IPOs, Acquisitions and the Use of Convertible Securities in Venture Capital

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Abstract

This paper provides a new explanation for the use of convertible preferred equity in venture capital. It explains that the key feature of the convertible security is to create different cash flow rights for acquisitions and IPOs. It shows how the convertible security implements an optimal trade-off between the need to allocate cash flows to the venture capitalist and the desire to make efficient exit decisions. This approach also explains some puzzling contract features, such as automatic conversion in case of an IPO, or the use of participating preferred equity, where conversion never benefits the investors.

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1 Introduction

When we think about success in start-ups and venture capital, we typically think of successful stock-market listed companies: Amazon, Intel, Yahoo, and the likes. There are many other successful venture capital investments that receive considerably less attention, namely when start-ups get acquired. According to VentureOne (2000) there were more exits by acquisition than exits by IPO in three out of the last four years.\footnote{Specifically, VentureOne reports 234, 135, 77 and 248 VC-backed IPOs for 1996-1999, compared to 201, 231, 230 and 256 acquisitions. Note that acquisitions and IPOs are called “exit events” in the venture capital industry, because they allow the venture capitalist to sell their investments. In case of an IPO there is typically a lock-up period before the venture capitalists can sell.} Many acquired companies are very successful, the most spectacular being a little known company called Cerent, that was acquired by Cisco for $6.9 billion. The question we address in this paper is what determines whether a successful start-up gets acquired or whether it remains an independent company and goes for an IPO?

Fundamentally, this is a question about the optimal boundaries of the firm, and what ownership structure generates the most value. There is a large literature on optimal ownership and the boundaries of the firm, which takes the perspective of a single asset owner that determines ex-ante an optimal ownership structure. The interesting aspect about exit decisions in venture capital is that the optimal ownership structure is not known ex-ante. And when it comes to making a decision between getting acquired versus remaining independent, the entrepreneur and the venture capitalist may not always agree. How are exit decisions made in the presence of
disagreements?

When contracting ex-ante, the entrepreneur and the venture capitalist can anticipate how they want to make future decisions. Sahlman (1990), Gompers (1997) and Kaplan and Strömberg (2000) provide detailed evidence on how venture capital contracts allocate control rights and cash flow rights between the entrepreneur and the venture capitalist. They note that there is variation in the allocation of control rights, that entrepreneurs usually get straight equity and that venture capitalists tend to get some type of convertible preferred security. The use of convertible securities has peaked the interest of a few theorists, including Berglöf (1994), Cornelli and Yosha (1999), Repullo and Suarez (1998) or Schmidt (1999). This theoretical literature, however, seems to have missed two relevant empirical facts. First, the convertible preferred security automatically converts to common stock in case of an IPO. Both Gompers and Kaplan and Strömberg find that well over 90% of all contracts in their samples have automatic conversion in case of IPO. The subtlety of automatic conversion is that it allocates different cash flow rights for different exit decisions: in case of an acquisition, investors own a preferred instrument, but in case of an IPO they simply own common stock. Second, the literature only examines one type of convertible preferred equity, where the investor can choose between either taking preferred dividends, or else converting to common stock and participate in the common dividends. Figure 1 shows the payout pattern of this security, which we will call “simple convertible preferred equity.” But there are other types of convertible se-
curities, most notably so-called “participating” convertible preferred equity. Kaplan and Strömberg report that 45% of all convertible preferred equity in their sample is “participating.” Prior to conversion, this security receives a preferred dividend and it participates in the common dividends too. After conversion, the preferred rights vanish, so that it only participates in the common dividends. See figure 2. The curious aspect of this security is that an investor would never want to convert, since this means loosing the preferred dividends without gaining anything in return. We cannot make sense of this security unless we explicitly consider the fact that there are two types of exit, and that there is a forced conversion for one of them.

In this paper we therefore provide a joint analysis of the venture capitalists’ exit decision and the use of convertible securities. We examine how venture capital contracts optimally allocate control rights and cash flow rights in a world where there is some uncertainty about whether exit should occur through an acquisition or an IPO. We identify a fundamental trade-off between the investor’s need for cash flows and the desire to achieve efficient exit decisions. Within this framework we derive the use of convertible preferred securities, including their automatic conversion in case of IPO.

Our analysis is based on two fundamental assumptions. First, the type of exit must affect the value of the venture. It may be higher or lower for an acquisition. All that matters is that there is some ex-ante uncertainty about what the optimal exit decision will be. Second, we assume that if the company stays independent, the

\[2\] This security is essentially the same as convertible debt, except that the firm is not required to make regular dividend/coupon payments. See Bartlett (1995) for a detailed discussion of how convertible securities are structured.
entrepreneur plays a significant role in the company, that allows her to extract a certain fraction of the rents. In an independent company the entrepreneur is likely to play a major role, such as being its CEO or chair(wo)man of the board. The company will thus give her some stake in the company, as a compensation for the value that she contributes. In case of acquisition, however, the power of the entrepreneur is diminished. This assumption is natural, since the purpose of the acquisition is to bring the venture under the acquirer’s management control.⁴

With these two assumptions, we find that the entrepreneur and the venture capitalist may disagree about the optimal exit decision. This happens in a range of parameters where, even though the total value is higher in case of an IPO, the value that a venture capitalist can extract from the venture is higher in case of an acquisition. We show that despite renegotiation, the allocation of control rights affects the exit decision. The optimal ex-ante contract must balance the venture capitalist’s need to receive a competitive return with the desire to avoid inefficient exit decisions, specifically inefficient acquisitions.

The model predicts the following three ranges of outcomes. For a loose finance constraint all parties may hold straight equity, control rights don’t matter and exit decisions are always efficient. For an intermediate finance constraint it is optimal to separate cash flow rights and control rights. The entrepreneur retains control rights, which ensures that no inefficient exit decisions are taken. The venture capitalist receives a convertible preferred security that allows him to capture additional cash

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³For example, in a case study of Symantec’s acquisitions, Blackburn et. al. (1996) found that the founders of the acquired firms typically left within a short period, precisely because they had lost their previous role.
flows in case of an acquisition. This security automatically converts into common stock in case of an IPO. Finally, for a tight finance constraint it is optimal to allocate both control rights and preferred cash flow rights to the venture capitalist. Again, convertible preferred equity with automatic conversion is optimal, but now we also have some inefficient acquisitions. We also derive the type of convertible security. We show that for an intermediate finance constraint, the optimal contract can always be implemented with simple convertible preferred equity (figure 1). For a tight finance constraint the optimal contract can be implemented with a security that closely resembles participating convertible preferred equity (figure 2).

We also extend the model in a number of ways that explore the role of the entrepreneur. We show that the larger the entrepreneur’s hold-up potential, the less efficient the outcome, and the smaller the range of ventures that can be financed. We discuss how bringing in a professional manager as CEO reduces, though not necessarily eliminates, the entrepreneur’s hold-up potential. We also show that it doesn’t matter whether the manager of the acquired division is also able to capture some of the rents. All that matters is that the power of the entrepreneur is diminished in case of an acquisition.

The current paper contributes to the literature on convertible securities. Seminal papers on this topic include Green (1984) and Stein (1992). Recently, a few theory papers have examined the use of convertible securities in venture capital contracts. Most of these papers provide an incentive-based explanation. Schmidt (1999), for example, examines a sequential investment problem where the entrepreneur completes
her private investment before the investor makes his. The convertible security provides an optimal incentive mechanism that ensures that the investor converts if and only if the entrepreneur has invested efficiently. Cornelli and Yosha (1999) examine incentives of the entrepreneur for short-term window dressing and show how a convertible security might alleviate this problem. Berglöf (1994) examines an incomplete contract model where a monopolistic acquirer could take advantage of the investor by only bargaining with the entrepreneur. Under some specific assumptions about the distribution of private benefits, convertible debt protects the investor from asset stripping and the entrepreneur from a loss of private benefits. Repullo and Suarez (1998) examine a two-sided moral hazard problem where there is a complementarity between financing and advising, and find that the optimal security resembles convertible preferred equity. Other incentive-based models include Marx (1998), Bergemann and Hege (1998) and Dessi (1999). We view these incentive-based models as complementary to our control-based approach. Incentive-based models are good for explaining the use of debt-like securities, and some of the models also provide an explanation for convertibility. Berglöf (1994) is the only paper that recognizes the existence of two exit options, and Schmidt (1999) is the only one to include a brief discussion of automatic conversion. None of the papers can account for the more puzzling features, such as participating preferred equity. Most important, this paper provides a new and remarkably simple explanation for convertible securities, that provides a more comprehensive analysis for how convertible securities are used in venture capital contracts.
The current paper also builds on Aghion and Bolton (1992), both by incorporating wealth constraints into the analysis and by exploring the possibility of contingent control. Zingales (1995) examines a different interaction between IPOs and acquisitions, namely how an IPO can serve as a prelude to an acquisition. Finally, the paper is related to the growing literature on security design (Harris and Raviv (1992), Zender (1991)).

