Structural Models in Credit Valuation: The KMV experience

Oldrich Alfons Vasicek
NYU Stern, November 2012
KMV Corporation

• A financial technology firm pioneering the use of structural models for credit valuation
• Founded in 1989 in San Francisco by
  – Stephen Kealhofer
  – John McQuown
  – Oldrich Vasicek
• Soon joined by two other partners
KMV mission

• Develop and implement a model for valuation of debt securities based on modern financial theory of derivative asset pricing
• Validate the model through comprehensive empirical testing
• Extend the model to portfolio level, accounting for asset correlations
• Support and foster the continuing evolution of the debt markets
KMV development

• Grew to a firm with 250 employees
• Over 150 clients worldwide
• 70% of world’s 50 largest banks are clients
• Annual revenue of US $80 million
• Bought by Moody’s Corporation in 2002 for US $210 million
• KMV technology continues to be available through Moody’s Analytics
KMV main products

• Credit Monitor
  – Measures credit risk of publicly traded firms

• Portfolio Manager
  – Characterizes the return and risk of a debt portfolio
  – Determines optimal buy/sell/hold transactions

• Credit Edge
  – Provides EDF Implied Option Adjusted Spread
  – Prices debt securities and derivatives
KMV clients

- Banks/Investment banks
- Fund managers
- Insurance/Reinsurance companies
- Others
  - Big accounting firms
  - Large corporations
  - Government and regulatory agencies
Credit Monitor

• Default probabilities for over 25,000 publicly traded firms worldwide
  – Probability of default is called the Expected Default Frequency (EDF)
• Updated daily
Traditional approaches to credit valuation

• Traditional approaches, such as agency ratings, involve a detailed examination of:
  – company’s operations
  – projection of cash flows
  – measures of leverage and coverage
  – assessment of the firm’s future earning power
Contrast with traditional approaches

• An assessment of the company’s future has already been made by all market participants and is reflected in the firm’s current market value

• Both current and prospective investors constantly perform this analysis, and their actions set the price
Credit Valuation Model

• Measure credit risk in terms of probabilities rather than ordinal ratings
• Based on a causal relationship between the state of the firm and the probability of the firm defaulting
• Utilize market information
• Provide frequent updates and early warning of deterioration (or improvement) of credit quality
Loan default

• A loan defaults if the market value of borrower’s assets at loan maturity is less than the amount due
• The asset value is the worth of the firm’s ongoing business
Determining asset value

• If all liabilities were traded, the market value of assets could be obtained as the sum of the market value of liabilities

• Typically, only the equity has observable price. The asset value must be inferred from equity value alone

• This can be done by the derivative asset pricing theory of Merton (the options pricing theory)
Derivative asset pricing

• The value of an asset is equal to the expected value of its cashflows discounted at the riskless short rate, the expectation being taken with respect to an equivalent pricing measure.

• The pricing measure, often called the risk-neutral measure, is such that the expected rate of return on any asset is the short riskless rate.

• For derivative assets, the value as a function of the underlying asset is subject to Merton’s PDE.
Merton’s model

• Merton’s equation:

\[
\frac{\partial S}{\partial t} + (rA - c_A) \frac{\partial S}{\partial A} + \frac{1}{2} \sigma_A^2 A^2 \frac{\partial^2 S}{\partial A^2} - rS + c_S = 0
\]

• Black/Scholes is a special case for very simple firms

• For real firms, we need to solve Merton’s equation to accommodate:
  – Realistic description of the firm’s liabilities
  – Cashflows: Interest payments and dividends
  – Convertibility, callability, etc.
Asset volatility

- The market value of assets changes as the firm’s future prospects change
- The volatility $\sigma_A$ of the asset value measures the firm’s business risk
- The asset volatility needs to be estimated simultaneously with asset value from stock price and stock volatility
Default point

- The default point $D^*$ is the cumulative amount of obligations payable within the given time frame.
- If the asset value falls below the default point, the firm does not have the resources to repay its debt obligations.
Probability of default
Distance to Default

• Asset value at loan maturity:

\[
\log A(T) = \log A - c_A T / A + \mu_A T - \frac{1}{2} \sigma_A^2 T + \sigma_A \sqrt{T} X
\]

• Calculate the Distance to Default (DD):

\[
Z = \frac{\log A - \log D^* - c_A T / A + \mu_A T - \frac{1}{2} \sigma_A^2 T}{\sigma_A \sqrt{T}}
\]
Probability of default (EDF)

