanikolaou (2012)'s analysis and add two novel insights. First, we abstract away from investment-specific shocks and highlight the role of the intertemporal composition of productivity risk, i.e. long- and short-run risks, to explain cross-sectional returns. Second, our model suggests that the term structure of equity may change slope at different maturity horizons.

Hansen and Scheinkman (2010), Jaroslav et al. (2011) and Hansen (2011) develop novel methods to quantify the time profile of risk exposure of cash flows to macroeconomic shocks. Borovička and Hansen (2011) provide novel tools to analyze both risk exposure and risk compensation of cash flows at different horizons in nonlinear models. Specifically, Borovička and Hansen (2011) apply their methods to study the risk profile of tangible and intangible capital in the context of the Ai et al. (2012) model. We differ from them because of our focus on the term structure of zero-coupon equity returns, i.e., the closest model counterpart of what is measured in Binsbergen et al. (2012a, b), and Boguth et al. (2012).

3 General Formulation of Our Model

We consider three classes of models: (i) a real business cycle model with homogenous capital; (ii) a vintage capital model where new investments are less exposed to aggregate productivity shocks than capital of older vintages; and (iii) an intangible capital model with heterogenous vintages of physical capital. In this section, we describe a general framework that incorporates all three economies as special cases.

Across all the economies that we consider, equilibrium quantities in the decentralized economy coincide with those obtained under the central planner's problem. For this reason, in this section we describe only the key elements of the Pareto problem of our economies. Under the assumption that markets are complete, asset prices are recovered from the planner's shadow valuations.

Preferences. Time is discrete and infinite, $t = 1, 2, 3, \cdots$. The representative agent has Kreps and Porteus (1978) preferences, as in Epstein and Zin (1989):

$$V_t = \left\{ (1-\beta)\, u\left(C_t, N_t\right)^{1-\frac{1}{\psi}} + \beta \left(E_t \left[V_{t+1}^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}}, \tag{1}$$

where C_t and N_t denote, respectively, the total consumption and total hours worked at time t. For simplicity, we consider a Cobb-Douglas aggregator for consumption and leisure:

$$u\left(C_t, N_t\right) = C_t^o (1-N_t)^{1-o}.$$

We normalize $N_t = 1$ in the case of inelastic labor supply, i.e., when $o = 1$.

Production Technology. Total output, Y_t, is produced according to a neoclassical Cobb-Douglas production technology:

$$Y_t = K_t^{\alpha} (A_t N_t)^{1-\alpha}, \tag{2}$$

where A_t is the labor augmenting productivity shock, and K_t is the stock of physical capital. In economies with heterogenous capital vintages, A_t is interpreted as the productivity of the initial generation of capital vintage, and K_t is interpreted as the productivity-adjusted capital stock.

Total output can be used for consumption, C_t, investment in physical capital, I_t, and investment in intangible capital (if any), J_t:

$$C_t + I_t + J_t = Y_t. \tag{3}$$

The law of motion of the productivity process is specified as in Croce (2008) and captures both short-run and long-run productivity risks:

$$\log \frac{A_{t+1}}{A_t} \equiv \Delta a_{t+1} = \mu + x_t + \sigma_a \varepsilon_{a,t+1}, \tag{4}$$
$$x_{t+1} = \rho x_t + \sigma_x \varepsilon_{x,t+1},$$
$$\begin{bmatrix} \varepsilon_{a,t+1} \\ \varepsilon_{x,t+1} \end{bmatrix} \sim i.i.d.N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right), \quad t = 0, 1, 2, \cdots .$$

According to the above specification, short-run productivity shocks, $\varepsilon_{a,t+1}$, affect contemporaneous output directly, but have no effect on future productivity growth. Shocks to long-run productivity, represented by $\varepsilon_{x,t+1}$, carry news about future productivity growth rates, but do not affect current output.

Capital Accumulation Technologies. We first specify the law of motion of intangible capital. Let S_t denote the total stock of intangibles available at time time t. We follow Ai (2007) in modeling intangible capital as a stock of growth options:

$$S_{t+1} = [S_t - G(I_t, S_t)] \times (1 - \delta_S) + H(J_t, K_t), \tag{5}$$

where $H(J_t, K_t)$ is a concave, constant return to scale adjustment cost function parameterized as follows:

$$H(J_t, K_t) = \left[\frac{a_1}{1 - 1/\xi} \left(\frac{J_t}{K_t} \right)^{1-1/\xi} + a_2 \right] K_t,$$

and the parameters a_1 and a_2 are determined so that at the steady state $\overline{H} = \overline{J}$ and $\overline{\partial H / \partial J} = 1$.

Each growth option can be used to build one unit of physical capital at a cost that depends on an i.i.d. shock distributed according to f. As shown in Ai (2007), under the optimal option exercise rule, the total amount of physical capital created at time t depends only on I_t and S_t and can be represented as a concave and constant return to scale aggregator, $G(I_t, S_t)$. Because each growth option can be used to construct exactly one unit of physical capital, $G(I_t, S_t)$ is also the total amount of growth options exercised at time t.

Equation (5) can therefore be interpreted as follows. At time t, the agent has a mass S_t of available growth options. If options are exercised optimally and the total amount of investment goods used to exercise options is I_t, then $[S_t - G(I_t, S_t)] \times (1 - \delta_S)$ is the total amount of growth options left at the end of the period after depreciation. $H(J_t, K_t)$ is the amount of growth options newly produced in period t.

Following Ai et al. (2012), we focus on the class of cost distributions f that generate an aggregator G with constant elasticity of substitution between physical investment and intangible capital:

$$G(I, S) = \left(\nu I^{1-\frac{1}{\eta}} + (1-\nu) S^{1-\frac{1}{\eta}}\right)^{\frac{1}{1-1/\eta}}. \tag{6}$$

Ai (2007) shows that for each combination of $\nu \in (0,1)$ and $\eta > 0$, there exists a well defined cost distribution f. Ai et al. (2012) show that their benchmark f distribution conforms well with data on the cross-section of book-to-market ratios.

We now turn our attention to tangible capital. We allow investments in different vintages of tangible capital to have heterogenous exposure to aggregate productivity shocks. The productivity processes are specified as follows. First, we assume that the log growth rate of the productivity process for the initial generation of production units, Δa_{t+1}, is given by equation (4).

Second, we impose that the growth rate of the productivity of capital vintage of age $j = 0, 1, ..., t-1$ is given by

$$\frac{A_{t+1}^{t-j}}{A_t^{t-j}} = e^{\mu + \phi_j (\Delta a_{t+1} - \mu)}. \tag{7}$$

Under the above specification, production units of all generations have the same unconditional expected growth rate. We also set $A_t^t = A_t$ to ensure that new production units are on average as productive as older ones. Heterogeneity is driven solely by differences in aggregate productivity risk exposure, ϕ_j.

The empirical findings in Ai et al. (2012) suggests that ϕ_j is increasing in j, that is, older production units are more exposed to aggregate productivity shocks than younger ones. To capture this empirical fact, we adopt a parsimonious specification of the ϕ_j function as follows:

$$\phi_j = \begin{cases} 0 & j = 0 \\ 1 & j = 1, 2, \cdots. \end{cases}$$

That is, new production units are not exposed to aggregate productivity shocks in the initial period of their life, and afterwards their exposure to aggregate productivity shocks is identical to that of all other existing generations.

Let K_t denote the productivity adjusted physical capital stock. As proven in Ai et al. (2012), aggregate production can be represented as a function of K_t and N_t as in equation (2) despite the heterogeneity in productivity. In addition, the law of motion of K_t can be written as:

$$K_{t+1} = (1-\delta_K) K_t + \varpi_{t+1} M_t, \quad t = 1, 2, \cdots \tag{8}$$

$$\varpi_{t+1} = \left(\frac{A_{t+1}^t}{A_{t+1}}\right)^{\frac{1-\alpha}{\alpha}} = e^{-\frac{1-\alpha}{\alpha}(x_t+\sigma_a \varepsilon_{a,t+1})(1-\phi_0)} \quad \forall t,$$

where M_t is the total mass of new vintage capital produced at time t, and ϖ_{t+1} is an endogenous process that accounts for the productivity gap between the newest capital vintage and all older vintages. Note that when $\phi_0 = 1$, the new capital vintage has the same exposure to aggregate productivity shocks as older ones. In this case, $\varpi_{t+1} = 1$ for all t and plays no role.

The three classes of models considered in this article differ in terms of the dependence of M_t on investment I_t and for our assumptions on ϕ_0. We detail these conditions in what follows.

Three Economies. First, we consider a classical real business cycle (RBC) model with Jermann (1998)'s convex adjustment costs and homogenous vintage capital ($\phi_0 = 1$):

$$\begin{aligned} M_t &= \left[\frac{\alpha_1}{1-\frac{1}{\tau}}\left(\frac{I_t}{K_t}\right)^{1-\frac{1}{\tau}} + \alpha_0\right] K_t, \\ \varpi_{t+1} &= 1. \end{aligned} \tag{9}$$

The case of no adjustment costs corresponds to the parameter specification $\tau = \infty$. In this setting, the production of new investment goods does not require exercising growth options. At the equilibrium, there is no need to accumulate any intangible capital ($S_t = J_t = 0$ for all t), and hence physical capital is the only relevant stock.

