Structural Models in Credit Valuation: The KMV experience

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KMV Corporation

- A financial technology firm pioneering the use of structural models for credit valuation
- Founded in 1989 in San Francisco by
  - Stephen Kealhofer
  - John McQuown
  - Oldrich Vasicek
- Soon joined by two other partners

KMV mission

- Develop and implement a model for valuation of debt securities based on modern financial theory of derivative asset pricing
- Validate the model through comprehensive empirical testing
- Extend the model to portfolio level, accounting for asset correlations
- Support and foster the continuing evolution of the debt markets

KMV development

- Grew to a firm with 250 employees
- Over 150 clients worldwide
- 70% of world's 50 largest banks are clients
- Annual revenue of US $80 million
- Bought by Moody's Corporation in 2002 for US $210 million
- KMV technology continues to be available through Moody's Analytics

KMV main products

- Credit Monitor
  - Measures credit risk of publicly traded firms
- Portfolio Manager
  - Characterizes the return and risk of a debt portfolio
  - Determines optimal buy/sell/hold transactions
- Credit Edge
  - Provides EDF Implied Option Adjusted Spread
  - Prices debt securities and derivatives

KMV clients

- Banks/Investment banks
- Fund managers
- Insurance/Reinsurance companies
- Others
  - Big accounting firms
  - Large corporations
  - Government and regulatory agencies

Credit Monitor

- Default probabilities for over 25,000 publicly traded firms worldwide
  - Probability of default is called the Expected Default Frequency (EDF)
- Updated daily

Traditional approaches to credit valuation

- Traditional approaches, such as agency ratings, involve a detailed examination of:
  - company's operations
  - projection of cash flows
  - measures of leverage and coverage
  - assessment of the firm's future earning power

Contrast with traditional approaches

- An assessment of the company’s future has already been made by all market participants and is reflected in the firm’s current market value
- Both current and prospective investors constantly perform this analysis, and their actions set the price

Credit Valuation Model

- Measure credit risk in terms of probabilities rather than ordinal ratings
- Based on a causal relationship between the state of the firm and the probability of the firm defaulting
- Utilize market information
- Provide frequent updates and early warning of deterioration (or improvement) of credit quality

Loan default

- A loan defaults if the market value of borrower's assets at loan maturity is less than the amount due
- The asset value is the worth of the firm's ongoing business

Determining asset value

- If all liabilities were traded, the market value of assets could be obtained as the sum of the market value of liabilities
- Typically, only the equity has observable price. The asset value must be inferred from equity value alone
- This can be done by the *derivative asset pricing theory* of Merton (the options pricing theory)

Derivative asset pricing

- The value of an asset is equal to the expected value of its cashflows discounted at the riskless short rate, the expectation being taken with respect to an equivalent pricing measure
- The pricing measure, often called the risk-neutral measure, is such that the expected rate of return on any asset is the short riskless rate
- For derivative assets, the value as a function of the underlying asset is subject to Merton's PDE

Merton's model

- Merton's equation:

$$\frac{\partial S}{\partial t} + (rA - c_A)\frac{\partial S}{\partial A} + \tfrac{1}{2}\sigma_A^2 A^2 \frac{\partial^2 S}{\partial A^2} - rS + c_S = 0$$

- Black/Scholes is a special case for very simple firms
- For real firms, we need to solve Merton's equation to accommodate:
  - Realistic description of the firm's liabilities
  - Cashflows: Interest payments and dividends
  - Convertibility, callability, etc.

