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Abstract

I present a theory of the optimal capital structure and dividend policy for family business groups expanding in countries where the private control benefit is substantial and the market for corporate control is active. Here, issuing debt overcomes the limited dilution capacity of equity, but reduces the expected control benefit by increasing the probability of default. When family business groups grow, they may either (i) establish a subsidiary of an existing parent firm (Pyramidal structure), or (ii) establish a new stand-alone entity (Horizontal structure). The pyramidal structure intrinsically generates greater availability of internal capital, but limits the family’s ability to extract rent from the subsidiary. Therefore, in equilibrium, a large scale, less profitable firm emerges as a pyramidal subsidiary, and the subsidiary should be less leveraged than a stand-alone firm directly controlled by the family. Furthermore, the pyramidal structure implies (i) the parent firm can augment its own cash flow with dividends paid by subsidiaries (Co-insurance benefit), and (ii) control on subsidiaries is exercisable only if the parent firm is solvent (Control optionality). These two properties imply that the parent firm should be more leveraged than its subsidiaries. Therefore, the theory predicts decreasing leverage ratio from the top to the bottom along the pyramid, and this capital structure decision is supported by a dividend policy such that the parent firm should pay out less to maximize group internal capital while subsidiaries should pay out more to service the parent firm’s debt. Using a unique comprehensive data set of Korean business groups, Chaebols, I provide empirical evidence that is consistent with the theoretical predictions. Together, the empirical results and theory suggest that the controlling family of Korean Chaebol strategically designs the business group structure by making capital structure decision and dividend policy to optimize the control efficiency of the business group.

Key words: Family firms, private control benefit, internal capital, business group structure, capital structure, dividend policy
JEL classification: G32, G34

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1 Introduction

In many countries, family business groups contribute significantly to economic activity. More than 30% of large public firms worldwide belong to family business groups and the firms in such groups are usually controlled by a few founding families using stock pyramid, dual-class share and cross-shareholding ties to consolidate corporate control (La Porta et al., 1999). These equity-based control enhancing mechanisms enable a controlling family to exercise control influence over group firms with minimal cash flow rights in the firms, which, in turn, drives a substantial wedge between ownership and control. For example where a family uses stock pyramid such that the family directly owns 50% of a firm, which in turn owns 50% of another firm, the family achieves control of the latter with a cash-flow right of only 25%.

The separation between cash-flow and control rights incurs agency problems (Jensen and Meckling, 1976), and the incurred agency cost can be extreme, especially when investor protection rights are weak. The extreme agency cost under a weak protection for minorities is well-documented in law and finance literature (La Porta et al., 2002, Faccio et al., 2002, and Claessens et al., 2000). In emerging markets and continental Europe where the protection for minorities is known to be weak, more than 26% of large public firms in these countries belong to pyramids (La Portal et al., 1999).

Nenova, 2003, and Dyck and Zingales, 2004, document evidence that private control benefit in markets with poor investor protection is substantial suggesting that weak protection rights for minorities drive a high control premium. These studies suggest that with weak investor protection rights, a controlling family can generate a large private control benefit which fuels the incentive to adopt control enhancing mechanisms that secure control over group firms.

To secure control over group firms, issuing debt- instead of voting equities- is known to also function as a control enhancing device given that each share carries one voting right. The role of debt as an anti-takeover device has been introduced in several studies in the market for corporate control literature (Harris and Raviv, 1988, Israel, 1991, and Aghion and Bolton, 1992). These studies theoretically argue that a wealth-constrained incumbent management -whose managerial skill is relatively poor to a market rival- can extend its financing capacity by issuing debt without incurring further dilution of its equity stake in the firm, which could jeopardize the family’s control in the event of takeover. However, debt comes at a cost. More debt increases the probability of default and loss of control benefit. Thus, the entrenched management tries to minimize the use of debt. The literature suggests that the optimal debt level should be closely related to the level of personal wealth of the incumbent management.

Personal wealth of the incumbent improves control security. More wealth implies that the incumbent can minimize the use of external capital, which in turn enables the incumbent to reduce the threat of other economic agents that purchase corporate securities and may try to exercise their control influence through the securities that they hold.

A recent study by Almeida and Wolfenzon, 2006, suggests that the availability of internal capital is closely related to the business group structure. They compare two usual business group structures - the vertical pyramid and the horizontal structure, and identify the ad-
vantage in internal capital in the pyramidal structure. They argue that under the pyramidal structure, the controlling family of a business group can use the entire free cash in the parent firm (a central firm hereafter) to subsidize the new investment of its subsidiary, whereas under the horizontal structure, the family can only use its personal wealth to support the new investment of a stand-alone group entity. According to this theory, an internal capital market formed under a pyramid can generate greater control efficiency for the controlling family than that under the horizontal structure.

In this paper, I investigate how the controlling family should design the business group structure to achieve control efficiency. I examine the relationship between the capital structure decision and dividend policy and determine how these decisions are linked to the choice of the business group structure in equilibrium. To this end, I ask the following research questions:

**Research Question 1:** When does the family prefer the pyramidal structure over the horizontal structure? How are different business group structure choices related to the control enhancing incentive of the family?

**Research Question 2:** What is the optimal capital structure for a subsidiary under the pyramid and for a stand-alone group entity directly controlled by the family under the horizontal structure? Under the pyramidal structure, should a central firm be more leveraged than its subsidiaries?

**Research Question 3:** Are the dividend policies for a central firm and its subsidiary different? How are they related to the capital structure decisions of the two firms in equilibrium?

The existing literature on these topics are scarce empirically and theoretically. I provide a theory to address these questions, and document empirical evidence that is consistent with the theoretical predictions using dynamic panel data for Korean family business groups known as Chaebols. This paper helps explain how a controlling family strategically designs the business group structure to maximize control, identifying a relationship between capital structure decision and family business group structure from the control efficiency perspective. The proposed relationship between capital structure and dividend policy in this paper can explain the puzzling phenomena that group firms with high expected agency cost tend to pay out more (Faccio et al., 2000).

The outline of the paper is as follows: Section 2 previews the theory and summarize the main results, and section 3 provides a brief review of the existing literature. Section 4 provides a baseline model that emphasizes how different availability of internal capital under the pyramid and the horizontal structure are related to the capital structure decisions for a pyramidal subsidiary and a stand-alone group entity. In section 5, I extend the baseline model to analyze the capital structure decision of the two firms along the pyramid-a central firm and a subsidiary. In section 6, I discuss the optimality of the conglomerate over the two business group structures -the pyramid and the horizontal structure. In section 7, I develop constituent firms (i.e., subsidiaries hereafter) by controlling a parent firm (central in control to subsidiaries, a central firm hereafter) that in turn, controls the subsidiaries. The horizontal structure is referred to a case where a family directly owns shares of group firms and controls them directly by its own equity stakes.
and test empirical implications from the theory. In this section, I also describe Korean Chaebols data and explain the group structure metrics that I use to test the predictions. And, in section 8, I conclude.

2 Preview of the Model and Main Results

The model starts with a major premise that private control benefit of a corporation is substantially valuable, and thus a controlling family of a business group is control-driven. With this premise, I introduce a risk-neutral family entrepreneur that owns and controls an existing firm and wants to set up a new firm based on a single new project. The family structures the new firm as either a pyramidal subsidiary (pyramidal structure) or a stand-alone entity that is independent from the established firm (the horizontal structure). The new project delivers a positive NPV with a large private control benefit exclusively to the controller of the new firm, and the family optimally chooses one of the two business group structures that maximizes its payoff - sum of a security benefit and a private control benefit.

When one share carries one voting right, external equity financing is limited when there is a takeover threat from a rival management. Due to this takeover threat, the family is concerned about securing its control over the new firm, and this concern is more severe for a family whose managerial ability is relatively poor compared to that of the rival. The family wants to find a group structure that minimizes the loss in the private control benefit, and thus prefers a structure that enables minimal use of external debt which carries the risk that control is taken over by creditors. Therefore, a business group structure that provides access to more internal capital is preferred.

The pyramidal structure intrinsically generates a greater availability of internal cash resource since the family uses the existing firm to invest in the new subsidiary. In contrast, under the horizontal structure, the family can only use its own cash stake in the existing firm as internal resource since it structures the new horizontal entity independently from the existing firm. Internal financing advantage under a pyramid implies a lower level of required debt, and thus leads to advantageous expected control benefit.

However, the indirect equity investment under the pyramid implies that the existing firm does not pay out dividends, which forces the family to share rent from the new subsidiary with existing firm’s minority shareholders who effectively provide the extra internal capital. On the other hand, the family receives full rent from the new firm under the horizontal structure because it sets up the firm by using its own wealth.

As a result, the pyramidal structure can be characterized by a relative advantage in the expected control benefit due to the greater availability of the internal capital but a disadvantage in rent compared to the horizontal structure. Given these properties, one can predict that a more capital-intensive, less profitable, and less leveraged firm emerges as a pyramidal subsidiary rather than a horizontal stand-alone entity in equilibrium. The lower leverage ratio implies the higher expected control benefit, and thus the leverage ratio of a firm functions as an important determinant of business group structure choice. Moreover, a relatively poorly skilled family finds that the internal capital advantage under the pyramid is useful, and the family’s ownership of a pyramidal subsidiary tends to be concentrated since its poor managerial skill limits access to outside equity capital.
In addition, I also analyze whether under the pyramid, it is optimal to re-capitalizes a central firm rather than issuing debt only from a subsidiary. Under the pyramid, debt can be issued from the central firm as additional capital to subsidize the investment of the subsidiary. From a financing perspective, this recapitalization of the central firm is optimal for the following reason: the pyramid allows two types of assets for the central firm, its own operating asset and the control shares in the subsidiary. Legally monetizing this position, and issuing additional debt on this extended collateral can improve the family’s debt financing efficiency. Due to consolidating control over the subsidiary, the family can not liquidate the control shares, and thus monetizing this position involves using dividend channels to transfer the capital from the subsidiary to the central firm. This extra capital from the subsidiary that allows the family to extend the central firm’s debt capacity, thereby further reducing the subsidiary’s debt is referred to as the co-insurance benefit. This is one of two important structural properties of the pyramid.

The financing advantage of the *co-insurance benefit*, however, can not entirely justify the optimality of recapitalization. To analyze the cost of recapitalization, I propose another important structural property of the pyramid - control optionality. Under the pyramid, the family controls the subsidiary indirectly through its control over the central firm. This implies that control over the subsidiary is contingent upon the survival of the central firm. This property makes the central firm’s debt financing more costly. Therefore, a highly leveraged central firm and a least leveraged subsidiary emerge in equilibrium only if the family effectively minimizes the group default probability, and the amount of private control benefit from the firm is moderate. This condition is more likely to be met when a central firm is more profitable, less capital-intensive, and larger. This prediction is consistent with the findings in many of empirical studies showing that business groups pursue profit stability for survival rather than profit maximization.

Therefore, the model proposes that a control efficient fully levered pyramid should imply decreasing leverage ratio\(^3\) from top to bottom, suggesting the leverage ratio is closely related to the position and centrality\(^4\) of a group firm. Furthermore, the model suggests an increasing dividend payout from top to bottom of the pyramid such that central firms tend to pay out less to ensure a greater availability of group internal capital while subsidiaries tend to pay out more to extend the central firm’s debt capacity.

### Related Literatures

There are a wealth of studies in law and corporate finance literature that investigate the relationship between legal enforcement and the ownership structures of business groups. In countries with poor investor protection rights, pyramids and dual class share structures are commonly used, and pyramidal group firms are roughly 26% of the publicly listed firms worldwide (La Porta et al., 1999, Faccio and Lang, 2002, and Claessens et al., 2000). Studies in this literature suggest that the weak investor protection right implies 1) lower valuation of

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\(^3\)Leverage ratio of a firm is measured as the ratio of total debt to the firm’s own operating assets

\(^4\)The definition of position and centrality will be introduced in section. Explaining the definitions briefly, the position of a given firm is roughly the same as the distance of the firm from the family, and a firm controlling a number of subsidiaries has high centrality.
corporations (La Porta et al., 1997), 2) less developed capital markets (Shleifer and Wolfenzon, 2002), 3) higher private benefit of control (Dyck and Zingales, 2004, and Nenova, 2003). These studies share the idea that a weak legal system induces lower cost of corporate resource diversion, and thus the expected resource diversion tends to be extreme in countries with weak legal protection for minorities.

External debt as a control-enhancing device in this paper owes ideas from studies in the literature on market for corporate control and capital structure (Harris and Raviv, 1988, Israel, 1991, and Aghion and Bolton, 1992). As briefly explained in section [1], the basic idea of the studies is that when a wealth-constrained incumbent management needs to raise external capitals, by issuing debt-non-voting equity-, it can increase financing capacity without incurring control loss. My model links the level of personal wealth of the incumbent to the business group structure choice problem, and proposes that the strategic choice of business group structure matters from a control efficiency perspective.

In this paper, I view a pyramid as limited recourse equity project financing. The primitives of my model inherit many similarities with those in Chemmanur and Kose, 1996. By highlighting three structural properties of the pyramid - 1) internal capital market, 2) control optionality, and 3) co-insurance benefit-, this paper extends project financing intuition to family business group literature.

Business group structure choice and a rationale for a pyramid are the focus of this paper. Most pyramid theories take an agency theoretical approach to analyze the consequences of the pyramid. Agency theory of outside equities was first introduced by Jensen and Meckling, 1976. La Porta et al., 2002, modified this framework to show how the degree of agency problem depends upon a condition of legal system. Bebchuk et al., 2000, analyzed how the managerial incentive can be distorted given that a pyramid is already in place, but they did not justify the existence and the popularity of pyramids worldwide. Conversely, Almeida and Wolfenzon, 2006, showed why such a problematic pyramidal structure can be commonly used in countries with a weak legal system. Their argument was subject to the agency issue, and focused on equity ownership. I modify and extend the model to analyze optimal capital structure decision for pyramidal family business groups, and justify the existence of pyramids from a different angle - control efficiency.

Although studies on the optimal capital structure decision for family business groups are relatively sparse empirically and theoretically, there are few papers which directly investigate this issue. Bianco and Nicodano, 2006, propose a theory of capital structure under the pyramidal structure together with empirical evidence from Italian family business groups consistent with my prediction. They found a decreasing trend of leverage ratios from top to bottom of the pyramid. Their model lacks in addressing investment, financing, and dividend policy of the firms in a pyramidal business group, which I focus on this paper. A recent paper by Luciano and Nicodano, 2009, proposes a capital structure theory for a pyramidal business group from a financial synergy perspective. They show that the holding should be optimally zero-leveraged while the subsidiary highly leveraged, but this result is mainly driven by an assumption that the holding can rescue the subsidiary’s default by providing capital through dividends. But, in reality, without any ownership link, dividends from one firm can not be legally transferred to the other firm without cost. Hence, their theory is for cross-shareholding ties. Without any legal link of dividends, their results are arguable. The theory may be more appropriate to capital structure decision in private equity
(PE)/leverage buy out (LBO) industry where control concern is rare due to well-established corporate governance in those industries.

The model in this paper concerns dividend policy, and thus is related to the literature of corporate governance and dividend policy. La Porta et al., 2000, first introduced the two competing hypotheses on dividend policy- 1) the Expropriation Hypothesis and 2) the Substitution Hypothesis. The former predicts that a corporation exposed to severe agency problem tends to pay out less when the investor protection right is poor while the latter predicts that the dominant controller of business group tends to pay out more to establish a reputation for decent treatment of minority shareholders. La Porta et al., 2000, find empirical evidence supporting Expropriation Hypothesis. Contrary, Faccio et al., 2001, present evidence consistent with the Substitution Hypothesis. A recent study by De Jong et al., 2009, propose a new hypothesis that the pyramidal subsidiaries should pay out more dividends to service the holding’s debt (Debt Service Hypothesis). The dividend policy in my model is quite similar to this Debt Service Hypothesis, which I support with theoretical justification. The dividend policy in my model suggests that for financial support of group member firms, dividend channel is important to legally transfer capitals from one group firm to the other, consistent with Goplan et al., 2006.

One last strand of relevant research area is literature on internal capital market. Theories of the dark-side of internal capital market (Jensen, 1986, and Scharfstein and Stein, 2000) predict a low profitability and low valuation, and inefficient investments in business groups. Ferris et al., 2003, and Shin and Park, 1999, provide evidence that Korean family business groups prefer Survival not efficiency, and this survival preference is also documented in Japanese business groups, Keiretsus (Nakatani, 1984, and Prowse, 1992). However, these empirical studies lack in documentation of how these problems vary within a business group. A recent paper by Almeida et al., 2009, develop group structure metrics systematically. By using these metrics, they report the causal relationship between the profitability/investment policy and the position of a group firm. I use the data from Almeida et al., 2009, and test predictions from my theory using their group metrics. Contrary to their study, I focus on the capital structure and dividends policy within a business group.

4 Baseline Model

In this section, I formally present a family entrepreneur’s decision problem. When the family already owns a firm A, and tries to set up a new group member firm B based on a newly arisen investment opportunity, it needs to make a business group structure choice and a financing decision accordingly.

4.1 Model Setting

Economic Environment:

The economic environment of the model is characterized by the following: 1) weak investor protection rights, 2) one-share-one-vote rule, and 3) absolute priority rule (AP) in bankruptcy. Hence, dual class share structure is excluded from possible ownership structure choices. Due
to AP, a corporate control is taken over by creditors when the firm defaults. For simplicity, market risk-free rate is assumed to be zero in this economy, and corporate securities are priced based upon the market investors’ distributional belief on the cash flow from the securities.

**Agents and Preferences:**

The model consists of 3 types of agents, all of whom are risk-neutral - 1) a family entrepreneur, 2) a rival management who contends for a corporate control against the incumbent family, and 3) a large number of atomistic passive investors -. The passive investors purchase corporate securities and vote in the corporate election. Hence, they are passive in the sense that they do not contend for corporate control. All the agents are payoff maximizers due to risk-neutrality of their preferences.

**Managerial Ability and Information Structure:**

The family and a rival each have their own managerial abilities on the new project, which are assumed to be project-specific, and thus do not depend upon the incorporation choice nor the financing decision of the new firm. I denote the managerial ability of the family by $p$, and $p$ is assumed to be uniformly distributed on the interval, $[0, 1]$. The rival’s managerial ability is also uniformly distributed on $[0, 1]$, and it is assumed be randomly drawn from the distribution. The family’s managerial ability level, $p$, and the distributional assumption on the managerial abilities of both the family and a rival are assumed to be a common knowledge in this economy. I assume that any agent is limited to this knowledge, and thus the information structure on the managerial ability is symmetric.

**Technology:**

A project available to the family is summarized by a triple, $(\bar{Y}, i, K^*)$. $i$ is an amount of investment outlay, and $\bar{Y} = \bar{Y}(p, i)$ is a stochastic cash flow of the project which depends on $p$ and $i$. The mean cash flow of the project, $\bar{Y}$ is given as $\bar{Y} = (1 + pr(i))i$ where $r(i)$ is the net mean return of the project cash flow when $p = 1$, i.e., net return when the family is strictly better than a rival in its managerial ability), and thus the mean cash flow increases in $p$. The NPV of the project, $\bar{Y} - i$, is assumed to be non-negative for $\forall i, p \in [0, 1]$. Moreover, I assume that managerial ability only affects $\bar{Y}$ but not the risk profile of cash flow. Thus, denoting the stochastic component of the project cash flow by $\epsilon$, $\bar{Y} = \bar{Y} + \epsilon$ where $\epsilon$ follows a c.d.f. of $F(\epsilon)$. $F(\epsilon)$ is assumed to be in the increasing hazard rate family (IHR). Hence, $f(\epsilon)/(1 - F(\epsilon))$, increases in $\epsilon$. Lastly, $K = K(i)$ denotes a deterministic amount of control benefit from the new project, exclusively available to the corporate controller at the end

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5WLOG, I normalize the NPV of the project cash flow when $p = 0$ to be zero, and thus $\bar{Y}(p = 0, i) - i = 0$. Therefore, a project with a higher $p$ first-order stochastically dominates a project with a lower $p$.

6This assumption is made to guarantee a unique interior optimality of debt financing decision of a given firm, and in fact, is not restrictive since it is valid for many probability distributions such as the (truncated) normal, (truncated) exponential, uniform, and Laplace. The gamma and Weibull distribution with degrees of freedom parameter larger than 1 also satisfy this. I can simplify this assumption by assuming that cash flows in the model are normally distributed. For more details on the increasing hazard rate family (IHR), see Barlow and Proschan(1975). This assumption is also used in Chemmanur and John(1996) and Gârleanu and Zwiebel(2009).
of project. I assume that $K(i)$ does not depend on the managerial ability, but only on $i$. I assume $\frac{\partial K}{\partial i} > 0$. This implies that the absolute amount of private control benefit for a larger scale project is greater than that for a smaller one. And from $\frac{\partial Y}{\partial p} > 0$ for $vi$, we know $\frac{\partial K/Y}{\partial p} < 0$ for $\forall i$, which implies that for a given $i$, the relative importance of private control benefit gets substantial when the family’s managerial ability is poor (La Porta, Shleifer, and Vishny, 2002).

**Dates of Actions:**

The model has three dates: date-0, -1, and -2. At date-0, a family entrepreneur with zero wealth starts up a firm A based on a positive NPV project, $(\tilde{Y}_A, i_A, K_A)$ with $p_A$ and c.d.f of $F_A(.)$, by selling $(1 - \alpha)$ fraction of the firm’s share. The firm generates cash flows and private control benefits at both date-1 and date-2. Suppose that at date-1, firm A generates cash flow, $C$ and private control benefit, $K_A$. The family consumes $K_A$, and a new investment opportunity, $(\tilde{Y}_B, i_B, K_B)$ with $p_B$ and $F_B(.)$, arises. Given the established ownership, $\alpha$, in firm A in addition to the firm’s retaining earning of $C$, the family tries to set up a new group firm B based on this new project, at which point, it should decide how to structure the new project and how to finance it. The new group member firm B will be either structured as a subsidiary of the existing firm A (Pyramidal structure) or a stand-alone entity that is independent from firm A and controlled directly by the family (Horizontal structure). The family can finance the new firm with any mix of cash, zero-coupon debt, and equity under each business group structure. There is a market for corporate control which is assumed to be open immediately after the family makes the business group structure and financing decision. In the market for corporate control, a rival management emerges, and contends against the family for control over firm B. The rival is assumed to initially have no equities in firm B, and emerges with zero wealth for simplicity. This implies that the rival contends for corporate control over firm B solely relying on the proxy votes that it can win from passive investors in the corporate election. Once the corporate election results are in, the winner of the election will control firm B and implement the new project. At date-2, the project cash flows in both firm A and B will be realized, and the payoff to the family will be determined. The payoff to the family at date-2 under each of the two business

8Clearly $K$ depends on the strength of investor protection rights. If I denote it by $s$ where $0 \leq s \leq 1$, then the higher $s$, the more minority investors in the economy are protected legally, and thus $\frac{\partial K(i,s)}{\partial s} < 0$ holds for $\forall i$

9This date-0 all equity financing assumption for firm A is taken to simplify the analysis of the model. I can relax this assumption and allow the family to possibly issue 1-period zero-coupon debt that matures at date-1. This assumption is not crucial for the main results of the model. The family’s zero wealth assumption can also be relaxed.