The remainder of the paper is organized as follows. Section 2 introduces the base model. Section 3 derives the optimal venture capital contract and explains its various features. Section 4 examines a number of extension related to rent capture. It is followed by a brief conclusion. All proofs are in the appendix. Moreover, the appendix also contains a series of numerical examples that further illustrate the structure of the main findings.

2 The base model

Suppose there is an entrepreneur, denoted by $E$, that wants to start a new venture. She is wealth constrained, seeking to raise an amount $K$ from a set of perfectly competitive venture capitalists, denoted by $V$. There are also established corporations, denoted by $C$, that may acquire the new venture and integrate it with their existing business. And there is a large set of stock market investors that may buy listed shares. All parties are risk-neutral and there is no discounting. Moreover, there is symmetric information throughout the game.

There are three stages: the contracting stage (date 0), the pre-IPO stage (date 1)
and the IPO stage (date 2). At date 0, $E$ and $V$ write a contract. At date 1 new information becomes available about the expected returns of the new venture. At this time a decision has to be made whether the new venture should be sold to $C$, or whether it should be developed as an independent company. We call this the exit decision. We denote the decision to get acquired by $A$ and the decision to remain independent by $I$. $E$ or $V$ may hold the control right over this decision, and they may always renegotiate. For simplicity sake, we assume that in case of renegotiation $V$ has all the bargaining power. We will relax this assumption later. If the venture is acquired, $C$ manages it and collects all cash flows after date 2. If the company remains independent, $E$ manages the venture. At date 2 the company may go public and cash flows are received thereafter.

There are two key assumptions in the model. First, we assume that different ownership structures generate different returns, and that it is not known ex-ante which ownership structure will be optimal. The idea is that an acquisition changes the strategy or structure of the new venture, and that the value of the venture will depend on its ownership structure. A large and growing literature examines the costs and benefits of integration. See Hart (1995) or Bolton and Scharfstein (1999) for some useful surveys. The differences in values may thus depend on a multitude of fundamental factors, such as the underlying technology and market of the new venture. Moreover, it could also depend on factors such as the demand in the IPO market.\footnote{We do not have to think of the ownership decision as permanent. An acquired unit, for example, may always be spun-off at a later stage. All that matters here is that at date 1 the ownership decision - whether temporary or permanent - has an effect on the expected returns of the new venture. Note} A strength of this model is that we do not need to specify these underlying
factors. Instead we allow for a very general distribution of expected returns. At date 1, \( E \) and \( V \) receive information about the expected values of the new venture. We denoted the expected values by \( \pi \) in case of an IPO and by \( p \) in case of acquisition. Our assumption of competition among acquirers ensures that the acquisition price always equals \( p \). Ex-ante, \( \pi \) and \( p \) have a joint distribution \( \Omega(\pi, p) \) over an interval \([\underline{\pi}, \overline{\pi}] \times [\underline{p}, \overline{p}]\). We don’t need to impose any particular restriction on \( \Omega(\pi, p) \). All that matters is that there is some ex-ante uncertainty about the optimal ownership structure.

The second key assumption is that \( E \) can always capture some of the rents if the company remains independent, but has no role if the company gets acquired. Pursuing an independent strategy allows \( E \) to manage the company, which puts her in a position of power. Her human capital becomes valuable to the success of the venture. It also makes it more difficult to by-pass her. To retain her and align her incentives, the company will thus need to give her some minimal stake (equity and stock options) in the company. We can think of this stake as a compensation for the value that she contributes and/or as a rent that she can hold up by virtue of being in a position of power (see Jensen and Mecklin (1976), Shleifer and Vishny (1989), Hart and Moore (1994)).

\(^5\)Baker and Gompers (1999) find that at the time of the IPO, founders own on average about 21% of the equity (14% for venture capital backed companies, and 25% for non-venture capital backed companies). Not all of this equity may be necessary to retain and compensate the entrepreneur, but Baker and Gompers’ result also suggest that newly appointed CEOs receive approximately 10% of the equity. Their findings clearly suggest that key personnel, such as the CEO, always receive some non-negligible stake in the company.
 Specifically, we assume that if $I$ is chosen, $E$ can always capture a fraction $\alpha$ of the total value $\pi$ of the venture. Put differently, because of the need to retain and incentivize $E$, $V$ cannot extract more than $(1 - \alpha) \equiv \overline{\pi}$ of the total value $\pi$. We contrast this with the case of an acquisition. In this case, $C$ manages the venture, making $E$ less important to the success of the new venture. For simplicity, we assume that in case of $A$, $E$ has no role at all. We further discuss this assumption in section 4.

A third assumption will be intuitive and useful, although not strictly necessary. While $\pi$ and $p$ are observable, they are not necessarily verifiable. Instead, verifiability is established through market transactions. It is well known that it is extremely difficult to establish valuations for privately-held companies, and this is especially true for young, high-technology companies, where assets are mainly intangible. Valuations, however, arise naturally in a course of a transaction, such as an IPO listing or an acquisition. At date 1, the acquisitions bids are clearly verifiable. Similarly, at date 2, an IPO also establishes a verifiable market valuation. However, at the time of the acquisition, it might be very difficult to verify what the valuation of the firm would be if it were to pursue an independent strategy. We therefore assume that at date 1, $\pi$ is observable but not verifiable. We will later relax this assumption.

To complete the model description, we need to explain the contract parameters. At date 0, $E$ and $V$ design an optimal contract that must satisfy $V$’s zero profit condition. The contract allocates control rights and cash flow rights. Control rights in this model pertain to the decision of $A$ versus $I$, and we distinguish between
$E - control$ and $V - control$.\textsuperscript{6} Cash flow rights may distinguish between $A$ and $I$.

We denote $V$’s claims on cash flows by $f(p)$ for $A$, and $\phi(\pi)$ for $I$.\textsuperscript{7}

It is useful to identify some common securities that will play an important role in the analysis. We use the abbreviation CPE for convertible preferred equity.

- \textit{Straight equity}: $\phi(\pi) = \tilde{\phi}\pi$ and $f(p) = \tilde{\phi}p$ where $\tilde{\phi}$ is a constant, measuring $V$’s percentage equity.

- \textit{Straight debt}: $\phi(\pi) = \text{Min}[\pi, D]$ and $f(p) = \text{Min}[p, D]$ where $D$ is the total value of debt.

- \textit{Simple CPE (figure 1)}: $\phi(\pi) = \tilde{\phi}\pi$ and $f(p) = \text{Max}[\text{Min}[p, D], \tilde{\phi}p]$ where $\tilde{\phi}$ is a constant, measuring $V$’s percentage equity after conversion, and $D$ the total value of the preferred dividends prior to conversion.

- \textit{Participating CPE (figure 2)}: $\phi(\pi) = \tilde{\phi}\pi$ and $f(p) = \text{Max}[\text{Min}[p, D+\tilde{\phi}p], \tilde{\phi}p] = \text{Min}[p, D+\tilde{\phi}p]$ where $\tilde{\phi}$ is a constant, measuring $V$’s percentage equity after conversion, and $D$ the total value of the preferred dividends prior to conversion.

- \textit{Generalized CPE (figure 3)}: $\phi(\pi) = \tilde{\phi}\pi$ and $f(p) = \text{Max}[\tilde{f}(p), \tilde{\phi}p]$ where $\tilde{\phi}$ is a constant, measuring $V$’s percentage equity after conversion, and $\tilde{f}(p)$ is the total value of the preferred security prior to conversion.