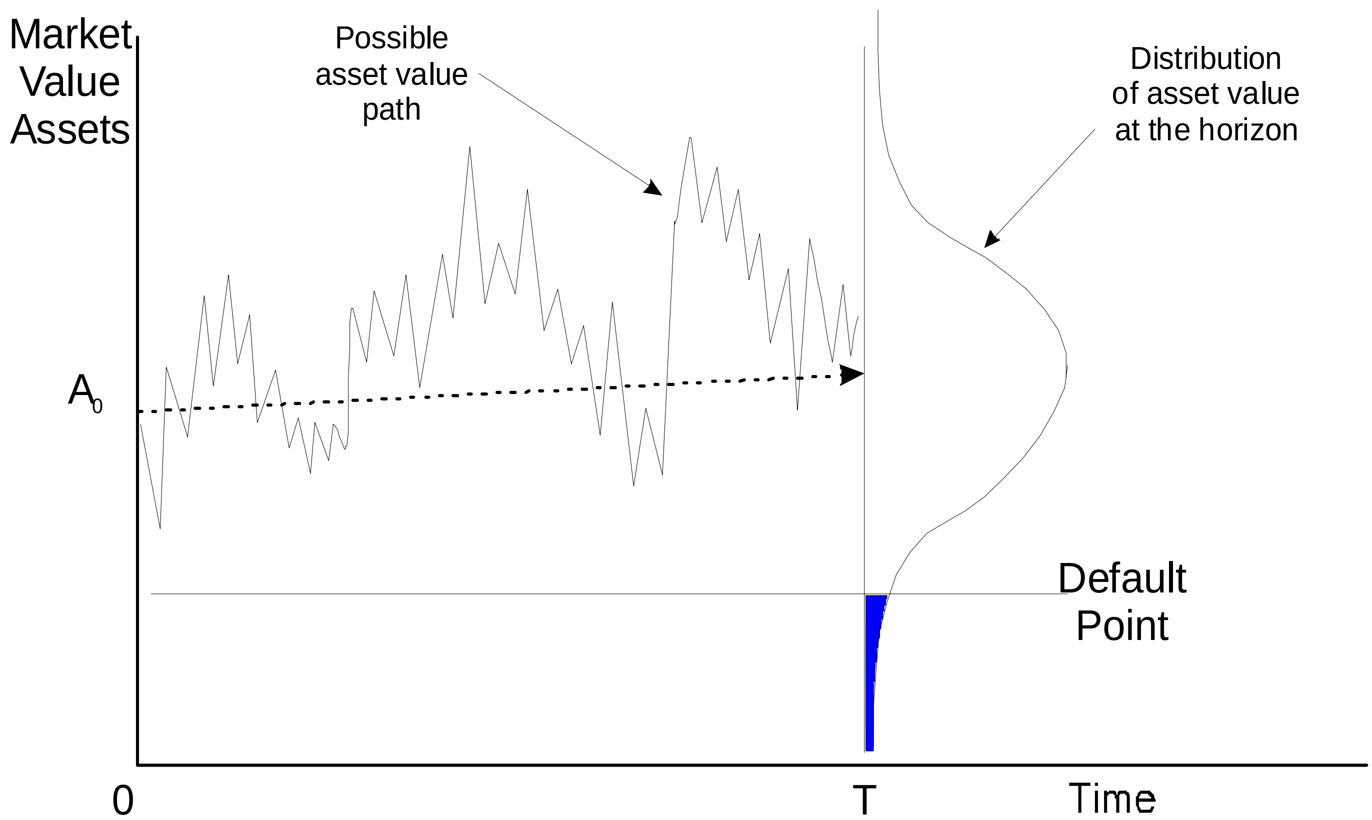
Asset volatility

- The market value of assets changes as the firm's future prospects change
- The volatility σ_A of the asset value measures the firm's business risk
- The asset volatility needs to be estimated simultaneously with asset value from stock price and stock volatility

Default point

- The default point D^* is the cumulative amount of obligations payable within the given time frame
- If the asset value falls below the default point, the firm does not have the resources to repay its debt obligations

Probability of default