The second class of economies that we consider abstracts away from both adjustment costs and intangible capital, but allows for vintage capital. Specifically, we continue to assume that physical capital accumulation does not require growth options and set $\tau = \infty$ to remove adjustment costs. In this setting, we assume $\phi_0 = 0$, so that new investments are less exposed to capital of older vintages. The law of motion of tangible capital is written as:

$$K_{t+1} = (1 - \delta_K) K_t + \varpi_{t+1} I_t. \tag{10}$$

The third class of economies retains heterogenous risk exposure of physical capital vintages ($\phi_0 = 0$) and incorporates intangible capital. Specifically, we assume that the accumulation of an extra unit of physical capital requires the exercise of a growth option, i.e., a unit of intangible capital. In this setting, $M_t = G(I_t, S_t)$ and the accumulation of tangible capital is tightly related to the stock of growth options specified in equation (5):

$$K_{t+1} = (1 - \delta_K) K_t + \varpi_{t+1} G(I_t, S_t).$$

Term Structures. Given a sequence of cash flows, $\{CF_t\}_{t=0}^{\infty}$, the time-t present value of a cash flow available at time $t+n$ is denoted by $P_{t,t+n}$ and can be computed as follows:

$$P_{t,t+n} = E_t[\Lambda_{t,t+n} CF_{t+n}] \quad n = 1, 2, \ldots,$$

where $\Lambda_{t,t+n} = \Lambda_{t,t+1} \times \Lambda_{t+1,t+2} \times \cdots \times \Lambda_{t+n-1,t+n}$ is the n-step ahead discount factor, and the one-step ahead stochastic discount factor is

$$\Lambda_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-1} \left(\frac{u_{t+1}}{u_t} \right)^{1-\frac{1}{\psi}} \left(\frac{V_{t+1}}{E_t \left[V_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma}. \tag{11}$$

The one-period return of the claim to CF_{t+n} is simply $\frac{P_{t+1,t+n}}{P_{t,t+n}}$. We are interested in studying the unconditional risk premium, $RP(n)$, on this return for different maturities n:

$$RP\left(n\right) = E\left[\frac{P_{t+1,t+n}}{P_{t,t+n}} - r_t^f\right], \quad n = 1, 2, \cdots,$$

where $r_t^f = \frac{1}{E[\Lambda_{t,t+1}]}$ is the one-period risk-free interest rate. The term structure of a cash flow sequence $\{CF_t\}_{t=0}^{\infty}$ refers to the link between $RP(n)$ and n.

We study the term structure of both the consumption claim and the dividend payout of the representative firm. In our setting, dividends equal capital income net of total investment costs, that is, $\alpha Y_t - I_t - J_t$. Given this consideration, to better understand the composition of dividends risk, we also study the term structure of a claim to total investment, $I_t + J_t$. In addition, we report the implications for the term structure of real interest rates so that we can better disentangle dividends risk premia from bond risk premia over different horizons. In summary, in the rest of the paper we focus on the following cash flow processes:

$$CF_t = \begin{cases} A_t & \text{Productivity} \\ Y_t & \text{Output} \\ C_t & \text{Consumption} \\ I_t + J_t & \text{Total investment} \\ \alpha Y_t - I_t - J_t & \text{Dividends} \\ 1 & \text{Real Bond.} \end{cases}$$

4 Neoclassical Business Cycle Models

In this section, we focus on specifications of the neoclassical real business cycle model featuring different capital accumulation formulations and risk structures. All these specifications produce an upward sloping term structure of equity, in contrast to the empirical evidence provided by Binsbergen et al. (2012a). This step of our analysis shows that the term structure anomaly documented by Binsbergen et al. (2012a) in endowment economy applies to standard RBC models as well.

We compare three models featuring different productivity processes and adjustment cost parameters in order to examine their implications on the shape of the term structure of dividends. We report the calibrations adopted in our study in Table 1, and summarize the main statistics produced by the three formulations of the RBC model in Table 2.

Table 1: Calibrated parameter values

		RBC Models			Vintage Capital Models		
		(1)	(2)	(3)	(4)	(5)	(6)
Preference parameters							
Discount factor	β	0.974	0.974	0.974	0.9705	0.9705	0.986
Labor-adjusted risk aversion	$\gamma \cdot o$	10	10	10	10	10	10
Intertemporal elasticity of substitution	ψ	2.0	2.0	2.0	2.0	2.0	2.0
Leisure weight	o	1	1	1	1	0.345	0.345
Technology parameters							
Capital share	α	0.3	0.3	0.3	0.35	0.35	0.3
Depreciation rate of physical capital	δ_K	9.1%	9.1%	9.1%	8%	8%	11%
Adjustment Costs on tangibles	τ	–	3.25	3.25	–	–	–
Depreciation rate of intangible capital	δ_S	–	–	–	–	–	11%
Weight on physical investment	ν	–	–	–	–	–	0.93
Elasticity of substitution	η	–	–	–	2.5	2.5	3.5
Adjustment Costs on Intangibles	ξ	–	–	–	–	–	5
Total factor productivity parameters							
Risk exposure of new investment	ϕ_0	1	1	1	0	0	0
Average growth rate	μ	2.0%	2.0%	2.0%	2.0%	2.0%	2.0%
Volatility of short-run risk	σ_a	5.08%	5.08%	5.08%	5.08%	5.08%	4.4%
Volatility of long-run risk	σ_x	–	–	0.86%	0.86%	0.86%	0.75%
Autocorrelation of expected growth	ρ	–	–	0.925	0.925	0.925	0.925

This table reports the parameter values used for our annual calibrations. Model (1) is a neoclassical real business cycle (RBC) model with short-run risk only. Model (2) features adjustment costs. Model (3) is a real business cycle model with both adjustment costs and long-run productivity shocks. Models (4)–(6) incorporate vintage capital. Model (5)–(6) feature endogenous labor supply. Model (6) is our benchmark model with intangible capital.

Table 2: Neoclassical Real Business Cycle Models

	Data	Model (1)	Model (2)	Model (3)
Short-Run Risk		Y	Y	Y
Adj. Costs			Y	Y
Long-Run Risk				Y
E[I/Y]	00.15	00.27	00.26	00.29
$\sigma(\Delta c)$	02.31	01.83	02.76	03.22
$\sigma(\Delta i)$	16.40	10.05	05.58	06.15
$E[r^f]$	00.89	02.79	02.47	00.86
$E[r^{ex}]$	05.70	00.60	02.90	04.00

All entries for the models are obtained from repetitions of small samples. Data refer to the U.S. and include pre-World War II observations (1930–2007). Our annual calibrations are reported in Table 1. Excess returns are levered by a factor of three, consistent with Garca-Feijo and Jorgensen (2010).

In our model (1) productivity growth is assumed to be i.i.d., and adjustment costs are null. We incorporate adjustment costs in model (2) by setting $\tau > 0$. In model (3), we further incorporate long-run productivity shocks. In this section, we assume that the labor supply is inelastic across all models. Relaxing this assumption affects only marginally the term structure of equity returns and does not alter its positive slope.

Short-run risk only. Model (1) refers to the neoclassical RBC model in its most essential form (without any investment frictions and subject only to short-run shocks). Even though this model produces counterfactual implications for asset prices, it is a useful starting point to understand key features of the term structures in production economies.

We plot the term structure of consumption, investment, dividends, output, and productivity in Figure 1. Several features of the term structures are salient. First, the long-end of all term structures converges to the same level because of the cointegration relationship imposed by our general equilibrium model. Over long horizons, balanced growth implies that the dynamics of all quantities should be determined by productivity. As a result, all risk premia must converge to that of the exogenous productivity process. Under our calibration, the long-end of the terms structures is about 1.5% per year. This sizeable figure is due to the fact that our shocks are permanent in productivity levels.

Second, looking at the short-end of the term structures, we can see very different levels across macroe-

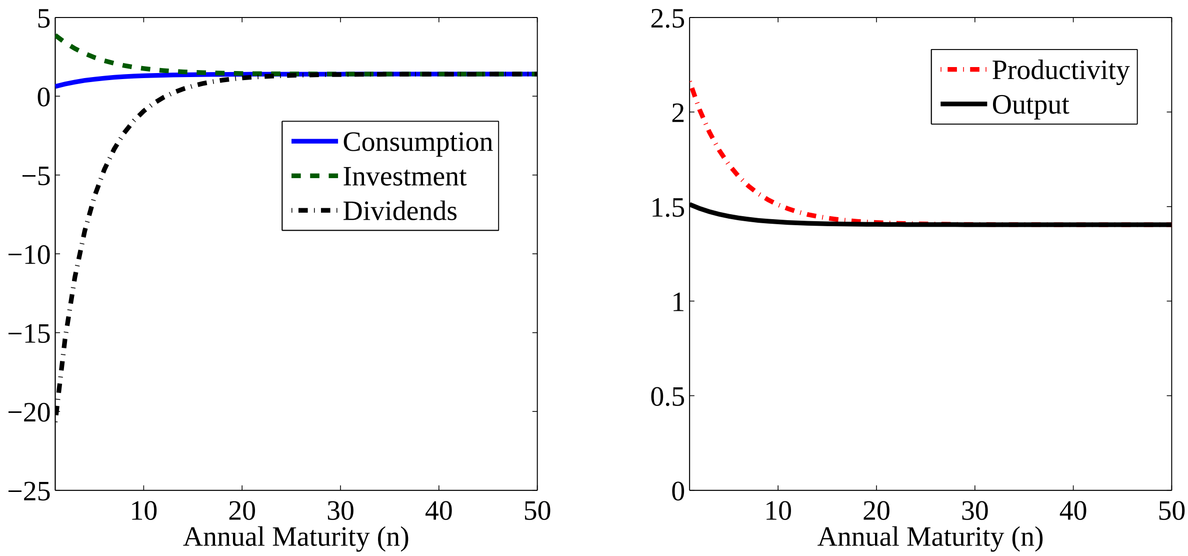

Fig. 1: Term Structures in Model (1)

This figure shows annual excess returns of zero coupon equities for different maturities associated with different cash-flows. Excess returns are multiplied by 100. These results are based on the calibration used for model 1, as reported in Table 1.

conomic quantities. This pattern is due to short-run consumption smoothing. When a positive short-run shock materializes, output, consumption and investment simultaneously increase. The response of consumption is moderate, whereas the investment response is much stronger, consistent with the volatility figures reported in Table 2. As a result, investment cash-flows are more risky in the short-run than over long maturities and generate a downward sloping term structure. The opposite is true for the consumption cash-flow, as the claim to consumption is a position long in output and short in investment, $C_t = Y_t - I_t$.