Straight equity and straight debt are non-contingent securities that do not distinguish between the type of exit. With the automatic conversion in case of an IPO,

\textsuperscript{6}It is easy to show that joint control (where both $E$ and $V$ have to agree to a decision) is very similar to $E - control$.

\textsuperscript{7}In theory, it is possible to have $\phi(\pi, p)$. In the appendix we show that this is never necessary. We therefore retain the simpler notation $\phi(\pi)$ throughout the main body of the paper.
however, the above CPE’s are contingent on the type of exit. This creates a natural
distinction between the cash flow claims in case of an acquisition (where the investor
has the option between the preferred terms or conversion) versus an IPO (where
there are no preferred terms). The differences between the various CPE’s concern
their value prior to conversion. The simple CPE essentially looks like debt prior to
conversion, whereas the participating preferred looks like a combination of debt and
equity. There are many other types of CPEs, with ranges and limits for the preferred
dividends and a variety of other adjustments. We therefore define the generalized
CPE. It allows for any specification of the preferred terms prior to conversion, as
expressed by the general functional form $f(p)$. The security allocates preferential
cash flow rights to $V$ in case of $A$ (i.e., $V$ gets the better of $f(p)$ or $\phi p$). And it
converts to common equity ($V$ gets $\phi \pi$) in case of $I$. Our definition of the generalized
CPE thus encompasses the entire family of convertible securities that automatically
convert to straight equity in case of an IPO. Obviously, there are many other possible
securities. For the analysis we will thus start with the set of all feasible contracts. We
then derive the properties of the optimal contract. Finally, we show how the above
securities implement this optimal contract.

3 Optimal venture capital financing

3.1 Bargaining over ownership choices

Central to the model is the exit decision at date 1. This decision is influenced by the
original contract which allocates control rights and cash flow rights. Moreover, the
two parties may always renegotiate the original contract.

We will say that an acquisition is “efficient” whenever \( p \geq \pi \), and “inefficient” whenever \( p < \pi \). We thus use the term “efficient” to denote joint value maximization between \( E \) and \( V \).\(^8\) The following proposition examines the equilibrium outcomes of the renegotiation game.

**Proposition 1: (Ex-post bargaining)**

(i) Under \( E \) - control, \( A \) is chosen if and only if it is efficient.

(ii) Under \( V \) - control, \( A \) is chosen if it is efficient, and if it is inefficient but \( \bar{\pi} \pi < f(p) < \pi \).

All proofs are in the appendix. The appendix also contains a numerical example that illustrates this proposition. Under \( E \) - control, \( A \) is chosen whenever it maximizes the joint surplus. Suppose that \( E \) were to make an inefficient exit decision. \( V \) faces no wealth constraint. He can thus always pay to convince \( E \) to a more efficient outcome. Under \( V \) - control, however, renegotiation might be impeded by two binding constraints. First, \( E \) faces a binding wealth constraint. Second, she cannot commit not to hold up a fraction \( \alpha \) of the returns once \( I \) is chosen. Suppose now that \( V \) prefers \( A \) even though it is inefficient. If \( \bar{\pi} \pi < f(p) \), then \( E \) has no means to make any offer to \( V \) that would make \( V \) better off choosing \( I \). Hence \( V \) chooses \( A \) even though this is inefficient.

This establishes that inefficient exit decisions may occur if \( V \) has control. Under

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\(^8\)This may not coincide with notions of social efficiency, where there may be other parties that are also affected by the ownership decision. See also our discussion in section 4.
what circumstances would $E$ and $V$ write a contract that leads to such inefficient outcomes? In the next subsection we derive the optimal contract and show when inefficient exit decisions occur.

### 3.2 The optimal contract

Throughout the paper we use the following notation: an indicator variable $\mathbb{I}(x)$ is a variable that takes the value $1$ if $\pi < x$ and $\mathbb{I}(x) = 0$ if $\pi \geq x$. Moreover, we also use the notation $\mathbb{I}(x) = 1 - \mathbb{I}(x)$.

To examine the optimal contract, we define

\[
K^* = \int_{p,\pi} \left[ \alpha p \mathbb{I}(p) + \alpha \pi \mathbb{I}(p) \right] d\Omega \\
K^{**} = \int_{p,\pi} \left[ (p - \alpha \pi) \mathbb{I}(p) + \alpha \pi \mathbb{I}(p) \right] d\Omega \\
\bar{K} = \int_{p,\pi} \left[ p \mathbb{I}\left(\frac{p}{1-\alpha}\right) + \alpha \pi \mathbb{I}\left(\frac{p}{1-\alpha}\right) \right] d\Omega
\]

$K^*$ is the level at which $V$ makes zero profits if he always receives a constant fraction $\alpha$ of the returns, and the efficient exit decision is always taken. $K^{**}$ is the level at which $V$ makes zero profits if $E$ always receives exactly $\alpha \pi$ and the efficient exit decision is always taken. $\bar{K}$ is the level at which $V$ makes zero profits if he receives all the returns that can possibly be extracted from the new venture, even if it means inefficient exit decisions are taken. It is easy to verify that $K^* < K^{**} < \bar{K}$.

**Proposition 2**

**Part 1: The optimal contract with a lax finance constraint**

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Suppose $K \leq K^*$ then we can find optimal contracts with the following properties:

(i) Control rights may be held by either $V$ or $E$.

(ii) Cash flows are not contingent on exit, and $\phi(x) = f(x) \leq \pi x$, where $x = \pi, p$.

(iii) The exit decision is always efficient.

**Part 2: The optimal contract with an intermediate finance constraint**

Suppose $K^* < K \leq K^{**}$ then any optimal contract satisfies the following properties:

(i) Control rights are held by $E$.

(ii) Cash flow rights satisfy $f(p) > \pi p$ for at least some $p$.

(iii) The exit decision is always efficient.

**Part 3: The optimal contract with a tight finance constraint**

Suppose $K^{**} < K \leq \bar{K}$ then the optimal contract satisfies the following properties:

(i) Control rights are held by $V$.

(ii) Cash flow rights satisfy $\phi(\pi) = \pi \pi$ and $f(p) > \pi p$ for all $p$.

(iii) In some states of the world the exit decision is inefficient. Specifically, $A$ is chosen even though $I$ is efficient.

The appendix contains an extended numerical example that illustrates the structure of the optimal contract. Proposition 2 shows that there are three distinct contracting regions, depending on the tightness of the financing constraint. If the finance constraint is lax ($K \leq K^*$), financing is relatively straightforward. The incentives of $E$ and $V$ are perfectly aligned with a simple non-contingent contact, and either $E$ or $V$ may retain control. There are no disagreements about ownership and thus no inefficient exit decisions. Note that the optimal contract is not unique since there is
considerable leeway for choosing the functional firm of $\phi(x)$.

For an intermediate finance constraint ($K^* < K < K^{**}$), $V$ requires additional cash flow rights. $V$’s returns in case of $I$ are capped by $\phi(\pi) \leq \overline{\pi} \pi$. In case of $A$, however, $V$ could obtain additional cash flow rights. The optimal contract allocates additional cash flows to $V$ in case of $A$, so that $f(p) > \overline{\pi} p$ for at least some $p$. While $V$ obtains these preferential cash flow rights, he does not, however, obtain control. Proposition 1 then directly implies that the exit decision is always efficient. Again, the optimal contract is not unique, since there are many different ways of allocating enough cash flow rights to $V$.

For a sufficiently tight finance constraint ($K > K^{**}$), the financing problem cannot be solved with cash flow rights alone. $V$ also requires control rights to secure enough returns. The economic cost of this are inefficient exit decisions. As shown in proposition 1, the combination of giving control rights to $V$ and setting $f(p) > \overline{\pi} p$ leads to some inefficient $A$ decisions. Figure 4 shows how an optimal contract implements a pattern of exit decisions in the $(\pi, p)$ space. The optimal contract trades off the benefit of allocating returns to $V$ with the cost of inefficient $A$ decisions. This optimal contract is generically unique.