Distance to Default

- Asset value at loan maturity:

$$\log A(T) = \log A - c_A T / A + \mu_A T - \tfrac{1}{2}\sigma_A^2 T + \sigma_A \sqrt{T}\, X$$

- Calculate the Distance to Default (DD):

$$Z = \frac{\log A - \log D^* - c_A T / A + \mu_A T - \tfrac{1}{2}\sigma_A^2\, T}{\sigma_A \sqrt{T}}$$

Probability of default (EDF)

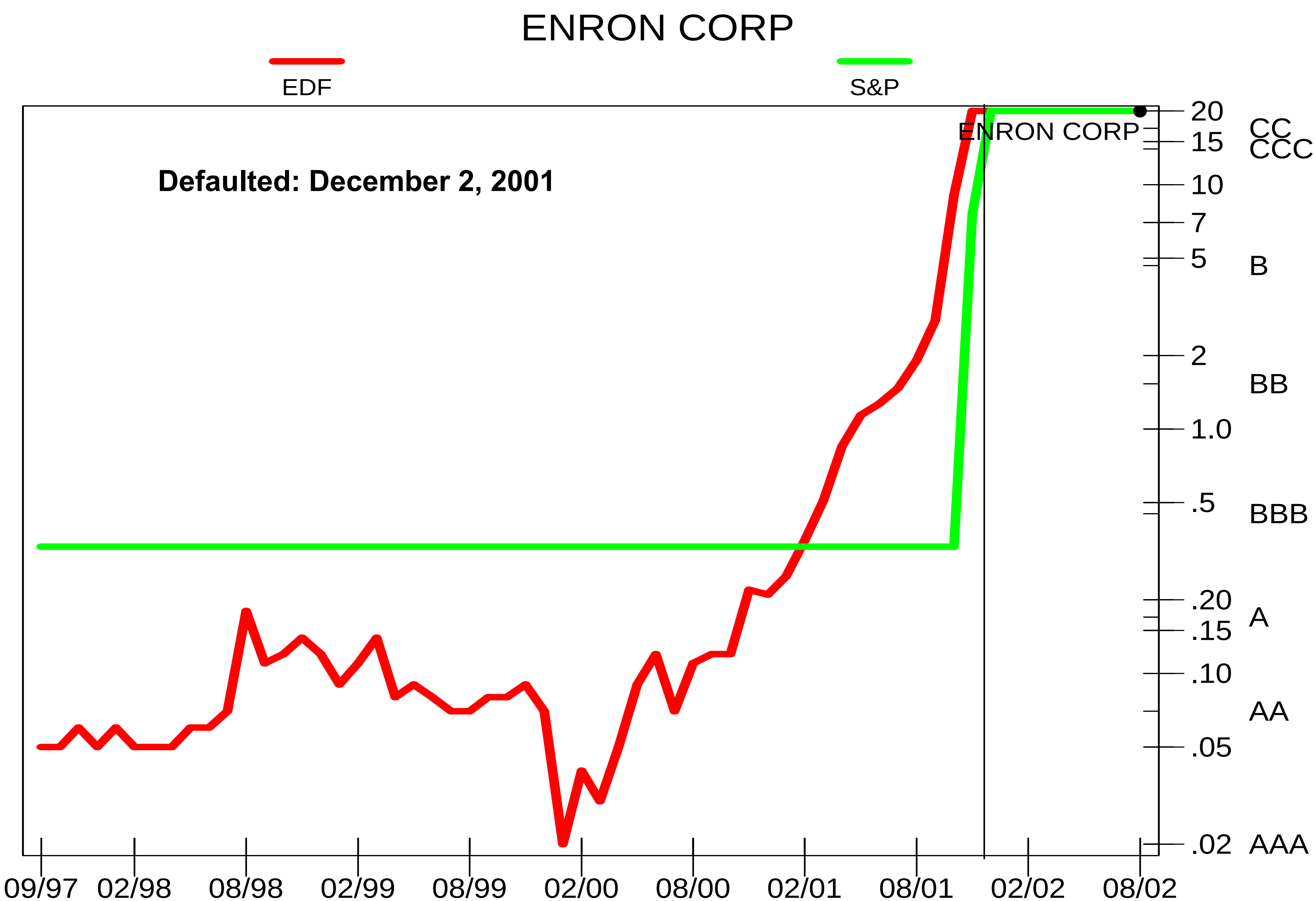
- Probability of default is

$$p = \mathrm{P}[A(T) < D^{*}] = \mathrm{N}(-Z)$$

- In practice, the normal distribution function N needs to be replaced by an empirically determined distribution function