10If there is any 1-period zero-coupon debt issued by firm A at date-0, then $C$ is a cash flow net of debt repayment.

11In addition to these two business group structure choices, the family can adopt the new project as a new division of the existing firm A (i.e., Conglomerate structure hereafter), but for the first part of the model, I focus on the pyramidal and horizontal structure choices, and later compare the two structures to the conglomerate structure in section 6.

12In this model, the actual take-over by a rival does not occur in equilibrium since the family strategically chooses its ownership level in order to ensure its control over a firm.
group structures - the pyramidal structure and the horizontal structure - consists of a sum of cash flow and private control benefit from both firm A and B.

**Valuable Corporate Control and Voting Behaviors of Agents:**

**Assumption 1 (Valuable corporate control).** *The right to control a corporation is valuable, and thus the incumbent management is not willing to give up the corporate control even if any rival management offers better security benefits.*

The deterministic amount of private control benefit, $K$, possibly from managerial perquisites, any diverted corporate resources through accounting fraud, or reputation effects, can not be contracted away in a cost-free way. Since I consider an environment with weak investor protection rights, the amount of benefit is expected to be substantial.

Assumption 1 induces the family and a rival to be control-driven. Hence, in the corporate control contest, the family will vote for itself even if it knows that it has inferior managerial skills than any rival. Since the rival emerges with zero wealth and no equities in a given firm, it contends for control over the firm solely to receive the private control benefit, and also votes for itself.

The passive investors, who hold the floating shares of outside equities, vote for either the family or a rival, based on who they believe is best-able to run a corporation, and their decision is not affected by any financing or dividend policies.

The fraction of votes that the family is expected to win from the passive investors is denoted by $\xi(p)$, and I assume that the family can win more votes from the passive investors when its managerial ability level is higher (i.e., higher $p$). Hence, $\frac{\partial \xi(p)}{\partial p} > 0$ holds.

At date-1, the family compares financing decisions under each of the two business group structures, and chooses to set up the new firm B under the structure that provides a higher expected payoff at date-1. The date-1 problem of the family under each business group structure will be defined in the following sections provided that the ownership structure of firm A, $\alpha$, the retained earning in firm A, $C$, and the two project profiles for firm A and B are given. The joint c.d.f of the two projects is denoted by $F_{A,B}(.,.)$. p.d.f.s are denote by lowercase $f(.)$.

### 4.2 Horizontal Structure

Under the horizontal structure choice, the family has a personal wealth of $\alpha C$ to spend for the new firm B since the firm is independently incorporated from firm A. Let’s denote the 13This behavior of the family and the rival can be formally stated as the following: Let’s denote the value of equity of a given firm with debt of face value of $D$ and the incumbent’s managerial ability level of $p \in [0, 1]$ by $S(p, D)$, and the expected control benefit with debt of face value of $D$ by $k(D)$. Then, both the family and the rival behave control driven if $S(p = 0, D) + k(D) \geq S(p = 1, D)$ for $\forall D \geq 0$. This implies that even if the family is relatively poor-skilled than any rival for sure, it is not likely to give up control to the rival in order to receive higher security benefit from its equity stake. I implicitly exclude a possibility that the rival offers a strategic tender to the family at the bid price of the sum of its security and control benefits (Grossman and Hart, 1988).

14When the firm is highly leveraged or pays out substantial amount of dividends, the firm is more vulnerable to any strategic tender offer made by a rival. I exclude this possibility in this model. Thus, dividend policy and capital structure decision do not affect the outcome of corporate control contest.
amount of cash financing, fraction of equities held by the family, and the amount of zero-coupon debt for firm B under this structure by \( t^H, \beta^H, \) and \( D_B^H \), and \( 0 \leq t^H \leq \alpha C \) holds. The total amount of capitals raised for firm B under this structure by internal and external financing is denoted by \( M_B^H \), and \( M_B^H = t^H + (1 - \beta^H)S_B(D_B^H) + B_B(D_B^H) \) where \( S_B(D_B^H) \) and \( B_B(D_B^H) \) denote the date-1 values of firm B’s equity and debt with face value of \( D_B^H \). I define a stochastic total cash flow in firm B at date-2 before any debt repayment, \( \tilde{X}_B^H \) where \( \tilde{X}_B^H = M_B^H - i_B + \tilde{Y}_B. \) Then, the family’s stochastic payoff at date-1 under the horizontal structure, \( \tilde{\Pi}^H \), is given as

\[
\tilde{\Pi}^H = \alpha C + \alpha \tilde{Y}_A + K_A + \left[ -t^H + \beta^H (\tilde{X}_B - D_B^H)^+ \right] + K_B1_{\{\tilde{X}_B^H \geq D_B^H\}}
\]

When the family sells \((1 - \beta^H)\) shares of firm B, it wants to ensure control over firm B with \( \beta^H \) fraction of equities. In the corporate election, \((1 - \beta^H)\) fraction of equities held by the passive investors will be won either by the family or a rival. By definition of \( \xi(p) \), \( \xi(p_B)\)-fraction of passive investors’ votes is expected to be won by the family in the corporate election. This implies that the family can maintain control right over firm B if

\[
\beta^H + \xi(p_B)(1 - \beta^H) \geq (1 - \beta^H)(1 - \xi(p_B))
\]

Therefore, the family’s equity financing capacity is given as

\[
\max\left(\frac{1-2\xi}{1-\xi}, 0\right) \leq \beta^H \leq 1
\]

\[
\Leftrightarrow \gamma(p_B) \leq \beta^H \leq 1
\]

The lower bound of equity financing capacity, \( \gamma(p_B) \), satisfies \( \frac{\partial \gamma(p_B)}{\partial p_B} < 0 \) since \( \frac{\partial \xi(p)}{\partial p} > 0. \) Hence, for a higher \( p_B \), the family can sell more shares of firm B to the external capital market. I denote this lower limit of equity financing parameter, \( \gamma(p_B) \) by \( \gamma_B \) for notational simplicity hereafter.

Because investors are risk-neutral and break even in equilibrium, the following should hold:

\[
M_B^H = t^H + (1 - \beta^H)S_B(D_B^H) + B_B(D_B^H)
\]

\[
\Rightarrow -t^H + \beta^H E_F[\tilde{X}_B^H - D_B^H]^+ = \tilde{Y}_B - i_B
\]

Using this condition, I can write the date-1 expected payoff to the family under the horizontal structure, \( \Pi^H \equiv E^{F_{A,B}}[\tilde{\Pi}^H] \), as

\[
\Pi^H = \alpha C + \alpha \tilde{Y}_A + \tilde{Y}_B - i_B + K_A + K_B[1 - F_B(\tilde{X}_B < D_B^H)]
\]

The family’s date-1 program under the horizontal structure is given as

\[
\max_{(t^H, \beta^H, D_B^H)} \Pi^H(t^H, \beta^H, D_B^H) \quad \text{s.t.} \quad \gamma_B \leq \beta^H \leq 1, \quad M_B^H \geq i_B, \quad 0 \leq t^H \leq \alpha C
\]

Lemma 1. [Optimality of Tight Financing] It is optimal for the family to set \( M_B^H = i_B \).

When the investment outlay can be supported only by cash and equity financing, any surplus in the raised fund does not help increase the family’s payoff. When debt should be issued, a marginal dollar increase in the face value of debt provides additional capital strictly less than a dollar since the debt is risky. Therefore, any fund surplus in this case reduces the family’s payoff, and thus is not optimal. This tight financing condition can be applied to any choice of business group structure. Therefore, I apply this tight financing condition throughout the paper hereafter.

Imposing the tight financing condition from Lemma [1], the date-1 program under the horizontal structure is given as

\[
\begin{align*}
\text{Max}_{(t^H, \beta^H, D^H_B)} & \quad \alpha C + \alpha \bar{Y}_A + \bar{Y}_B - i_B + K_A + K_B[1 - F_B(D^H_B)] \\
s.t. & \quad \gamma_B \leq \beta^H \leq 1 \\
& \quad t^H + \bar{Y}_B - \beta^H S_B(D^H_B) = i_B \\
& \quad 0 \leq t^H \leq \alpha C
\end{align*}
\]

(1)

where

\[ S_B(D^H_B) = E^{F_B} [\tilde{Y}_B - D^H_B] \]

The first constraint is the equity financing constraint, and the second is the budget constraint. The third is the cash financing constraint.

4.3 Pyramidal Structure

When the pyramidal structure is chosen, the availability of the internal cash resource of the family is extended to the entire retained earnings, \(C\), in firm A since the family allows firm A to invest in the new subsidiary B. Let’s denote the amount of cash financing, fraction of equities held by the family, and the amount of zero-coupon debt for firm B under this structure by \(t^P_B\), \(\beta^P\), and \(D^P_B\), and \(0 \leq t^P_B \leq C\) holds. In this baseline model, I exclude the possibility that the family can raise debt through firm A, and subsidize firm B’s investment by using this extra debt capital. This possibility will be analyzed in section [5].

By imposing tight financing condition, the family’s date-2 stochastic payoff under the pyramidal structure, \(\tilde{\Pi}^P\) is given as

\[
\tilde{\Pi}^P = \alpha \left[ C - t^P + \bar{Y}_A + \beta^P (\bar{Y}_B - D^P_B)^+ \right] + K_A + K_B 1_{\{\bar{Y}_B \geq D^P_B\}}
\]

Since the managerial ability of the family is assumed to not depend on the business group structure choice, the equity financing capacity of the family under the pyramidal structure is the same as under the horizontal structure. This implies

\[ \gamma_B \leq \beta^P \leq 1 \]

By imposing the breakeven condition, the date-1 program of the family under the pyramidal structure is given as:

\[
\begin{align*}
\text{Max}_{(t^P, \beta^P, D^P_B)} & \quad \alpha C + \alpha \bar{Y}_A + \alpha (\bar{Y}_B - i_B) + K_A + K_B[1 - F_B(D^P_B)] \\
s.t. & \quad \gamma_B \leq \beta^P \leq 1 \\
& \quad t^P + \bar{Y}_B - \beta^P S_B(D^P_B) = i_B \\
& \quad 0 \leq t^P \leq C
\end{align*}
\]

(2)
where
\[ S_B(D^P_B) = E^{F_B} [\tilde{Y}_B - D^P_B] + \]

The first constraint is the equity financing constraint, and the second is the budget constraint. The third is the cash financing constraint.\(^{15}\)

4.4 Results

Lemma 2. Under the horizontal structure,

\[
(t^H, \beta^H, D^H_B) \begin{cases} 
(t, \beta, 0) & \text{if } i_B \leq \alpha C + (1 - \gamma_B)\tilde{Y}_B \\
\text{where } t \in [0, \alpha C], \beta \in [\gamma_B, 1] \text{ s.t. } t + (1 - \beta)\tilde{Y}_B = i_B \\
(aC, \gamma_B, \bar{D}^H_B) & \text{if } i_B > \alpha C + (1 - \gamma_B)\tilde{Y}_B \\
\text{where } S_B(\bar{D}^H_B) = \frac{\alpha C + \tilde{Y}_B - i_B}{1 - \gamma_B}
\end{cases}
\]

Under the pyramidal structure,

\[
(t^P, \beta^P, D^P_B) \begin{cases} 
(t, \beta, 0) & \text{if } i_B \leq C + (1 - \gamma_B)\tilde{Y}_B \\
\text{where } t \in [0, C], \beta \in [\gamma_B, 1] \text{ s.t. } t + (1 - \beta)\tilde{Y}_B = i_B \\
(C, \gamma_B, \bar{D}^P_B) & \text{if } i_B > C + (1 - \gamma_B)\tilde{Y}_B \\
\text{where } S_B(\bar{D}^P_B) = \frac{C + \tilde{Y}_B - i_B}{1 - \gamma_B}
\end{cases}
\]

Proof. Since risky debt reduces the expected control benefit due to the increase in default probability, the family does not want to issue risky debt when the the maximal amount of cash and equity financing is sufficient for \(i_B\). Risky debt is issued only if \(i_B\) exceeds the maximal cash and equity financing amount, and the issued amount of risky debt should be minimal that satisfies \(M^j_B = i_B\) for \(\forall j = P, H\).

Lemma 3. For a given \(i_B\), the leverage ratio for the firm \(B\) under pyramidal structure measured as a ratio of book value of total debt to book value of stand-alone asset is lower than that under horizontal structure.

Proof. From Proposition 2, we have \(S_B(D^H_B) = \frac{\alpha C + \tilde{Y}_B - i_B}{\gamma_B}\) and \(S_B(D^P_B) = \frac{C + \tilde{Y}_B - i_B}{\gamma_B}\). This implies that \(\bar{D}^H_B > \bar{D}^P_B\). Hence, \(\frac{D^H_B}{\tilde{Y}_B} > \frac{D^P_B}{\tilde{Y}_B}\).

When the family chooses the pyramidal structure, the leverage ratio of the subsidiary \(B\) is less than that of a stand-alone entity \(B\) under the horizontal structure due to the greater availability of internal capital. The internal financing advantage under the pyramid enables the family to minimize the use of debt, and thus can lead to the greater amount of expected

\(^{15}\)I assume that under the pyramidal structure, there is no rival contending for control over firm \(A\) against the family at date-1. In other words, the family optimally choose the level of its ownership in firm \(A\), \(\alpha\) at date-0 to ensure its control influence over firm \(A\) at both date-1 and date-2 with the given \(\alpha\). This assumption does not derive the main results in this paper, and thus can be relaxed, if necessary.
control benefit. Therefore, there is a relative control benefit advantage under the pyramid, but the advantage does not come up at zero-cost in equilibrium. The following Proposition describes the condition where the pyramidal structure is optimal in equilibrium.

**Proposition 1.** The pyramidal structure is chosen as the optimal business group structure in equilibrium only if

\[
i_B \geq \alpha C + (1 - \gamma)\bar{Y}_B \quad \text{and} \quad K_B \left[ F_B(\bar{D}_H^B) - F_B(\bar{D}_P^B) \right] \geq (1 - \alpha) \left( \bar{Y}_B - i_B \right)
\]

(3)

**Proof.** In Appendix B.2.

Equation (3) in Proposition 1 implies that the more likely the pyramidal structure is optimally chosen in equilibrium if 1) the new project is capital-intensive, and 2) the project delivers lower rent and higher control benefit, and thus the cash capital becomes useful. There is a trade-off for internal cash - control benefit advantage and rent disadvantage. Hence, the greater availability of internal capital is useful only if control benefit advantage dominates rent disadvantage. Moreover, the scale of investment matters. For a small scale investment, the family can support the required investment outlay without hurting the expected control benefit even under the horizontal structure. Therefore, for a small scale project, there is no benefit to have a greater internal capital assessibility. But when the investment scale is large, and the family can not finance the project with only cash and equities under the horizontal structure, the greater availability of internal capital under the pyramidal structure becomes valuable. Therefore, for a large scale project, the family finds the pyramidal structure favored to the horizontal structure, and the pyramidal structure becomes optimal in equilibrium only if the relative control advantage of the pyramidal structure dominates the relative rent advantage of the horizontal structure. These results are consistent with the existing theories on the pyramid such as Bebchuck et al., 2002, and Almeida and Wolfenzon, 2006.

The greater availability of internal capital under the pyramidal structure directly comes from the central firm A’s dividend policy that the family does not pay out any of the retained earnings of the firm to ensure the internal slack at the maximum availability. Therefore, the relative control advantage and relative rent disadvantage under the pyramid are closely related to the conservative dividend payout policy of the central firm A.

Proposition 3 suggests that the amount of debt in firm B is as an important determinant of the optimal business group structure. The debt amount determines the expected control benefit, and thus for a given amount of control benefit, \(K_B\), the lower leveraged a firm is, the more likely the firm is adopted as a pyramidal subsidiary in equilibrium. This implies that in equilibrium, the expected control premium of a pyramidal subsidiary should be higher than that of a firm controlled directly by the family. This result is consistent with the prediction from the traditional argument on the separation between cash flow and control rights in agency theories. But, in my model, this is the equilibrium outcome, not a consequence of the pyramid in place.

**Lemma 4.** A relatively poor-skilled family finds a pyramidal structure optimal in equilibrium. Conditional that the pyramidal structure is chosen, it is more likely that the family’s managerial ability, \(p_B\), is below the average, 0.5. This implies that in equilibrium, the ownership of a pyramidal subsidiary tends to be concentrated.
Proof. In Appendix B.3.

Lemma 4 indicates that the internal capital advantage is useful especially when the family has inferior equity financing capacity due to poor managerial skill. This implies that the pyramidal structure is favored to the horizontal structure if the family is relatively less able than a rival. The relatively poor managerial skill of the family for a pyramidal subsidiary implies that, in equilibrium, the ownership of a pyramidal subsidiary is more concentrated than a stand-alone entity directly controlled by the family since $\frac{\partial \gamma(p)}{\partial p} < 0$. This result resolves a puzzling empirical evidence that the separation between cash flow and control rights under the pyramid is not as extreme as documented by Frank and Mayer, 2000.

5 Extension: A Fully Levered Pyramid

The baseline model proposes how the capital structure decision for a pyramidal subsidiary differs from a stand-alone entity under the horizontal structure. In this model, whether or not recapitalization of firm A under the pyramidal structure is beneficial to the family was not analyzed.

Under the horizontal structure, any new debt issuance from firm A at date-1 is not desirable since 1) the capital raised from the new debt can not be used to support firm B’s investment due to the independence of firm B from firm A and 2) any increase in default probability of firm A reduces the expected control benefit from the firm, which is ensured without the debt.

But, under the pyramidal structure, it is not straightforward whether issuing debt from firm A at date-1 is optimal. When the family adopts the pyramidal structure, the central firm A invests in the subsidiary’s equities. This implies that the firm acquires a new asset, control shares in firm B. Hence, under the pyramidal structure, the central firm A has two types of assets - 1) its own existing operating asset, and 2) the newly acquired control shares in firm B. Therefore, if the control shares can be used as an additional collateral for firm A’s debt, the family can further reduce the debt amount that firm B should issue for the new investment. This extended debt capacity of firm A leads to better control efficiency of the business group if the cost of firm A’s debt is not high. Given these conditions, I analyze the optimality of recapitalization of firm A under the pyramid.

To simplify the analysis, I assume that at the current ownership of the family in firm A, $\alpha$, the family’s control over the firm is tightly secured, and there is no further equity financing in firm A at date-1. "tightly secured” means that the family can not sell any more equities of firm A due to takeover threat. This assumption can be relaxed, and does not derive the main implication of the model in this section.

---

*I implicitly exclude a possibility that the family raises capital by issuing debt from firm A, and transfer the capital to firm B by paying it as dividends. Interchanging the cash flow of firm A from date-2 to date-1 does not affect the main results in this section except the capital-intensity prediction of the central firm in Proposition 3. This prediction becomes conditional if I allow the possibility. However, it is not feasible to transfer capital from firm A to B if the family can issue the debt from firm A only after it pays out the firm’s retaining earning, C, as dividends.*
5.1 Pyramidal Structure: Date-1 Program Reformulation

At date-1, the family also considers raising capital for firm B by issuing zero-coupon debt with face value of \( D_A^P \) from firm A. In this case, both the entire retained earning, \( C \), and the newly raised capital by the debt in firm A subsidize firm B’s new investment.

Let’s denote the amount of additional capital raised by firm A’s debt by \( B_A(D_A^P, D_B^P, \delta). \)

I denote the stochastic total cash flow in firm B before debt repayment at date-2 by \( \tilde{X}_B = M_B^p - i_B + \tilde{Y}_B \) where \( M_B^p \) is the total capital raised by internal and external financing for firm B. \( \delta \) is a fraction of cash flow claim of firm B’s equity (i.e., \( [\tilde{X}_B^P - D_B^P]^+ \)), paid out as dividends. When firm B pays out dividends, firm A receives \( \beta^P \delta[\tilde{X}_B^P - D_B^P]^+ \) as a non-operating profit. Therefore, the total cash flow in firm A before any debt repayment at date-2, \( \tilde{X}_A(\delta) \), is written as

\[
\tilde{X}_A(\delta) = C + B_A(D_A^P, D_B^P, \delta) - t^P + \tilde{Y}_A + \beta^P \delta[\tilde{X}_B^P - D_B^P]^+ \]

The family need to repay \( D_A^P \) at date-2 by only using \( \tilde{X}_A(\delta) \) since \( \tilde{X}_A(\delta) \) is the actual amount of cash that the family has access to for the debt repayment. The default of firm A is defined by \( 1_{\{\tilde{X}_A^P(\delta) < D_A^P \}} \) with \( \delta \leq 1 \). Then, firm A’s equity holder has a claim on \( [\tilde{X}_A^P(\delta = 1) - D_A^P]^+1_{\{\tilde{X}_A^P(\delta) \geq D_A^P \}} \). Default trigger is defined with \( \delta \), but the claim amount in solvency is defined with \( \delta = 1 \).

Therefore, the date-2 stochastic payoff to the family under the pyramid is given as

\[
\Pi(\delta) = \alpha[\tilde{X}_A^P(\delta = 1) - D_A^P]^+1_{\{\tilde{X}_A^P(\delta) \geq D_A^P \}} + K_A 1_{\{\tilde{X}_A^P(\delta) \geq D_A^P \}} + K_B 1_{\{\tilde{X}_A^P(\delta) \geq D_A^P, \tilde{X}_B^P \geq D_B^P \}} \tag{4}
\]

Equation (4) highlights an intrinsic property of the pyramidal structure. The third line in Equation (4) indicates that the family’s control over firm B is subject to the solvency of firm A. This emphasizes why control over firm A is central in control to the subsidiary B. When firm A defaults, the control right of the firm is transferred to creditors of the firm, which implies that the creditors of firm A become the effective controller of firm B in the event. Hence, there is intrinsic control optionality under a pyramid.