Proposition 2 establishes the basic structure of the optimal contract. An interesting point to note is that cash flow rights and control rights are separable. For a low finance constraint, a simple non-contingent contract makes control rights unimportant. For an intermediate finance constraint the model predicts a separation of cash flow and control rights. $V$ obtains preferential cash flow rights in case of $A$, but
E retains control. And for a sufficiently tight finance constraint the model predicts the bundling of preferential cash flow rights and control rights, in the hands of V. Interestingly, it is only then that inefficient exit decisions occur.

3.3 The use of convertible preferred equity

In the previous subsection we derived the properties of the optimal contract without reference to any particular securities. The premise of the security design literature is to first derive optimal contracts, and then show how these optimal contracts map into a set of intuitive securities that are used in practice. In this section we therefore examine how securities that are commonly observed in venture capital implement the optimal contract. We immediately state:

**Proposition 3: (Implementing the optimal contract)**

The optimal contract can always be implemented as follows. E receives straight equity and

(i) For $K \leq K^*$, V receives straight equity.

(ii) For $K^* < K \leq K^{**}$, V receives simple convertible preferred equity that automatically converts in case of an IPO.

(iii) For $K^{**} < K \leq K$, V receives generalized convertible preferred equity that automatically converts in case of an IPO.

Proposition 3 shows how the optimal contract can be implemented with simple and intuitive securities that are actually used in venture capital. If the finance con-
straint is lax \((K \leq K^*)\), then straight equity works fine. With straight equity, \(V\) always gets a constant fraction of the returns, and there are no conflicts of interest between \(V\) and \(E\).

For an intermediate finance constraint \((K^* < K < K^{**})\) the constraint \(\phi(\pi) \leq \overline{\pi}\pi\) becomes binding. The optimal contract needs to allocate additional cash flows to \(V\) in case of \(A\). We noted before that the optimal contract is not unique. Proposition 3, however, shows it is always possible to implement an optimal contract using simple CPE. The intuition is that in case of \(I\), \(V\) receives the largest possible incentive-compatible stake \(\phi(\pi) = \overline{\pi}\pi\). In case of \(A\), \(V\) receives some additional cash flow rights in the form of preferred dividends prior to conversion. We can then always choose the terms of the preferred dividends such that \(V\)’s participation constraint is satisfied.

For a tight financing constraint \((K^{**} < K < K)\) the generalized CPE always implements the optimal contract. The structure of the terms prior to conversion are chosen to optimally trade off the benefits of allocating rents to \(V\) with the cost of inefficient acquisitions (see figure 4). The optimal shape of \(f(p)\) may be non-linear as it depends on the two-dimensional distribution of \((\pi, p)\). The most interesting property is that \(f(p) > \overline{\pi}p\) for all \(p\). This says that the cash flow rights are always strictly higher before conversion than after conversion. This property captures the paradoxical nature of participating CPE. Remember that participating CPE has the puzzling property that conversion always implies an absolute loss of cash flow rights. Our optimal security has exactly that same property. The optimal security thus
closely resembles participating CPE. The only difference between the optimal security
and participating CPE is the possible non-linearity of the generalized CPE (see figure
3). This emanates from our very general specification of the distribution \( \Omega(\pi, p) \). We
can think of participating CPE as a linear approximation to our generalized CPE.

3.4 Further discussion of the optimal securities

Propositions 2 and 3 provide a new and joint explanation for the choice between
acquisitions and IPO, the allocation of control rights, and the use of convertible
securities in venture capital. In this subsection we will discuss some further properties
of the optimal securities from Proposition 3.

The fundamental logic of the security design literature is to map an optimal
contract into a set of securities commonly used in practice. This mapping, however,
is rarely unique. First, the optimal contract itself may not be unique. But even for a
given optimal contract, (i.e., a given allocation of cash flow and control rights), there
are typically several ways of combining standard securities to implement the same
optimal contract. This also applies to our analysis. For example, instead of using
a CPE, it would also be possible to use a set of state-contingent options, that gives
\( V \) more (or \( E \) fewer) cash flow rights in case of \( A \). The point, however, is that such
a solution yields identical cash flow and control rights than the solution with CPE.
These are thus different labels for the same solution. It is more natural to therefore
describe this solution in terms of the securities actually used in the venture capital
industry.

A closely related question is whether we can find even simpler securities that
also implement the optimal contract. In particular, could we implement the optimal contract using simple debt or equity contracts that have no convertibility clauses? There are two problems with these types of securities. First, in many cases, they fail to implement the optimal contract. With a simple debt contract, for example, the condition \( f(p) = \text{Min}[D, p] > \alpha p \) is always violated for sufficiently large \( p \). Second, even if we can find non-contingent securities that implement the optimal solution, this solution always entail unnecessary renegotiation. Consider a generic non-contingent security with \( f(x) = \phi(x) > \alpha x \) \((x = p, \pi)\). In case of \( A \), this security allocates cash flow rights beyond \( \alpha x \) to \( V \), which is fine. The problem, however, is that it does the same thing in case of \( I \), which violates the incentive-compatibility constraint. To choose \( I \) it is then always necessary to renegotiate the original contract. Introducing a convertible feature anticipates this problem in the initial contract and thus eliminates the need for renegotiation. Convertible securities are thus more elegant and more robust than their non-contingent counterparts. In fact, we can state an even stronger result.

**Proposition 4: (Ease of Implementation)**

With the securities of proposition 3, no renegotiation is ever necessary in equilibrium.

The main intuition for this result is that, whenever \( I \) is chosen, the CPE with its automatic conversion in case of an IPO naturally anticipates any renegotiation. The only other renegotiation we may have to worry about is if \( E \) has control and prefers \( I \), even though \( A \) is efficient. In the appendix we show how renegotiation may
be avoided in this case too. The idea here is that $C$ can naturally anticipate this renegotiation and prevent it through the structure of the acquisition offer itself.

We noted above that straight debt is inappropriate in this model. However, the optimal CPE is likely to include some debt component prior to conversion. There are two reasons for this: liquidations, and entrepreneurial moral hazard. Consider first liquidation. A risk in many start-ups is that the project is simply not viable, and that it has to be liquidated. We can represent this risk as a mass point in the distribution of $\Omega(\pi, p)$. Specifically, assume that there is a mass point of probability $q$ at $\pi = 0$ and $p = L$, where $L$ is the liquidation value. We immediately state:

**Proposition 5: (A debt component prior to conversion)**

There exists $\hat{q}$, so that for all $q \geq \hat{q} \geq 0$ the optimal CPE always has some debt component prior to conversion, i.e., $f(p) = p$ for all $p \leq D$ where $D \geq L$.

This proposition shows that the existence of a liquidation state directly implies some debt component prior to conversion. The intuition is simply that a debt component allows $V$ to collect all the proceeds in case of liquidation, which is always efficient. Moreover, in the proof of this proposition we show that in many cases $\hat{q} = 0$, so that the use of debt components prior to conversion is indeed very general.

The other argument for the use of some debt component relates to moral hazard considerations not formally modelled here. But it is worth mentioning that adding entrepreneurial moral hazard provides further reasons for the use of debt components prior to conversion. For example, we noted before that for an intermediate finance constraint, there are multiple optimal contracts. If we add entrepreneurial moral
hazard, then it is easy to see that simple CPE is likely to dominate all other contracts. The simple CPE concentrates all the additional cash flow rights in the lower end of the distribution, i.e., for low values of $p$. This helps to combat moral hazard, since the entrepreneur only gets rewarded for sufficiently high value realizations. While this paper deliberately focuses on control problems (which have received less attention in the literature on convertible securities), it is comforting to see that adding moral hazard considerations only strengthens the results.

3.5 Contingent contracting

Propositions 2 and 3 explain some of the core features of a typical venture capital contract. Gompers (1997) and Kaplan and Strömberg (2000) show that venture capital contracts also include a variety of contingency clauses that shift cash flow rights and control rights, depending on some milestones. In this section we extend the model to account for the use of such additional contingency clauses.