Probability of default as a function of Distance from Default

Distance to default	Normal distribution	Empirical distribution
2	0.0455	0.0490
4	0.0001	0.0097
6	0.0000	0.0025
8	0.0000	0.0008
10	0.0000	0.0003



ENRON CORP
EDF
S&P
Defaulted: December 2, 2001
ENRON CORP
20
15
10
7
5
2
1.0
.5
.20
.15
.10
.05
.02
CC
CCC
B
BB
BBB
A
AA
AAA
09/97
02/98
08/98
02/99
08/99
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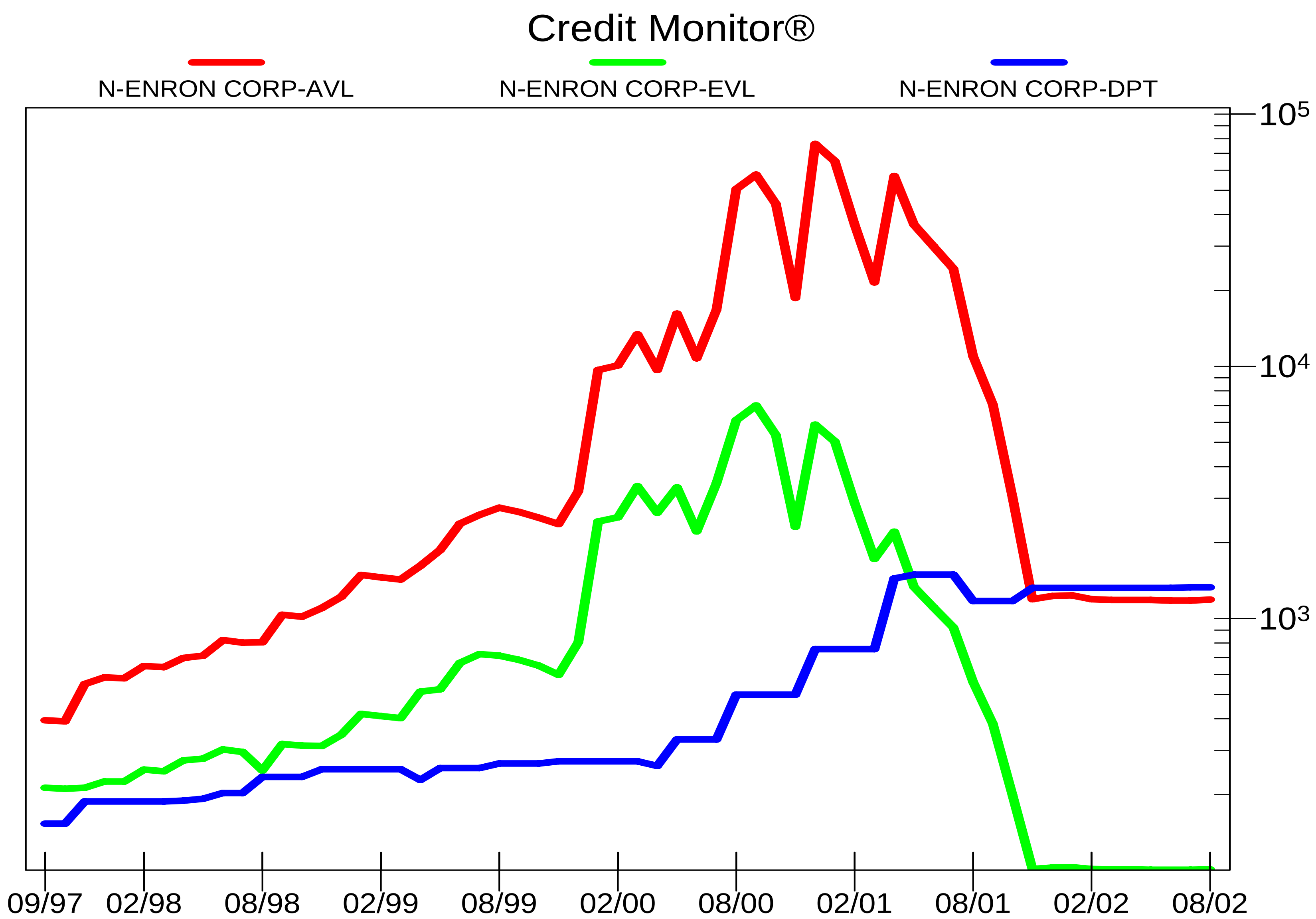
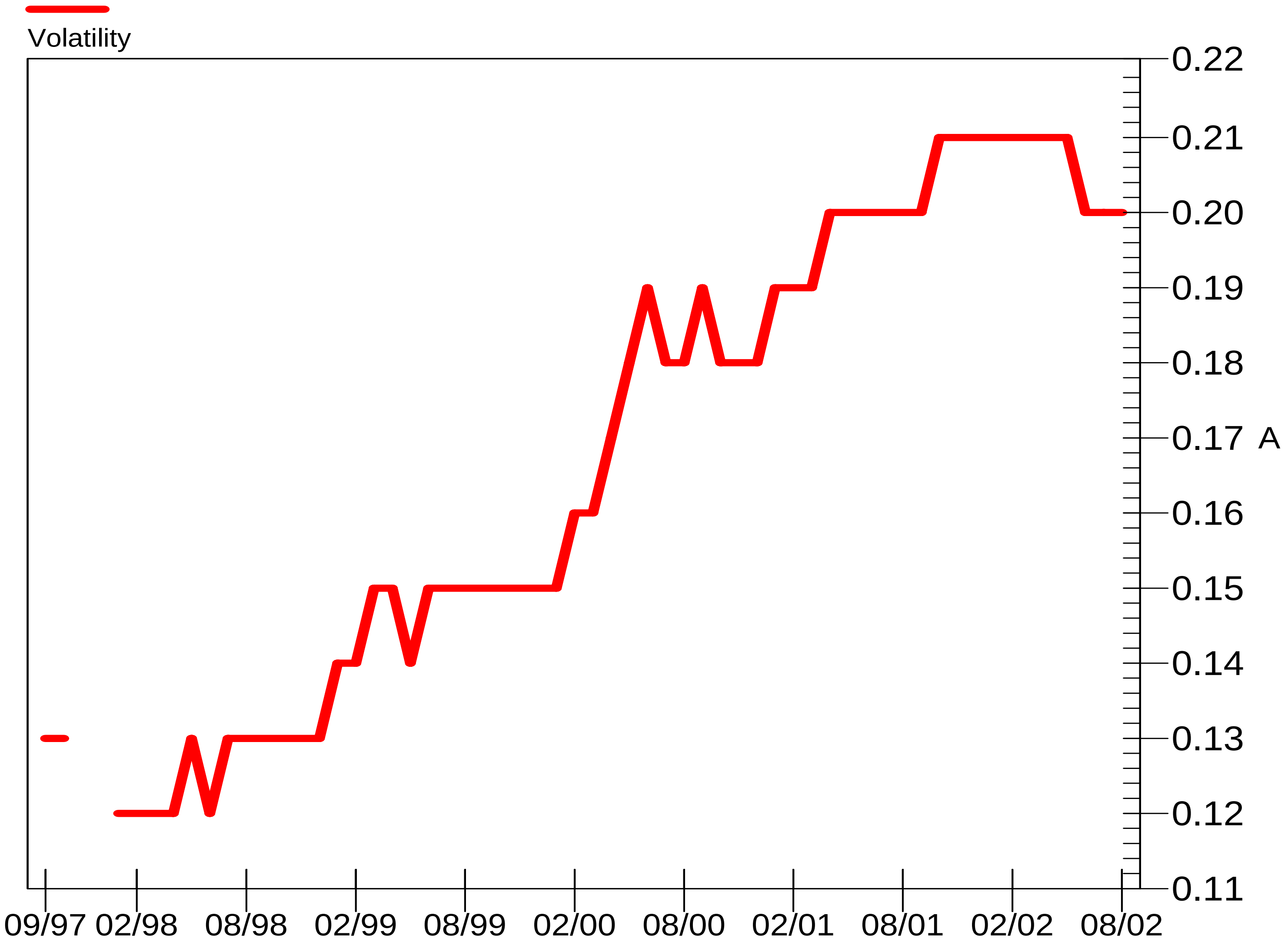
08/01
02/02
08/02