Third, the term structure of dividend has a steep positive slope. Like for consumption, the dividends claim is a position short on investment and long on capital income, $D_t = \alpha Y_t - I_t$. Because the share of capital income, α, is close to the average investment-output ratio, the leverage effect from investment is much stronger than in the consumption case. As a result, the risk premium on short-term dividends is actually negative, as short-term dividends provide a strong hedge against short-run productivity shocks. The negative intercept of the dividends term structure and the low level of its long-end reflect the well-known difficulty in generating a high equity premium in a standard RBC model (Rouwenhorst 1995).

The role of adjustment costs. A common way to improve the asset pricing implications of a production economy is to introduce capital adjustment costs, as they allow for fluctuations in the marginal value of capital. We add convex adjustment costs to the neoclassical RBC setting in our model (2).

As documented by Croce (2008), this class of adjustment costs is unsatisfactory in the long-run risk framework because it poses a strong trade-off between the equity premium and investment volatility. Specifically, adding adjustment costs reduces significantly the response of investment to productivity shocks. In model (2), for example, when we calibrate the adjustment costs to achieve a levered equity premium of almost 3%, the volatility of investment growth becomes three times smaller than in the data.

We depict the term structures generated with and without adjustment costs in Figure 2. Compared to the model without adjustment cost, the term structure of investment in model (2) is much flatter, precisely because investment is less responsive to productivity shocks. As a result, the term structures of consumption and dividends are less steep than those obtained in model (1).

In the top-left panel of Figure 2, we depict the term structure of real bonds. As in the case of dividends, consumption and investment, adjustment costs make the term structure of real bonds flatter, but do not alter the basic pattern obtained in the standard RBC model. In contrast to the term structure of consumption and dividends, our term structure of real bonds is downward sloping, implying that our dividends term structure is not a mere reflection of the behavior of long-term interest rates.

This result is standard in endowment economies with long-run risk. Indeed, even though our productivity growth is i.i.d., our consumption process features an endogenous predictable component due to capital accumulation. High expected consumption growth rates are associated with higher interest rates and low valuation of real bonds. In these states of the world, the representative agent also experiences low marginal utilities under preference for early resolution of uncertainty. As a result, real bonds provide insurance and require a negative risk premium. This effect is stronger for long duration bonds because their price is more sensitive to interest rates fluctuations.

The role of long-run risk. In model (3), we incorporate long-run productivity shocks and show that they increase the equity risk premium by making the term structure of equity returns even more upward sloping, i.e., more inconsistent with the empirical results of Binsbergen et al. (2012a).

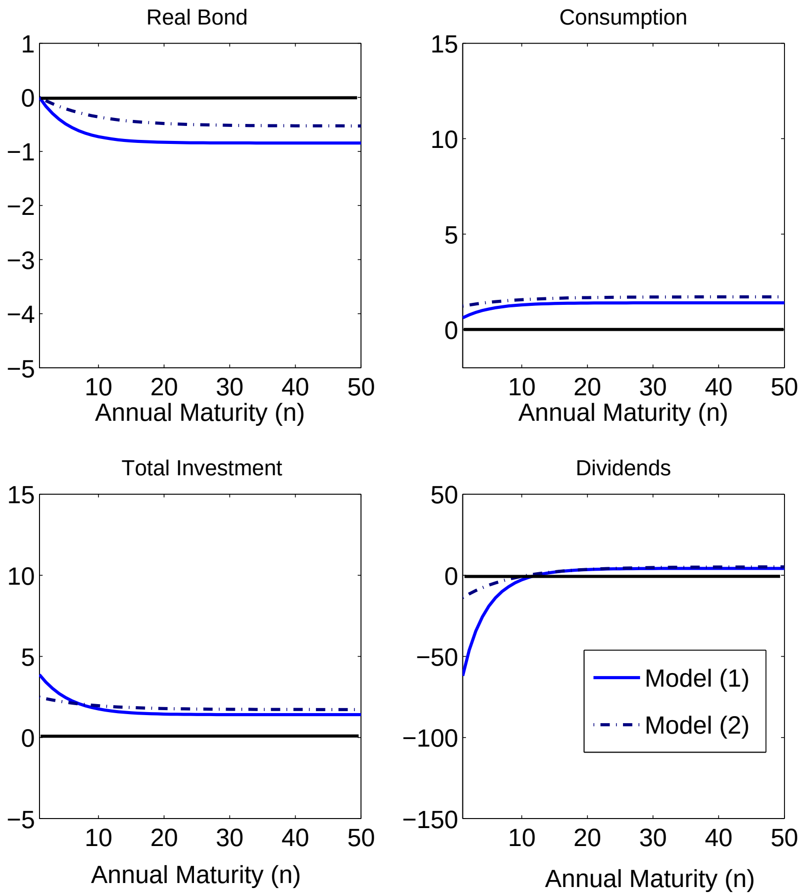

Fig. 2: The Role of Adjustment Costs on the Term Structures.
This figure shows annual excess returns of zero coupon equities for different maturities associated with different cash-flows. Excess returns are multiplied by 100. Dividend returns are levered by a factor of three. Model (1) is the neoclassical RBC model with short-run shocks only. Model (2) features convex adjustment costs. All the parameters are calibrated to the values reported in Table 1.

We plot the term structure of output, consumption, investment, dividends and real bonds for both model (2) and (3) in Figure 3 and make several remarks. First, the overall equity premium in model (3) is higher than that in model (2) due to the introduction of long-run risk. In fact, as shown in Table 2, the average risk premium increases to 4% under Model (3), as long-run productivity growth shocks translate into long-run risks in consumption and dividends (Croce 2008).

Second, the term structure of consumption, investment, and output are all upward sloping. This feature of the model is a consequence of the introduction of long-run risk in productivity. On the one

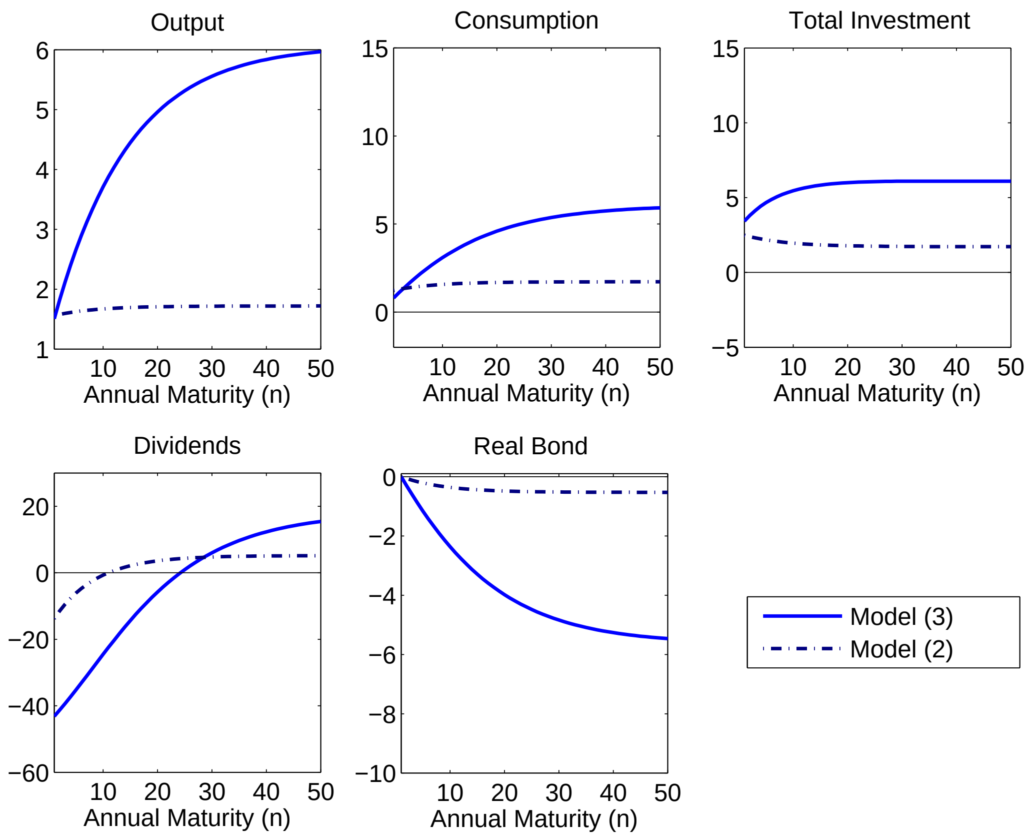

Fig. 3: The Role of Long-Run Risk on the Term Structures

This figure shows annual excess returns of zero coupon equities for different maturities associated with different cash-flows. Excess returns are multiplied by 100. Dividend returns are levered by a factor of three. Model (2) features short-run productivity shocks and convex adjustment costs. Model (3) is augmented with long-run productivity risk. All the parameters are calibrated to the values reported in Table 2.

hand, high expected productivity growth is associated with high expected consumption growth and high interest rates. Keeping everything else constant, this should imply less upward sloping curves, as for the real bonds term structure.