Lemma 5 (Ex-post optimal dividend policy for firm B under the pyramid). For a given \( D_A^P > 0, D_B^P \geq 0, \) and \( t^P \), if \( \tilde{X}_A^P(\delta = 0) < D_A^P \) and \( \tilde{X}_A^P(\delta = 1) \geq D_A^P \), then WLOG, it is optimal to set \( \delta^* = 1 \)

Proof. In Appendix B.4

Lemma 5 says that when the family can not repay firm A’s debt with the firm’s own operating cash flow, it has an incentive to transfer capital from firm B to firm A. Due to the non-marketability of firm A’s control stakes in firm B, the capital should be legally transferred through dividends. Lemma 5 introduces another intrinsic property of the pyramid called co-insurance benefit. The asset composition of the central firm A and the optimal dividend policy for the subsidiary B imply that there is a co-insurance benefit by augmenting cash flows from the two assets to service the central firm’s debt.
By imposing Lemma 5, the date-2 stochastic payoff to the family under the pyramid is given as

\[ \tilde{\Pi}^P = \alpha [\tilde{X}_A^P(\delta^* = 1) - D_A^P]^+ \\
+ K_A \text{Prob}\{\tilde{X}_A^P(\delta^* = 1) \geq D_A^P\} \\
+ K_B \text{Prob}\{\tilde{X}_A^P(\delta^* = 1) \geq D_A^P, \tilde{X}_B^P \geq D_B^P\} \]

Because investors are risk-neutral and break even in equilibrium,

\[ EF_{A,B}[\tilde{X}_A^P(\delta^* = 1) - D_A^P]^+ = C + \bar{Y}_A + \bar{Y}_B - i_B \]

Using this condition, the date-1 expected payoff to the family under the pyramidal structure with \( \delta^* = 1 \), \( \Pi^P = EF_{A,B}[\tilde{\Pi}^P] \), is given as following:

\[ \Pi^P = \alpha C + \alpha \bar{Y}_A + \alpha (\bar{Y}_B - i_B) \\
+ K_A \text{Prob}\{\tilde{X}_A^P(\delta^* = 1) \geq D_A^P\} \\
+ K_B \text{Prob}\{\tilde{X}_A^P(\delta^* = 1) \geq D_A^P, \tilde{X}_B^P \geq D_B^P\} \]

The family’s date-1 program under the pyramidal structure with \( \delta^* = 1 \) is finally written as:

\[
\begin{align*}
\max_{(t^P, \beta^P, D_A^P, D_B^P)} \quad & \Pi^P(t^P, \beta^P, D_A^P, D_B^P; \delta^* = 1) \\
\text{s.t.} \quad & \gamma_B \leq \beta^P \leq 1 \\
& M_B^P \geq i_B \\
& 0 \leq t^P \leq C + B_A(D_A^P, D_B^P; \delta^* = 1) 
\end{align*}
\]

where

\[ B_A(D_A^P, D_B^P; \delta^* = 1) = EF_{A,B} \left[ \min \left\{ D_A^P, \tilde{X}_A^P(\delta^* = 1) \right\} \right] \]

The first two constraints are the equity financing constraint and the budget constraint. And, the third is the cash financing constraint.

**Lemma 6.** For \( i_B > C + (1 - \gamma_B) \bar{Y}_B \) (i.e., \( \tilde{D}_B^H > 0 \)), it is optimal to set \( t^P = C + B_A(D_A^P, D_B^P) \) and \( M_B^P = i_B \) where \( M_B^P = t^P + \bar{Y}_B - \beta^P S_B(D_B^P) \)

**Proof.** In Appendix B.5

Imposing Lemma 6 gives

\[ t^P = C + \bar{Y}_A + \beta^P EF_{A,B}[\bar{Y}_B - D_B^P]^+ \\
- EF_{A,B} \left[ \bar{Y}_A + \beta^P[\bar{Y}_B - D_B^P]^+ - D_A^P \right]^+ \]

Plugging this into \( t^P + \bar{Y}_B - \beta^P S_B(D_B^P) = i_B \) gives a new group budget constraint

\[ C + \bar{Y}_A + \bar{Y}_B - EF_{A,B} \left[ \bar{Y}_A + \beta^P[\bar{Y}_B - D_B^P]^+ - D_A^P \right]^+ = i_B \]

This new group budget constraint is useful to illustrate how recapitalization of firm A is beneficial for the family. \( EF_{A,B} \left[ \bar{Y}_A + \beta^P[\bar{Y}_B - D_B^P]^+ - D_A^P \right]^+ \leq EF_A[\bar{Y}_A - D_A^P]^+ + \beta^P EF_{B}[\bar{Y}_B - D_B^P]^+ \) always holds since it is known that the value of sum of two call options is
always greater than the value of a call on call option. This is true for any correlation value between \( \bar{Y}_A \) and \( \bar{Y}_B \). This implies that if \( D^P_A > 0 \),

\[
C + \bar{Y}_A + \bar{Y}_B - E^{F_A,B} \left[ \bar{Y}_A + \beta^P [\bar{Y}_B - D^P_A] + D^P_A \right]^+ \\
> C + \left( \bar{Y}_A - E^{F_A}[\bar{Y}_A - D^P_A] \right) + \bar{Y}_B - \beta^P E^{F_B} \left[ \bar{Y}_B - D^P_B \right]^+ \\
> C + \bar{Y}_B - \beta^P E^{F_B} \left[ \bar{Y}_B - D^P_B \right]^+ 
\]

The first line in Equation (5) is a total funds available for the family when the family recapitalizes the central firm A, and the third line is a total funds available for the family when the family does not re-capitalize the central firm A, which is the baseline case. The strict inequality implies that the recapitalization of firm A ensures a better financing capacity to the family, and the advantage of recapitalization directly comes from the term, \( \left( \bar{Y}_A - E^{F_A}[\bar{Y}_A - D^P_A] \right) \).

This indicates that the more the family issues debt from firm A, the greater amount of funds available to the family for a given level of \( D^P_B \), which, in turn, implies that the family can reduce \( D^P_B \) by appropriately issuing \( D^P_A \). Therefore, unless the cost of firm A’s debt is not high, then this advantage can lead to a greater control benefit to the family under the pyramid, and thus recapitalization of firm A becomes optimal in equilibrium.

Focusing on the case where \( D^P_B > 0 \), the date-1 program of the family under the pyramidal structure with Lemma 6 is written as

\[
\text{Max}\,(D^P_A, D^P_B) \quad \Pi^P(D^P_A, D^P_B) \\
\text{s.t.} \quad C + \bar{Y}_A + \bar{Y}_B - S_A(D^P_A, D^P_B) = i_B 
\]

where

\[
\Pi^P = \alpha C + \alpha \bar{Y}_A + \alpha (\bar{Y}_B - i_B) \\
+ K_A \text{Prob}\{ \bar{X}_A \geq D^P_A \} + K_B \text{Prob}\{ \bar{X}_A \geq D^P_A, \bar{Y}_B \geq D^P_B \} \\
\bar{X}_A = \bar{Y}_A + \gamma_B [\bar{Y}_B - D^P_B]^+ \\
S_A(D^P_A, D^P_B) = E^{F_A,B} \left[ \bar{Y}_A + \gamma_B [\bar{Y}_B - D^P_B]^+ - D^P_A \right]^+ 
\]

Since we assume \( D^P_B > 0 \), \( \beta^{P*} = \gamma_B \). The given constraint is the new budget constraint.

### 5.2 Results

The solution of Program [6] depends on the correlation between \( \bar{Y}_A \) and \( \bar{Y}_B \). A limitation on the mathematical tractability is that firm A’s equity value, \( S_A(D^P_A, D^P_B) \), cannot be expressed in simple closed form formula since the term, \( [\bar{Y}_B - D^P_B]^+ \) is truncated at zero. But, if I restrict the correlation between \( \bar{Y}_A \) and \( \bar{Y}_B \) to be 1, then I can drive a nice intuitive characterization of the optimality. Hence, to illustrate the main economic intuition of the model, I restrict the correlation between the two cash flows to be 1 in this section. I will relax this correlation restriction and show that the results derived in this section are not
changed for the general correlation cases in the following section 5.3. The same intuition will be carried over to the general correlation cases.

When the cash flows from both firm A and B are perfectly correlated, \( \hat{Y}_A = \hat{Y}_A + \epsilon \) and \( \hat{Y}_B = \hat{Y}_B + \epsilon \) where \( \epsilon \sim F(\epsilon) \). Hence, I can write \( \hat{Y}_A = \hat{Y}_B + Q \) where \( Q = \hat{Y}_A - \hat{Y}_B \). Let’s denote the security benefit, \( \alpha C + \alpha \hat{Y}_A + (\alpha(\hat{Y}_B - i_B)) \), by \( Z \). Then, from equation (7), we have

\[
\Pi^P = Z + (K_A + K_B) \text{Prob}\{\hat{Y}_B(1 + \beta^P) \geq (D^P_A - Q) + \beta^P \hat{D}^P_B, \hat{Y}_B \geq D^P_B}\]
\[
+ K_A \text{Prob}\{\hat{Y}_B \geq (D^P_A - Q), \hat{Y}_B < D^P_B\}
\]
\[
= Z + (K_A + K_B) \text{Max}\{\frac{(D^P_A - Q) + \beta^P \hat{D}^P_B}{1 + \beta^P}, D^P_B\}
\]

\[
+ K_A \text{Prob}\{(D^P_A - Q) < \hat{Y}_B, \hat{Y}_B < D^P_B\}
\]

This can be summarized as

\[
\Pi^P = \begin{cases} 
Z + (K_A + K_B)[1 - F(D^P_G)] & \text{if } D^P_A \geq D^P_B \\
Z + (K_A + K_B) - K_A F(D^P_A) - K_B F(D^P_B) & \text{if } D^P_A < D^P_B 
\end{cases}
\]

(9)

where

\[
D^P_A = \frac{D^P_A - Q}{D^P_B + \beta^P \hat{D}^P_B} \\
D^P_G = \frac{D^P_A + \beta^P \hat{D}^P_B}{1 + \beta^P}
\]

\( D^P_G \) is an effective group debt amount under the pyramid, and it lies between \( D^P_A \) and \( D^P_B \) by its definition. If \( D^P_A \geq D^P_B \), then the family’s control default on the entire business group is synchronized, but the event is defined at \( D^P_G \), which is strictly less than \( D^P_B \).

To illustrate how useful recapitalization of the pyramid is, I consider the following two cases where two projects are identical (i.e., \( \hat{Y}_A = \hat{Y}_B = \hat{Y} \)). One case is \( D^P_A = \hat{D}^P_B, D^P_B = 0 \), and the other is \( D^P_A = 0, D^P_B = \hat{D}^P_B \). The second case was the solution of baseline model.

Let’s compare the new budget constraint to the baseline one as following:

\[
C + \hat{Y} + \hat{Y} - E[(1 + \gamma_B)\hat{Y} - \hat{D}^P_B]^+ \\
= C + \left(\hat{Y} - E[\hat{Y} - \frac{D^P_B}{1 + \gamma_B}]^+\right) + \left(\hat{Y} - \gamma_B E[\hat{Y} - \frac{D^P_B}{1 + \gamma_B}]^+\right) \\
> C + \hat{Y} - \gamma_B E[\hat{Y} - \frac{D^P_B}{1 + \gamma_B}]^+
\]

Above algebra tells us that the family can reduce group-wise required debt amount dramatically through recapitalization.

However, given that the optimal payoff under the pyramid in the baseline case is \( Z + K_A + K_B[1 - F_B(\hat{D}^P_B)] \), there is a trade off of the recapitalization - 1) the extra gain in expected control benefit of firm B, \( K_B [F_B(\hat{D}^P_B) - F_B(D^P_G)] \), and 2) the extra loss in expected control benefit of firm A, \( K_A F_B(D^P_G) \). Therefore, if the extra gain in expected control benefit of firm B exceeds the extra loss in firm A, the family has an incentive to re-capitalise the pyramid.

The following Lemma summarizes when either \( D^P_A \geq D^P_B \) or \( D^P_A < D^P_B \) is an optimal recapitalization of the pyramid.

**Lemma 7.** When the cash flows of firm A and B are perfectly correlated, in an interior equilibrium (i.e., \( D^P_A^*, D^P_B^* > 0 \)),
1. The optimal debt amounts, \( D_A^{P*} \) and \( D_B^{P*} \) satisfy
\[
\begin{cases}
  D_A^{P*} \geq D_B^{P*} & \text{if } K_A \leq \frac{K_B}{\gamma_B} \\
  D_A^{P*} < D_B^{P*} & \text{if } K_A > \frac{K_B}{\gamma_B}
\end{cases}
\]
And, they are characterized as following:
\[
\begin{cases}
  \text{if } K_A \leq \frac{K_B}{\gamma_B} & D_A^{P*} \geq D_G^* \geq D_B^{P*} \\
  \text{if } K_A > \frac{K_B}{\gamma_B} & D_A^{P*} < D_B^{P*}
\end{cases}
\]
where
\[
E^{F_A}[\tilde{Y}_B - D_A^{P*}]^+ = \frac{C + Y_A + Y_B - i_B}{1+\gamma_B}
\]
and
\[
\begin{align*}
E^{F_A}[\tilde{Y}_B - D_A^{P*}]^+ + \gamma_B E^{F_B}[\tilde{Y}_B - D_B^{P*}]^+ &= C + \tilde{Y}_A + \tilde{Y}_B - i_B \\
&= D_G^* + \gamma_B E^{F_B}[\tilde{Y}_B - D_B^{P*}]^+
\end{align*}
\]
(10)

2. The optimal payoff to the family, \( \Pi^{P*} \) satisfies
\[
\begin{cases}
  \text{if } K_A \leq \frac{K_B}{\gamma_B} & \Pi^{P*} = Z + (K_A + K_B) \left[ 1 - F(D_G^{P*}) \right] \\
  \text{if } K_A > \frac{K_B}{\gamma_B} & \Pi^{P*} = Z + (K_A + K_B) - K_A F(D_A^{P*}) - K_B F(D_B^{P*})
\end{cases}
\]
(11)

Proof. In Appendix B.6

The hazard rate function, \( K_A H(D_A^{P*}) \), is the cost-to-benefit ratio of firm A’s debt, which summarizes the marginal cost of reducing the control benefit by issuing debt from firm A to the marginal benefit in financing by issuing debt from firm A. The same intuition holds for the scaled hazard rate function, \( \frac{K_B}{\gamma_B} H(D_B^{P*}) \), which is the cost-to-benefit ratio of firm B’s debt. Lemma 7 says that in an interior equilibrium, the cost to benefit ratios of the optimal debt amounts for both firms should equate. Therefore, if \( \frac{K_B}{K_A\gamma_B} < 1 \), IHR property of \( H(.) \) implies \( D_A^{P*} < D_B^{P*} \). And, otherwise, \( D_A^{P*} \geq D_B^{P*} \) is optimal.

Proposition 2.  

- In equilibrium, when a fully levered pyramid emerges in equilibrium, \( D_A^{P*} \geq D_B^{P*} \) is more likely to be observed than \( D_A^{P*} < D_B^{P*} \).
- Therefore, the leverage ratio for the central firm A measured as a ratio of book value of total debt to book value of stand-alone asset tends to be greater than that of the subsidiary B in equilibrium when the project profiles of the two firms are identical.

Proof. In Appendix B.7

The intuition behind Proposition 2 is the following: the family has an incentive to recapitalize the pyramid only if it delivers a greater payoff than what it receives in baseline case. When \( K_A \) is much greater than \( K_B \), recapitalization gain is likely to be dominated by recapitalization loss, and the optimal solution is degenerated to that of baseline case. This implies that if a fully levered pyramid is observed in equilibrium, \( D_A^{P*} \geq D_B^{P*} \) is more likely.
Proposition 3. A fully levered pyramid is favored to the horizontal structure in equilibrium if

\[ i_B \geq \alpha C + (1 - \gamma_B)\bar{Y}_B \]

and

\[ K_B \left[ F(\bar{D}_B^H) - F(D_G^{P_*}) \right] \geq (1 - \alpha)(\bar{Y}_B - i_B) + K_A F(D_G^{P_*}) \]

(12)

Proof. In Appendix B.8

In equilibrium, the fully levered pyramidal structure is chosen over the horizontal structure only if recapitalization of the pyramid enhances relative control advantage, where \( K_A F(D_G^{P_*}) \) is minimized. The equilibrium condition of the fully levered pyramid is more likely to be met, for a given \( i_B, K_B = K(i_B), C, \) and \( \alpha \), if 1) the lower \( K_A = K(i_A) \) is, and 2) the smaller \( D_G^{P_*} \) is. The first condition implies that the fully levered pyramid emerges in equilibrium only if firm A delivers smaller private control benefit. Therefore, the investment of firm A at date-0 tends to be smaller when we observe the fully leveraged pyramid at date-1 in equilibrium.

The second condition implies that a fully leveraged pyramid is optimal only if the family successfully minimizes the probability of group default. This intuition is consistent with the empirical finding in the conglomerate literature that found that Korean family business groups, Chaebols pursue profit stability to assure the groups’ survival (Ferris et al., 2003). The same tendency is found in Japanese business groups, Japanese Keiretsus (Nakatani, 1984, and Prowse, 1992). These studies argue that cross-entity subsidization within a business group, which is motivated from the profit stability tendency, makes the business groups possess greater debt capacity. But, I propose that even without cross-subsidization or diversification, the structural properties of the pyramid - control optionality and co-insurance benefit- can generate extended debt financing capacity of a pyramidal business group. The results may be significant considering that cross-subsidization across different legal entities in a business group is legally prohibited in Korea.

One last remark is that a small value of \( D_G^{P_*} \) does not imply that the leverage ratio of a pyramidal business group should be lower than that of a horizontal business group. The central firm A’s debt amount, \( D_A^{P_*} \), under a fully leverage pyramid can exceed \( \bar{D}_B^H \) which is the optimal debt amount of firm B under the horizontal structure.

The following lemma summarizes a condition where the family can successfully minimize effective group debt amount.

Lemma 8. A fully levered pyramid is more likely to be observed in equilibrium if the size of the central firm A is large (i.e., \( \bar{Y}_A \) is large). This condition implies that the managerial ability of the family for the central firm A is better than that for the subsidiary B under the fully levered pyramid.

Proof. \( E^F[\bar{Y}_B - D_G^{P_*}]^+ = \frac{C + \bar{Y}_A + \bar{Y}_B - i_B}{1 + \gamma_B} \) from Proposition 7. This implies that for a given project profile of \( (\bar{Y}_B, i_B, K_B), D_A^{P_*} \) decrease in \( \bar{Y}_A \), which in turn, implies a higher value of \( p_A \).

To make the fully leveraged pyramid favored to the horizontal structure, the family needs to effectively minimize the probability of group-wise control default, and thus \( D_G^{P_*} \). For a given project profile of firm B, this is more likely when the existing central firm is profitable.
Hence, a fully levered pyramid is more likely to emerge when the family’s managerial ability in firm A is proficient.

5.3 Robustness of Results: General Correlations

In this section, I numerically analyze the optimal leverage ratios for general correlation cases. The numerical results in this section suggest that a general correlation value does not change the main results in section 5.1 for reasonable parameter space, which, in turn, implies that the effect of correlation on the main results in the previous section is not first-order.

Now, I provide a numerical example with bivariate normally distributed cash flows for the two firms. To highlight the effect of correlation on the optimal leverage ratios, I assume that the two projects are identical in the mean cash flow and the size of investment. I vary the correlation from -0.9 to 0.9. Bivariate normal distribution for $\tilde{Y}_j$ for $j = A, B$ implies $(\tilde{Y}_A, \tilde{Y}_B)^T \sim N(\bar{Y}_A, \bar{Y}_B)^T, \Sigma)$ where $\Sigma = \begin{bmatrix} \sigma_A^2 & \rho \sigma_A \sigma_B \\ \rho \sigma_A \sigma_B & \sigma_B^2 \end{bmatrix}$ with $\bar{Y}_A = \bar{Y}_B = \bar{Y}$ and $\sigma_A = \sigma_B = \sigma$. The definitions of the main numerical objects are defined in Appendix A.

The parameters used in the numerical analysis are given in Table 1:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{Y}$</td>
<td>102</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>30.6</td>
</tr>
<tr>
<td>$i_B$</td>
<td>100</td>
</tr>
<tr>
<td>$K_A$</td>
<td>30.9</td>
</tr>
<tr>
<td>$K_B$</td>
<td>30.9</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.25</td>
</tr>
<tr>
<td>$\gamma_B$</td>
<td>0.4</td>
</tr>
<tr>
<td>$C$</td>
<td>5</td>
</tr>
</tbody>
</table>

The time period between date-1 and date-2 is set to be 1-year. Hence, in Table 1, projects have a return of 3% per annum, and risks are 30% per annum of the cashflows. The control benefit amount for both firms are taken as 30% of a firm value for both projects. This value of private control benefit is roughly true for countries such as Mexico, Italy, and Argentina based on Dyck and Zingales, 2004. I set $\gamma_B = 0.4$ to ensure an interior solution, and can be relaxed. These parameter values are taken from the summary statistics of the data that I use in the empirical analysis. With these parameter values, I solve the optimization program (6) for a general correlation, $\rho \in [-0.9, +0.9]$ by increasing the correlation by 0.3 for each optimization. Hence, each optimization requires 7 total iterations. The results are attached in Appendix A.

First, to illustrate the extent to which the required debt amount is sensitive to a correlation, I set $D_B^p = 0$ and compute $D_A^p$ that satisfies the new group budget constraint.