Credit Monitor®
N-ENRON CORP-AVL
N-ENRON CORP-EVL
N-ENRON CORP-DPT
10⁵
10⁴
10³
09/97
02/98
08/98
02/99
08/99
02/00
08/00
02/01
08/01
02/02
08/02

ENRON CORP


Volatility
0.22
0.21
0.20
0.19
0.18
0.17
A
0.16
0.15
0.14
0.13
0.12
0.11
09/97
02/98
08/98
02/99
08/99
02/00
08/00
02/01
08/01
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08/02

ENRON CORP [ENRNQ]

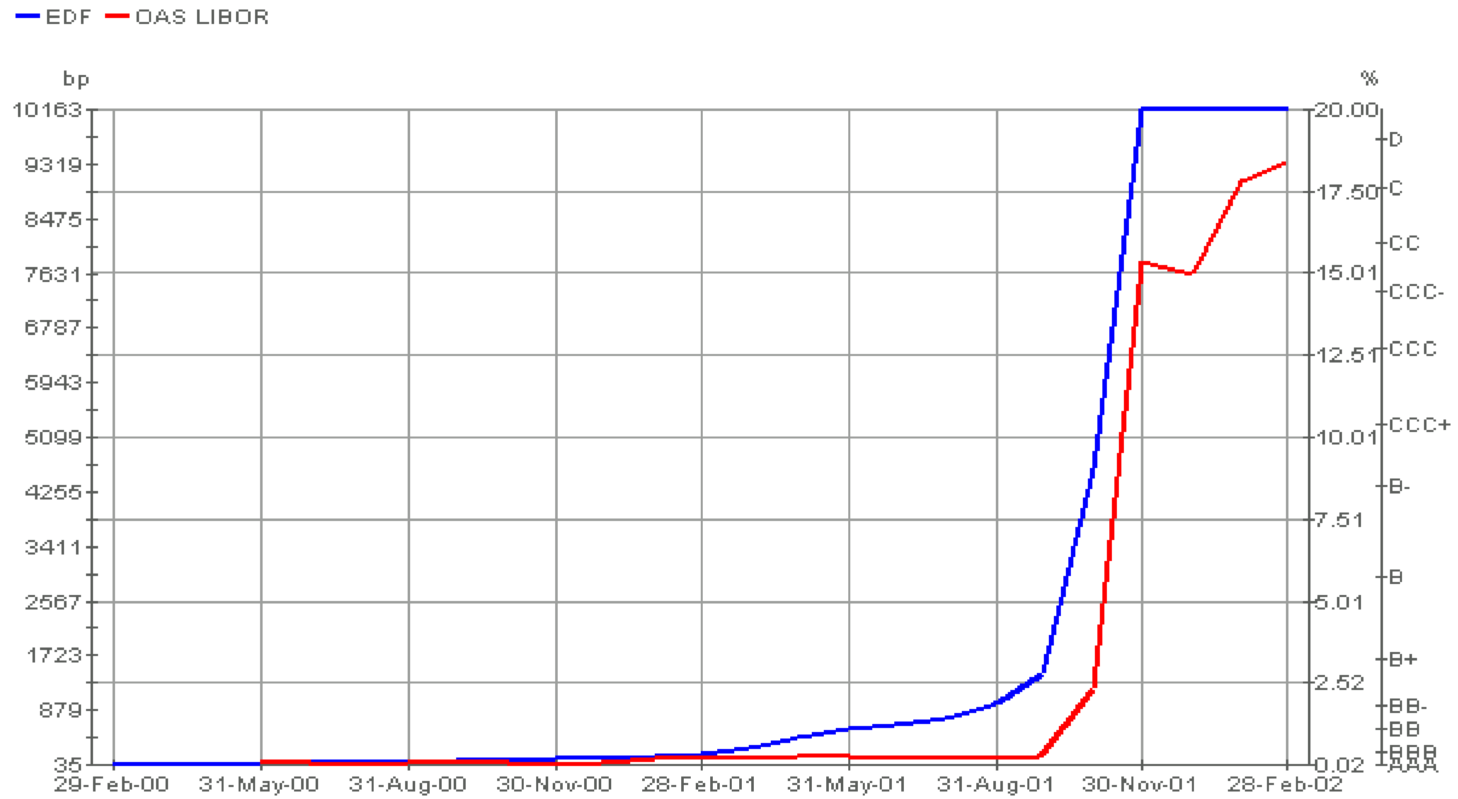
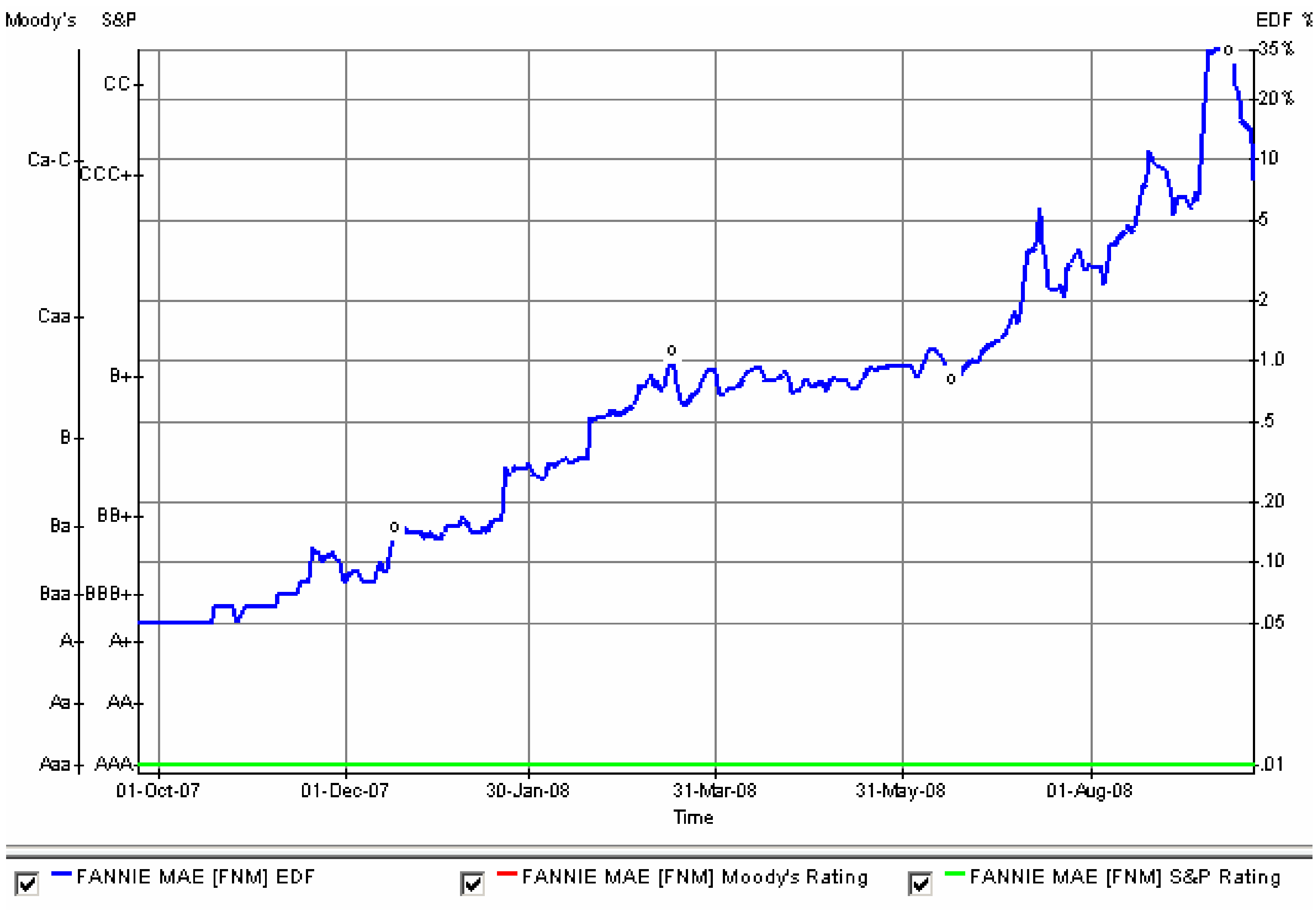