• Probability of default is

\[ p = P[A(T) < D^*] = N(-Z) \]

• In practice, the normal distribution function \( N \) needs to be replaced by an empirically determined distribution function
Probability of default as a function of Distance from Default

<table>
<thead>
<tr>
<th>Distance to default</th>
<th>Normal distribution</th>
<th>Empirical distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.0455</td>
<td>0.0490</td>
</tr>
<tr>
<td>4</td>
<td>0.0001</td>
<td>0.0097</td>
</tr>
<tr>
<td>6</td>
<td>0.0000</td>
<td>0.0025</td>
</tr>
<tr>
<td>8</td>
<td>0.0000</td>
<td>0.0008</td>
</tr>
<tr>
<td>10</td>
<td>0.0000</td>
<td>0.0003</td>
</tr>
</tbody>
</table>
ENRON CORP

EDF  S&P

Defaulted: December 2, 2001

ENRON CORP

09/97 02/98 08/98 02/99 08/99 02/00 08/00 02/01 08/01 02/02 08/02

0.02 0.05 0.10 0.15 0.20 0.5 1.0 2.0 5 10 15 20
ENRON CORP [ENRNQ]

Chart Type: EDF vs. OAS LIBOR

<table>
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<tr>
<th>CUSIP</th>
<th>29357WAA5</th>
<th>Coupon</th>
<th>8.38%</th>
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<tr>
<td>ISIN</td>
<td>US29357WAA53</td>
<td>Maturity</td>
<td>23-May-05</td>
</tr>
<tr>
<td>Amt Outstanding</td>
<td>175.000</td>
<td>Yield to Maturity</td>
<td>95.39%</td>
</tr>
<tr>
<td>Debt Type</td>
<td>Sr Unsec Nt</td>
<td>S &amp; P Rating</td>
<td>BBB+</td>
</tr>
<tr>
<td>Date Issued</td>
<td>23-May-00</td>
<td>EDF</td>
<td>20.00% D</td>
</tr>
<tr>
<td>OAS LIBOR</td>
<td>9338 bp</td>
<td>OAS Treasury</td>
<td>9193 bp</td>
</tr>
<tr>
<td>Price</td>
<td>16.000</td>
<td>Current Yield</td>
<td>52.34%</td>
</tr>
<tr>
<td>Last Traded</td>
<td>28-Feb-02</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fannie Mae EDF and Agency Rating
How much warning does EDF give?
Distributions of EDFs: Global Firms

25th, 50th and 75th percentiles of EDF values, Firms defaulted between Oct07 and Dec08
Total Number of unique firms: 205
Portfolio Manager

- Characterizes the return and risk of a debt portfolio
- Determines optimal buy/sell/hold transactions
- Provides the probability distribution of portfolio losses
Debt portfolio risk

• Portfolio characteristics:
  – Expected loss
  – Standard deviation of loss (Unexpected loss)
  – Value-at-Risk
  – Measures of diversification/concentration
  – Tail risk contribution
  – Change in portfolio value due to credit migration
  – Required economic capital

• These characteristics are determined by the probability distribution of the portfolio value
Portfolio value distribution

• What is the distribution of portfolio losses?
• What is the distribution of portfolio market value at horizon date due to credit migration?
• What is the risk-neutral portfolio distribution?
  – needed for pricing portfolio derivatives, such as CDOs
Asymptotic distribution of portfolio loss

- The loss on a homogeneous loan portfolio converges to a limiting distribution as the portfolio size increases.
- In the limit, the distribution function of portfolio loss is

\[
P[L \leq x] = N\left(\sqrt{1 - \rho} \frac{N^{-1}(x) - N^{-1}(p)}{\sqrt{\rho}}\right)
\]

where \(p\) is default probability and \(\rho\) is the correlation between firms’ assets.
Loan Losses as a Fraction of Total Loan Portfolio
(Commercial/Industrial loans, banks < $300 million assets, 1984-1991)
Loan loss percentage points as multiples of standard deviation

<table>
<thead>
<tr>
<th>Expected Loss</th>
<th>Asset Correlation</th>
<th>Percentage Point</th>
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<tr>
<td></td>
<td></td>
<td>1%</td>
</tr>
<tr>
<td>1%</td>
<td>0.1</td>
<td>3.8</td>
</tr>
<tr>
<td>1%</td>
<td>0.4</td>
<td>4.5</td>
</tr>
<tr>
<td>10 bp</td>
<td>0.1</td>
<td>4.1</td>
</tr>
<tr>
<td>10 bp</td>
<td>0.4</td>
<td>3.2</td>
</tr>
<tr>
<td>Normal</td>
<td></td>
<td>2.3</td>
</tr>
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Actual portfolio