\footnote{The two extreme correlation values, -1 and 1, are excluded in this analysis due to the singularity.}
Hence, this is the upper bound of firm A’s debt amount that the family needs to issue for re-capitalizing firm A. Panel (a) of Figure 1 shows the results. The panel tells us that the maximal required debt amount, $D^p_A$, does not vary greatly across correlations. This implies that the correlation does not affect the debt amount at first-order. $D^p_A$ is smallest and has a value of 33.80 when the correlation is -0.9, and gradually increases as the correlation increases from -0.9 to +0.9. When the correlation is +0.9, $D^p_A$ has a maximal value of 33.87. Therefore, from $\rho = -1$ to $\rho = 1$, the required debt amount of firm A is increased minimal just by 0.07. I also report the required debt amount issued by firm B without any recapitalization of firm A (i.e., the solution of the baseline case, $\bar{D}^p_B$) to demonstrate how the family can minimize the required amount of debt through the recapitalization. In the panel, $D^p_A$ is much smaller than $\bar{D}^p_B$, and $D^p_A$ is almost 36.5% of $\bar{D}^p_B$. Hence, we can see that there is a great amount of financing advantage in recapitalization.

Now, I relax the restriction, $D^p_B = 0$, and solve the optimal recapitalization program. Panel (b) of Figure 1 shows the results. The leverage ratio in this panel is defined as the amount of debt of each firm divided by $\bar{Y}$, the value of each firm’s own operating asset. The leverage ratios of the two firms do not vary significantly across different correlations. As the correlation varies from -0.9 to +0.9, the leverage ratio of firm A increases from 29.14% to 29.20%, and firm B’s increases from 11.41% to 11.45%. The ratio of firm A’s leverage ratio to firm B’s is roughly 2.5010 on average. This is close to $\frac{K_B}{K_A \gamma B}$, which has a value of $\frac{1}{0.4} = 2.5$ in this numerical analysis. This implies that the optimal debt amounts for firm A and B are determined mostly by $\frac{K_B}{K_A \gamma B}$ consistent with the results in Proposition 7 where I assumed perfect correlation. This suggests that the intuition in the previous section is carried over the general correlation cases.

Panel (c) of Figure 1 shows how optimal payoff varies with correlations, and how much the family gains from the recapitalization. The results show that the family receives a greater expected payoff with the recapitalization. Moreover, the expected payoff tends to increase as the correlation value gets lower. This is because the family can reduce the group default probability through a diversification due to low correlation. The expected payoff decreases from 61.1517 to 60.8657 as the correlation increases from -0.9 to +0.9. But, this diversification effect on the expected payoff to the family is not at first-order, either. The effect of recapitalization is clearly at the first-order, and the difference between the two payoffs, with and without recapitalization, is 11.4623 on average, which is equivalent to 23.11% of the expected payoff that the family receives in the baseline case. This payoff enhancement is substantial. The recapitalization benefit directly results from effectively minimized group default probability, and thus it is optimal to re-capitalize the pyramid to utilize the two intrinsic properties - co-insurance benefit and control-optionality- of the pyramid.

6 Discussion: Why not conglomerate?

So far I assume that the family should structure the new group member firm B by either establishing as a pyramidal subsidiary or a stand-alone entity under the horizontal structure. But, clearly, the family can take the new project as a new business unit in firm A (i.e., conglomerate structure hereafter), and this conglomerate structure may be advantageous over
both the pyramid or the horizontal structure. When the family chooses the conglomerate structure, the rent from the new project is also shared with the existing firm A’s old shareholders. This makes the conglomerate structure quite similar to the pyramid except that the family needs to secure control over the conglomerate solely by using its own equity stake. In this section, I analyze the conglomerate structure compared to the pyramidal business group structure, and provides when this structure is optimal over the pyramid. A short discussion on the comparison between the conglomerate and the horizontal structure will be provided at the end of this section.

6.1 The date-1 program of the family under the conglomerate structure

At date-1, suppose that the family adopts the new project, \((\tilde{Y}_B, i_B, K_B)\), as a new division of firm A. The family’s ownership in firm A, \(\alpha\), and a free cash flow of \(C\) in the firm at date-1 are given. Since dividends policy does not affect the family’s monetary part of the payoff, WLOG, I assume that the family does not pay out any dividends and use the internal slack at the maximum, \(C\). Hence, the internal financing amount under the conglomerate, \(t^C\), satisfies \(0 \leq t^C \leq C\). In addition to this internal slack, the family issues \((1 - \alpha')\)-fraction of new shares, and raise zero-coupon debt with a face value of \(D^C_A\). After the new share issuance, the family’s ownership in the conglomerate is diluted to \(\alpha'\). Suppose that the family’s managerial ability on the combined businesses in the conglomerate, \(p^C\), is different from that on the new project, \(p_B\). In this case, due to the take-over threat on the conglomerate, the family has an equity financing constraint that satisfies \(\min\left(\gamma\left(p^C\right)\alpha, 1\right) \leq \alpha' \leq 1\) for a notational simplicity, I denote \(\min\left(\gamma\left(p^C\right)\alpha, 1\right)\) by \(V(p^C, \alpha)\), and express the new variable by \(V^C\) hereafter.

Let’s denote the total capital raised by the internal cash, external debt, and equities by \(M^C_B\). It satisfies \(M^C_B = t^C + (1 - \alpha')S_A(D^C_A) + B_A(D^C_A)\) where \(S_A(D^C_A)\) and \(B_A(D^C_A)\) are the date-1 value of firm A’s equity and debt. Let’s denote the stochastic total cash flow in firm A at date-2 before any debt repayment by \(\tilde{X}^C_A\) where \(\tilde{X}^C_A = \tilde{Y}_A + M^C_B - i_B + \tilde{Y}_B\). With these, the date-2 stochastic payoff to the family under the conglomerate structure, \(\tilde{\Pi}^C\), is given as

\[
\tilde{\Pi}^C = \alpha \left[ C - t^C + \alpha' [\tilde{X}^C_A - D^C_A]^+] \right] + (K_A + K_B)1_{\{\tilde{X}^C_A \geq D_A^C\}}
\]

Breakeven condition and tight financing condition imply the date-1 expected payoff to the family under this structure, \(\Pi^C = E_{F_{A,B}}[\tilde{\Pi}^C]\), is given as

\[
\Pi^C = \alpha C + \alpha \tilde{Y}_A + \alpha(\tilde{Y}_B - i_B) + (K_A + K_B)[1 - F(D^C_A)]
\]

The family’s date-1 program under the conglomerate structure is given as

\[
\max_{(t^C, \alpha', D^C_A)} \Pi^C = Z + (K_A + K_B)[1 - F(D^C_A)] \quad \text{s.t.} \quad \begin{align*}
V^C & \leq \alpha' \leq 1 \\
0 & \leq t^C \leq C \\
t^C + \tilde{Y}_A + \tilde{Y}_B - \alpha' S(D^C_A) & = i_B
\end{align*}
\]

(13)
where
\[ Z = \alpha C + \alpha \bar{Y}_A + \alpha (\bar{Y}_B - i_B) \]
\[ S(D^C_A) = E^{F_A,B}[\bar{Y}_A + \bar{Y}_B - D^C_A]^+ \]

The first constraint is an equity financing constraint, and the second is a budget constraint. The third is a cash financing constraint.

This program resembles the pyramidal structure problem, but it differs chiefly in its equity financing constraint and the value of equity. The lower limit of the equity ownership under the conglomerate is higher than that of the pyramid, if the managerial ability of the family over the conglomerate, \( p_C \), stays the same as that over the single new project, \( p_B \) (i.e., \( p_C = p_B \)). However, the value of equity under the conglomerate structure, \( S(D^C_A) \), is greater than \( S(D^P_A, D^P_B) \) in Equation (7). These two properties will differentiate the conglomerate structure from the pyramid.

6.2 The optimal solution of the conglomerate structure program

**Lemma 9.** The optimal financing decision for the conglomerate is as following:

\[
(i^C_A, \alpha', D^C_A) = \begin{cases} 
(t, w, 0) & \text{if } i_B \leq C + (1 - V_C)(\bar{Y}_A + \bar{Y}_B) \\
(C, V_C, \bar{D}_A^C) & \text{if } i_B > C + (1 - V_C)(\bar{Y}_A + \bar{Y}_B) 
\end{cases}
\]

where
\[ \frac{\partial \Pi^C}{\partial D^C_A} < 0 \] for \( D^C_A > 0 \), and zero otherwise.

**Proof.** In Appendix B.9.

With this financing decision, the optimal date-1 expected payoff to the family under the multi-divisional structure is given as

\[
\Pi^C = \begin{cases} 
Z + K_A + K_B & \text{if } i_B \leq C + (1 - V_C)(\bar{Y}_A + \bar{Y}_B) \\
Z + (K_A + K_B)[1 - F(\bar{D}_A^C)] & \text{if } i_B > C + (1 - V_C)(\bar{Y}_A + \bar{Y}_B) 
\end{cases}
\] (14)

6.3 Discussion: Pyramid vs. Conglomerate

Suppose that the family’s control over the existing firm A is tightly secured at \( \alpha \), and the family can not improve its managerial ability by combining the two businesses, \( \bar{Y}_A \) and \( \bar{Y}_B \). This implies \( \gamma(p_C) \geq \alpha \), and thus the family can not issue any new shares from the conglomerate(i.e., \( V_C = 1 \)) since there are outstanding old shares of firm A.

**Proposition 4.** A fully leveraged pyramid strictly dominates a conglomerate structure for the new project, \( (\bar{Y}_B, i_B, K_B) \) with \( i_B > C \) if there is no managerial synergy for the family to run both projects, \( (\bar{Y}_A, i_A, K_A) \) and \( (\bar{Y}_B, i_B, K_B) \) under the conglomerate structure.

**Proof.** In Appendix B.9
To finance the new project, the pyramidal structure requires smaller amount of debt than the conglomerate. This can be seen as the following: under the pyramid, the family can sell additional $(1 - \gamma_B)$-fraction of firm B’s shares to finance the new project whereas under the conglomerate structure, it can’t sell any shares without any managerial synergy. This extra equity financing capacity under the pyramid is obtained by indirect control of the subsidiary. Unless the family loses control over the central firm A, it can sell $(1 - \gamma_B)$-fraction of shares of firm B.

However, under the conglomerate structure, the family needs to secure control over both projects by using its own equity stake, and thus it faces additional limitation in equity financing capacity. Therefore, by effectively reducing the required debt amount by selling equities of the subsidiary, there is relative control advantage in the pyramid over the conglomerate. Since both structures are limited to receive rent from the new project, the relative control advantage under the pyramid directly leads to the optimality of the pyramid over the conglomerate in equilibrium. Unless the new investment’s scale is small and supportable by internal capital, $C$, the pyramidal structure is strictly preferred to the conglomerate if the family can not improve its managerial ability on the conglomerate through synergy. This implies that the pyramid dominates the conglomerate for a capital-intensive project.

Moreover, the strict preference of the pyramid over the conglomerate is present only if there is private control benefit, $K_A$ and $K_B$, which are strictly positive. This implies that the pyramidal structure should be more commonly used than the conglomerate in countries with weak investor protection rights. When the private control benefits are negligible, both structures pay off the same to the family, and thus are equally likely to be observed in equilibrium.

Suppose that the family is better able to run the combined businesses, $\tilde{Y}_A$ and $\tilde{Y}_B$, than the existing business in firm A, $\tilde{Y}_A$. This implies $\gamma(p_C) < \alpha$, and thus $V_C < 1$. Then, the family can sell $(1 - V_C)$ more shares of the conglomerate. Since the value of unlevered equity on the combined businesses, $\tilde{Y}_A + \tilde{Y}_B$, is greater than that on the single business, $\tilde{Y}_B$, the family may be able to raise enough capital by selling the equities of the conglomerate (i.e., Using large size of a firm to maximize control, call size effect hereafter). Hence, if the maximal amount of equity financing under the conglomerate without debt, $(1 - V_C)(\tilde{Y}_A + \tilde{Y}_B)$, is greater than that under the unlevered pyramid, $(1 - \gamma_B)\tilde{Y}_B$, then the conglomerate is favored to the pyramid. But, if the investment outlay, $i_B$, is sufficiently large such that the family needs to issue debt under the conglomerate, then the analysis becomes complicated, and the optimality of the conglomerate structure over the pyramid depends whether the diversification benefit of the combined cash flows under the conglomerate is greater than that under the pyramid. I do not discuss this case in this paper.

Based on the discussion made so far, we know that the pyramidal structure is attractive when the family can not improve its managerial ability through synergy. This implies that a pyramidal structure is more likely to emerge in equilibrium when the family’s ownership on the existing firm A is significantly diluted such that the family can not raise capital through any new share offering. Therefore, the family has a strong incentive to adopt the pyramidal structure when the size of the existing firm A becomes sufficiently large, and thus size effect of the firm is not effective to deter a take-over threat. This suggests that the central firm under the pyramid tends to be large. This intuition is consistent with the result in Lemma S

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6.4 Discussion: Horizontal spin-off vs. Conglomerate

From the previous section, we know that the family needs to share rent from the new project with the existing firm’s old shareholders under a conglomerate structure. This sharing rule of rent implies that the family does not maximize the profitability under the conglomerate in contrast to the horizontal structure where it maximizes the profitability. Therefore, a stand-alone entity directly controlled by the family under the horizontal structure should be more profitable than the conglomerate. Since the pyramidal structure emerges when the family can not improve its managerial skill by combining multiple businesses, the central firm under the pyramid resembles a conglomerate when the pyramidal structure is observed.

The expansion of a family business group can be summarized by the following: The family expands the existing firm under a conglomerate structure until the firm becomes large and profitable, and then it starts expanding the business group -1) by spinning off a stand-alone entity, if the scale of the new project is small, and the project is very profitable or 2) by forming a pyramid, if the new project is capital-intensive and less profitable.

In other words, the profitability of group member firms should be ordered in the following way: 1) the most profitable spun-off firm under the horizontal structure, 2) more or less profitable but large central firms under the pyramid, and 3) the least profitable small pyramidal subsidiaries.

7 Empirical Implications and Results

7.1 Testing Hypotheses

The model predicts the following testable hypotheses:

H1 The more capital-intensive with a larger private control benefit, less profitable, and the less leveraged a firm is, the firm is more likely to be a pyramidal subsidiary in equilibrium. Moreover, the subsidiary tends to be younger than a central firm, and the family’s ownership of the subsidiary is likely to be concentrated:

The more capital-intensive a firm is, the family faces the more severe financial constraint when it chooses the horizontal group structure. A greater amount of internal capital under the pyramidal structure enables the family to minimize the reliance on external debt financing, and thus the family can receive the higher expected control benefit. This relative advantage in expected control benefit under the pyramid is optimal when the extra internal capital carries a low cost, which is implied by a low rent from the subsidiary. Therefore, when the project delivers a large private control benefit and is less profitable, the pyramidal structure emerges as the optimal business group structure in equilibrium, and the subsidiary should be less leveraged. The internal capital market formed in the central firm under the pyramid is important to make the structure optimal over the subsidiary in equilibrium. This implies that a pyramidal subsidiary can only emerge when there is an established central firm. Hence, the pyramidal subsidiary tends to be be younger. The internal capital under the pyramid becomes more useful when the family can not raise enough capital through equity financing. Moreover, the cost of the internal capital is minimized when the project is
less profitable. This implies that a relatively poorly skilled family finds the pyramidal structure attractive, and thus the ownership of the family in the pyramidal subsidiary should be concentrated.

**H2** The less capital-intensive with a smaller private control benefit, and the more leveraged a firm is, the firm is more likely to be a central firm under the pyramid in equilibrium. Moreover, the central firm tends to be old and large, and the family’s ownership of the firm is lower than that of subsidiaries:

The central firm can extend its debt financing capacity by combining its own operating cash flow with the non-operating cash flow - the dividends paid by its subsidiaries. To utilize the coinsurance benefit of the dividends, the family optimally lever up the central firm. By doing so, the family can effectively minimize the group-wise default probability while extending the central firm’s debt capacity. The default probability of the central firm is effectively minimized if the firm operates on debt-favored projects. The cost of debt financing of the central firm is low when the private control benefit of the firm is low. This condition is more likely to be satisfied when the firm is less capital-intensive. Moreover, the central firm is more likely to adopt a new subsidiary instead of expanding through the conglomerate structure if the family can not sell any more equities due to the dilution effect of the existing old shares, and thus can not strategically use the large size of the conglomerate as an anti-takeover device. This implies that under the pyramid, the central firm tends to be old and large, and the family’s ownership of the central firm tends to be lower than that of subsidiaries.

**H3** Central firms tend to pay out less to ensure the greater availability of group internal capital to support the capital-intensive projects taken by their subsidiaries:

When the pyramidal structure is chosen, the choice is made to minimize the amount of external capital, especially external debt amount. To ensure the greater availability of group internal capital, the family decides not to pay out any dividends from central firms and allocates the internal capital into group firms. Thus, a firm with lots of subsidiaries is more likely to keep its retained earning as internal financing resource and work as a headquarter of an internal capital market. This implies that the lower dividends should be paid out from central firms under the pyramid.

**H4** Pyramidal subsidiaries tend to pay out more to support the debt of highly leveraged central firms:

The family pushes the subsidiaries to pay out as much as possible to service central firm’s debt. Since the controlling shares in the subsidiaries can not be monetized due to non-marketability, the only way to utilize the position is to receive dividends from subsidiaries. Thus, the optimal payout policy for subsidiaries is to fully pay out their earnings whenever the dividends can help rescue the central firm from default. By commitment to this ex-post optimal payout policy, the family can extend debt financing capacity of the central firm, which implies an efficient debt financing in the entire business group. Therefore, the pyramidal subsidiaries tend to pay out more.
# 7.2 Data and Variable Descriptions

The data set that I use is from Almedia et al., 2009. The data set consists of the ownership and financial information for all non-financial group firms in Korean family business groups, Chaebols, between 1998-2004. The data do not include any Chaebol groups controlled by the Korean government, and thus they are purely family-controlled Korean business groups. The Korean government established the Korean Fair Trade Commission (KFTC) in 1981 to regulate the market competition. Each year the KFTC defines a list of Chaebol groups and collects the complete ownership data for all group firms for each Chaebol as a part of the regulatory purpose.

The ownership information is in detail to pin down what fraction of shares are owned by the family owner, its relatives, and the group affiliates. I define the fraction of common shares owned by the family owner and its relatives for each firm as the ownership of controlling family. The definition of family relatives that KFTC uses is broad, and include large shareholders who might co-operate with the controlling family to exercise control. The ownership information is collected as of April 30th each year.

The KFTC also collects the financial information of these firms, making them publicly available upon request. In addition to the data from the KFTC, the financial information is augmented with the financial data from two other databases: Korea Listed Companies Association (KLCA) and Korea Investors Service (KIS). These two databases contain information for both listed public firms and private firms in each Chaebol, which are subject to external audit. Hence, the data used in this paper provide a comprehensive dynamic panel of Korean Chaebols including ownership structure and financial information.

The data period, 1998-2004, is ideal to test the model predictions. The Korean economy experienced 1997 Asian financial crisis, and during this data period, many business groups underwent active restructuring. Due to the active restructuring across all Chaebol groups, there were massive mergers and acquisitions initiated both by the domestic investors, which are the other competing Chaebol groups or foreign investors such as private equities (PEs) or leverage buy-outs (LBOs), making this period as a time when Chaebol groups were concerned about control retention over group firms. Hence, the data period provides an ideal opportunity to test how business groups are restructured, and corporate financing decisions are made to ensure control influences over the entire business group.

One complexity in the empirical analysis comes from the complex ownership structure of Korean Chaebols. Almost every Chaebol uses circular ownership chain on top of the vertical pyramid. Since Chaebols are legally prohibited from using cross-share holdings, the circular ownership is commonly found, and each circular ownership chain intrinsically includes a minimum of 3 firms. The complex ownership links between group firms make it difficult to define whether any given firm in a Chaebol is a pyramidal subsidiary, or a group central firm, or a stand-alone firm since each group firm belongs to the multiple pyramidal ownership chains. To identify the type of a firm, I use the group metrics developed by Almeida et al., 2009, who use the dividend algorithm to compute the position and centrality of a group firm in each Chaebol. They define three main group structure metrics - Position, Centrality, and Loop. The definitions are briefly discussed below, and the mathematical definitions are attached in Appendix C.
7.2.1 Group Structure Metrics

**Position:**

One of the main group structure metrics is $\text{Position}_{i,t}$, which measures the distance between the family owner and a given firm-$i$ in the group at year-$t$. $\text{Position}_{i,t}$ is defined as a cash flow weighted distances of a given firm-$i$ from the controlling family. Its mathematical definition is given in Equation (2) in Appendix C. A simple example is useful to explain how this metric works. When a business group consists of two firms, and the two firms belong to a single pyramidal ownership chain, then the central firm at the top of the pyramid is 1-distance away from the family, and the subsidiary is 2-distances away from the family. The two firms belong to a single vertical ownership chain, and thus the top firm has a $\text{Position}_{i,t}$ value of 1, and the bottom firm has a $\text{Position}_{i,t}$ value of 2 because in this simple example, the weight assigned to the single pyramidal ownership chain is 1. Therefore, the distance of a given firm-$i$ at year-$t$ from a controlling family is equivalent to $\text{Position}_{i,t}$ in this case.

To illustrate how $\text{Position}_{i,t}$ is defined for more complex case, suppose that a given subsidiary-$i$ belongs to multiple pyramidal ownership chains. In this case, the distance of a given firm-$i$ from the family depends upon which pyramidal ownership chain we take into account to measure the distance of the firm. To define a firm’s position in this multiple pyramidal chains, Almeida et al., 2009, propose a method to quantify the weight assigned to each pyramidal chain. They first compute the dividends that the family can receive from firm-$i$ from each of pyramidal ownership chains if the firm pays a dollar dividend. To compute the amount of dividends that the family receives from firm-$i$, they assume that any intermediate firms between the family and firm-$i$ pay out full in dividends that they receive from firm-$i$. Due to the possible circular ownership chain, there are a number of multiple pyramidal ownership chains to which a given firm belongs.

To deal with the circular ownership chain, they introduce two ownership matrices - 1) inter-corporate holding matrix and 2) the family’s direct equity stake in each group firm. Using these two ownership matrices, they compute the family’s cash flow from firm-$i$ for each of all possible pyramidal ownership chains to which the firm belongs. The inter-corporate holding matrix, $A$, represents how group member firms are linked through the equity ownership to each other. The dimensions of $A$ are $N \times N$ where, $N$ denotes total number of group firms in each business group. $s_{i,j}$, an $(i, j)$-element of $A$, represents how much control equity stakes that a group firm-$i$ directly has in another group firm-$j$. By definition, $s_{i,i} = 0$ for $\forall i = 1, 2, ..., N$. In Korea, due to legal prohibition of direct cross-shareholding between any two group firms, $A$ is not symmetric in general (i.e., $s_{i,j} \neq s_{j,i}$ for $\forall i, j = 1, 2, ..., N$). The second ownership matrix is a $N \times 1$-column vector, $f$, whose $i$th-row element, $f_i$, denotes the fraction of control shares that the controlling family directly has in firm-$i$. Using these two ownership matrices, Almeida et al. compute the dividends that the family receives from firm-$i$ for each of multiple pyramidal chains.