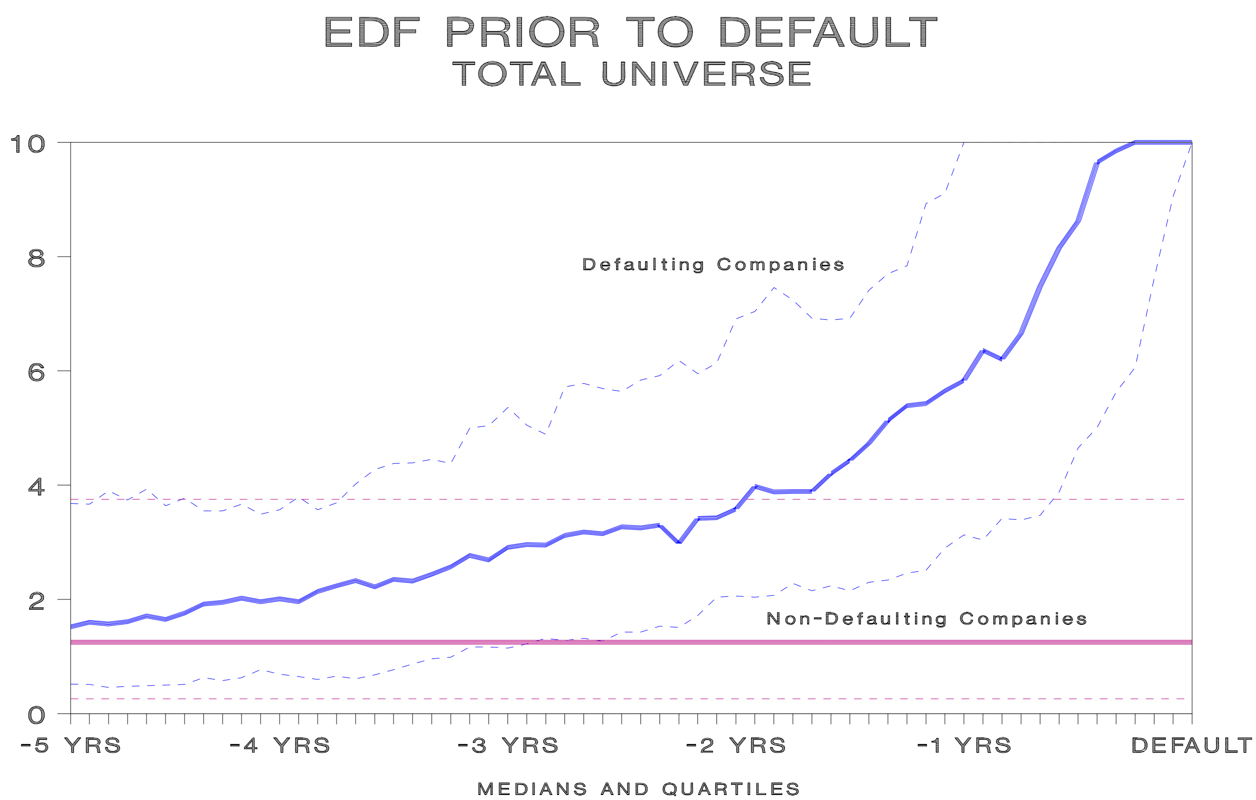
Chart Type: EDF vs. OAS LIBOR

CUSIP	29357WAA5	**Coupon**	8.38%	**OAS LIBOR**	9338 bp
ISIN	US29357WAA53	**Maturity**	23-May-05	**OAS Treasury**	9193 bp
Amt Outstanding	175.000	**Yield to Maturity**	95.39%	**Price**	16.000
Debt Type	Sr Unsec Nt	**S & P Rating**	BBB+	**Current Yield**	52.34%
Date Issued	23-May-00	**EDF**	20.00% D	**Last Traded**	28-Feb-02