• Approximate the portfolio loss distribution by the limiting distribution with the same first two moments
  – Calculate the actual portfolio expected loss and variance of loss
  – Determine the parameters of the limiting distribution to have the same mean and variance
Expected portfolio loss

The expected portfolio loss is

\[ E L = \sum_{i=1}^{n} w_i p_i \]

where \( w_i \) are the portfolio weights (amounts at risk)
Variance of portfolio loss

Calculated from the covariances of loan losses:

\[ \text{Var} L = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \text{Cov}(L_i, L_j) \]

\[ \text{Cov}(L_i, L_j) = N_2 \left( \frac{N^{-1}(p_i), N^{-1}(p_j), \rho_{ij}}{\sqrt{T_i T_j}} \right) - p_i p_j \]
Determination of bank capital adequacy

• Bank rating corresponds to the probability of default for the bank:
  – AAA : 2 bp bank default probability
  – AA  : 5 bp
  – A   : 10 bp
  – BBB : 20 bp etc.

• To maintain a desired rating, the bank must have enough capital so that the probability of loss larger than capital is that corresponding to the rating
Determining required capital

\[ EL = 1\%, \quad p = .4 \]

<table>
<thead>
<tr>
<th>Percentage Loss</th>
<th>Cumulative Probability</th>
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<tbody>
<tr>
<td>5.00%</td>
<td>1.16%</td>
</tr>
<tr>
<td>6.00%</td>
<td>0.80%</td>
</tr>
<tr>
<td>7.00%</td>
<td>0.56%</td>
</tr>
<tr>
<td>8.00%</td>
<td>0.41%</td>
</tr>
<tr>
<td>9.00%</td>
<td>0.30%</td>
</tr>
<tr>
<td>10.00%</td>
<td>0.22%</td>
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<tr>
<td>11.00%</td>
<td>0.16%</td>
</tr>
<tr>
<td>12.00%</td>
<td>0.12%</td>
</tr>
<tr>
<td><strong>12.62%</strong></td>
<td><strong>0.10%</strong></td>
</tr>
<tr>
<td>13.00%</td>
<td>0.09%</td>
</tr>
<tr>
<td>14.00%</td>
<td>0.07%</td>
</tr>
<tr>
<td>15.00%</td>
<td>0.05%</td>
</tr>
</tbody>
</table>
Percentiles of the loss distribution, $\alpha = .999$

<table>
<thead>
<tr>
<th>Average EDF</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
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<tbody>
<tr>
<td>0.10%</td>
<td>0.52%</td>
<td>1.12%</td>
<td>1.90%</td>
<td>2.85%</td>
<td>4.01%</td>
<td>5.41%</td>
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<tr>
<td>0.20%</td>
<td>0.90%</td>
<td>1.89%</td>
<td>3.13%</td>
<td>4.66%</td>
<td>6.54%</td>
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</tr>
<tr>
<td>0.30%</td>
<td>1.24%</td>
<td>2.54%</td>
<td>4.14%</td>
<td>6.11%</td>
<td>8.52%</td>
<td>11.51%</td>
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<tr>
<td>0.40%</td>
<td>1.55%</td>
<td>3.11%</td>
<td>5.03%</td>
<td>7.36%</td>
<td>10.18%</td>
<td>13.66%</td>
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<tr>
<td>0.50%</td>
<td>1.84%</td>
<td>3.64%</td>
<td>5.82%</td>
<td>8.45%</td>
<td>11.61%</td>
<td>15.47%</td>
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<tr>
<td>0.60%</td>
<td>2.11%</td>
<td>4.13%</td>
<td>6.55%</td>
<td>9.43%</td>
<td>12.87%</td>
<td>17.03%</td>
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<tr>
<td>0.70%</td>
<td>2.37%</td>
<td>4.59%</td>
<td>7.21%</td>
<td>10.32%</td>
<td>14.01%</td>
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<tr>
<td>0.80%</td>
<td>2.63%</td>
<td>5.02%</td>
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<td>22.95%</td>
<td>28.18%</td>
</tr>
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</table>
Conclusions

• EDFs quantify credit risk and allow pricing of debt

• Portfolio value distribution can be used to measure portfolio risk, optimize portfolio composition, determine required capital, and structure and price credit derivatives