Here is an example. Suppose that firm-1 pays 1-dollar dividend. Let’s denote a basis column vector by $d_1$, which has a single non-zero element, 1, in the first row, and zeros elsewhere. Then $f^T d_1$ represents how much cash flow the family directly receives from firm-1. Clearly, the family receives nothing unless the firm-$i$ is directly controlled by the family. Hence, $f^T d_1 = f^T A^{(1-1)} d_1$ represents dividends that the family receives by 1-distance
dividend channel. Similarly, the $i$th-row element of $Ad_1$, a $N \times 1$ column vector, represents how much cash flow that another group firm-$i$ directly receives from firm-1 if the firm-1 pays 1-dollar dividend. Hence, $f^TAd_1 = f^TA^{(2^{-1})}d_1$ represents the amount of cash flow that the family receives from firm-1 by 2-distances dividend channel. If firm-1 belongs to any 2-distances pyramidal ownership chain, then the family will receive $f^TAd_1$ amount of cash flow from firm-1, and the firm's distance from the family in this ownership chain is 2. This procedure is repeated for all possible ownership links between the family and firm-1. Therefore, $Position_{i,t}$ in Equation (2) in Appendix C can be interpreted as a cash flow weighted distances of a given firm-$i$ from the family at year-$t$, and the value of $Position_{i,t}$ will approximate the distance between the firm and the family in the most significant ownership link to which the firm belongs. This property of $Position_{i,t}$ directly relates to the weighting scheme used in the definition of $Position_{i,t}$.

Centrality:
The other group structure metric is $Centrality_{i,t}$, which measures how important a given firm is in ensuring control over entire group firms. A high value of $Centrality_{i,t}$ implies that a group firm-$i$ at year-$t$ is central in control to other member firms, which, in turn, implies that the firm has a number of subsidiaries. To define this group metric, Almeida et al., 2009, define a variable named Critical control threshold ($CC_{i,t}$, on forth) for firm-$i$ at year-$t$. By its name, $CC_{i,t}$ is a critical value of control threshold. The definition of a control threshold, $T$, is necessary to define $CC_{i,t}$. $T$ determines whether a given firm is under the family’s control or not, which is determined if the family has a fraction of voting shares either directly or indirectly in a given firm more than $T$. This definition of the family’s control over a firm with $T$ is described in Assumption 2 in Appendix C. Hence, for a given value of $T$, a set of firms are defined as under the family’s control (see Definition 3 in Appendix C). As we increase the control threshold value, $T$, the more parsimoniously the set of the firms are defined. This implies that there is a maximal value of $T$ for each group firms such that for a given value of $T$, firm-$i$ is treated as under the family’s control. Hence, the maximal value of $T$ defines the family’s control over firm-$i$, and thus $CC_{i,t}$ is set to be the maximal value of $T$. By its definition, $CC_{i,t}$ can be interpreted as an upper bound of minimum required fraction of shares to define firm-$i$ under the family’s control at year-$t$.

Once $CC_{i,t}$ for firm-$i$ at year-$t$ is defined for all group firms (i.e., $\forall i = 1, 2, ..., N$), a centrality of the firm, $Centrality_{i,t}$, is defined based upon $CC_{i,t}$s for $\forall i$. Almeida et al., 2009, define $Centrality_{i,t}$ as the average decrease in $CC_{i,t}$ across all firms at year-$t$ if a given firm-$i$ were excluded from the group. This definition of $Centrality_{i,t}$ is intuitive to define how important the family’s ownership in firm-$i$ is in terms of its control influence over the entire business group. When the family loses its ownership link to a group central firm, the family loses indirect voting shares for all of the firm’s subsidiaries. To define that a given subsidiary is under the family’s control, the control threshold of the subsidiary should be decreased. This implies that there is a large drop in $CC$ of a subsidiary when the family loses control link to the central firm which controls the subsidiary. And, the average drop of subsidiaries’ $CC$ increase as the family loses control link over the firm which controls lost of subsidiaries.

Here is an example. Consider a business group structured with two firms through a 2-step single pyramidal ownership chain. Suppose that the family owns 40% of top firm’s shares,
and the top firm owns 50% of the bottom firm. With this structure given, both firms are defined under the family’s control with any $T \leq 40\%$ since the family’s control over the top firm is established with $T \leq 40\%$, and unless the family’s control over the top firm is lost, the family has indirect votes of 50% in the bottom firm through control over the top firm. For $T > 40\%$, neither of the two firms are not treated under the family’s control since the family’s control over the top firm is not defined with $T > 40\%$, and thus the family has no voting shares in the bottom firm. By definition of $CC$, both firms have $CC$ of 40% in this case. Now, we exclude the top firm from the group. Then the family’s control over the bottom firm is only established with $T$ of zero since the family has no ownership link to the bottom firm when it loses the ownership link to the top firm. Hence, $CC_{i,t}$ for the bottom firm, a subsidiary, lowers down to zero from 40% when we exclude the top firm. From Equation (5) in Appendix C, 40% is assigned to $Centrality$ of the top firm in this example. Now, suppose that we exclude the bottom firm from the business group. Since the family’s total voting fraction in the top firm does not change even after eliminating the bottom firm, $CC_{i,t}$ for the top firm stays at the same level of 40%, which in turn, implies that $Centrality$ of the subsidiary is zero. From this simple example, we know that $CC$s of other member firms decrease when we exclude a central firm from the business group. This property of $Centrality_{i,t}$ intuitively defines how central a given firm-$i$ exercises control over the business group at year-$t$.

**Loop:**

The third group structure metric is $Loop_{i,t}$. The idea to define this metric is that if a firm belongs to a circular ownership chain, then any 1-dollar dividend paid from the firm will be eventually received by itself in a finite number of steps. For example, suppose that a firm A owns firm B, which owns firm C, which owns the firm A. Then, any dividend paid from firm A will be received by itself in 3-steps, which is finite.

But, in the previous simple 2-step single pyramidal ownership example, any dividend paid either from the top firm or from the subsidiary will not return to the firm itself in any finite number of steps. Thus, Almedia et al. define $Loop_{i,t}$ to be a dummy variable, which takes value of 1 only if the minimum number of steps is finite. I define $Loop_{i,t} = 0$ if any dividends paid from firm-$i$ do not return to the firm itself within $N$-steps where $N$ denotes total number of group firms in a business group.

Table 2 summarizes the statistics of the group structure metrics and several characteristics of the firms in the data. Table 3 summarizes the correlations between these variables. The data consists of 3442 firm-year observations. It has 1072 firms for 47 Chaebol groups for 7 years from 1998 to 2004. The average $Position$ value of firms in data is 2.11, and the standard deviation of the variable is 0.816. These statistics of $Position$ imply that the depth of the pyramidal structure for each Chaebol is shallow, and each Chaebol group is typically structured as a 3-tiered pyramidal structure.

The average $Centrality$ value is 0.150 with a standard deviation of 0.049. Using the average value of $Centrality_{i,t}$, 0.015, I define a dummy variable, $Central_{i,t}$, which denotes whether or not a given firm-$i$ at year-$t$ is a central firm. The definition of $Central_{i,t}$ is given as:

$$Central_{i,t} = \begin{cases} 1 & \text{if } Centrality_{i,t} \geq 0.015 \\ 0 & \text{if } Centrality_{i,t} < 0.015 \end{cases}$$
The dummy variable, Central_{i,t}, will be used as a dependent variable in testing the two empirical implications—1) H2 and 2) H3. The main reason that I use this dummy variable as a main regressand in the regression analysis, instead of using the continuous variable, Centrality_{i,t}, is that 1) Centrality_{i,t} is too right skewed and 2) more than 50% of Centrality_{i,t} takes value of zero. This extremely right skewed variable might underestimate the correlation between leverage ratio and Centrality_{i,t}. To be cautious for this possibility, I split the firms in the sample into two groups.

The average Position for central firms (i.e., Central_{i,t} = 1) is 1.610 with a standard deviation of 0.600. The 25-percentile and 75-percentile of Position values for central firms are 1.10 and 2.01, respectively. This implies that roughly 75% of central firms are positioned at more than 1.10-distances away from the family. And a half of the central firms have position value between 1.10 and 2.01.

Moreover, the average Position for firms belonging to a circular ownership chain (i.e., Loop = 1) is 1.859 with a standard deviation of 0.764. The median of firms belong to a circular ownership chain is 1.710. Hence, roughly 50% of central firms belong to circular ownership link, and these firms are positioned between 1.10- and 2.01-distances away from the family owner. Since I do not have a clear prediction on the correlation between leverage ratio and circular ownership, I try to be cautious to deal with these firms. In the test, I run the regression both with and without including these firms, and show how the results can be affected by these firms.

In Table 2, I summarize the statistics of Top Firms’ Centrality. Top Firms are defined as firms whose Positions are less than 1.1. The median Centrality of Top Firms is 0.002, and the 75-percentile is 0.040. These statistics suggest that more than 50% of Top Firms are not central firms, and thus more than half of top firms are stand-alone entities directly controlled by the family. And these firms are least likely to belong to a circular ownership chain.

Therefore, I define another dummy variable, Pyramid_{i,t}, and identify whether a given firm-i is a pyramidal subsidiary or a stand-alone group entity by using this variable. The definition of the dummy variable is:

\[
Pyramid_{i,t} = \begin{cases} 
1 & \text{if } Position_{i,t} \geq 2.0 \\
0 & \text{if } Position_{i,t} \leq 1.1
\end{cases}
\]

This variable will be used to test H1 and H4. Pyramid_{i,t} = 1 implies that the firm is located more than 2-distances away from the family and is a subsidiary. And firms with Pyramid_{i,t} = 0 are stand-alone firms directly controlled by the family.

Less than 50% of firms with Pyramid_{i,t} = 0 may be also central firms, but I do not exclude them from the testing sample in order to make sure that subsidiaries do not greatly outnumber stand-alone firms. When subsidiaries outnumber stand-alone entities in the testing sample, then the different relationship between the leverage and the firm’s status for the two types of firms are likely to be blurred since the relationship will be driven by the covariation of the two variables in pyramidal subsidiaries. To avoid any bias caused by the observation imbalance between the two types of firms, I do not further categorize Top Firms into sub-categories such as Central Top Firms and Non-central Top Firms. Furthermore, the theory suggests that both central firms and stand-alone entities should be more leveraged.
than subsidiaries. Hence, testing $H1$ with this dummy variable is consistent with theoretical prediction.

Table 2 also reports the statistics of additional three variables - Public, Employees, and Firm Age. Public is a dummy variable taking value of 1 if a firm is publicly listed, and zero otherwise. Based on the mean value of Public, 26% of the firms in the data are public, which accounts for 279 firms. Employees is the number of employees of a firm, and Firm Age is measured in years since the year of foundation. The average firm in the sample has 1204 employees and has been in business for 17 years.

7.2.2 Financial Variables

Now I describe the definitions of financial variables. Since the theoretical predictions for leverage ratio are derived for the ratios based on each firm’s own operating assets, first, I quantify the book value of a firm’s operating assets. The book value of Total Assets of a firm consists of the two components- 1) Operating Assets and 2) Non-operating Assets (i.e., the firm’s control stakes in the other group member firms). The accounting information in the data enables me to identify each of the two types of assets in a firm. The accounting information in the data is constructed by an accounting method called Equity Method. Equity Method is an accounting technique used by firms to assess the profits earned by their investments in other companies. The firm reports the income earned on the investment on its income statement and the reported value is based on the firm’s share of the company assets. The reported profit is proportional to the size of the equity investment. This is the standard technique used when one company has significant influence over another. Based on this accounting method, there are three important items which enable me to quantify assets and profits from a firm’s own operation and those from a firm’s investment in the other member firms’ equities - 1) Equity Method Stock, 2) Gains from Equity Method, and 3) Losses from Equity Method.

Equity Method Stock refers to aggregate book value of equity stakes held in the other group member firms, and thus this variable quantifies the book value of Non-operating Assets. And, Gains (or Losses) from Equity Method refers to how much profits (or losses) a given firm makes from its equity investments in the other group firms. Clearly, Gains (or Losses) from Equity Method is reported based on their book values. Using these three items, I quantify two financial variables, Operating Assets and Operating Profits as

\[
\begin{align*}
\text{Operating Assets} &= \text{Total Assets} - \text{Equity Method Stock} \\
\text{Operating Profits} &= \text{Total Profits} - \text{Gains from Equity Method} + \text{Losses from Equity Method}
\end{align*}
\]

Using these items, I define book leverage ratio of firm-$i$ at year-$t$, $LEV_{i,t}$, as a ratio of book value of debt to book value of Operating Assets. Book value of debt is computed as a sum of the following three: Short-term debt, Long-term debt in current liability, and long-term debt. Similar to the definition of $LEV_{i,t}$, I define a measure of capital expenditure, $CAPEX_{A,t}$, as a ratio of book value of total capital expenditure to Operating Assets. Using Operating Profits, I define a measure of a firm’s operating profitability, OPROA, as a ratio of Operating Profits to Operating Assets. I also define a measure for dividends payout ratio, DIVA, as a ratio of book value of total dividends paid out to Operating Assets.
I define three additional financial variables based on Total Assets, instead of Operating Assets. A total profitability measure of a firm, ROA (i.e., the return on Total Assets), is defined as a ratio of book value of ordinary income to Total Assets. And, CAPEXUNA is defined as a ratio of total capital expenditure to Total Assets. For dividends payout ratio, I define DIVUNA as a ratio of total dividends paid out to Total Assets.

The summary statistics of these financial variables and the correlations between them are reported in Table 4 and 5, respectively. From Table 4 the average leverage ratio is 20.3% on the operating asset basis, and a standard deviation 38.2%. Firms in data has 2.50%, rate of return on assets, on average. Capital expenditure is around 5.60% of book value of operating assets on average, and total paid-out dividends is 80 bps of book value of operating assets. Leverage ratio is negatively correlated with Pyramid, which implies that subsidiaries tend to have lower leverage ratios than stand-alone firms. And, the leverage ratio is positively correlated with Central, which implies that central firms are more leveraged than non-central firms. Capital expenditure tends to be larger for subsidiaries than stand-alone firms, and smaller for central firms than non-central firms. Relatively smaller dividends are paid out from central firms than non-central firms whereas pyramidal subsidiaries tend to pay out more than stand-alone entities. These are just correlations, and can not be interpreted as any causal evidence for the theoretical predictions that I test. But, the correlations reported in Table 5 show the consistency with theoretical predictions of the model in their signs.

One last variable that I introduce is industry code. In the regression analysis provided in the following sections, I control for industry fixed effects. To this end, I use industry classification code corresponding roughly to a 2-digit SIC classification in the US. There are 45 different industries in the sample, and each firm in the sample belongs to one of 45 industries.

### 7.3 Main Empirical Results

#### 7.3.1 H1: Position and leverage

To test the hypothesis, H1, I estimate the following model:

$$
Pyramid_{i,t} = a_0 + a_1 LEV_{i,t-1} + a_2 OPROA_{i,t-1} + a_3 CAPEXA_{i,t-1} + \Gamma_1 Controls + \epsilon_{i,t}
$$

Equation (15) is a direct implication of the hypothesis that as a firm is less leveraged, more capital-intensive, and less profitable, the more likely the firm emerges as a pyramidal subsidiary. Hence, we expect to see $a_1, a_2 < 0$, and $a_3 > 0$. The main RHS variables are LEV, the leverage ratio, OPROA, the operating profitability, and CAPEXA, the capital-intensity. The theory predicts a contemporaneous relationship between Pyramid$_{i,t}$ and these three RHS variables. But, corporate governance literature suggests that a pyramidal subsidiary is likely to suffer from a low profitability due to a severe agency problem. Moreover, the entrenched family is likely to have an empire-building incentive when the agency problem is extreme under the pyramid. Therefore, there is a possible reverse causality problem in Equation (15) when I use the contemporaneous regressors in the RHS. I try to control for the problem by using lagged variables as main regressors in the RHS. I also run the same
regression with the contemporaneous variables, and report the results. The more detailed
discussion on the reverse causality issue will be provided in section 7.4.

I control for FirmAge, Public, and Ln(Assets). Ln(Assets) is defined as a natural log of
book value of Total Assets. By including this variable as a control, I can control for
any non-linear effect of a firm’s size. Since pyramidal subsidiaries tend to be younger, less
established, and smaller, the expected signs of these three control variables are negative.
I do not control for all three variables at the same time since these variables are highly
correlated to each other\textsuperscript{18} and thus there is a possible collinearity issue if I control for all
of them simultaneously. To avoid the collinearity effect on the point estimates of the main
coefficients, I control for each of the three variables separately. I use the scaled FirmAge\textsubscript{t−1}
by 1000 in the regression to adjust the magnitude of the coefficient of the variable. Usual
controls for Year and Industry fixed-effects are included in the regression. In addition to
these two, I also control for group fixed effect. Standard errors are clustered at each firm
level to correct for any possible within-firm correlation.

The results are reported in Table 6. First three columns in Table 6 report the results
when I use the lagged variables for all regressors in the RHS of Equation (15). In column
(4), I show the results when I use the contemporaneous regressors. The coefficients of the
leverage ratio, LEV\textsubscript{i,t−1}, in the first three columns are negative, and they are statistically
significant at less than 1% level in the first two and 5% level in the third. A lower leverage
ratio implies a higher expected control benefit, and thus this result confirms that a less
leveraged firm is more likely to be a pyramidal subsidiary positioned at the bottom of the
pyramid. The coefficient of OPROA\textsubscript{i,t−1} has a negative sign. It is statistically significant
at less than 10% level in column (2) and (3). This sign and significance of \( a_2 \) confirms
the theoretical prediction that a less profitable firm is more likely to be a subsidiary. The
coefficient of CAPEX\textsubscript{i,t−1} is positive and statistically significant in column (2) and (3) at
less than 5% and 10% level, respectively. The positive coefficient, \( a_3 \), implies that the more
capital-intensive a firm is, the more likely the firm emerges as a subsidiary.

The coefficients of control variables come up with right signs, and all of them are strongly
significant at less than 1% level. This suggests that business groups grow over time, and a
young and less established small sized firm tends to be a pyramidal subsidiary.

In column (4), I report the regression result when I change the variables from lagged to
contemporaneous. The results are mostly the same, but the statistical significances of the
coefficients of OROPA and CAPEX are lost. The significantly negative coefficient of LEV
confirms that the leverage ratio of a firm plays an important role to determine the firm’s
location in the business group.

In column (5), I replace the dummy LHS variable, Pyramid\textsubscript{i,t}, with a continuous coun-
terpart of the variable, Position\textsubscript{i,t}. One might argue that the results in the first four columns
are possibly driven by the generic definition of the dummy variable, Pyramid\textsubscript{i,t}. To show
the robustness of the results, I run the regression with Position\textsubscript{i,t} as a dependent variable.
The results in column (5) show that LEV\textsubscript{i,t−1} and FirmAge\textsubscript{t−1} are still significant. The signs of the coefficients of these two variables are consistent with the results in the other four
columns, but the statistical significance of the coefficient of LEV\textsubscript{i,t−1} is reduced to 5% level
from 1% level. However, this is expected since I also include firms located between Position\textsubscript{i,t}

\textsuperscript{18}From Table 2 we can see that FirmAge and Public are correlated roughly at 60%.
of 1.1 and 2.0. These are intermediate central firms that tend to belong to circular ownership link. They tend to have mixed characteristics between a central firm and a subsidiary. If these firms act like central firms, then they tend to have a higher leverage ratio, which possibly reduce the explanatory power of \( LEV_{i,t-1} \). \( LEV_{i,t} \) and \( Central_{i,t} \) actually have roughly 10.0% level of cross-correlation, and thus including the intermediate central firms in the test clearly works against the prediction of \( H1 \). However, the negative and significant coefficient of \( LEV_{i,t-1} \) even with the new continuous dependent variable suggests that the results in first four columns in Table 6 are not likely to be driven by the definition of \( Pyramid_{i,t} \).

In column (6), I run the same regression as in column (5) but only with firms that do not belong to any circular ownership link (i.e., Loop = 1). In this case, the coefficient of \( LEV_{i,t-1} \) is significant at less than 1% level. Therefore, the mixed firm characteristic of intermediate central firm belonging to circular ownership link is confirmed.

I do not report the regression results when I replace \( OPROA \) with \( ROA \) in the paper, but the results are essentially the same as in Table 6. Most of coefficients show the same magnitudes and statistical significances as reported in Table 6.

7.3.2 \( H2 \): Centrality and leverage

In this section, I test the hypothesis, \( H2 \), by estimating the following model:

\[
Central_{i,t} = b_0 + b_1 LEV_{i,t-1} + b_2 OPROA_{i,t-1} + b_3 CAPEXA_{i,t-1} + \Gamma_2 Controls + \epsilon_{i,t}
\]

(16)

The specification of the test is almost the same as in Equation (15) in section 7.3.1. The same controls as in Equation (15) are included in the regression. Similar to what is done in the previous section, I control for \( FirmAge \), \( Public \), and \( Ln(Assets) \). \( Year \), \( Industry \), and group fixed-effects are also controlled in the regression. The standard errors are clustered at each firm level to correct for within-firm correlation.

The results are reported in Table 7. From the theory, I expect the sign of the coefficient of \( LEV_{i,t-1}, b_1 \), to be positive. The expected sign of \( b_3 \), the coefficient of \( CAPEXA_{i,t-1} \), is negative whereas the sign of \( b_2 \), the coefficient of \( OPROA_{i,t-1} \), is positive. The first three columns show the results when I regress the LHS variable, \( Central_{i,t} \), on the lagged regressors. In all three columns, \( b_1 \) turns out to be positive and is strongly significant at less than 1% level. Hence, a higher leverage ratio for a central firm is confirmed empirically. This result together with the result from the previous section 7.3.1 imply that there is a decreasing tendency of a leverage ratio from top to bottom of the pyramid in Korean Chaebols. This is consistent with the model’s prediction.