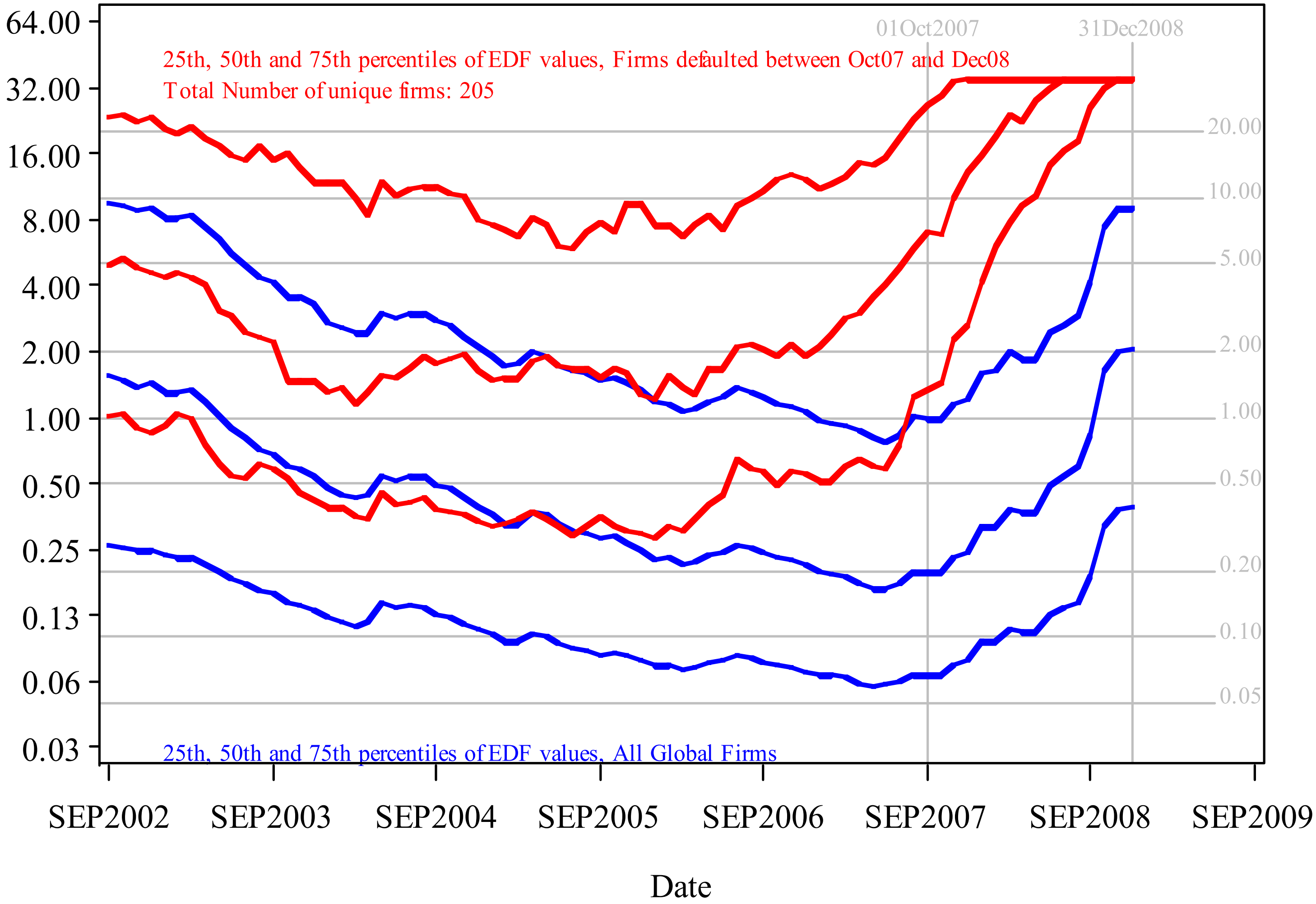
Fannie Mae EDF and Agency Rating



How much warning does EDF give?



Distributions of EDFs: Global Firms



Portfolio Manager

- Characterizes the return and risk of a debt portfolio
- Determines optimal buy/sell/hold transactions
- Provides the probability distribution of portfolio losses

Debt portfolio risk

- Portfolio characteristics:
  - Expected loss
  - Standard deviation of loss (Unexpected loss)
  - Value-at-Risk
  - Measures of diversification/concentration
  - Tail risk contribution
  - Change in portfolio value due to credit migration
  - Required economic capital
- These characteristics are determined by the probability distribution of the portfolio value

Portfolio value distribution

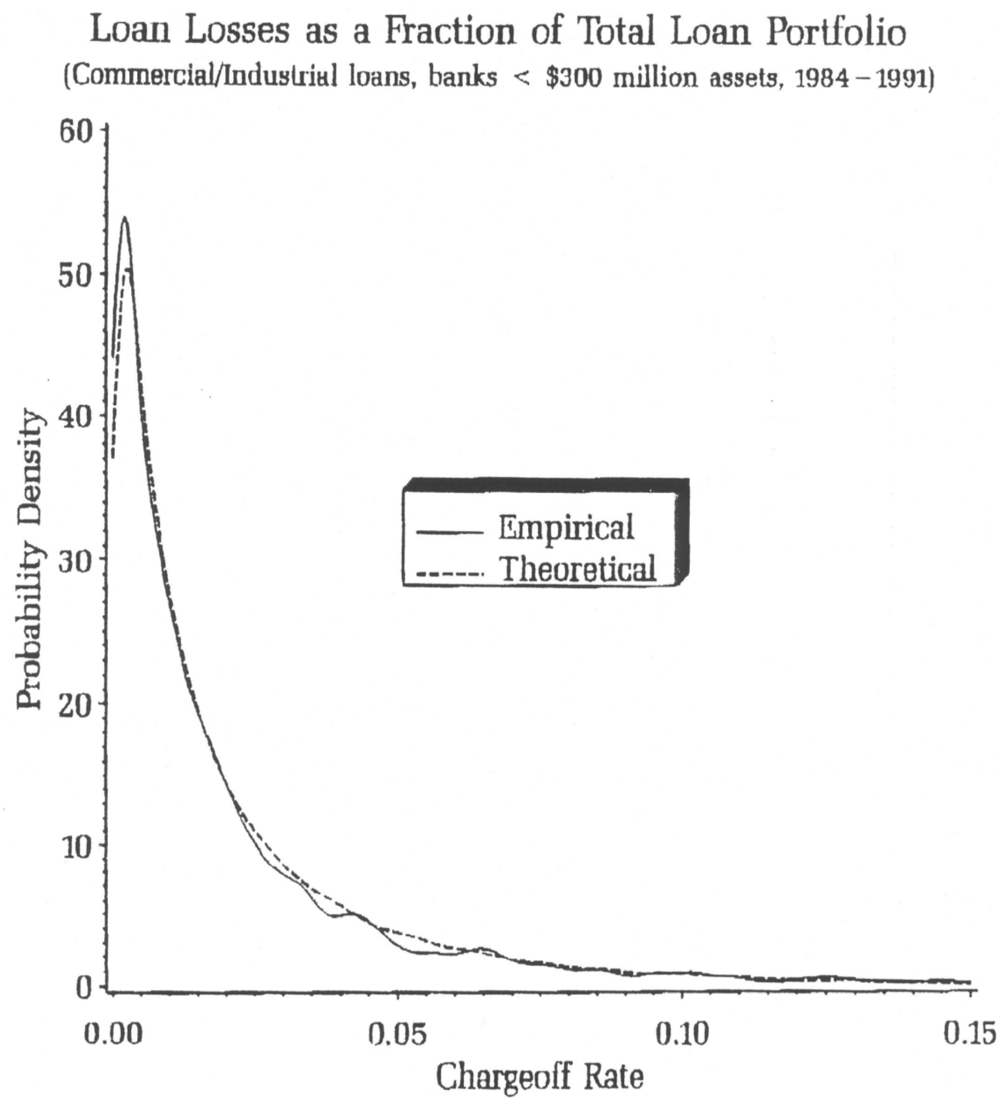
- What is the distribution of portfolio losses?
- What is the distribution of portfolio market value at horizon date due to credit migration?
- What is the risk-neutral portfolio distribution?
  - needed for pricing portfolio derivatives, such as CDOs