On the other hand, however, long-run productivity shocks also imply high growth of expected cash flows for consumption, investment, and output. When the agent has a preference for early resolution for uncertainty ($\gamma - 1/\psi > 0$), the cash flow effect dominates the discount effect and valuation ratios increase with positive long-run productivity shocks. This effect is stronger for long duration cash flows because their value is more sensitive to expected growth rate shocks. As a result, the compensation required for long-run risk is positive for the cash flows associated with output, consumption, investment and dividends. Most importantly, risk premia increase with the maturity of all these cash flows.

Third, the slope of the term structure of investment is considerably smaller than that of the output term structure. This is because short-term investment is more exposed to long-run productivity shocks than consumption and output. When the IES is high enough, positive news about future productivity increases contemporaneous investment because the substitution effect dominates the income effect (Croce 2008). Since long-run news has no impact on current output, consumption has to drop upon the realization of long-run shocks (due to the resource constraint). At the equilibrium, short-term consumption provides a hedge against long-run shocks, whereas short-term investment provides a risky payoff. This implies that the intercept of the consumption term structure has to be lower than that of the investment term structure. Since all term structures converge to the same level over the long-run, the slope of the consumption term structure is more pronounced.

Fourth, the term structure of dividends is even more upward sloping than that of consumption. As mentioned before, the dividends cash flow corresponds to a position long in capital income and short in investment. Since the leverage effect of investment on dividends is stronger, dividends end up providing substantial insurance over short maturities.

In summary, adding long-run risk does not help in reconciling the model with Binsbergen et al. (2012a)'s findings on the term structure of equity returns. In the next section we provide a resolution of the term structure anomaly.

5 Models with Vintage Capital

In this section, we consider models that incorporate the empirical fact that new investments are less exposed to aggregate productivity shocks than capital of older vintages. This empirical evidence is discussed in detail in Ai et al. (2012). In model (4), we remove adjustment costs ($\tau = \infty$) and incorporate heterogenous productivity of vintage capital by setting $\phi_0 = 0$, consistent with our equation (10). We show that our model with heterogenous capital vintages generates a V-shaped term structure of equity returns. In particular, the dividends risk premium decreases with maturity in the short term, consistent with the empirical evidence documented in Binsbergen et al. (2012a). We first discuss the optimality conditions of the social planner's problem, and then we study the implications of our model.

Optimality Conditions. Let $q_{K,t}$ and $p_{K,t}$ denote the ex- and cum-dividends price of one unit of productivity-adjusted aggregate physical capital, respectively. Given equilibrium quantities, $q_{K,t}$ and $p_{K,t}$ are jointly determined by the following recursions:

$$\begin{aligned} p_{K,t} &= \alpha \frac{Y_t}{K_t} + (1-\delta) q_{K,t}, \\ q_{K,t} &= E_t \left[\Lambda_{t+1} p_{K,t+1} \right]. \end{aligned} \tag{12}$$

Optimality of investment implies

$$1 = E_t[\Lambda_{t+1} p_{K,t+1} \varpi_{t+1}]. \tag{13}$$

The left hand side of equation (13) measures the marginal cost of investment. In the absence of adjustment costs, the marginal cost is one. The right hand side of this equation shows the marginal benefit of investment. In our vintage capital economy, the newly established capital vintage is less exposed to aggregate productivity shocks than older vintages. This heterogeneity creates a wedge between the productivity of new and older capital generations measured by ϖ_{t+1}.

Specifically, after accounting for productivity differences, one unit of new vintage capital is equivalent to ϖ_{t+1} units of the existing capital stock, where the expression for ϖ_{t+1} is given by equation (8). Therefore, the marginal benefit of investment measured in terms of time-t consumption numeraire is $E_t[\Lambda_{t+1} p_{k,t+1} \varpi_{t+1}]$. This is the key feature that distinguishes our model from the standard RBC model discussed in the last section. Heterogenous exposure to risk across capital vintages is what improves our asset pricing implications.

Impulse Response Functions. To understand the implications of vintage capital, we study the impulse response functions of quantities and prices of both the vintage capital model and the RBC model with adjustment costs (model (4) and model (3), respectively). Figure 4 depicts the response of both asset prices and quantities to short-run shocks. The responses to long-run productivity shocks are shown

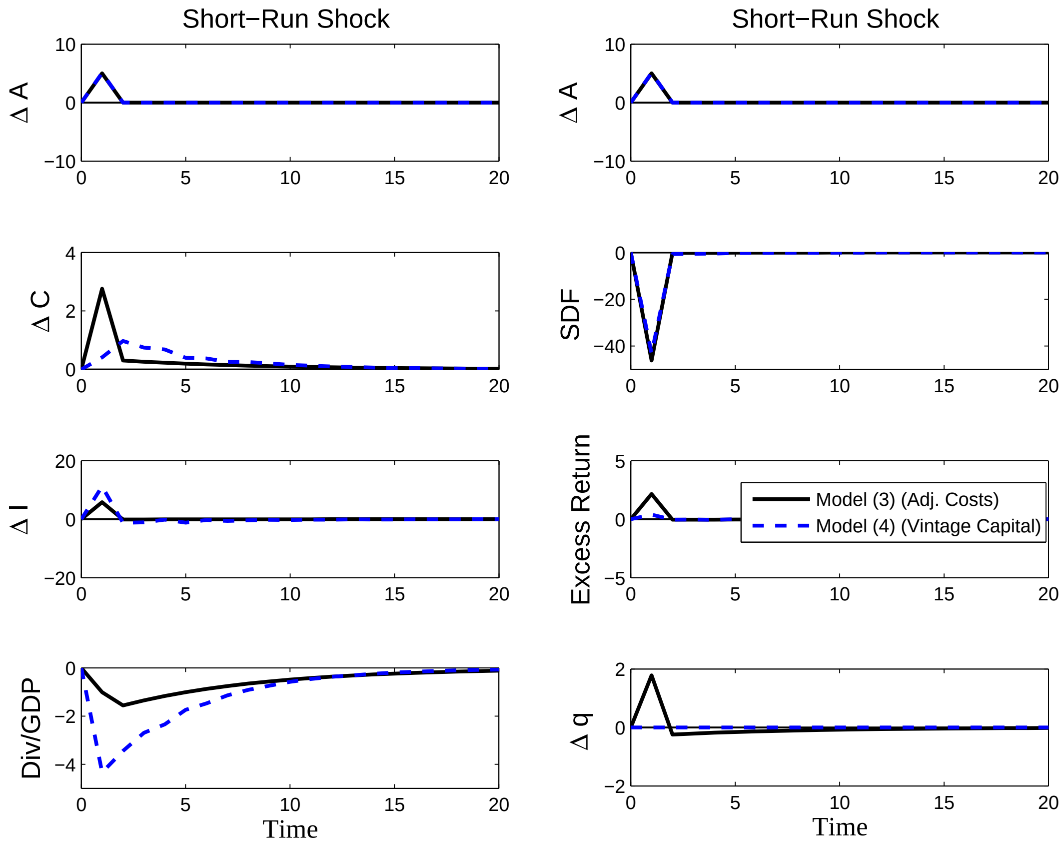

Fig. 4: Impulse Response Functions for Model (4) (SRR).

This figure shows percentage annual log-deviations from the steady state upon the realization of a positive short-run shock. Tangible capital returns are not levered. Both model (3) and (4) feature short- and long-run productivity risk. In model (3) we assume the existence of convex adjustment costs. Model (4) features heterogeneous capital vintages. For model (3), the deviations of the dividends-gdp ratio are multiplied by 10 to be visible. All the parameters are calibrated to the values reported in Table 1.

in Figure 5.

The differences in the impulse response functions with respect to short-run shocks in models (3) and (4) are due to the presence of adjustment costs. Because adjustment costs limit the agent's ability to smooth consumption, investment in model (3) responds less to short-run productivity shocks than in model (4). As a result, both the return and price of physical capital respond strongly to short-run productivity shocks in model (3), while these responses are basically null in model (4).

Because $D_t = \alpha Y_t - I_t$, the dividends-to-output ratio drops upon short-run productivity shocks in both model (3) and (4). This drop, however, has a much smaller magnitude in model (3) because of the less pronounced adjustment of investment. These patterns imply that physical capital requires a higher risk compensation for short-run productivity shocks in the adjustment cost model than in the vintage capital model.

Consistent with the statistics reported in Table 2, Figure 4 shows that a substantial share of the high equity premium obtained under model (3) is due to short-run shocks. As already mentioned, this strong performance on the equity premium side comes at the expense of a counter-factually low level of volatility of investment associated with adjustment costs. As shown in Table 3, matching the volatility of investment growth is not a problem in the context of our model (4). The reason for this result is that short-run i.i.d. growth shocks have almost no effect on investment decisions in the presence of heterogenous exposure to aggregate productivity shocks. In model (4), therefore, investment growth increases almost as much as in the case of a frictionless RBC economy (model (1)).