The coefficient of \( OPROA_{i,t-1}, b_2 \), is positive in the first three columns although it is not statistically significant. The sign confirms that a firm is more likely to adopt a new subsidiary when the firm realizes a profitable return from its existing business. In all three columns, the coefficient of \( CAPEXA_{i,t-1}, b_3 \), confirms negative expected sign, and is significant especially in column (2) and (3) at less than 1% level. The theory says that the family has an incentive to recapitalize the central firm only if the cost of the firm’s debt is not high. The cost of recapitalization is likely to be high when the central firm generates substantial private control benefit, which is expected to be higher for a large scale project. Therefore, a firm
tends to lever itself up, and serves as a central firm when the firm does not invest in a large scale project. The negative sign and significance of \( b_3 \) confirms this prediction.

All the control variables, \( \text{FirmAge} \), \( \text{Public} \), and \( \text{Ln(Assets)} \) come up with the right signs of their coefficients, and they are significant at less than 1% level. In column (3), the firm size variable, \( \text{Ln(Assets)}_{i,t-1} \) turns out to be very significant at less than 1% level. This is consistent with the theoretical prediction that an established large firm is more likely to serve as a central firm since the large size helps minimize the group default probability. The positive coefficients of \( \text{FirmAge}_{i,t-1} \) and \( \text{Public}_{i,t-1} \) confirm that \textit{Chaebols} grow over time, and that an established old firm is more likely to be a central firm.

In column (4), I report the result when I replace the lagged regressors with the contemporaneous ones. The significance of the coefficient of leverage ratio, \( \text{LEV}_{i,t} \) is still at less than 1% level, and the magnitude of the coefficient is more or less the same as that in column (1). The sign and statistical significance of \( \text{FirmAge}_{i,t-1} \) stays the same as that in the first column, but the significance of the coefficient of \( \text{CAPEX} \) is absent.

In column (5), to show that the results in first four columns are not driven by the generic definition of \( \text{Central}_{i,t} \), I use a continuous variable, \( \text{Centrality}_{i,t} \), as a dependent variable. The significance of \( \text{LEV}_{i,t-1} \) is reduced much to 10% level, but the positive sign of its coefficient is confirmed. One possible reason for the lower significance of the \( \text{LEV}_{i,t-1} \) is that the distribution of \( \text{Centrality}_{i,t} \) is too right-skewed, and there are many zero values below the median \( \text{Centrality}_{i,t} \). Hence, the coefficient of \( \text{LEV}_{i,t} \) can be underestimated due to the left truncated distribution of \( \text{Centrality}_{i,t} \). The other possible reason to explain this is that part of central firms also belong to circular ownership chains, and they have mixed characteristic between a pyramidal subsidiary and a central firm as discussed in the previous section. Therefore, in column (6), I run the same regression as in column (5) only with firms that do not belong to a circular ownership link. The coefficient of \( \text{LEV}_{i,t-1} \) becomes strongly significant at less than 1% level, and thus the mixed characteristic of the firms belonging to circular ownership link is confirmed.

7.3.3 H3: Centrality and Dividend Payout

Another important feature of the pyramidal structure is greater availability of internal capital. The greater availability of internal capital is ensured because central firms tend to keep their earnings by not paying out as much dividends, and instead can use them for the investment in subsidiaries. To test whether central firms serve as an internal capital market in the business group, I run the following regression:

\[
\text{DIV} A_{i,t} (or \text{DIVUNA}_{i,t}) = c_0 + c_1 \text{Central}_{i,t} + c_2 \text{OPROA}_{i,t} + c_3 \text{CAPEX}_{i,t} + c_4 \text{LEV}_{i,t} + c_5 \text{Public}_{i,t} + \Gamma_3 \text{Controls} + \epsilon_{i,t}
\]

The causal link implied in Equation (17) is that when a firm becomes a central firm, it tends to pay out less dividends. This conservative dividend policy of a central firm is more likely to be a consequence of the business group structure in place, but less likely to cause the next year centrality of the firm in the business group. If central firms form an internal capital market within a business group, and provide the internal cash for group subsidiaries, then the sign of \( c_1 \) should be negative. I include the measures of profitability, \( \text{OPROA}_{i,t} \), capital

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expenditure, \( CAPEXA_{i,t} \), and leverage ratio, \( LEV_{i,t} \), in the RHS since all of these variables are perceived to be closely related to the amount of dividends paid out from a firm. The more profitable a firm is, the more dividends the firm is expected to pay out, which implies that the expected sign of \( c_2 \) is positive. \( CAPEXA_{i,t} \) and \( LEV_{i,t} \) are related to the cash out-flows from a firm. Hence, high value of \( CAPEXA_{i,t} \) limits the resource that can be used in dividends payout. Similarly, a high value of \( LEV_{i,t} \) implies that a large fraction of the firm’s earnings will be used to pay interests of the outstanding debt of the firm. For these reasons, both \( c_3 \) and \( c_4 \) are expected to be negative. I also control for \( Public \) since a publicly listed firm may tend to pay out more dividends to appreciate its share price. I expect to see a positive sign of the coefficient of \( Public \).

The results are reported in Table 8. In the first two columns, I use \( DIVA_{i,t} \) as a dependent variable, and in the latter two columns, I use \( DIVUNA_{i,t} \) as a dependent variable. In all four columns, one finds a strong tendency for central firms to pay out less dividends. The significance of \( c_1 \) in all four columns are at less than 1% level, and the magnitude of the coefficient is roughly -30 bps. This tells us that a central firm pays out less by around 30 bps of book value of its operating assets. \( c_2 \), \( c_3 \), and \( c_4 \) come up with their expected signs. \( OPROA_{i,t} \) and \( LEV_{i,t} \) seem to be significantly correlated with the amount of dividends paid out from a firm, and the significance is at less than 1% level for \( OPROA_{i,t} \) and 10% level for \( LEV_{i,t} \). \( c_3 \) shows a negative expected sign, but is not statistically significant.

\( DIVA_{i,t} \) and \( DIVUNA_{i,t} \) have book value of operating assets and total assets as their denominators. Hence, the positive correlation between \( Central_{i,t} \) and \( LEV_{i,t} \) may cause the payout ratios to be lower for the central firms. To show a robustness of the results, in column (5), I use \( Dividends_{i,t} \), book value of total dividends paid out, as a dependent variable. In this column, I use \( OrdIncome_{i,t} \), book value of operating profit, \( Capex_{i,t} \), book value of total capital expenditures, \( Public_{i,t} \), and \( Liabilities_{i,t} \), book value of total liabilities as independent variables. To control for size effect, I control for \( Assets_{i,t} \), book value of total assets, and \( Assets_{i,t}^2 \), square of \( Assets_{i,t} \). The results are mostly the same as in the other four columns.

The results in Table 8 support that the internal capital market exists in Korean \textit{Chaebols}, and that the central firms of Korean \textit{Chaebols} form the market by implementing conservative dividends payout policies. The result of this conservative dividends payout policy in Table 8 supports the empirical finding in Table 6 where I report that a low leveraged firm tends to be a pyramidal subsidiary. The theory suggest that these subsidiaries should be less leveraged due to the internal capital advantage. Hence, the results in Table 6 and Table 8 are consistent with the theoretical prediction of the model.

### 7.3.4 H4: Pyramid and Dividend Payout

The theory predicts that a central firm can be highly leveraged due to the control optionality and the co-insurance benefit of the dividends from subsidiaries. Hence, we are expected to see extensive dividends payout from subsidiaries to central firms in a business group. To test this prediction, I run the following regression:

\[
DIVA_{i,t} (\text{or } DIVUNA_{i,t}) = d_0 + d_1 Pyramid_{i,t} + d_2 OPROA_{i,t} + d_3 CAPEXA_{i,t} + d_4 LEV_{i,t} + d_5 Public_{i,t} + \Gamma_4 Controls + \epsilon_{i,t}
\]  

(18)
I use a dummy variable, $Pyramid_{i,t}$, instead of using a continuous variable, $Position_{i,t}$, in the main specification since theory predicts that a central firm’s dividend payout policy is opposite to a subsidiary’s. Including intermediate central firms located between $Position_{i,t}$ value of 1.1 and 2.0 can possibly make the empirical result ambiguous. But, to show the robustness of the results, I also run the regression with $Position_{i,t}$. I run the regression with contemporaneous variables in the RHS of Equation (18) for the same reason that I discussed in the previous section - it is more likely that dividend amounts are a consequence of the business group structure in place rather than a cause. The specification is similar to Equation (17). I control for $Public$, and the usual $Year$ and $Industry$ fixed-effects are controlled. Standard errors are clustered at each firm level.

According to theory, if a central firm can extend its debt capacity by aggressively pushing its subsidiaries to pay out dividends, the sign of $d_1$ should be positive. The expected signs of the coefficients of $OPROA_{i,t}$, $CAPEXA_{i,t}$, and $LEV_{i,t}$ should be the same as in Equation (17). $d_2$ should be positive, and both $d_3$ and $d_4$ negative.

The results are reported in Table 9. First two columns show the results when I use $DIVA_{i,t}$, book value of total dividends divided by book value of operating assets, as a dependent variable. The other two columns show the regression results with $DIVUNA_{i,t}$ as a dependent variable, which is book value of total dividends divided by book value of total assets. The sign of $d_1$, the coefficient of $Pyramid_{i,t}$, is positive and statistically significant at less than 1% level for column (1), and 5% level for column (2), (3), and (4). The average magnitude of the coefficients tells that a pyramidal subsidiary pays out more than the other group firms by around 28 bps of book value of its operating assets, and around 29 bps of book value of its total assets. $d_2$ confirms its expected sign, positive, and is statistically significant at less than 1% level. The coefficient of $LEV_{i,t}$, $d_4$, also confirms its expected negative sign in both column (2) and (4), but is significant at less than 10% level only in column (2). The results confirm that the more profitable and the less leveraged a firm is, the more the firm pays out. The sign of $d_3$, the coefficient of $CAPEXA_{i,t}$, is confirmed even though it is not statistically significant. The positive sign of the coefficient of $Public$ is also confirmed though not significant.

In the last column (5), I use $Position_{i,t}$ as a dependent variable. I use the same independent variables as in Equation (17) from the previous section. The coefficient of $Position_{i,t}$ is significant at less than 5% level. Hence, neither the generic definition of $Pyramid_{i,t}$ nor the possible issue in the denominators of $DIVA_{i,t}$ and $DIVUNA_{i,t}$ derive the results.

In summary, the results in Table 9 imply that subsidiaries tend to pay out more, and the paid-out dividends will be transferred to the central firm as the firm’s non-operating profit, which helps increase the firm’s leverage ratios as shown in Table 7. The two results in Table 9 and 7 confirm the theoretical prediction that the dividends paid from subsidiaries are used to extend the debt capacity of the central firm -co-insurance benefit. De Jong et al., 2009, report empirical evidence on dividends policy of French family business group that is similar to the results in this paper. They document that in French pyramids, more dividends are paid out from subsidiaries to service central firms’ debt, and the central firms tend to be highly leveraged.

Benevolent dividends payout in pyramidal subsidiaries are also documented in Faccio et al., 2001. They argue that by commitment to the benevolent dividends payout, the controlling family can achieve better firm valuation of group firms since it helps build a reputation
for less incentive to divert corporate resources and better treatment to minority shareholders. They document the empirical evidence for this hypothesis by using family business groups data in western Europe and Asia. Their hypothesis is grounded on a corporate governance perspective, and lacks in establishing the relationship between dividend policy and capital structure decision. Hence, the theory and the empirical evidence in this paper can provide a more comprehensive picture of how the two policies are linked together to generate a control efficient family business group.

Lastly, all the empirical results so far support the main prediction of the theory - a decreasing tendency of the leverage ratio from top to bottom of the pyramid, which is consistently implied by the increasing tendency of dividends payout from top to bottom of a pyramid.

7.4 Robustness Checks: Is leverage ratio a determinant of a business group structure?

The group structure metrics such as $\text{Position}_{i,t}$ and $\text{Centrality}_{i,t}$ are relatively persistent. Moreover, leverage ratio is known to be persistent. Even though it is not reported, here, indeed, the AR(1) coefficient of the variable is high and takes a value of 0.53, and the coefficient is statistically significant at less than 1% level. This implies that the empirical results in Table 6 and 7 may be subject to the severe reverse causality problem, and thus the causal link between leverage ratio, $\text{LEV}_{i,t-1}$, and $\text{Position}_{i,t}$/$\text{Centrality}_{i,t}$ is not clearly proven by the results in the two tables. To provide further evidence that a low leveraged firm is more likely to be a pyramidal subsidiary and a high leveraged firm to a group central firm, I run two additional tests utilizing the dynamic property of my data. Even though $\text{Position}$/$\text{Centrality}$ of a firm tend to vary little year by year, there are a number of firms in the sample which show a significant change in its $\text{Position}$/Centrality in two consecutive years. These relatively large changes in $\text{Position}$/Centrality enable me to further investigate the causal relationship between leverage ratio and group structure metrics.

7.4.1 Robustness Checks: Does a low leverage ratio predict a large Position increase?

First, by using a threshold value of change in $\text{Position}$, I identify firms that become pyramidal subsidiaries by increasing their $\text{Positions}$ more than the threshold value. Similarly, I can also identify firms that become stand-alone entities by decreasing their $\text{Positions}$ more than the threshold value. To define a threshold value of large change in $\text{Position}$, I define a new variable, $D\text{Position}_{i,t}$, as

$$D\text{Position}_{i,t} = \text{Position}_{i,t} - \text{Position}_{i,t-1}$$ (19)

$D\text{Position}_{i,t}$ measures change in position of a firm-$i$ from year-$t-1$ to year $t$. A detail of summary statistics of $D\text{Position}_{i,t}$ is given in Table 10. The average and standard deviation of $D\text{Position}$ are 0.027 and 0.035, respectively. The 25-percentile and 75-percentile of the variable are -0.024 and 0.047, respectively, and the median value is zero. The slightly positively skewed distribution of the variable might be because a few firms become new members
of a business group each year, and thus the inclusion of these new member firms into the sample results in a slightly positive average value of \( D\text{Position} \). But, the zero median of the variable suggests that most firms in the sample tend to stay at the same \( \text{Position} \) year by year.

I pick up various threshold values, \( T_j^k \)'s, \( j = \text{Up, Down} \), \( k = 1, 2 \), and define an event that a given firm-\( i \) experiences a significant change in \( \text{Position}_{i,t} \) from year-\( t-1 \) to year-\( t \) by using these variables. The main reason why I focus only on a significant change in \( \text{Position} \) in this test is that leverage ratios are persistent, and thus using a small increase/decrease in \( \text{Position}_{i,t} \) can not help resolve the reverse causality issue. Moreover, a very small change in \( \text{Position}_{i,t} \) is possibly related to an indirect effect from the restructuring of the other member firms, and thus for these small changes, the causal link from the leverage ratio to the firm’s new \( \text{Position} \) decision can not be clearly seen. Hence, I try to collect the cases where a firm has experienced a major \( \text{Position} \) change either up or down. By using these subsamples, I check the robustness of results in Table 6.

Using a pair of threshold values for up and down movement in \( \text{Position} \), \( (T^\text{Up}_k, T^\text{Down}_k) \) for \( k = 1, 2 \), I define a dummy variable, \( \text{BCPyramid}^k_{i,t} \), as following:

\[
\text{BCPyramid}^k_{i,t} = \begin{cases} 
1 & \text{if } D\text{Position}_{i,t} \geq T^\text{Up}_k, \text{ Pyramid}_{i,t} = 1 \\
0 & \text{if } D\text{Position}_{i,t} \leq T^\text{Down}_k, \text{ Pyramid}_{i,t} = 0 \\
\text{Not Assigned} & \text{O.W.} 
\end{cases} 
\]

(20)

The dummy variable, \( \text{BCPyramid}^k_{i,t} \), identifies two events- 1) A firm-\( i \) becomes a subsidiary located at the bottom of pyramid at year-\( t \) by changing its position value up from year-\( t-1 \) by more than \( T^\text{Up}_k \) (i.e., \( \text{BCPyramid}^k_{i,t} = 1 \)), or 2) A firm-\( i \) becomes a stand-alone entity located at the top of the group by changing its position value down from year-\( t-1 \) by more than \( T^\text{Down}_k \) (i.e., \( \text{BCPyramid}^k_{i,t} = 0 \)). I do not impose a restriction that the type of a firm-\( i \) in two consecutive years should also change (i.e., \( \text{Pyramid}_{i,t} \neq \text{Pyramid}_{i,t-1} \)). Hence, several firms in the testing sample might preserve its type from year-\( t-1 \) to year-\( t \) even after experiencing a large increase/decrease in its \( \text{Position} \) value. Imposing this restriction reduces the testing sample size too much. Therefore, to ensure a reasonable size of the testing sample, I do not impose the restriction, and focus on a large change in each firm’s \( \text{Position}_{i,t} \). However, I check whether a firm-\( i \) at year-\( t \) is a pyramidal subsidiary (or a stand-alone entity) after a large \( \text{Position} \) up (or down) movement to ensure that the large up (or down) movement of a firm’s \( \text{Position} \) results in the firm being a pyramidal subsidiary (or a stand-alone entity) at year-\( t \).

To complete the definition of \( \text{BCPyramid}^k_{i,t} \), I define the threshold values of \( D\text{Position}_{i,t} \) as

\[
T^\text{Up}_1 = 0.027 , \quad T^\text{Down}_1 = -0.027 \quad \text{for } k = 1 \\
T^\text{Up}_2 = 0.047 , \quad T^\text{Down}_2 = -0.024 \quad \text{for } k = 2
\]

(21)

For \( k = 1 \), I use the average value of \( D\text{Position} \), 0.027, as thresholds for both \( \text{Position} \) increase and decrease. In this case, firms whose absolute value of position increase/decrease from year-\( t-1 \) to year-\( t \) is greater than 0.027 are included in the testing sample. Among these firms, only a firm with \( \text{Pyramid}_{i,t} = 1 \) after \( \text{Position} \) increase takes \( \text{BCPyramid}^1_{i,t} \) of 1. Similarly, only a firm with \( \text{Pyramid}_{i,t} = 0 \) after \( \text{Position} \) decrease takes \( \text{BCPyramid}^1_{i,t} \) of 0.

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For $k = 2$, I consider the asymmetric nature of $DPosition$’s distribution, and use two different levels of threshold values for $Position$ up and down movements. In this case, I use 0.047 as the threshold value for $Position$ increase, which is the 75-percentile of $DPosition$. And, I use -0.024, the 25-percentile of $DPosition$, as the threshold value for $Position$ decrease. In this case, I assign $BCPyramid^2_{i,t} = 1$ only to the firms that satisfy $Pyramid_{i,t} = 1$, and $BCPyramid^2_{i,t} = 0$ to the firms satisfying $Pyramid_{i,t} = 0$. The sample size decreases in $k$ since the absolute value of the threshold increases in $k$. Hence, for $k = 1$, I have 313 observations, and for $k = 2$, 282.

With $BCPyramid^k_{i,t}$ for $k = 1, 2$, I run the following causal regression:

$$BCPyramid^k_{i,t} = f_0 + f_1LEV_{i,t-1} + f_2OPROA_{i,t-1} + f_3CAPEX_{i,t-1} + \Gamma_5Controls + \epsilon_{i,t} \quad \text{for } k = 1, 2$$

Equation (22) should be much less restrictive to the reverse causality problem since it is least likely that any large change in $Position$ from year-$t - 1$ to year-$t$ reverse-causes the leverage ratio decision made at year-$t - 1$. The same argument applies to the other regressors in the RHS of Equation (22), operating profitability, capital expenditures. Hence, if the expected negative sign of $f_1$ is confirmed with this new testing sample, it implies that a low leverage ratio is indeed a determinant of a pyramidal subsidiary. I control for $Ln(Assets)_{i,t-1}$. I do not control for the other two variables, $FirmAge$ and $Public$, in this regression since both of them are highly correlated with $Ln(Assets)_{i,t-1}$. Any effect of these factors may be captured by $Ln(Assets)_{i,t-1}$. The usual Industry and Year fixed-effects are controlled, and the standard errors are clustered at each firm level.

The results are reported in Table [11]. The first two columns show the results when I use the average value of $DPosition$, 0.027, as a threshold value for up/down position movements. $f_1$ has a negative value of -0.053 in column (1) and -0.058 in column (2), and both are statistically significant at less than 1% level. Hence, a low leverage ratio causes a large $Position$ value increase of a firm, and the firm with a low leverage ratio is likely to be a pyramidal subsidiary in the following year. The negative value of $f_2$ in column (1) and (2) confirms the theoretical prediction that as profitability of a firm decreases, the likelihood to be a pyramidal subsidiary increases. But, it is statistically significant at 10% level only in column (2). In column (1), the positive and significant coefficient of $CAPEX_{i,t-1}$ confirms the capital-intensity result in Table [6]. In column (2), the significance of the coefficient is not present, but the sign is still confirmed. The control variable, $Ln(Assets)_{i,t-1}$, is still a significant determinant of a pyramidal subsidiary. The negative sign tells us that a smaller firm tends to be a pyramidal subsidiary.

The results in column (3) and (4) are very similar to those in column (1) and (2). In column (3) and (4), I use asymmetric threshold values for $DPosition_{i,t}$. They are 0.047 and -0.024 for $Position$ increase and decrease, respectively. Even with this more stringent definition of $BCPyramid^2_{i,t}$, the leverage ratio remains to be an important determinant of the business group structure. The coefficients of the other variables show similar magnitude and significance to those in column (1) and (2).
7.4.2 Robustness Checks: Does a highly levered firm become a group central firm?