Contingency clauses must be defined on verifiable information. The analysis so far assumes that $\pi$ is not verifiable at date 1. We now consider the case where $\pi$ is contractible. This allows us to think about more sophisticated contracts where cash flow and control rights are contingent on the state $(\pi, p)$ at date 1. In the context of the model, contingency clauses are potentially useful only when there are
inefficiencies, i.e., for $K > K^{**}$.

Define

$$K^{**} = \int_{p,\pi} [p \mathbb{I}(p) + \bar{\alpha} \pi \mathbb{I}(p)] \, d\Omega$$

$K^{***}$ is the level at which $V$ makes zero profits if he receives the maximum available cash flows, and the efficient exit decision is always taken. It is easy to verify that $K^{**} < K^{***} < \bar{K}$.

**Proposition 6: (Contingent control rights)**

Suppose $K > K^{**}$ and suppose that $\pi$ is always verifiable at date 1. Let $\lambda \in [\bar{\alpha}, 1]$ be a constant. The optimal contract has the following properties:

(i) $V$ has control if $\lambda \pi < p$ and $E$ has control if $\lambda \pi \geq p$.

(ii) $V$ receives simple CPE.

(iii) For $K \leq K^{***}$, we have $\lambda = 1$ and there are no inefficient exit decisions.

(iv) For $K > K^{***}$, we have $\lambda < 1$ and there are some inefficient $A$ decisions.

The numerical example in the appendix provides some further intuition for this result. Ideally, it is possible to allocate control to $E$ when $I$ is efficient and allocate control to $V$ when $A$ is efficient. This corresponds to contingent control with $\lambda = 1$.

For $K \leq K^{***}$, preferential cash flow rights to $V$ ensure that $V$ obtains sufficient returns. For $K > K^{***}$, $V$ requires additional cash flows. This can only be achieved by extending the range where $V$ has control, i.e., by decreasing $\lambda$. While the optimal contract thus has $\lambda < 1$, we find that there are some inefficient $A$ decisions. Figure

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In a real world situation, there are clearly many other reasons for contingent contracting that for simplicity we leave out of the analysis here.
5 shows the pattern of exit decisions for this case. Even if they do not eliminate all inefficient A decisions, the additional contingent control rights improve contract efficiency. Without them there is the problem that increasing V’s cash flow rights in case of A also increases the range where inefficient A decisions occur. With the additional contingency clauses, it is possible to separately contract on the cash flow rights in case of A, and the range where inefficient A decisions occur.

Proposition 6 looks at contingent control rights, as a way to implement the optimal contract. An alternative way of achieving the same outcome is to use contingent cash flow rights.

**Proposition 7: (Contingent cash flow rights)**

The outcome of proposition 6 can also be achieved with V − control and CPE that automatically converts into straight equity if the contingency $\lambda \pi \geq p$ is satisfied.

The intuition for this result is straightforward. Instead of giving E control to ensure that I is chosen for $\lambda \pi \geq p$, we may alternatively eliminate V’s incentive to choose A by forcing an early conversion of the CPE into straight equity. Gompers (1997) documents that 38% of contracts had some form of early conversion clauses.

Thus, our model can also explain the use of contingent cash flow and/or contingent control rights. The main difficulty with these contingency clauses is that at the time of the initial contract, the two parties need to have a high degree of confidence that the contingencies are indeed verifiable and objective without unfair manipulation. Direct verification, such accounting-based valuations are often very difficult, especially in the context of early-stage start-ups, where assets are mostly intangible.
A more natural way of obtaining a verifiable signal of $\pi$ would be through some market transaction, such as a follow-up financing round. Issuing securities to informed outsiders would generate a verifiable signal of $\pi$. But there may be difficulties with this. The acquisition offers may occur at a time when there is no need to raise funds, so that selling new shares may be inefficient, and selling insider shares may be suspicious for the usual reasons. It may also be that there are no informed outsiders. In fact, $V$ has a vested interest in that. And since $E$ and $V$’s returns are discontinuous at $p = \lambda \pi$, there is ample room for manipulation of bids and signals.\(^{10}\)

4 The role of rent capture

4.1 Rent capture in an independent company

Central to proposition 2 is the assumption that if $I$ is chosen, $E$ can capture some of the rents of the venture. This generates a need for CPE and possibly $V - control$. In this section we further examine the importance of rent capture. We begin by examining some comparative statics. A simple way of assessing the efficiency of contracting is to consider $K^{**}$. The lower $K^{**}$, the less efficient contracts are, in the sense that inefficient exit decisions occur over a larger range of parameters.\(^{11}\) Closely related, $\overline{K}$ measures the feasible range of contracting, so that a lower $\overline{K}$ means fewer

\(^{10}\)Admati and Pfleiderer (1994) show that if there is an informed inside investor, and a set of uninformed outside investors, then a “constant proportion contract” will induce the insider to truthfully reveal his information. Their solution requires that all investors hold straight equity, and that there is a single exit option. The Admati-Pfleiderer result thus does not extend to this model, where different ownership decisions imply different valuations and where the insider has an incentive to strategically price the security issue to obtain control. Finally, note that we cannot use revelation games as in Moore and Repullo (1988) or Maskin and Tirole (1999) to reveal the true state of nature. These games assume either no renegotiation, or risk aversion and no wealth constraints.

\(^{11}\)It is easy to see that the lower $K^{**}$, the bigger also the efficiency loss for every $K \in [K^{**}, \overline{K}]$. 
ventures get funded.

The natural question to ask is how the rent capture problem affects the efficiency and feasibility of contracting. If $E$ creates more rents and then captures them, this will increase her utility without affecting $V$. We will focus instead on unproductive rent capture, where for a given value $\pi$, $E$ is able to capture a larger fraction $\alpha$. A closely related question is how bargaining power might influence the efficiency and feasibility of contracting. So far we have assumed that $V$ has the bargaining power in case of renegotiation. We may ask what happens if $E$ has the bargaining power in case of renegotiation? Yet a third question turns out to be closely related. So far, we have focused on $E$’s ability to hold up some of the cash flows of the venture. One may also ask what the role of non-monetary private benefits might be, such as the prestige of running a company? The following proposition addresses all of these concerns.

**Proposition 8: (Comparative statics on rent capture)**

(i) The larger $E$’s rent capture (i.e., larger $\alpha$), the lower the efficiency of contracting (i.e., lower $K^{**}$) and the lower the feasibility of contracting (i.e., lower $\overline{K}$).

(ii) If $E$, rather than $V$, has the bargaining power in case of renegotiation, then contracting is less efficient (i.e., lower $K^{**}$), but the feasible range is unaffected (i.e., same $\overline{K}$).

(iii) If $E$ enjoys some private benefits $B$ of running the company, then $K^{**}$ is decreasing in $B$, although $\overline{K}$ is independent of $B$.

This proposition provides some intuitive result on rent capture. The larger the
ability to hold up cash flows, as measured by $\alpha$, the more likely that $V$ needs to hold control rights. We either find contracts with more inefficient exit decisions, or else no financing at all. A similar result obtains for bargaining power. If $E$ can extract all the rents from renegotiation, then contracting becomes less efficient. Interestingly though, bargaining power does not affect the feasible region. This is because unlike the hold-up problem, the bargaining power problem can be resolved by allocating control rights to $V$.\textsuperscript{12} The proposition also shows that private non-monetary benefits behave very similar to bargaining power. The important effect of private benefits is to make $E$ less willing to sell the company, even if it makes financial sense. $V$ thus requires control more often, and contracting becomes less efficient. And again we find that giving $V$ the control rights solves this problem, so that the feasible range is not affected by private benefits.

One way of limiting $E$’s hold-up power is to replace her in the position of CEO with a professional manager. In a sample of young Silicon Valley firms, Hellmann and Puri (2000) find that a little over 50% of all founders get replaced with professional managers. Obviously, replacing $E$ may only transfer to the opportunity to capture rents from $E$ to the newly appointed CEO, but how would this affect our results?