As shown in Table 3, model (4) inherits the success of the standard frictionless RBC model in replicating key features of the dynamics of quantities. Furthermore, our model with vintage capital produces a sizeable equity premium. A key message of Figure 4 is that the risk premium generated by short-run shocks is quantitatively unimportant in the vintage capital model. In order to understand the higher equity risk premium reported in Table 3 for model (4), we need to focus on the interaction between long-run shocks and the heterogeneous productivity of capital vintages.

Turning our attention to Figure 5, we can see that the impulse response functions with respect to long-run shocks are significantly different across the model with and without vintage capital. With a one-standard-deviation long-run productivity shock, the unlevered excess return on physical capital in model (4) increases by about 1.5%, whereas the change in the adjustment cost model is much smaller. This is the reason why the equity premium in the vintage capital model is significantly higher than that in model (3).

The realized return on physical capital, $r_{K,t+1}$, is given by:

$$1 + r_{K,t+1} = \frac{\alpha \left(\frac{K_{t+1}}{A_{t+1}}\right)^{-(1-\alpha)} + (1-\delta) q_{K,t+1}}{q_{K,t}},$$

and in the model with vintage capital, the response of $r_{K,t+1}$ is almost entirely driven by the spike of

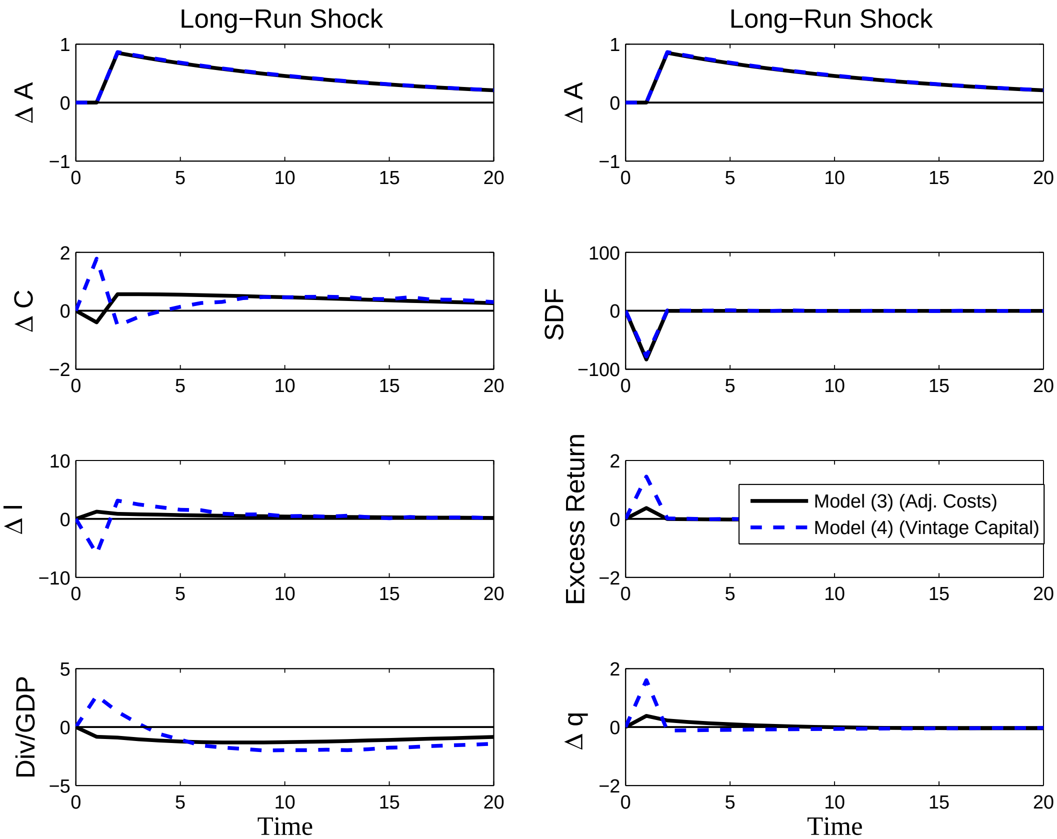

Fig. 5: Impulse Response Functions for Model (4) (LRR).
This figure shows percentage annual log-deviations from the steady state upon the realization of a positive long-run shock. Tangible capital returns are not levered. Both model (3) and (4) feature short- and long-run productivity risk. In model (3) we assume the existence of convex adjustment costs. Model (4) features heterogeneous capital vintages. For model (3), the deviations of the dividends-gdp ratio are multiplied by 10 to be visible. All the parameters are calibrated to the values reported in Table 1.

ex-dividend price of physical capital, $q_{K,t+1}$ (see Figure 5, bottom-right panel).

Iterating equation (12) forward, we can express $q_{K,t}$ as the present value of the infinite sum of all future payoffs:

$$q_{K,t} = \sum_{j=1}^{\infty} (1-\delta_K)^j \, E_{t+1}\left[\Lambda_{t+1,t+1+j}\alpha\left(\frac{K_{t+1+j}}{A_{t+1+j}}\right)^{\alpha-1}\right]. \tag{14}$$

Equation (14) implies that the price of an existing unit of physical capital, $q_{K,t}$, is the present value of the marginal product of capital in all future periods. A positive innovation in the long-run productivity

Table 3: Vintage Capital Models

	Data	Model (4)	Model (5)	Model (6)
Endogenous Labor			Y	Y
Intangible Capital				Y
$\mathrm{E}[I/Y]$	00.15 (00.05)	00.33	00.31	00.18
$\sigma(\Delta c)$	02.53 (00.56)	02.74	03.05	02.58
$\sigma(\Delta i)/\sigma(\Delta c)$	05.29 (00.50)	05.20	06.91	05.84
$\sigma(\Delta j)/\sigma(\Delta i)$	00.50 (00.07)	01.00	01.00	00.61
$\sigma(\Delta n)$	02.07 (00.21)	–	03.38	01.96
$\rho_{\Delta C,\Delta n}$	00.28 (00.07)	–	-00.31	00.05
$E[r^f]$	00.89 (00.97)	01.38	02.65	01.19
$E[r^{ex}]$	05.70 (02.25)	04.32	03.94	04.17
$E[r^L_K - r^L_S]$	04.32 (01.39)	–	–	04.76
$\alpha_K - \alpha_S$	04.01 (01.77)	–	–	05.98
$\Delta_K - \Delta_S$	-48.00 (—.—)	–	–	-13

All entries for the models are obtained from repetitions of small samples. Data refer to the U.S. and include pre-World War II observations (1930–2007). Numbers in parentheses are GMM Newey-West adjusted standard errors. $E\left[r^L_K - r^L_S\right]$ and $E\left[r^{L,ex}_M\right]$ measure the levered spread between tangible and intangible capital returns, and the market premium, respectively. Our leverage coefficient is three, consistent with Garca-Feijo and Jorgensen (2010). The difference in the intercept of the CAPM regression for tangible and intangible returns is denoted by $\alpha_K - \alpha_S$. The difference in the duration of cash flows of tangible and intangible capital is denoted by $\Delta_K - \Delta_S$ and is expressed in number of years. The empirical duration spread is from Dechow et al. (2004). Our annual calibrations are reported in Table 1.

component, x_{t+1}, has two effects on the future marginal product of physical capital. The first is a direct effect: keeping quantities constant, an increase in x_{t+1} raises the marginal product of physical capital by increasing all future A_{t+1+j} for $j = 1, 2, \cdots$. The second is a general equilibrium effect: an increase in the productivity of capital also triggers more investment, which augments K_{t+1+j} in all future periods. Because of the decreasing returns to scale ($\alpha < 1$), an increase in K_{t+1+j} mitigates the direct effect.

Without adjustment costs and heterogenous vintage capital, investment responds elastically to long-run shocks and the general equilibrium effect completely offsets the direct effect. In this case, optimality requires that the agent continues to invest until $q_{K,t+1} = 1 = E_t[\Lambda_{t,t+1} p_{K,t+1}]$. As a result, long-run productivity shocks are completely absorbed by the adjustment in the quantity of capital, and $q_{K,t+1}$ remains unchanged.

The convex adjustment cost in model (3) moderates the response of both investment and capital accumulation and allows $q_{K,t+1}$ to react positively to long-run shocks. As noted by Croce (2008), convex adjustment costs mitigate the general equilibrium effect and enhance the equity premium, but at the cost of excessively reducing the volatility of investment.

In contrast to adjustment costs, the heterogenous risk exposure of vintage capital in model (4) creates a productivity wedge between new investments and existing capital stock. This wedge sharply affects the market value of existing capital, q_K. Specifically, a positive long-run shock increases the productivity of all existing generations of capital, but affects the newly established vintage only with a delay. In relative terms, new vintages are less productive than old vintages. This effect is captured by the persistent decline of ϖ_{t+1} upon the realization of good long-run news.

On the one hand, all existing capital vintages are expected to be more productive, and the associated income effect encourages consumption. On the other hand, new investments do not benefit from these shocks immediately and the substitution effect in the current period is small. Because news shocks do not affect contemporaneous output, consumption increases and investment drops on impact. Aggregate investment recovers only after a few periods, when the productivity gap among young and older vintages becomes less severe.

On the quantity side, the negative response of investment with respect to news shocks is consistent with recent empirical findings of Barsky and Sims (2011) and Kurmann and Otrok (2010). On the pricing side, the general equilibrium effect reduces investment, boosts future marginal product of capital, and hence reinforces the direct effect of productivity. As a result, the return on physical capital is highly sensitive to long-run shocks and requires a large risk premium. This result is achieved without any counterfactual dynamics for macroeconomic quantities.