Similar to what is done with Position,

I define two events for firm-

1) It becomes a group central firm at year-

2) It stays as a non-central firm from year-

This restriction is made to exclude cases where a pyramidal subsidiary becomes an intermediate central firm located at the bottom of the pyramid. Since the new intermediate central firms located far from the family tend to have mixed characteristics between a central firm and a subsidiary, including these firms clearly makes the analysis ambiguous. Hence, I focus only on firms located relatively close to the controlling family of a business group by imposing a restriction that the new central firms should be positioned at the top-tier of the business group (i.e, Position,

As a first step to define a dummy variable to identify the two main events, I compute the change in

I denote it by a new variable,

which measures increase/decrease in

It is defined as

A detail of summary statistics of

The average and standard deviation of

The median, 25-percentile, and 75-percentile of the variable are all close to zero. This implies that the change in

To define a significant change in

I use 1-standard deviation of

as a threshold value. By using this threshold, I determine whether a given firm-

experiences a radical increase in

in two consecutive years, t − 1 and t. The value of 0.024 in

change is almost close to 95-percentile of

and thus it implies a substantial change in

By using this threshold for

I define a dummy variable,

The new dummy variable,

takes value of 1 if a non-central firm-

b) becomes a central firm at year-

(i.e., Central, Central, −1 = 1) by increasing its centrality by more than 0.024. The zero value of this variable indicates that a firm-

does not change its

much from year-

1 to year-

and stays as a non-central firm in the two consecutive years. Since I focus on firms that experience significant

increase, the sample size becomes small, and the test sample consists of 369 observations. With this sample, I run the following regression:

\[ BCCentral_{i,t} = g_0 + g_1 LEV_{i,t-1} + g_2 OROA_{i,t-1} + g_3 CAPEX_{A_{i,t-1}} + \Gamma_6 Controls + \epsilon_{i,t} \]
Equation (25) directly tests whether leverage ratio, operating profitability, and capital expenditure are the determinant of a central firm. We expect to get $g_1 > 0$, $g_2 > 0$, $g_3 < 0$. I control for $\text{FirmAge}$, $\text{Public}$, and $\ln(\text{Assets})$, and the usual $\text{Industry}$ and $\text{Year}$ fixed-effects are controlled. Standard errors are clustered at each firm level.

The results of Equation (25) are reported in Table 12. In all four columns, a high leverage ratio predicts that the firm is more likely to be a new central firm in the following year. The sign of $g_1$ is positive and statistically significant at least less than 5% level except in column (4) where it is significant at 5% level. Positive and statistically significant $g_2$ indicates that a firm is more likely to be a new central firm when the firm realizes a profitable return from its existing business. Except in column (1), the coefficient is significant at less than 10% level. The sign of $\text{CAPEXA}_{i,t-1}$ is positive in the first two columns and negative in the latter two. But, all of them are insignificant, and the magnitudes are also small. The signs of coefficients of $\text{FirmAge}_{i,t-1}$ and $\text{Public}_{i,t-1}$ suggest that an old public firm tends to be a new central firm, and that finding is consistent with the theoretical argument - established firms can use its internal capital as a financing resource of a new subsidiary, and thus are more likely to be central firms-. They are significant at least at 5% level. Lastly, a larger firm seems more likely to be a new central firm. The positive and significant at 1% level coefficient of $\ln(\text{Assets})_{i,t-1}$ support this.

8 Conclusions

I propose a theory of how the control-driven family designs a business group structure and debt financing decision to achieve efficient control of the group. By combining the settings in two well-known papers -Harris and Raviv, 1988, and Almeida and Wolfenzon, 2006-, I propose several novel predictions about pyramidal family business groups - decreasing leverage ratio and increasing dividend payouts from top to bottom of the pyramid. The internal capital market formed at the top of the pyramid and two intrinsic properties of the pyramid- 1) control optionality and 2) co-insurance benefit- derive the decreasing leverage ratios from top to bottom of the pyramid to be optimal. This decreasing leverage ratios are supported by the increasing dividend payouts from top to bottom of the pyramid.

To access internal capital market formed in central firms at the top of a pyramid, the family conservatively pays out dividends from the firms while it utilizes the co-insurance benefit by legally transferring capital from the subsidiary to the central firm through dividends. Control optionality and co-insurance benefit under the pyramid implies that a central firm’s debt financing is financially more efficient than a subsidiary’s, and thus the family has an incentive to re-capitalize the central by issuing more debt from the firm than the subsidiary. Recapitalization of the central firm leads to control optimality of the pyramid in equilibrium only if the family successfully minimizes the group default probability. Therefore, a fully leveraged pyramid with decreasing leverage ratios from top to bottom emerges in equilibrium if the central firm is large and less capital-intensive.

Using unique Korean family business groups, Chaebols’ 7-years panel data used in Almeida et al., 2009, I provide empirical evidence consistent with these model predictions. Leverage ratio seems to be closely related to the business group structure choice. Dividends are used as the means to transfer resource from one group firm to another to financially support each
other.

The theory and empirical evidence support that controlling families of Korean family business groups, *Chaebols*, strategically design business group structure and make financing decisions to maximize control benefit.
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A Numerical Analysis: Quadrature Definitions

The joint p.d.f of the cash flows for firm A and B is denoted by \( f_{A,B}(\tilde{Y}_A, \tilde{Y}_B) \). The payoff to the family under the pyramidal structure, \( \Pi^P \), is written as

\[
\Pi^P = Z + (K_A + K_B) \int_{D_B^P}^{+\infty} \int_{D_A^P - \beta^P (y - D_B^P)}^{+\infty} f(x, y) dx dy \\
+ K_A \int_{-\infty}^{D_B^P} \int_{D_A^P}^{+\infty} f(x, y) dx dy
\]

And, the equity values of firm A and firm B are

\[
S_A(D_A^P, D_B^P) = \int_{D_B^P}^{+\infty} \int_{D_A^P - \beta^P (\tilde{Y}_B - D_B^P)}^{+\infty} \left[ \tilde{Y}_A + \beta^P (\tilde{Y}_B - D_B^P) - D_A^P \right] f(\tilde{Y}_A, \tilde{Y}_B) d\tilde{Y}_A d\tilde{Y}_B \\
+ \int_{-\infty}^{D_B^P} \int_{D_A^P}^{+\infty} (\tilde{Y}_A - D_A^P) f(\tilde{Y}_A, \tilde{Y}_B) d\tilde{Y}_A d\tilde{Y}_B
\]

\[
S_B(D_B^P) = \int_{D_B^P}^{+\infty} (\tilde{Y}_B - D_B^P) f(\tilde{Y}_B) d\tilde{Y}_B
\]

The payoff and equity value functions are expressed with the double-integration on a non-square integration region. To compute these quantities, I use a numerical quadrature. Each double-integration is computed for \([-5\sigma, 5\sigma] \) range of \( \tilde{Y} \), and the computation delivers a precision level of \( 10^{-9} \).
Figure 1: Symmetric Projects, $K_A = K_B$, 30% of a Firm Value
B Proofs

B.1 Proof of Lemma 1

As long as \( i_B \leq \alpha C + (1 - \gamma_B)\bar{Y}_B \), \( \Pi^H(t^H, \beta^H, 0) = \alpha C + \alpha \bar{Y}_A + \bar{Y}_B - i_B + K_A + K_B \), which is constant for all \( t^H \leq \alpha C \), \( \forall \beta^H \geq \gamma_B \). Hence, raising an excessive fund by cash and equity financing beyond \( i_B \) does not increase the payoff, \( \Pi^H(t^H, \beta^H, 0) \). But, using risky debt is costly since \( \frac{\partial \Pi^H(M_B^H = i_B)}{\partial D_B^H} < 0 \) for \( D_B^H > 0 \).

\[
\frac{\partial (\text{Prob}(M_B^H = i_B))}{\partial D_B^H} = \frac{\partial (1 - F_B(D_B^H - M_B^H + i_B))}{\partial D_B^H} = -f_B(D_B^H - M_B^H + i_B)(M_B^H = i_B) \frac{\partial (D_B^H - M_B^H + i_B)}{\partial D_B^H}(M_B^H = i_B) \\
= - \left(1 - \frac{\partial (C + \bar{Y}_B - \beta^H E_B[\bar{Y}_B - D_B^{H, +}])}{\partial D_B^H}\right) f_B(D_B^H) = - \left(1 + \beta^H \frac{\partial E_B[\bar{Y}_B - D_B^{H, +}]}{\partial D_B^H}\right) f_B(D_B^H) \\
= - \left(1 - \beta^H \int_{D_B^H} f_B(\bar{Y}_B)d\bar{Y}_B \right) f_B(D_B^H) \\
\leq 0
\]

A marginal dollar face value increase in the risky debt returns capital, which is strictly less than a dollar. Therefore, \( t^H + (1 - \beta^H)S_B(D_B^H)^2 + B_B(D_B^H) \) should exactly equate \( i_B \) for \( i_B > \alpha C + (1 - \gamma_B)\bar{Y}_B \).

B.2 Proof of Proposition 1

By imposing the optimal financing policy from Proposition 2, we have the optimal payoff to the family under the horizontal structure given as:

\[
\Pi^{H*} = \begin{cases} 
\alpha C + \alpha \bar{Y}_A + (\bar{Y}_B - i_B) + K_A + K_B & \text{if } i_B \leq \alpha C + (1 - \gamma_B)\bar{Y}_B \\
\alpha C + \alpha \bar{Y}_A + (\bar{Y}_B - i_B) + K_A + K_B [1 - F_B(D_B^H)] & \text{if } i_B > \alpha C + (1 - \gamma_B)\bar{Y}_B
\end{cases}
\]

Similarly, the optimal payoff to the family under the pyramidal structure is

\[
\Pi^{P*} = \begin{cases} 
\alpha C + \alpha \bar{Y}_A + \alpha (\bar{Y}_B - i_B) + K_A + K_B & \text{if } i_B \leq C + (1 - \gamma_B)\bar{Y}_B \\
\alpha C + \alpha \bar{Y}_A + \alpha (\bar{Y}_B - i_B) + K_A + K_B [1 - F_B(D_B^P)] & \text{if } i_B > C + (1 - \gamma_B)\bar{Y}_B
\end{cases}
\]

\( \bar{D}_B^H > \bar{D}_B^P \) from Lemma 3. Since \( \bar{Y}_B - i_B \geq 0 \), the horizontal structure is always chosen if \( i_B \leq \alpha C + (1 - \gamma_B)\bar{Y}_B \). For \( i_B > C + (1 - \gamma_B)\bar{Y}_B \), the pyramidal structure is chosen if

\[
\Pi^{P*} \geq \Pi^{H*} \iff K_B [F_B(D_B^H) - F_B(D_B^P)] \geq (1 - \alpha)(\bar{Y}_B - i_B)
\]

\[\blacksquare\]

B.3 Proof of Lemma 4

Now, I express the dependence of \( \gamma_B \) on \( p_B \) denoting it by \( \gamma(p_B) \). \( \frac{\partial (1 - \gamma(p_B))\bar{Y}_B}{\partial p_B} < 0 \) since \( \frac{\partial \gamma(p_B)}{\partial p_B} > 0 \) and \( \frac{\partial \bar{Y}_B}{\partial p_B} > 0 \). Moreover, \( K_B [F_B(D_B^H)) - F_B(D_B^P)] \geq (1 - \alpha)(\bar{Y}_B - i_B) \) is more...
likely met since at \( p_B = 0 \), the condition is clearly met, and \( \frac{\partial K_B/\gamma_B}{\partial p_B} < 0 \) and \( \frac{\gamma_B - i_B)/\gamma_B}{p_B} > 0 \) hold. At \( p_B = 1 \), \( i_B \leq \alpha C + \tilde{Y}_h(i_B) \) for \( \forall i_B \) since \( \gamma(p_B) = 0 \) and \( \tilde{Y}_h(i_B) - i_B > 0 \). Hence, there is \( \tilde{p} < 1 \) such that the two conditions in Proposition 5 hold at strict equality. This implies that when the pyramidal structure is chosen (i.e., \( 0 \leq p_B \leq \tilde{p} \)), the likelihood of \( p_B < 0.5 \) is greater than that \( p_B > 0.5 \) by \( 1 - \tilde{p} \) if \( p \) is uniformly distributed over \([0, 1]\).

### B.4 Proof of Lemma 5

From the conditions given in Lemma 5, we know that there is \( 0 < \tilde{\delta} < 1 \) s.t. \( \tilde{X}_A^p(\tilde{\delta}) = D_A^p \). Then, the date-2 stochastic payoff to the family under the pyramid is summarized as

\[
\tilde{\Pi}^p = \begin{cases} 
0 & \text{if } \delta < \tilde{\delta} \\
\alpha \left[ \tilde{X}_A^p(\delta = 1) - D_A^p \right]^+ + K_A + K_B & \text{if } \delta \geq \tilde{\delta}
\end{cases}
\]

Since \( \alpha \left[ \tilde{X}_A^p(\delta = 1) - D_A^p \right]^+ + K_A + K_B > 0 \), it is optimal to set \( \delta \geq \tilde{\delta} \). This implies that WLOG, \( \delta^* = 1 \) is optimal.

### B.5 Proof of Lemma 6

I need to show \( \frac{\partial \Pi^p(M_B = i_B, t^p = C + B(D_A^p, D_B^p))}{\partial D_A^p} < 0 \), \( \frac{\partial \Pi^p(M_B = i_B, t^p = C + B(D_A^p, D_B^p))}{\partial D_B^p} < 0 \) for \( \forall D_A^p, D_B^p > 0 \). Let’s denote \( \alpha C + \alpha \tilde{Y}_A + \alpha (\tilde{Y}_B - i_B) \) by \( Z \), and by using a joint p.d.f of \( \tilde{Y}_A \) and \( \tilde{Y}_B \), define \( P_{r1} \) and \( P_{r2} \) as following:

\[
P_{r1} = \text{Prob}\{\tilde{Y}_A + \beta^p (\tilde{Y}_B - D_B^p) + D_A^p \}
= \text{Prob}\{\tilde{Y}_A > D_A^p, \tilde{Y}_B < D_B^p \} + P_{r2}
= \int_{D_B^p}^{D_B^p} \int_{D_A^p}^{D_A^p} f_{A,B}(x,y)dy + P_{r2}
\]

\[
P_{r2} = \text{Prob}\{\tilde{Y}_A + \beta^p (\tilde{Y}_B - D_B^p) \geq D_A^p, \tilde{Y}_B \geq D_B^p \}
= \int_{D_B^p}^{D_B^p} \int_{D_A^p}^{D_A^p} f_{A,B}(x,y)dy
\]

Then,

\[
\Pi^p(M_B^p = i_B, t^p = C + B(D_A^p, D_B^p)) = Z + K_A P_{r1} + K_B P_{r2}
\]

\[
\frac{\partial P_{r1}}{\partial D_A^p} = -\int_{D_B^p}^{D_B^p} f_{A,B}(D_A^p, y)dy + \frac{\partial P_{r2}}{\partial D_A^p}
\]

\[
\frac{\partial P_{r2}}{\partial D_A^p} = -\int_{D_B^p}^{D_B^p} f_{A,B}(D_A^p - \beta^p (y - D_B^p), y)dy
\]

This implies \( \frac{\partial P_{r2}}{\partial D_A^p} < 0 \) for \( D_A^p, D_B^p > 0 \). Moreover, \( \frac{\partial P_{r1}}{\partial D_B^p} < 0 \) for \( D_A^p, D_B^p > 0 \) since

\[
-\int_{D_B^p}^{D_B^p} f_{A,B}(D_A^p, y)dy < 0 \quad \text{and} \quad \frac{\partial P_{r2}}{\partial D_B^p} < 0 \quad \text{for} \quad D_A^p, D_B^p > 0.
\]

\[
\frac{\partial P_{r2}}{\partial D_B^p} = -\beta^p \int_{D_B^p}^{D_B^p} f_{A,B}(D_A^p - \beta^p (y - D_B^p), y)dy
- \int_{D_A^p}^{D_A^p} f_{A,B}(x, D_B^p)dx
\]
This implies $\frac{\partial P_A}{\partial D_B} < 0$ for $D_A^P, D_B^P > 0$.

Lastly,

$$\frac{\partial P_A}{\partial D_A} = \int_{D_B^P} f_{A,B}(x,D_B^P)dx + \frac{\partial P_A}{\partial D_B}$$

$$= -\beta^P \int_{D_B^P} f_{A,B}(D_A^P - \beta^P(y - D_B^P), y)dy$$

And thus, $\frac{\partial P_A}{\partial D_B} < 0$ for $D_A^P, D_B^P > 0$.

All these quantities imply the following:

$$\frac{\partial n}{\partial D_A} = -K_A \int_{D_B^P} f_{A,B}(D_A^P, y)dy$$

$$-[K_A + K_B] \int_{D_B^P} f_{A,B}(D_A^P - \beta^P(y - D_B^P), y)dy$$

$$\frac{\partial n}{\partial D_B} = -K_B \int_{D_B^P} f_{A,B}(x,D_B^P)dx$$

$$-[K_A + K_B] \int_{D_B^P} f_{A,B}(D_A^P - \beta^P(y - D_B^P), y)dy$$

And both are negative for $D_A^P, D_B^P > 0$. ■

### B.6 Proof of Lemma 7

For an interior equilibrium such that $D_A^{P*}, D_B^{P*} > 0$, $\beta^{P*} = \gamma_B$. First, suppose $D_A^{P*} < D_B^{P*}$. Then

$$S_A(D_A^P, D_B^P) = E^F[\tilde{Y}_B - D_A^{P*}] + \gamma_B E^F[\tilde{Y}_B - D_B^{P*}]$$

Therefore, the following holds:

$$\frac{\partial S_A}{\partial D_A} = -[1 - F(D_A^{P*})]$$

$$\frac{\partial S_A}{\partial D_B} = -\gamma_B[1 - F(D_B^{P*})]$$

Set the Lagrangian for the program with Equation (9) in the paper.

$$L(D_A^P, D_B^P, \mu) = Z + (K_A + K_B) - K_A F(D_A^P) - K_B F(D_B^P)$$

$$+ \mu \left[C + \tilde{Y}_A + \tilde{Y}_B - S_A(D_A^P, D_B^P) - i_B\right]$$

$\mu$ is a non-negative multiplier for the budget constraint in the program. The constraint qualification for the two control variables, $D_A^P, D_B^P$, is clearly met. Moreover, the convexity of $S_A(D_A^P, D_B^P)$ in $D_A^P$ and $D_B^P$, in addition to IHR assumption on $F(.)$, imply Kuhn and Tucker(KKT) condition is a sufficient and necessary for the optimality. Hence, the optimality is characterized by the following two first-order conditions:

$$\frac{\partial L}{\partial D_A^{P*}} = -K f(D_A^{P*}) + \mu (1 - F(D_A^{P*})) = 0$$

$$\frac{\partial L}{\partial D_B^{P*}} = -K f(D_B^{P*}) + \gamma_B \mu (1 - F(D_B^{P*})) = 0$$

These two conditions imply

$$K_A \frac{f(D_A^{P*})}{1 - F(D_A^{P*})} = H(D_A^{P*}) = \mu = \frac{K_B}{\gamma_B} \frac{f(D_B^{P*})}{1 - F(D_B^{P*})} = \frac{K_B}{\gamma_B} H(D_B^{P*})$$

Hence, if $K_A > \gamma_B K_B$, then by IHR assumption, $D_A^{P*} < D_B^{P*}$ at optimality, and they satisfy the budget constraint at strict equality. And thus, $E^F[\tilde{Y}_B - D_A^{P*}] + \gamma_B E^F[\tilde{Y}_B - D_B^{P*}]^+ =$
under the pyramidal structure is given as

\[ \Pi^{P*} = Z + K_A + K_B - K_A F(D_A^{P*}) - K_B F(D_B^{P*}) \]

Now, for \( D_A^{P*} \geq D_B^{P*} > 0 \), the following holds:

\[ S_A(D_A^{P*}, D_B^{P*}) = \left[ 1 + \gamma_B \right] (F[Y_B - D_B^{P*}] + S_A(D_B^{P*}) \]

This implies

\[ \frac{\partial S_A(D_B^{P*})}{\partial D_B^{P*}} = \left[ 1 + \gamma_B \right] (1 - F(D_B^{P*})) \]

Setting the Lagrangian for \( D_A^{P*} \geq D_B^{P*} > 0 \) gives

\[ L(D_B^{P*}, \mu) = Z + (K_A + K_B) \left[ 1 - F(D_B^{P*}) \right] + \mu \left[ C + \bar{Y}_A + \bar{Y}_B - S_A(D_B^{P*}) - i_B \right] \]

First-order sufficient and necessary condition by KKT is

\[ \frac{DL}{\partial D_B^{P*}} = -(K_A + K_B)f(D_B^{P*}) + \mu[1 + \gamma_B][1 - F(D_B^{P*})] = 0 \]

\[ \Rightarrow \frac{(K_A + K_B)}{1 + \gamma_B} H(D_B^{P*}) = \mu \]

And, at this \( D_B^{P*} \), the budget constraint is tight since \( \mu > 0 \). Thus we have

\[ [1 + \gamma_B]E[F[Y_B - D_B^{P*}]] = C + \bar{Y}_A + \bar{Y}_B - i_B \]

From the definition of \( D_B^{P*} \), \( D_B^{P*} < D_B^{P*} < D_A^{P*} \). At this optimality, the payoff to the family under the pyramidal structure is given as

\[ \Pi^{P*} = Z + (K_A + K_B)[1 - F(D_B^{P*})] \]

\[ \boxed{B.7 \ \text{Proof of Proposition 2}} \]

Let’s denote \( \frac{K_B}{K_A} \) by \( \Sigma \). From Equation [26] in Appendix B.2, \( \Pi_{BM}^{P*} \) is the optimal date-1 expected payoff to the family under the pyramid without recapitalization of firm A, when \( D_B^{P*} > 0 \), is

\[ \Pi_{BM}^{P*} = Z + \alpha(\bar{Y}_B - i_B) + K_A + K_B \left[ 1 - F_B(\bar{D}_B^P) \right] \]

where \( Z = \alpha C + \alpha \bar{Y}_A + \alpha(\bar{Y}_B - i_B) \) and \( C + \bar{Y}_B - \gamma_B E[F[\bar{Y}_B - \bar{D}_B^P]] = i_B \). The recapitalization is done only if the optimal date-1 expected payoff to the family under the pyramid with recapitalization of firm A, \( \Pi_{EM}^{P*} \), satisfies \( \Pi_{EM}^{P*} > \Pi_{BM}^{P*} \). When \( \Sigma \geq 1 \), \( \Pi_{EM}^{P*} > \Pi_{BM}^{P*} \) only if \( K_B \left[ F(\bar{D}_B^P) - F(D_B^P) \right] > K_A F(D_B^{P*}) \). Therefore, the smaller \( \frac{K_A}{K_B} \), the more likely the recapitalization of firm A can be optimal. When \( \Sigma < 1 \), \( \Pi_{EM}^{P*} > \Pi_{BM}^{P*} \) only if \( K_B \left[ F(\bar{D}_B^P) - F(D_B^{P*}) \right] > K_A F(D_A^{P*}) \). Therefore, the smaller \( \frac{K_A}{K_B} \), the more likely the recapitalization of firm A can be optimal. But, \( \Sigma < 1 \) implies \( \frac{K_A}{K_B} > \frac{1}{\gamma_B} K_B \) (i.e., \( K_A >> K_B \)), and thus \( K_B \left[ F(\bar{D}_B^P) - F(D_B^{P*}) \right] > K_A F(D_A^{P*}) \) is least likely to met. Therefore, \( D_A^{P*} \geq D_B^{P*} \) is more likely to be observed under the pyramid, in equilibrium. This implies that when the two project profiles are identical, and thus \( Q = \bar{Y}_A - \bar{Y}_B = 0 \), \( \frac{D_A^{P*}}{\bar{Y}_A} \geq \frac{D_B^{P*}}{\bar{Y}_B} \). \( \blacksquare \)
B.8 Proof of Proposition 3

Comparing the two optimal payoffs, $\Pi^P$ under a fully leveraged pyramid and $\Pi^H$ under the horizontal structure. For $i_B \geq \alpha \bar{C} + (1 - \gamma_B) \bar{Y}_B$,

$$\Pi^P - \Pi^H \geq 0 \iff \alpha (\bar{Y}_B - i_B) + (K_A + K_B)[1 - F(D^P_A)] - (\bar{Y}_B - i_B) - K_A - K_B[1 - F(D^H_B)] \geq 0$$

$$\iff K_B[F(D^H_B) - F(D^P_A)] - (1 - \alpha)(\bar{Y}_B - i_B) - K_A F(D^P_A) \geq 0$$

$$\iff K_B[F(D^H_B) - F(D^P_A)] \geq (1 - \alpha)(\bar{Y}_B - i_B) + K_A F(D^P_A)$$

For $i_B < \alpha C + (1 - \gamma_B) \bar{Y}_B$, $\bar{D}^H_B = 0$, and thus, a fully leveraged pyramid is never chosen in equilibrium.