Hellmann (1998) develops a formal model of founder replacement, and we adopt a simplified version here. In particular, assume that if the company remains independent, a better CEO can be identified with probability $m$. This new CEO increases the expected returns from $\pi$ to $\mu \pi$, but (s)he will also be able to extract a fraction

\textsuperscript{12}If $E$ and $V$ can determine ex-ante how they will renegotiate ex-post, then they will always give $V$ all the bargaining power. This motivates our default assumption that $V$ has all the bargaining power.
\( \alpha_{\text{new}} \) of those returns. The net improvement in returns to \( E \) and \( V \) is \((\mu \alpha_{\text{new}} - 1)\pi\), and we assume that \( \mu \alpha_{\text{new}} > 1 \), so that bringing in a new CEO is efficient. With probability \((1 - m)\) no better CEO is found.

**Proposition 9: (Replacing the entrepreneur)**

Suppose that under \( I \), there is a probability \( m \) that \( E \) is replaced with a professional manager. Proposition 2 continues to hold as long as \( m < 1 \). Higher values of \( m \) and \( \mu \) increase \( K^{**} \) and \( \overline{K} \), thus increasing efficiency and feasibility of contracting.

The intuition for this result is immediate. The fact that \( E \) may be replaced by a professional manager reduces her hold-up power of \( E \), but as long as \( m < 1 \), it does not eliminate it. The possibility of replacement, however, does improve the efficiency and feasibility of contracting. This can be seen from the fact that \( K^{**} \) and \( \overline{K} \) are increasing in \( m \) and \( \mu \).

Another issue is that even if the founder is replaced by a professional manager, (s)he may still have some residual hold-up power. Hellmann and Puri (2000) find that in about 40\% of all replacements, the founder retained some function in the company, such as Chief Technology Officer or Chair(wo)man of the Board. It seems likely that these positions continue to afford the founder with some on-going power. This is especially true if the founder becomes a company spokesperson: think, for example, of Jerry Yang, the co-founder of Yahoo.

Finally note that we have equated a strategy of being independent with an IPO. An independent company doesn’t have to go public, but could instead remain private. This, however, might pose a problem for the venture capitalist, who cares
about liquidity. Black and Gilson (1998) conjecture that the existence of an active stock market increases contracting efficiency, by giving the venture capitalist liquidity while allowing the entrepreneur to remain in control of her company. Our analysis formalizes their hypothesis, but also clarifies that even with an active stock market the first-best efficient outcome may not always be attained.

4.2 Rent capture in case of acquisitions

A different concern is that in case of an acquisition, $E$ may still have a limited amount of power, most notably during the transition period. Moreover, $C$ may not necessarily be able to fully solve the hold-up problem internally. There may be some internal rent capture, where the new division manager acquires power by virtue of managing the acquired unit.

We now include the possibility that in case of $A$, $E$ has some power. Specifically, we assume that she can extract a fraction $\alpha_A$ of the acquisition price $p$. Moreover, the new division manager is also able to extract some fraction of the rents. Let $P$ be the total value created by the acquisition and let $\beta$ be the fraction extracted by the division manager. The value of the acquisition to $C$ is then $\beta P$. This gets reflected in the acquisition price, which is now given by $p = \beta P$.

Proposition 10: (Hold-up power in acquisitions)

(i) As long as $\alpha_A < \alpha$, Proposition 2 continues to hold. Higher values of $\alpha_A$ decrease the efficiency and feasibility of contracting, i.e., they decrease $K^{**}$ and $\overline{K}$.

(ii) For a given distribution of $p$, $\beta$ has no effect on the optimal contract.
The possibility that $E$ may also capture some rents in case of acquisitions does not affect the main results, provided that the rent capture in case of $A$ is smaller than in case of $I$, i.e., $\alpha_A < \alpha$. Hold-up power in the transition to $A$ also makes financing the new venture more difficult, as witnessed by the decrease in $K^{**}$ and $\overline{K}$.

An interesting point to note is that for a given distribution of $p$, $\beta$ plays no role whatsoever in the optimal contract. This simply says that $C$'s extraction problem is not relevant to $E$ and $V$. All they care for is the acquisition price $p$. This price reflects the value to $C$'s shareholders, net of any rent capture to the division manager. Naturally, $\beta$ can have an effect on the optimal contract if it affects the distribution of $p$. For a given distribution of $P$, the larger $\beta$, the lower the expected returns to $E$ and $V$, and thus the lower $K^{**}$ and $\overline{K}$.\textsuperscript{13}

5 Conclusion

This paper asks how entrepreneurial companies choose between two distinct ownership solutions: get acquired or go public. Venture capital contracts allocate cash flow and control rights ex-ante to influence this ex-post exit decision. The paper derives the optimality of convertible preferred equity in this context. It also provides an explanation for some puzzling contract features, such as automatic conversion in case of an IPO, or the use of participating preferred equity.

\textsuperscript{13}It is worth noting that in the last two subsections we introduced a divergence between private and social efficiency. In the replacement model, $E$ and $V$ do not care about the rents captured by the new CEO. And with acquisitions, $E$ and $V$ do not care about the rents captured by the new division manager. In the first case $I$ becomes socially more valuable, and in the second case $A$ becomes more valuable. In a more general model there may be yet other parties whose utility is affected by the exit decision.
The analysis yields a number of new testable implications. The model makes some detailed predictions about the structure of venture capital contracts. It identifies three distinct contracting regimes, as a function of the finance constraint. For a mild finance constraint it is optimal to use straight equity, and control rights don’t matter. For an intermediate finance constraint, the investor obtains convertible preferred equity, but the entrepreneur retains control rights. For a tight finance constraint, the investor obtains both convertible preferred equity and control. The tighter the finance constraint, the stronger the preferred terms. The use of participating preferred equity is associated with a tight finance constraint. The model also makes predictions about the relative frequencies of exit through acquisition versus IPO. For example, the tighter the finance constraint, the more we would expect exit by acquisitions rather than IPO. And the model predicts different outcomes depending on the rent extraction potential for the entrepreneur. The easier it is to extract rents, the fewer ventures can get financed, the stronger the preferred terms, and the higher the proportion of acquisitions to IPOs.

The paper examines the problem of writing contracts that govern how future ownership decisions will be made. This is a significant departure from the standard assumption that exit decisions are made ex-ante. The notion that contracts can govern how future exit decisions are made opens up a whole new set of interesting research questions. What are the dynamics of exit decisions? When is there disagreement about the optimal timing of exit decisions? And how can contracts resolve these problems? We hope to address these issues in future research.
6 Appendix

6.1 Proof of Proposition 1

If $\pi < p$, then $E$ and $V$ can always choose $A$ and allocate the surplus between themselves. If $p < \alpha \pi$, then they can always choose $I$ and allocate the transferable surplus $\alpha \pi$ between themselves. The critical case is thus $\alpha \pi < p < \pi$, where $I$ yields the highest joint surplus, but $A$ has a higher transferable surplus. If $f(p) < \phi(\pi)$, then neither $E$ nor $V$ want to choose $A$ and $I$ occurs. But if $f(p) > \phi(\pi)$, then the outcome depends on who has control. Suppose first that $E$ has control. Since $\pi - \phi(\pi) > p - f(p)$, $E$ does not want to choose $A$, and since $I$ is efficient, there is no renegotiation. If $V$ has control, he wants to choose $A$. The highest utility he could obtain from negotiation to $I$ is $\alpha \pi$. Renegotiation is feasible iff $f(p) \leq \alpha \pi$. For $f(p) > \alpha \pi$, however, no renegotiation can make $V$ better off.

6.2 Proof of Proposition 2

For $K \leq K^*$ let $\phi(x) = f(x) \leq \alpha x$, $x = \pi, p$. Both $E$ or $V$ choose $A$ iff $p > \pi$ and since there is no disagreement between $E$ and $V$, there is never any need to renegotiate. $\phi(x)$ is chosen so that $V$’s ex-ante utility, denoted by $U_V$, satisfies

$$U_V(\phi) = \int_{\pi, \pi} [\phi(p) I(p) + \phi(\pi) I(\pi)] d\Omega - K = 0$$

This is feasible for $\phi(x) \leq \alpha x$. It is easy to verify that $U_V(\phi(x) = \alpha x) = 0$ at $K^*$.