Term Structure of Dividends. In Figure 6, we depict the term structure of output, consumption, investment, dividends and the real bond for both our adjustment cost-based and vintage capital-based models (models (3) and (4), respectively). Three features need to be recognized. First, the term structure of real interest rates in both models are very similar, indicating that the two models feature very similar dynamics of the stochastic discount factor. This is consistent with Figure 4 and 5, in which the impulse responses of the discount factor in models (3) and (4) are almost indistinguishable. The term structure

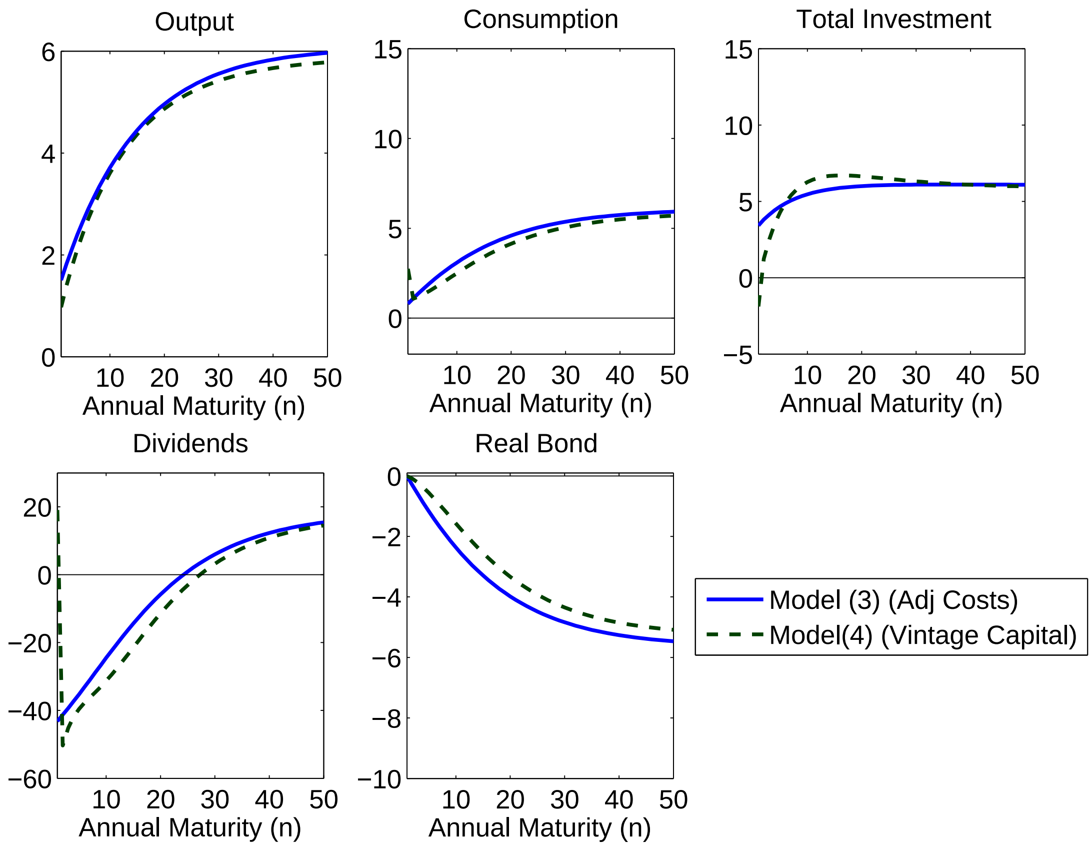

Fig. 6: The Role of Vintage Capital on the Term Structures.
This figure shows annual excess returns of zero coupon equities for different maturities associated with different cash-flows. Excess returns are multiplied by 100. Dividend returns are levered by a factor of three. Both model (3) and (4) feature short- and long-run productivity risk. In model (3) we assume the existence of convex adjustment costs. Model (4) features heterogeneous capital vintages. All the parameters are calibrated to the values reported in Table 1.

of output is also very similar across these two models, implying that vintage capital makes an important difference when we look at the subcomponents of output, as opposed to total production.

Second, in contrast to what is observed in model (3), the term structure of investments in the vintage capital model has a negative intercept and is strongly upward sloping over the maturities of the first ten years. The negative intercept of the term structure is due to the fact that the immediate response of investment to long-run productivity shocks is negative. In other words, short term investment has negative exposure to long-run shocks and thus carries an insurance premium.

Third, the term structures of both consumption and dividends are V-shaped. This pattern is more pronounced for dividends. The increasing segment of the term structure in the vintage capital model tracks fairly closely to that in model (3) and is consistent with standard long-run risk models. To understand the negative slope of the short-end of the term structure, recall that both consumption and dividends can be replicated by long positions in output and short positions in investment ($C_t = Y_t - I_t$, and $D_t = \alpha Y_t - I_t$). The negative response of investment with respect to long-run risks implies high risk exposure of short-term consumption claims to long-run shocks. This effect is stronger for dividends because investment, although quantitatively small compared to total output, accounts for a large fraction of dividends.

Figure 6 shows that vintage capital is sufficient to replicate the Binsbergen et al. (2012a)'s findings from a qualitative point of view. From a quantitative point of view, however, the decline of the dividends term structure is unsatisfactory, as it occurs only over a one-year maturity horizon. In the next subsection, we show that adding an elastic labor supply to our setting solves this problem by making the V-shape of the term structure of dividends smoother.

Endogenous labor. We conclude this section by exploring the role of endogenous labor. We incorporate elastic labor supply in model (5) and set $o = 0.345$ to match the volatility of hours worked over the business cycle. We plot the implied term structure of model (4) and the vintage capital model with endogenous labor supply in Figure 8.

All improvements obtained in the vintage capital model are preserved and actually enhanced. As shown in Figure 8, the possibility of gradually adjusting labor produces a smoother term structure of dividends, except for the very first maturity. With endogenous labor, the first maturity zero-coupon equity has an excess return of 60%, four times greater than before. This is because the income effect associated with positive news about the future immediately reduces the labor supply and hence, output. This strengthens the negative response of investment with respect to long-run shocks, making short-term dividends riskier than in the vintage capital model with inelastic labor supply. Most importantly for model (5), the term structure of equities becomes downward sloping over a maturity horizon of ten years, consistent with Binsbergen et al. (2012a).

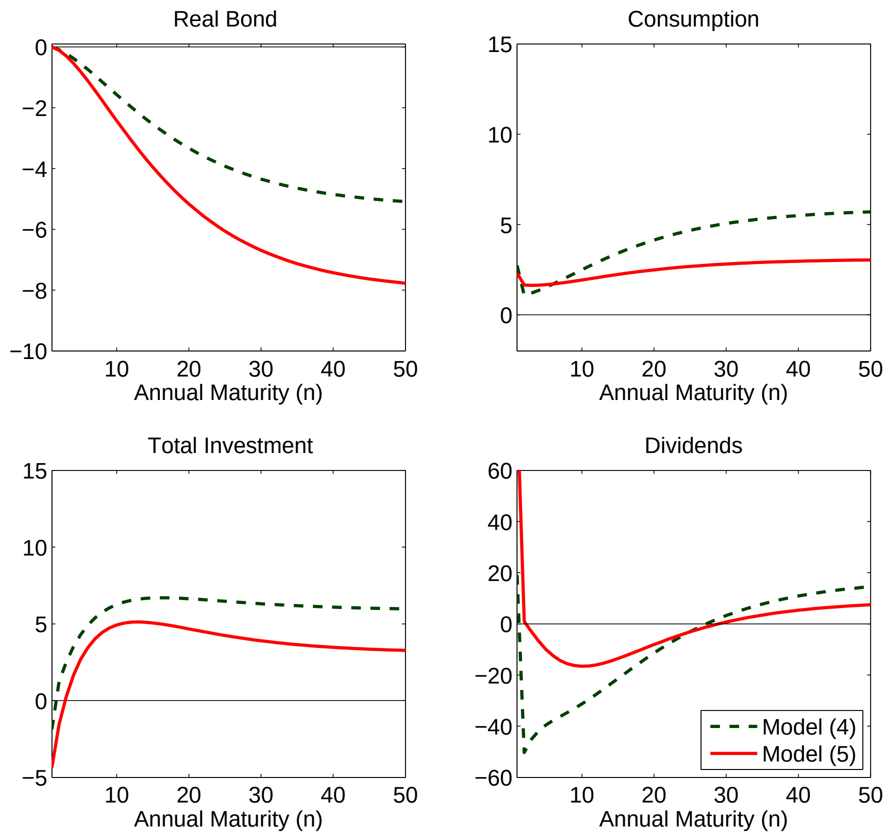

Fig. 7: Term Structures for Models (4) and (5): the Role of Labor.
This figure shows annual excess returns of zero coupon equities for different maturities associated with different cash-flows. Excess returns are multiplied by 100. Dividend returns are levered. Dashed lines refer to model (4), i.e, the economy with vintage capital and fixed labor supply. Solid lines refer to model (5), i.e., model (4) with endogenous labor. All the parameters are calibrated to the values reported in Table 2.

6 Intangible Capital

In this section, we incorporate intangible capital into our model as in Ai et al. (2012). This allows us to study the relationship between expected returns and timing of cash flows from both an aggregate and a cross-sectional perspective in a unified general equilibrium framework. We show that our model continues to produce positive results for the term structure of equity returns. Additionally, our setting replicates the negative relationship between cash flow duration and expected returns observed in the cross section (Dechow et al. 2004 and Da 2009). We first discuss the optimality conditions in the intangible capital model, and then show our results.