B.9 Proof of Proposition 4

Suppose that $D^P_B = 0$. Then, the family receives the optimal payoff, $\Pi^P(D^P_B = 0)$, from a fully leveraged pyramid such that $\Pi^P(D^P_B = 0) = Z + (K_A + K_A)[1 - F(D^P_A)]$ where $E^{F_{A,B}}[\bar{Y}_A + \gamma_B \bar{Y}_B - D^P_A]^+ = C + \bar{Y}_A + \bar{Y}_B - i_B$. And, $E^{F_{A,B}}[\bar{Y}_A + \gamma_B \bar{Y}_B - D^P_A]^+ \leq E^{F_{A,B}}[\bar{Y}_A + \bar{Y}_B - D^C_A]^+$ for $\forall D^P_A \geq 0$. Moreover, from Lemma 9 by plugging in $V_C = 1$, we have $C + \bar{Y}_A + \bar{Y}_B - E^{F_{A,B}}[\bar{Y}_A + \bar{Y}_B - D^C_A]^+ = i_B$. This implies $D^P_A \leq D^C_A$. Therefore, $\Pi^P(D^P_B = 0) \geq \Pi^C$. Since the optimal payoff under the fully leveraged pyramid, $\Pi^P$, satisfies $\Pi^P \geq \Pi^P(D^P_B = 0)$, we know $\Pi^P \geq \Pi^C$. ■
C Definitions of group metrics

The description of the group metrics in this section is from Almeida et al.(2009). Consider a business groups with $N$ firms. We define the matrix of inter-corporate holdings $A$ as follows:

$$A = \begin{bmatrix}
0 & s_{12} & \cdots & s_{1N} \\
s_{21} & 0 & \cdots & s_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
s_{N1} & \cdots & s_{N\,N-1} & 0
\end{bmatrix}$$

where $s_{ij}$ is the stake of firm $i$ in firm $j$. In other words, column $j$ contains the stakes of the corporate direct owners of firm $j$ in all other firms, $1, 2\ldots i, N, i \neq j$. We also define a vector with the direct stakes of the family in each of the $N$ firms:

$$f = \begin{bmatrix}
f_1 \\
f_2 \\
\vdots \\
f_N
\end{bmatrix}$$

**Definition 1** (Ultimate Ownership). The ultimate ownership of the family in each of the $n$ firms is given by $u = [u_1 \; u_2 \; \ldots \; u_N]'$:

$$u' = f'(I_N - A)^{-1}$$

where $I_N$ is the $N \times N$ identity matrix.

**Definition 2** (Position). The position of firm $i$ in the group is defined as:

$$\text{Position}_i = \sum_{n=1}^{\infty} n \frac{f' A^{n-1} d_i}{u_i}$$

Almeida et al.(2009) assume the following two for defining centrality:

**Assumption 2.** (Control Threshold) A family controls a firm if and only if it holds more than $T$ votes in it.

**Assumption 3.** The votes that a family holds in a firm is the sum of its direct votes plus all the direct votes of firms under family control, where control is defined in Assumption 1.

**Definition 3** (Set of firms under the family’s control). For a given threshold $T$, the set of firms controlled by the family is given by:

$$C(T) = \{ i \in N : f_i + \sum_{j \in C(T), j \neq i} s_{ji} \geq T \}.$$ 

**Definition 4** (Critical Control Threshold). For any firm $i \in N$, the critical control threshold is given by

$$CC_i = \max\{T \mid i \in C(T)\}$$
Definition 5 (Centrality). We define the centrality of a firm $i$ as:

$$Centrality_i = \frac{\sum_{j \neq i} CC_j - \sum_{j \neq i} CC_{-i}}{\#N - 1},$$

where $CC_{-i}$ is the critical control threshold of firm $j$, computed as if firm $i$ held no shares in the other group firms.

Definition 6 (Loop). Let

$$loop_i = \min\{n \mid n \geq 1 \text{ and } d_i^t A^n d_i > 0\},$$

then firm $i$ is in a loop if and only if $loop_i < \infty$. Hence, define $\text{Loop}_i = 1_{\{loop_i < \infty\}}$.
### Table 2: Summary Statistics of Group Structure Metrics

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>Stdev</th>
<th>Median</th>
<th>25%</th>
<th>75%</th>
<th>Obs.</th>
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<tr>
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<td>0.000</td>
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<td>0.000</td>
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<td>Employees</td>
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<td>3770</td>
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<td>45</td>
<td>846</td>
<td>3442</td>
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<td>17</td>
<td>14</td>
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### Table 3: Correlations between variables in Table 2

<table>
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<th>Variables</th>
<th>Position</th>
<th>Centrality</th>
<th>Loop</th>
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<th>Employees</th>
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<td>0.30</td>
<td>0.35</td>
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### Table 4: Summary Statistics of Financial Variables

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<th>Stdev</th>
<th>Median</th>
<th>25%</th>
<th>75%</th>
<th>Obs.</th>
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<tbody>
<tr>
<td>Total Assets (in KRW mil.)</td>
<td>905</td>
<td>3090</td>
<td>88.3</td>
<td>20.1</td>
<td>509</td>
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<tr>
<td>Operating Assets (in KRW mil.)</td>
<td>831</td>
<td>2870</td>
<td>85.3</td>
<td>19.3</td>
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<td>0.382</td>
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<td>0.000</td>
<td>0.315</td>
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<tr>
<td>ROA</td>
<td>0.025</td>
<td>0.113</td>
<td>0.280</td>
<td>-0.008</td>
<td>0.086</td>
<td>2974</td>
</tr>
<tr>
<td>OPROA</td>
<td>0.024</td>
<td>0.113</td>
<td>0.027</td>
<td>-0.009</td>
<td>0.085</td>
<td>2974</td>
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<td>0.148</td>
<td>0.029</td>
<td>0.008</td>
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<td>DIVA</td>
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<td>0.099</td>
<td>0.000</td>
<td>0.000</td>
<td>0.006</td>
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<td>0.070</td>
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### Table 5: Correlations between Pyramid, Central and main financial variables in Table 4

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<th>LEV</th>
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<th>OPROA</th>
<th>CAPEXA</th>
<th>DIVA</th>
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<td>-0.050</td>
<td>0.041</td>
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<td>0.262</td>
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Table 6: Pyramid vs. Leverage

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<th>(5)</th>
<th>(6)</th>
</tr>
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<tr>
<td>$Pyramid_{i,t}$</td>
<td>$-0.056^{***}$</td>
<td>$-0.049^{***}$</td>
<td>$-0.040^{**}$</td>
<td>$-0.060^{***}$</td>
<td>$-0.074^{**}$</td>
<td>$-0.097^{***}$</td>
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<td>(-2.15)</td>
<td>(-2.66)</td>
<td>(-2.01)</td>
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<td>$-0.169$</td>
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<td>$(t - 1)$</td>
<td>$(t - 1)$</td>
<td>$t$</td>
<td>$(t - 1)$</td>
<td>$(t - 1)$</td>
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<tr>
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<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
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</tr>
</tbody>
</table>

$^{***}0.01$, $^{**}0.05$, $^{*}0.1$, t-stats are shown in parentheses

Standard errors are clustered at each firm level

This Table shows results of the following regression:

$Pyramid_{i,t} = a_0 + a_1 LEV_{i,t-1} + a_2 OPROA_{i,t-1} + a_3 CAPEXA_{i,t-1} + \Gamma_1 Controls + \epsilon_{i,t}$

The first three columns report the results when the dummy variable, $Pyramid_{i,t}$, is a dependent variable, and all independent variables are lagged ones. The fourth column reports the results when the dummy variable, $Pyramid_{i,t}$, is a dependent variable, and all independent variables are contemporaneous ones. The fifth and sixth columns report the results when the continuous variable, $Position_{i,t}$, is a dependent variable, and all independent variables are lagged ones. In the sixth column, I exclude firms in circular ownership link (i.e., $Loop = 0$), whereas I do not impose the condition in the fifth column.
Table 7: Central vs. Leverage

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<th>(6)</th>
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<td>0.046***</td>
<td>0.049**</td>
<td>0.058***</td>
<td>0.004*</td>
<td>0.006***</td>
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<td>(2.19)</td>
<td>(2.98)</td>
<td>(1.93)</td>
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<td>-0.175**</td>
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<td>(t - 1)</td>
<td>(t - 1)</td>
<td>t</td>
<td>(t - 1)</td>
<td>(t - 1)</td>
</tr>
<tr>
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<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
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</tbody>
</table>

***0.01, **0.05, *0.1, t-stats are shown in parentheses
Standard errors are clustered at each firm level

This Table shows the results of the following regression:

\[
Central_{i,t} = b_0 + b_1 LEV_{i,t-1} + b_2 OPROA_{i,t-1} + b_3 \text{CAPEXA}_{i,t-1} + \Gamma_2 \text{Controls} + \epsilon_{i,t}
\]

The first three columns report the results when the dummy variable, Centralit, is a dependent variable, and all independent variables are lagged ones. The fourth column reports the results when the dummy variable, Centralit, is a dependent variable, and all independent variables are contemporaneous ones. The fifth and sixth columns report the results when the continuous variable, Centralityit, is a dependent variable, and all independent variables are lagged ones. In the sixth column, I exclude firms in circular ownership link (i.e., Loop = 0), whereas I do not impose the condition in the fifth column.
### Table 8: Payout vs Central

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<td>DIVA&lt;sub&gt;t&lt;/sub&gt;</td>
<td>DIVA&lt;sub&gt;t&lt;/sub&gt;</td>
<td>DIVUNA&lt;sub&gt;t&lt;/sub&gt;</td>
<td>DIVUNA&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Dividends&lt;sub&gt;t&lt;/sub&gt;</td>
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<td>-0.003***</td>
<td>-0.004***</td>
<td>-0.003***</td>
<td>Centrality&lt;sup&gt;(×10&lt;sup&gt;7&lt;/sup&gt;)&lt;/sup&gt;</td>
</tr>
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<td>(-3.66)</td>
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<td>0.047***</td>
<td>0.047***</td>
<td>0.045***</td>
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<td>(5.29)</td>
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<td>-0.002*</td>
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<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>Public&lt;sup&gt;(×10&lt;sup&gt;7&lt;/sup&gt;)&lt;/sup&gt;</td>
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<td>(0.80)</td>
<td>(0.59)</td>
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<td></td>
<td>Assets&lt;sup&gt;(×10&lt;sup&gt;−14&lt;/sup&gt;)&lt;/sup&gt;</td>
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</tr>
<tr>
<td>**Assets&lt;sup&gt;2&lt;/sup&gt;&lt;sup&gt;(×10&lt;sup&gt;−14&lt;/sup&gt;)&lt;/sup&gt;</td>
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### Industry, Year, Obs, Adj – R<sup>2</sup>, RHS Var

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***0.01, **0.05, *0.1, t-stats are shown in parentheses

Standard errors are clustered at each firm level

This Table shows the results of the following regression:

\[
DIVA_{i,t} (or \ DIVUNA_{i,t}) = c_0 + c_1 Central_{i,t} + c_2 OPROA_{i,t} + c_3 CAPEX_{A,i,t} + c_4 LEV_{i,t} + c_5 Public_{i,t} + \Gamma_3 Controls + \epsilon_{i,t}
\]

All RHS variables are contemporaneous ones. In the first two columns, I use DIVA<sub>t</sub>, book value of total dividends divided by book value of operating assets, as a dependent variable. And in the latter two columns, I use DIVUNA<sub>t</sub>, book value of total dividends divided by book value of total assets. In the fifth column, I use book value of total dividends, Dividends, as a dependent variable, and use a continuous variable, Centrality, as a main independent variable.
### Table 9: Payout vs Pyramid

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DIVA&lt;sub&gt;t&lt;/sub&gt;</td>
<td>DIVA&lt;sub&gt;t&lt;/sub&gt;</td>
<td>DIVUNA&lt;sub&gt;t&lt;/sub&gt;</td>
<td>DIVUNA&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Dividends&lt;sub&gt;t&lt;/sub&gt;</td>
</tr>
<tr>
<td>Pyramid</td>
<td>0.0028***</td>
<td>0.0027**</td>
<td>0.0030**</td>
<td>0.0028**</td>
<td>Position(×10&lt;sup&gt;7&lt;/sup&gt;) 0.118**</td>
</tr>
<tr>
<td></td>
<td>(2.22)</td>
<td>(2.16)</td>
<td>(2.33)</td>
<td>(2.27)</td>
<td>(2.31)</td>
</tr>
<tr>
<td>OPROA</td>
<td>0.052***</td>
<td>0.050***</td>
<td>0.050***</td>
<td>0.048***</td>
<td>OrdIncome 0.036***</td>
</tr>
<tr>
<td></td>
<td>(4.82)</td>
<td>(4.71)</td>
<td>(4.66)</td>
<td>(4.53)</td>
<td>(3.90)</td>
</tr>
<tr>
<td>CAPEXA</td>
<td>-0.0023</td>
<td>-0.003</td>
<td>-0.003</td>
<td>-0.003</td>
<td>Capex -0.003</td>
</tr>
<tr>
<td></td>
<td>(-1.02)</td>
<td>(-1.11)</td>
<td>(-1.09)</td>
<td>(-1.19)</td>
<td>(-0.27)</td>
</tr>
<tr>
<td>LEV</td>
<td>-0.002*</td>
<td>-0.003</td>
<td>-0.003</td>
<td>-0.003</td>
<td>Liabilities -0.017***</td>
</tr>
<tr>
<td></td>
<td>(-1.66)</td>
<td>(-1.64)</td>
<td></td>
<td></td>
<td>(-5.40)</td>
</tr>
<tr>
<td>Public</td>
<td>0.0001</td>
<td>-0.0002</td>
<td>-0.0003</td>
<td>-0.0005</td>
<td>Public(×10&lt;sup&gt;7&lt;/sup&gt;) -0.028</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(-0.12)</td>
<td>(-0.21)</td>
<td>(-0.28)</td>
<td>(0.28)</td>
</tr>
<tr>
<td>Industry</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Year</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Obs</td>
<td>1856</td>
<td>1856</td>
<td>1856</td>
<td>1856</td>
<td>2590</td>
</tr>
<tr>
<td>Adj – R&lt;sup&gt;2&lt;/sup&gt;</td>
<td>0.12</td>
<td>0.12</td>
<td>0.11</td>
<td>0.12</td>
<td>0.80</td>
</tr>
<tr>
<td>RHS Var</td>
<td>t</td>
<td>t</td>
<td>t</td>
<td>t</td>
<td>t</td>
</tr>
</tbody>
</table>

***0.01, **0.05, *0.1, t-stats are shown in parentheses
Standard errors are clustered at each firm level

This Table shows the results of the following regression:

\[
DIVA_{it}(\text{or } DIVUNA_{it}) = d_0 + d_1 Pyrami_{it} + d_2 OPROA_{it} + d_3 CAPEXA_{it} + d_4 LEV_{it} + d_5 Public_{it} + \Gamma_4 Controls + \epsilon_{it} 
\]

All RHS variables are contemporaneous ones. In the first two columns, I use DIVA<sub>t</sub>, book value of total dividends divided by book value of operating assets, as a dependent variable. And in the latter two columns, I use DIVUNA<sub>t</sub>, book value of total dividends divided by book value of total assets. In the fifth column, I use book value of total dividends, Dividends, as a dependent variable, and use a continuous variable, Position, as a main independent variable.

### Table 10: Summary Statistics of Change in Group Structure Metrics

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>Stdev</th>
<th>Median</th>
<th>25%</th>
<th>75%</th>
<th>Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>DPosition</td>
<td>0.027</td>
<td>0.350</td>
<td>0</td>
<td>-0.023</td>
<td>0.047</td>
<td>2370</td>
</tr>
<tr>
<td>DCentrality</td>
<td>0.002</td>
<td>0.024</td>
<td>0</td>
<td>-0.000</td>
<td>0.000</td>
<td>2350</td>
</tr>
</tbody>
</table>

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Table 11: Robustness Check: Pyramid vs. Leverage

<table>
<thead>
<tr>
<th></th>
<th>Dependent Variable: ( BCP_{Pyramid}^k )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( k = 1: ) Symmetric Thresholds ( k = 2: ) Asymmetric Thresholds</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>( LEV_{t-1} )</td>
<td>-0.053***</td>
</tr>
<tr>
<td></td>
<td>(-5.48)</td>
</tr>
<tr>
<td>( OPROA_{t-1} )</td>
<td>-0.260</td>
</tr>
<tr>
<td></td>
<td>(-1.61)</td>
</tr>
<tr>
<td>( CAPEXA_{t-1} )</td>
<td>0.230*</td>
</tr>
<tr>
<td></td>
<td>(1.96)</td>
</tr>
<tr>
<td>( Ln(Assets) )</td>
<td>-0.034***</td>
</tr>
<tr>
<td></td>
<td>(-2.62)</td>
</tr>
<tr>
<td>( Industry )</td>
<td>Yes</td>
</tr>
<tr>
<td>( Year )</td>
<td>Yes</td>
</tr>
<tr>
<td>( Obs )</td>
<td>313</td>
</tr>
<tr>
<td>( Adj \ - \ R^2 )</td>
<td>0.21</td>
</tr>
<tr>
<td>( Cluster Level )</td>
<td>Firm</td>
</tr>
</tbody>
</table>

***0.01, **0.05, *0.1, t-stats are shown in parentheses

Standard errors are clustered at each firm level

This Table shows the results of the following regression:

\[
BC_{Pyramid}^k_{i,t} = f_0 + f_1 LEV_{i,t-1} + f_2 OPROA_{i,t-1} + f_3 CAPEXA_{i,t-1} + \Gamma_5 Controls + \epsilon_{i,t}
\]

The dependent variable, \( BCP_{Pyramid}^k_{i,t} \), is a dummy variable taking value of 1 if a given firm-\( i \) increases its \( Position \) value from year-\( (t - 1) \) to year-\( t \) more than a given threshold level and becomes a pyramidal subsidiary at year-\( t \). It takes a value of 0 if a given firm-\( i \) decreases its \( Position \) value from year-\( (t - 1) \) to year-\( t \) more than a given threshold level and becomes a stand-alone entity at year-\( t \). See the section 7.4.1 for more details of the definition of \( BCP_{Pyramid}^k_{i,t} \). The first two columns show the results when I use the average \( Position \) change as thresholds for both increase and decrease of \( Position \). The other two columns show the results when I use the 25-percentile of \( Position \) change as a threshold for \( Position \) decrease, and the 75-percentile of \( Position \) change for \( Position \) increase.
Table 12: Robustness Check: Central vs. Leverage

<table>
<thead>
<tr>
<th></th>
<th>Dependent Variable: $BCCentral_{i,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>$LEV_{i-1}$</td>
<td>0.174**</td>
</tr>
<tr>
<td></td>
<td>(2.05)</td>
</tr>
<tr>
<td>$OPROA_{t-1}$</td>
<td>0.173</td>
</tr>
<tr>
<td></td>
<td>(1.58)</td>
</tr>
<tr>
<td>$CAPEX_{A_{t-1}}$</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
</tr>
<tr>
<td>$FirmAge_{t-1}$</td>
<td>2.54**</td>
</tr>
<tr>
<td></td>
<td>(2.05)</td>
</tr>
<tr>
<td>$Public_{t-1}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$Ln(Assets)_{t-1}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Industry</td>
<td>Yes</td>
</tr>
<tr>
<td>Year</td>
<td>Yes</td>
</tr>
<tr>
<td>Obs</td>
<td>369</td>
</tr>
<tr>
<td>$Adj - R^2$</td>
<td>0.13</td>
</tr>
</tbody>
</table>

***0.01, **0.05, *0.1, t-stats are shown in parentheses
Standard errors are clustered at each firm level

This Table shows the results of the following regression:

$BCCentral_{i,t} = g_0 + g_1 LEV_{i,t-1} + g_2 OPROA_{i,t-1} + g_3 CAPEX_{A_{i,t-1}} + \Gamma_6 Controls + \epsilon_{i,t}$

The dependent variable, $BCCentral_{i,t}$, is a dummy variable taking value of 1 if a given firm-$i$ becomes a new central firm at year-$t$ by increasing its centrality from year-$(t-1)$, and 0 otherwise. See the section [4.2.4] for more details of the definition of $BCCentral_{i,t}$. Test samples are restricted to a condition that Position of the firms should be less than equal to $T_p$. The regression focuses on firms within the top-tier of the business group.