For $K > K^*$ we can increase $V$’s utility by setting $\phi = \alpha$ and increasing $f(p)$.
beyond $\pi p$. Throughout the appendix we will use the notation

$$f(p) = \overline{\alpha} p + d(p)$$

so that $d(p)$ measures the strength of preferred terms. Under $E-\text{control}$ we continue to have $A$ iff $p > \pi$. Renegotiation occurs when $E$ inefficiently prefers $I$ over $A$, namely when $\pi < p$ and $\alpha \pi > p - f(p) = \alpha p - d(p) \iff \pi > p - \frac{d(p)}{\alpha}$. (In proposition 3 we will show how a minor modification allows us to obtain exactly the same outcome, but without renegotiation actually occurring in equilibrium.) In this case, $V$ obtains $\overline{\alpha} \pi + p - \pi$ after renegotiation, so that $V$’s returns are given by

$$U_V(d(p)) = \int_{p,\pi} \left[ (\overline{\alpha} p + d(p)) \mathbb{1}(p) \mathbb{1}(p - \frac{d(p)}{\alpha}) + (\overline{\alpha} \pi + p - \pi) \mathbb{1}(p) \mathbb{1}(p - \frac{d(p)}{\alpha}) + \overline{\alpha} \pi \mathbb{1}(p) \right] d\Omega - K$$

Since $\overline{\alpha} p + d(p) = \overline{\alpha} \pi + p - \pi$ at $\pi = p - \frac{d(p)}{\alpha}$, $U_V(d(p))$ is increasing in $d(p)$ for every value of $p$. Joint control is feasible up until the highest possible value of $d(p)$, given by $\overline{d}(p) = \alpha p$. It is easy to verify that $U_V(\overline{d}) = 0$ at $K^{**}$.

Note that for $K < K^{**}$, the exact shape of $d(p)$ does not matter, as long as it satisfies $U_V(d(p)) = 0$. Note that it is even possible (but not particularly useful) to have $d(p) < 0$ for some $p$, or $\phi(\pi) < \overline{\alpha} \pi$ for some $\pi$. The general point is that the optimal contract is not unique for $K^* < K < K^{**}$.

For $K > K^{**}$, $V-\text{control}$ is necessary. $V$ chooses $A$ whenever $\overline{\alpha} p + d(p) > \overline{\alpha} \pi$. As a consequence inefficient $A$ decisions occur for $p < \pi < p + \frac{d(p)}{1 - \alpha}$. $d(p)$ must satisfy $V$’s participation constraint.
\[ U_V(d(p)) = \int_{p,\pi} \left[ (\alpha p + d(p)) \Pi(p + \frac{d(p)}{1-\alpha}) + \frac{d(p)}{1-\alpha} \Pi(p + \frac{d(p)}{1-\alpha}) \right] d\Omega - K = 0 \]

\( U_V(d(p)) \) is increasing in \( d(p) \), and it is easy to verify that with \( V \)-control, \( U_V(\overline{d}(p)) = 0 \) at \( \overline{K} \). Note that for \( K^{**} < K < \overline{K} \) the optimal choice of \( d(p) \) can be found by maximizing

\[ U_E + U_V = \int_{p,\pi} \left[ p \Pi(p + \frac{d(p)}{1-\alpha}) + \frac{d(p)}{1-\alpha} \Pi(p + \frac{d(p)}{1-\alpha}) \right] d\Omega - K \]

w.r.t. \( d(p) \), subject to \( U_V = 0 \).

To see that \( f(p) > \overline{\pi}p \) for all \( p \), consider the effect of increasing \( f(p) = \overline{\pi}p + d(p) \) at \( d(p) = 0 \). We have

\[ \frac{\partial(U_E + U_V)}{\partial d(p)} = \frac{\bar{\lambda} \bar{\Omega}}{1-\alpha} - \frac{\omega(\bar{\pi},p)}{1-\alpha} (\bar{\pi} - p) = \frac{\bar{\lambda} \bar{\Omega}}{1-\alpha} - \frac{\omega(\bar{\pi},p)}{1-\alpha} \frac{d(p)}{1-\alpha} \]

where \( \bar{\pi} = \bar{\pi}(p) = p + \frac{d(p)}{1-\alpha}, \bar{\Omega} = \bar{\Omega}(p) = \int_{\pi=\bar{\pi}} d\Omega(\pi, p) \) and \( \bar{\lambda} \) the Lagrangian constant.

At \( d(p) = 0 \), we have \( \frac{\partial(U_E + U_V)}{\partial d(p)} \geq 0 \), so that one can only increase the efficiency by raising \( d(p) \). W.l.o.g. the optimal \( d(p) \) thus always satisfies \( d(p) > 0 \Leftrightarrow f(p) > \overline{\pi}p \).

If standard concavity is satisfied, we can characterize the optimal solution of \( d(p) \) by its first-order condition of the Lagrangian. For every \( p \), \( d(p) \) must satisfy

\[ \frac{\bar{\lambda} \bar{\Omega}}{1-\alpha} - \frac{\omega(\bar{\pi},p)}{1-\alpha} (\bar{\pi} - p) = 0 \]
The first term measures the increased rents that flow to \( V \) (for all \( \pi \in [0, \bar{\pi}] \)) whereas the second term measures cost of raising \( d(p) \) in terms of inefficient exit decisions. This thus shows the basic trade-off between the need to allocate rents to \( V \) and the cost of inefficient exit decisions. Note that we can also rewrite this as

\[
d(p) = \alpha \lambda \frac{\Omega}{\omega(\pi, p)}
\]

which shows that the optimal shape of \( d(p) \) depends on the distribution \( \Omega(\pi, p) \), and may be non-linear.

To finish the proof note that we have used \( \phi(\pi) \), but in theory it would be possible to also condition this on \( p \), i.e., \( \phi(\pi; p) \). Such conditioning, however, is never useful. For \( K < K^{**} \) the optimal contract already implements the first-best outcome. And for \( K > K^{**} \), \( \phi(\pi) \leq \overline{\pi} \pi \) is a binding constraint that is independent of \( p \).

6.3 Proof of Proposition 3

For \( K < K^* \), it is immediate that straight equity implements the optimal contract. For \( K^* < K < K^{**} \), we use the definition of simple CPE, \( f(p) = Max[Min[p, D], C\overline{p}] \), to get \( d(p) = Max[Min[p, D] - C\overline{p}, 0] \). For all values of \( p \), this is a continuous and increasing function of \( D \). Moreover, at \( D = 0 \), we have \( U_V(d(p)) = K^* \) and at \( D = \overline{p} \), we have \( U_V(d(p)) = K^{**} \). Using the continuity and monotonicity of \( U_V(d(p)) \) it follows that for every \( K \in [K^*, K^{**}] \) there exists some \( D \in [0, \overline{p}] \), so that \( U_V = K \). We can thus always implement the optimal contract with a simple CPE. Finally, for \( K > K^{**} \), the optimal contract requires \( f(p) > C\overline{p} \) and \( \phi(\pi) = \overline{\pi} \pi \).
Applying the definition of the generalized CPE at $\tilde{\phi} = \pi$, we have $\phi(\pi) = \pi \pi$ and $f(p) = Max[\tilde{f}(p), \pi p]$. We can thus choose $\tilde{f}(p) = f(p) > \pi p$ to implement the optimal contract.