Optimality Conditions. We continue to denote the cum-dividend and ex-dividend price of physical capital by p_K and q_K, respectively. We use p_S and q_S for the cum- and ex-dividend price of intangible capital. By definition, q_K and q_S can be computed from p_K and p_S through the present value relation:

$$q_{K,t} = E_t[\Lambda_{t+1} p_{K,t+1}]$$
$$q_{S,t} = E_t[\Lambda_{t+1} p_{S,t+1}].$$

The first order conditions imply that:

$$p_{K,t} = \alpha K_t^{\alpha-1} (A_t N_t)^{1-\alpha} + H_K(J_t, K_t) q_{S,t} + (1-\delta_K) q_{K,t} \tag{15}$$
$$p_{S,t} = \frac{1-\nu}{\nu} \left(\frac{I_t}{S_t}\right)^{\frac{1}{\eta}} + (1-\delta_S) q_{S,t}, \tag{16}$$

where $H_K(J,K)$ denotes the partial derivative of $H(J,K)$ with respect to K. Both equations (15) and (16) have an intuitive interpretation. The first two terms on the right hand side of equation (15) are the marginal product of physical capital: one additional unit of physical capital increases total output by $\alpha K_t^{\alpha-1} (A_t N_t)^{1-\alpha}$ and reduces the adjustment cost on intangible capital by $H_K(J_t, K_t)$. Similarly, the term $\frac{1-\nu}{\nu} \left(\frac{I_t}{S_t}\right)^{\frac{1}{\eta}}$ is the marginal product of intangible capital.

Equations (15) and (16) are key to the understanding of the returns on physical and intangible capital. The payoff of physical capital is increasing in A_t, i.e, is directly exposed to aggregate productivity shocks. The return of intangible capital, in contrast, does not depend directly on productivity. The cum-dividend value of intangible capital is increasing in I_t because states with high demand of options must be associated with high tangible investment, as this is required for option exercise. On the other side, $p_{S,t}$ declines in S_t because when the stock of intangible capital is large, there is an abundant supply of growth options. In our setup, $\frac{I_t}{S_t}$ is the relevant factor that fully determines the marginal product of intangible capital.

Optimality of investment in physical and intangible capital requires:

$$
\begin{aligned}
E_t \left[\Lambda_{t,t+1} \varpi_{t+1} p_{K,t+1} \right] - \frac{1}{G_{I,t}} &= (1-\delta_S)\, E_t \left[\Lambda_{t,t+1} p_{S,t+1} \right], && (17) \\
q_{S,t} &= 1/H_J(J_t, K_t). && (18)
\end{aligned}
$$

The left hand side of equation (17) measures the net marginal benefit of exercising an additional option: the representative investor obtains the present value of one additional unit of physical capital net of the $\frac{1}{G_{I,t}}$ cost due to tangible investment. The right hand side of equation (17) is, instead, the opportunity cost of exercising an additional option, i.e., the market value of the option adjusted for the death probability. Finally, equation (18) prescribes that intangible investment has to be set so the ex-dividend value of a marginal growth option equals its own marginal production cost.

Term Structure of Dividends. We calibrate our intangible capital model as in prior work (Ai et al. 2012) to match the first and second moments of macroeconomic quantities. We summarize key statistics on both quantities and asset prices in the last column of Table 3. Our intangible capital model largely inherits the success of our vintage capital economy in terms of low volatility of consumption growth, high volatility of investment, high equity risk premium and low and smooth risk-free interest rate. In addition, the mean and volatility of intangible investments produced by our model roughly matches the empirical evidence provided in Corrado et al. (2006).

In Figure 8, we plot the term structure of equity in our intangible capital model (model (6)). For comparison purpose, we also depict the term structures obtained from the vintage capital model with endogenous labor supply (model (5)) discussed in the last subsection. We make two remarks. First, both models feature a V-shaped term structure of aggregate equity returns. In the intangible capital model, the term structure starts at about 5% per year at the one-year horizon, slowly drops to about -32% at the ten year horizon, and gradually converges back to the long-run level of about 9% per year on the long-end of the curve. The V-shape is due to the heterogenous risk exposure of vintage capital, as explained in the last section. Second, the term structure in the intangible capital model is lower than one without intangible capital (model (5)). This is because in our model, growth options are less risky than assets

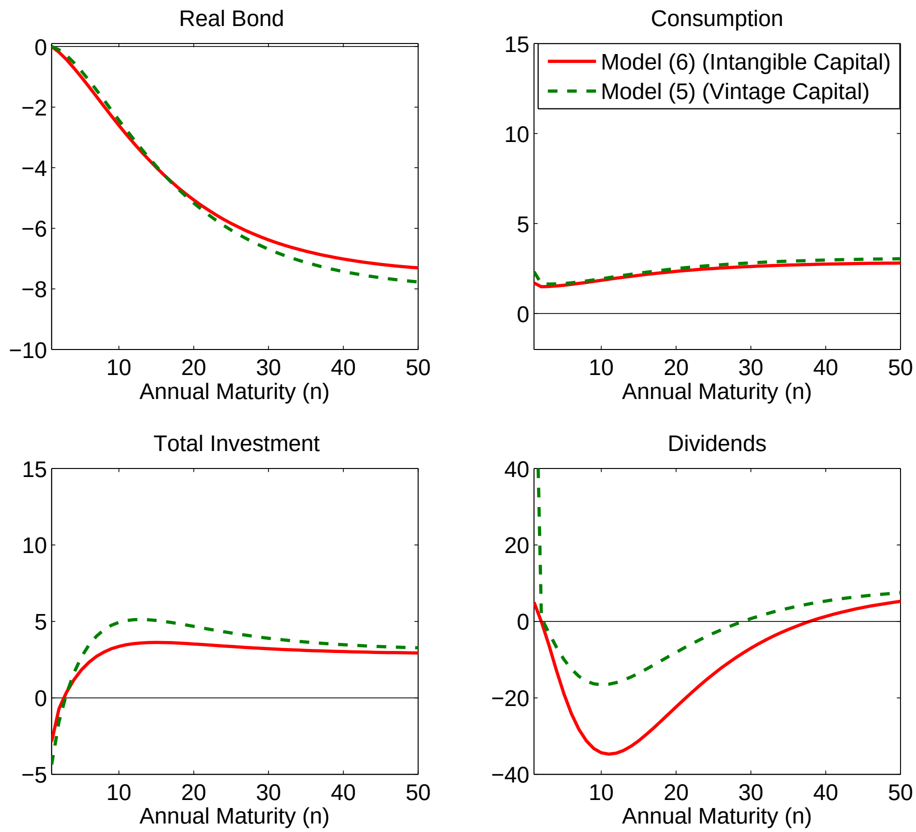

Fig. 8: Term Structures with Intangible Capital

This figure shows annual excess returns of zero coupon equities for different maturities associated with different cash-flows. Excess returns are multiplied by 100. Dividend returns are levered by a factor of three. Model (5) features heterogenous capital vintages and endogenous labor. In model (6) we add intangible capital to model (5). All the parameters are calibrated to the values reported in Table 1.

in place. The presence of intangible capital reduces the overall riskiness of aggregate dividends at all horizons.

Cash Flow Duration and Expected Returns. It is well documented that high book-to-market ratio stocks (value stocks) earn a higher average return than low book-to-market ratio stocks (growth stocks). Value stocks are also known to have shorter cash flow duration. The negative relationship between cash flow duration and expected returns in the cross-section is a challenge for general equilibrium asset pricing models. Leading asset pricing models, such as the habit model of Campbell and Cochrane (1999), the long-run risk model of Bansal and Yaron (2004), and the rare disaster model of Reitz (1988), Barro

(2006), and Gabaix (2009), typically imply a positive relationship between duration of cash flow and expected return in time series. The difficulty of generating a negative relationship between cash flow duration and expected return was emphasized in Santos and Veronesi (2010) and Croce et al. (2007), among others.

We note that the relationship between the duration of cash flow and expected return depends not only on the pricing kernel of the model, but also on the properties of the cash flow of the equity claim. Our production economy generates cash flows as an endogenous outcome of the model, and can be used to confront the empirical evidence from both the time-series and the cross-section perspective. We interpret book value as a measure of the physical capital stock of the firm. Because market price must incorporate the value of both physical and intangible capital of the firm, value stocks are physical capital intensive and growth stocks are intangible capital intensive. As in Hansen et al. (2005), we interpret the empirical evidence of the value premium as evidence for the difference in return on physical and intangible capital.

Our model is quantitatively consistent with the value premium evidence. As shown in the last column of our Table 3, the spread between the levered return on physical and intangible capital is 4.5% per year, fairly close to its empirical counterpart. Furthermore, since we work with a two-factor asset pricing model (long- and short-run risk), in our economy the CAPM fails as in the data. As documented in prior work, this is because the return on physical capital responds strongly to long-run productivity shocks, whereas the price of intangible capital reacts negatively to such shocks, consistent with the empirical evidence in Bansal et al. (2005), Bansal et al. (2007), and Hansen et al. (2008).

In fact, our intangible capital model features heterogeneous vintages as in model (5). Upon the realization of good news for the long-run, both aggregate investment and the demand of growth options decline. Consequently, the return of the growth options drops exactly when the agent has low marginal utility. Growth options, therefore, provide insurance against long-run shocks and require a lower premium in equilibrium.