Asymptotic distribution of portfolio loss

- The loss on a homogeneous loan portfolio converges to a limiting distribution as the portfolio size increases
- In the limit, the distribution function of portfolio loss is

$$\mathrm{P}[L \le x] = \mathrm{N}\left(\frac{\sqrt{1-\rho}\,\mathrm{N}^{-1}(x) - \mathrm{N}^{-1}(p)}{\sqrt{\rho}} \right)$$

where p is default probability and ρ is the correlation between firms' assets



Loan Losses as a Fraction of Total Loan Portfolio
(Commercial/Industrial loans, banks < $300 million assets, 1984 – 1991)
Probability Density
Chargeoff Rate
Empirical
Theoretical
0
10
20
30
40
50
60
0.00
0.05
0.10
0.15

Loan loss percentage points as multiples of standard deviation

Expected Loss	Asset Correlation	Percentage Point		
		1%	10 bp	1 bp
1%	0.1	3.8	7.0	10.7
1%	0.4	4.5	11.0	18.2
10 bp	0.1	4.1	8.8	15.4
10 bp	0.4	3.2	13.2	31.7
Normal		2.3	3.1	3.7

Actual portfolio

- Approximate the portfolio loss distribution by the limiting distribution with the same first two moments
  - Calculate the actual portfolio expected loss and variance of loss
  - Determine the parameters of the limiting distribution to have the same mean and variance

Expected portfolio loss

The expected portfolio loss is

$$\mathrm{E}\, L = \sum_{i=1}^{n} w_i\, p_i$$

where w_i are the portfolio weights (amounts at risk)

Variance of portfolio loss

Calculated from the covariances of loan losses:

$$\operatorname{Var} L = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \operatorname{Cov}(L_i, L_j)$$

$$\operatorname{Cov}(L_i, L_j) = \mathrm{N}_2\left(\mathrm{N}^{-1}(p_i), \mathrm{N}^{-1}(p_j), \rho_{ij} \frac{\min(T_i, T_j, H)}{\sqrt{T_i T_j}} \right) - p_i p_j$$

Determination of bank capital adequacy

- Bank rating corresponds to the probability of default for the bank:
  - AAA : 2 bp bank default probability
  - AA : 5 bp
  - A : 10 bp
  - BBB : 20 bp etc.
- To maintain a desired rating, the bank must have enough capital so that the probability of loss larger than capital is that corresponding to the rating

Determining required capital

E*L* = 1%, ρ = .4

Percentage Loss	**Cumulative Probability**
5.00%	1.16%
6.00%	0.80%
7.00%	0.56%
8.00%	0.41%
9.00%	0.30%
10.00%	0.22%
11.00%	0.16%
12.00%	0.12%
12.62%	**0.10%**
13.00%	0.09%
14.00%	0.07%
15.00%	0.05%

Percentiles of the loss distribution, $\alpha = .999$

	Average asset correlation					
Average EDF	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**
0.10%	0.52%	1.12%	1.90%	2.85%	4.01%	5.41%
0.20%	0.90%	1.89%	3.13%	4.66%	6.54%	8.87%
0.30%	1.24%	2.54%	4.14%	6.11%	8.52%	11.51%
0.40%	1.55%	3.11%	5.03%	7.36%	10.18%	13.66%
0.50%	1.84%	3.64%	5.82%	8.45%	11.61%	15.47%
0.60%	2.11%	4.13%	6.55%	9.43%	12.87%	17.03%
0.70%	2.37%	4.59%	7.21%	10.32%	14.01%	18.40%
0.80%	2.63%	5.02%	7.84%	11.15%	15.03%	19.62%
0.90%	2.87%	5.43%	8.42%	11.91%	15.97%	20.71%
1.00%	3.10%	5.82%	8.98%	**12.62%**	16.83%	21.70%
1.10%	3.33%	6.20%	9.50%	13.29%	17.63%	22.60%
1.20%	3.55%	6.56%	10.00%	13.92%	18.38%	23.42%
1.30%	3.76%	6.90%	10.47%	14.51%	19.07%	24.18%
1.40%	3.97%	7.24%	10.93%	15.08%	19.73%	24.88%
1.50%	4.17%	7.57%	11.36%	15.61%	20.34%	25.53%
1.60%	4.37%	7.88%	11.78%	16.13%	20.92%	26.13%
1.70%	4.57%	8.19%	12.19%	16.61%	21.47%	26.69%
1.80%	4.76%	8.48%	12.58%	17.08%	21.99%	27.22%
1.90%	4.95%	8.77%	12.95%	17.53%	22.48%	27.72%
2.00%	5.13%	9.05%	13.32%	17.96%	22.95%	28.18%

Conclusions

- EDFs quantify credit risk and allow pricing of debt
- Portfolio value distribution can be used to measure portfolio risk, optimize portfolio composition, determine required capital, and structure and price credit derivatives