In our model, physical capital also features shorter cash flow duration than intangible capital. Intangibles are claims to growth options, which pay off in the future after being exercised. Physical capital, instead, refers to installed assets in place that deliver immediate cash flows. To formalize our argument, we define the Macaulay duration, MD_t, of a stochastic cash flow process, CF_t, as:

$$MD_t = \frac{\sum_{s=1}^{\infty} s \cdot E_t \left[\Lambda_{t,t+s} CF_{t+s} \right]}{\sum_{s=1}^{\infty} E_t \left[\Lambda_{t,t+s} CF_{t+s} \right]}. \tag{19}$$

Under our calibration, the Macaulay duration of physical capital is about 17 years, and that of intangible is almost two times higher, about 30 years. As shown in our Table 3, the duration difference is qualitatively consistent with the Dechow et al. (2004) estimates.

In summary, our model is consistent with the negative relationship between cash flow duration and expected returns in the cross section. This is despite the fact that it is a long-run risk model and the term structure of a consumption claim is upward sloping. In contrast to what is done in endowment economy models, we do not assume directly that value and growth stocks are aggregate consumption claims with different maturities. We rely on our structural general equilibrium model for predictions on the cash flow associated with tangible and intangible capital.

7 Conclusion

Focusing on dividend strips with a maturity of up to ten years, Binsbergen et al. (2012a, b) and Boguth et al. (2012) document an inverse relation between zero-coupon equity expected returns and maturity. This empirical fact represents a puzzle for several leading endowment-based asset pricing models (Campbell and Cochrane 1999, Bansal and Yaron 2004, and Barro (2006)).

In this paper, we present a production-based general equilibrium model accounting for the downward sloping term structure of aggregate dividends over a maturity horizon of up to 10 years. In a production economy with recursive preferences, long-run productivity risk and vintage capital, we produce a V-shaped term structure of aggregate dividends that slopes up after the ten-year horizon.

Specifically, Ai et al. (2012) document that in the data, young investment vintages are less exposed to aggregate productivity shocks than older vintages. Embodying this source of heterogeneity across capital vintages radically changes the short-run response of investment to long-run shocks. As a result, short-run aggregate dividends become very exposed to long-run risk. Over longer maturities, in contrast, the term structure of dividends becomes upward sloping as in standard long-run risk models.

We also add intangible capital to our model in order to explore the link between the term-structure of aggregate dividends and the cross-section of equity returns. We model intangible capital as an endogenous stock of growth options. In our model, high book-to-market ratio (value) stocks are tangible capital-intensive and have both a higher long-run risk exposure (Bansal et al. 2005, Bansal et al. 2007, Hansen et al. 2008) and a shorter cash flow duration (Dechow et al. 2004 and Da 2009) than low book-to-market ratio (growth) stocks. This result reconciles the long-run risk and the duration literature.

The fact that new investments are less exposed to aggregate productivity shocks plays a crucial rule in our analysis. It is responsible for all of the following key features of our model: a high equity premium and a high volatility of investment, a V-shaped term structure of dividends, and a significant spread in the return on value and growth stocks. More broadly, our analysis connects a novel vintage-based way to think about capital accumulation (Ai et al. 2012), to the intertemporal distribution of dividends risk. Future research should provide micro-foundations of vintage capital.

References

Ai, H. The cross section of equity returns. Working Paper. November 2007.

Ai, H., M. M. Croce, and K. Li. 2012. Toward a Quantitative Genaral Equilibirum Asset Pricing Model with Intangible Capital. *Review of Financial Studies*. Forthcoming.

Bansal, R., and A. Yaron. 2004. Risk for the long run: A potential resolution of asset pricing puzzles. *Journal of Finance* 59:1481–509.

Bansal, R., R. Dittmar, and C. Lundblad. 2005. Consumption, dividends, and the cross section of equity returns. *Journal of Finance* 60:1639–672.

Bansal, R., R. F. Dittmar, and D. Kiku. 2007. Cointegration and consumption risks in asset returns. *Review of Financial Studies, forthcoming.*

Barro, R. August 2006. Rare disasters and asset markets in the twentieth century. *Quarterly Journal of Economics* 121(3):823–866.

Barsky, B., and E. Sims. 2011. News shocks and business cycles. *Journal of Monetary Economics* 58: 273–89.

Belo, F., P. Colin-Dufresne, and R. Goldstein. 2012. Endogenous dividend dynamics and the term structure of dividend strips. Working Paper.

Berk, J., R. C. Green, and V. Naik. 1999. Optimal investment, growth options, and security returns. *Journal of Finance* 54:1553–608.

Binsbergen, J., M. W. Brandt, and R. S. Koijen. 2012a. On the Timing and Pricing of Dividends. *American Economic Review*. Forthcoming.

Binsbergen, J., W. H. Hueskes, R. S. Koijen, and E. B. Vrugt. 2012b. Equity Yields. Working Paper.

Boguth, O., M. Carlson, A. J. Fisher, and M. Simutin. 2012. Leverage and the Limits of Arbitrage Pricing: Implications for Dividend Strips and the Term Structure of Equity Risk Premia. Working Paper.

Borovička, J., and L. Hansen. 2011. Examining macroeconomic models through the lens of asset pricing. Working Paper, Chicago University.

Borovička, J., L. Hansen, M. Hendricks, and J. Scheinkman. 2011. Risk-price dynamics. *Journal of Financial Econometrics* 9:3–65.

Campbell, J., and J. Cochrane. April 1999. By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy* 107:205–251.

Carlson, M., A. Fisher, and R. Giammarino. 2004. Corporate investment and asset price dynamics: Implications for the cross-section of returns. *Journal of Finance* 59:2577–603.

Corrado, C., C. Hulten, and D. Sichel. 2006. Intangible capital and economic growth. Working Paper, Federal Reserve Board.

Croce, M. M. 2008. Long-run productivity risk: A new hope for production-based asset pricing. Working Paper, Kenan-Flagler Business School, UNC.

Croce, M. M., M. Lettau, and S. C. Ludvigson. Investor information, long-run risk, and the duration of risky cash-flows. February 2007.

Da, Z. 2009. Cash flow, consumption risk, and the cross-section of stock returns. *Journal of Finance* 64: 923–56.

Dechow, P., R. Sloan, and M. Soliman. 2004. Implied equity duration: A new measure of equity risk. *Review of Accounting Studies* 9:197–228.

Epstein, L., and S. E. Zin. 1989. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica* 57:937–69.

Gabaix, X. 2009. Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance.

Garca-Feijo, L., and R. D. Jorgensen. 2010. Can operating leverage be the cause of the value premium? *Financial Management* 39:1127–54.

Gomes, J., L. Kogan, and L. Zhang. 2003. Equilibrium cross section of returns. *Journal of Political Economy* 111:693–732.

Hansen, L. P. 2011. Dynamic Value Decomposition within Stochastic Economies. *Econometrica.* Forthcoming.

Hansen, L. P., and J. A. Scheinkman. 2010. Long term risk: An operator approach. *Econometrica* 77(1): 177–234.

Hansen, L. P., J. C. Heaton, and N. Li. 2005. *Intangible risk?* Cumberland, RI: MIT Press.

Hansen, L. P., J. C. Heaton, and N. Li. 2008. Consumption strikes back? Measuring long-run risk. *Journal of Political Economy* 116:260–302.

Jaroslav, B., L. P. Hansen, M. Hendricks, and J. A. Scheinkman. 2011. Risk-price dynamics. *Journal of Financial Econometrics* 9(1):3–65.

Jermann, U. J. 1998. Asset pricing in production economies. *Journal of Monetary Economics* 41:257–75.

Kogan, L., and D. Papanikolaou. 2009. Growth opportunities, technology shocks and asset prices. Working Paper, MIT Sloan School.

Kogan, L., and D. Papanikolaou. 2010. Growth opportunities and technology shocks. *American Economic Review, P&P* 100:532:36.

Kogan, L., and D. Papanikolaou. 2011. Economic activity of firms and asset prices. Working Paper, MIT Sloan School.

Kogan, L., and D. Papanikolaou. 2012. A theory of firm characteristics and stock returns: The role of investment-specific technology shocks. Working Paper.

Kreps, D. M., and E. L. Porteus. 1978. Temporal resolution of uncertainty and dynamic choice theory. *Econometrica* 46:185–200.

Kurmann, A., and C. Otrok. 2010. News shocks and the slope of the term structure of interest rates. Working Paper, University of Missouri-Columbia.

Lettau, M., and J. A. Wachter. 2007. Why Is Long-Horizon Equity Less Risky? A Duration-Based Explanation of the Value Premium.

Lettau, M., and J. A. Wachter. 2011. The Term Structure of Equity and Interest Rates.

Li, N. 2009. Intangible capital and stock prices. Working Paper, Chicago University.

Papanikolaou, D. 2011. Investment-specific shocks and asset prices. *Journal of Political Economy*. Forthcoming.

Reitz, T. July 1988. The equity risk premium: A solution. *Journal of Monetary Economics* 22(1): 117–131.

Rouwenhorst, K. G. Asset pricing implications of equilibrium business cycle models. In Cooley, T. F., editor, *Frontiers of Business Cycle Research* chapter 10, pages 294–330. Princeton University Press Princeton, NJ 1995.

Santos, T., and P. Veronesi. 2010. Habit formation, the cross section of stock returns and the cash flow risk puzzle. *Journal of Financial Economics* 98:385–413.