### 6.4 Proof of Proposition 4

We immediately see that renegotiation never occurs in equilibrium for $K < K^*$, since the optimal contract itself involves no renegotiation. For $K^* < K < K^{**}$, our formulation of proposition 2 had some renegotiation. This renegotiation, however, occurs only in case of $A$, and consists of $V$ making some concessions to $E$, in order to induce $E$ to choose $A$ over $I$. This renegotiation can be avoided in equilibrium if we allow $C$ to make slightly more sophisticated acquisition offers. In particular, $C$ may offer two options: a standard acquisition at $p$, and an alternative offer. This alternative offer would also have a total price of $p$, but it would propose that $E$ and $V$ receive different shares than those specified by their original contract. In particular, the alternative offer simply specifies the returns to $E$ and $V$ that would obtain after renegotiation. $E$ and $V$ are thus willing to sign this alternative offer without any need for renegotiation. This slightly more sophisticated deal structure thus eliminates renegotiation by folding the outcome of any renegotiation into the original acquisition offer. Finally, for $K > K^{**}$, $V$ has control. Renegotiation can only occur if $E$ can propose a more efficient decision than the one preferred by $V$. $E$ will never want to renegotiate if $I$ is chosen. And if $A$ is chosen, then we know from $\phi(\pi) = \pi \pi$ that $E$ has no means to make $V$ a more attractive offer for choosing $I$. Hence no renegotiation occurs in equilibrium.
6.5 Proof of Proposition 5

Suppose that there is a probability \( q \) of failure, in which case the efficient decisions is to liquidate for \( p = L \). With probability \( (1 - q) \), however it is not a failure, and some distribution \( \Omega'(p, \pi) \) applies. For \( K^* < K < K^{**} \), the exact structure of \( f(p) \) does not matter, so an optimal contract can always include a debt component. For \( K^{**} < K < \overline{K} \) we simply revisit the maximization problem for \( d(p) \). As before, the optimal choice of \( d(p) \) maximizes \( U_E + U_V \), subject to \( U_V = 0 \). Suppose \( f(p) = p \) at \( p = L \), and consider a decrease in \( f(p) \). The net benefit of lowering \( f(p) \) (or equivalently, \( d(p) \)) at \( p = L \) is now given (after some simple transformations) by

\[
(1 - q) \frac{\omega'(\pi, p)}{1 - \alpha}(\pi - p) - \frac{\lambda}{1 - \alpha}(q + (1 - q)\overline{\Omega'})
\]

where \( \pi = \pi(p) = p + \frac{d(p)}{1 - \alpha} \) and \( \overline{\Omega'} = \overline{\Omega}'(p) = \int_{\pi=\pi}^{\pi=\pi} d\Omega'(\pi, p) \). This is a decreasing function of \( q \), and we find the critical level \( \hat{q} \) by equating the two terms to zero, so that

\[
\hat{q} = \text{Max}[0, \frac{\omega'(\pi, p)d(p) - \overline{\pi}\overline{\lambda}\overline{\Omega'}}{\omega'(\pi, p)d(p) + \overline{\pi}\overline{\lambda}(1 - \overline{\Omega'})}]
\]

where \( p \) evaluated at \( p = L \). For \( q > \hat{q} \), it is never optimal to set \( f(p) < p \) at \( p = L \). The optimal contract thus always satisfies \( f(p) = p \) for all \( p \leq D \), where \( D \geq L \).

In many case we might even expect that \( \hat{q} = 0 \). This is because it is unlikely that inefficient exit decisions occur in the neighborhood of \( D = L \). They can only occur in the interval \( \pi \in [L, \frac{L}{1 - \alpha}] \). But for such low values, it may well be impossible to create a viable stand-alone business. This is particularly true if we take into
consideration that in order to do an IPO, companies have to meet some minimum size requirement. Formally, a sufficient condition for $\hat{q} = 0$ is that $\omega'(\pi, L) = 0$ for all $\pi \in [L, \frac{L}{1-\alpha}]$.

### 6.6 Proof of Proposition 6

Consider a CPE with $\phi(\pi) = \pi$ and $f(p) \geq \pi p$. Suppose first that $\lambda = 1$. For $\pi \geq p$, $E$ always chooses the efficient outcome $I$ and for $\pi < p$, $V$ always chooses $A$. $U_V$ is given by

$$U_V = \int_{p, \pi} [f(p) \mathbb{1}(p) + \pi \mathbb{1}(p)] \ d\Omega - K$$

If $f(p) = p$, then $U_V = 0$ at $K = K^{**}$. For any $K \leq K^{**}$ we thus have no inefficient $A$ decisions. For $K > K^{**}$, however, the only way of increasing $V$’s utility is to decrease $\lambda$. We allocate maximal cash flows to $V$ by setting $f(p) = p$ and $\phi(\pi) = \pi$. Inefficient $A$ decisions occur for $p < \pi < \frac{p}{\lambda(p)}$. To show that the optimal $\lambda$ is a constant, we maximize

$$U_E + U_V = \int_{p, \pi} [p \mathbb{1}(\frac{p}{\lambda(p)}) + \pi \mathbb{1}(\frac{p}{\lambda(p)})] \ d\Omega - K$$

w.r.t. $\lambda(p)$, subject to

$$U_V = \int_{p, \pi} [p \mathbb{1}(\frac{p}{\lambda(p)}) + \pi \mathbb{1}(\frac{p}{\lambda(p)})] \ d\Omega - K = 0$$

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From the first-order condition of the Lagrangian, for every \( p \), \( \lambda(p) \) must satisfy

\[
\frac{\tilde{\lambda} p - \alpha \tilde{\pi}}{1 - \alpha} \omega(\tilde{\pi}, p) - \frac{\tilde{\pi} - p}{1 - \alpha} \omega(\tilde{\pi}, p) = 0
\]

where \( \tilde{\pi} = \frac{p}{\lambda(p)} \) and \( \tilde{\lambda} \) the Lagrangian multiplier. (Note that \( \lambda(p) \) and \( \tilde{\lambda} \) are two distinct variables!) After transformations we find \( \lambda(p) = \frac{1 + \alpha \tilde{\lambda}}{1 + \lambda} \), so that the optimal \( \lambda(p) \) is constant in \( p \). Moreover, for \( \tilde{\lambda} = 0 \) we have \( \lambda = 1 \) and for \( \tilde{\lambda} \to \infty \) we have \( \lambda \to \alpha \), showing that the tighter the finance constraint (as expressed by the Lagrangian \( \tilde{\lambda} \)) the lower is \( \lambda \).

### 6.7 Proof of Proposition 7

Consider a CPE that automatically converts to common equity whenever \( \lambda \pi \geq p \). We have \( \phi(\pi) = \alpha \pi, f(p) = p \) for \( \lambda \pi < p \) and \( f(p) = \alpha p \) for \( \lambda \pi \geq p \). Suppose has \( V \) has full control. He will choose \( A \) whenever \( \lambda \pi < p \), since \( f(p) = p > \lambda \pi \geq \alpha \pi = \phi(\pi) \). For \( \lambda \pi \geq p \), however, he will choose \( I \), since \( \phi(\pi) = \alpha \pi \geq \frac{\alpha \pi}{\lambda} \geq \alpha p = f(p) \). This implements exactly the same outcome as with contingent control rights.

### 6.8 Proof of Proposition 8

Part (i) is immediate from taking the derivatives of \( K^{**} \) and \( K^{*} \). For part (ii) we examine the case where \( E \) has all the bargaining power in case of renegotiation. For \( K < K^{*} \) there is never any renegotiation, so the model is unchanged. For \( K > K^{*} \), however, renegotiation occurs. Suppose that \( E \) has control. From proposition 1 we know that the efficient exit decision is always reached. Consider a contract
with \( f(p) = \alpha p + d(p) \) and \( \phi(\pi) = \pi \alpha \). Suppose that \( p > \pi \), so that \( A \) is chosen in equilibrium. \( E \), however, may not agree to this before renegotiation. Prior to renegotiation, \( E \) gets \( p - f(p) = \alpha p - d(p) \) in case of \( A \) and \( \alpha \pi \) in case of \( I \). For \( \alpha p - d(p) > \alpha \pi \Leftrightarrow \pi < p - \frac{d(p)}{\alpha} \) there is no renegotiation, but for \( \pi > p - \frac{d(p)}{\alpha} \), \( E \) can now extract all the surplus from renegotiation, giving her \( p - \pi \alpha \) and leaving \( V \) with \( \pi \alpha \). We thus have

\[
U_V (d(p)) = \int_{p, \pi} [((\alpha p + d(p)) I(p) \beta (p - \frac{d(p)}{\alpha}) + \pi \alpha \beta (p) \beta (p - \frac{d(p)}{\alpha}) + \pi \alpha \beta (p)] d\Omega - K
\]

Unlike before, increasing \( d(p) \) may not always increase \( U_V \): on the one hand it increase \( V \)'s returns if there is no renegotiation, but on the other hand it also increases the range where renegotiation occurs, which reduces \( V \)'s returns. We choose \( d(p) \) to maximize \( U_V (d(p)) \) (subject to \( d(p) \leq \alpha p \)), and denote the solution by \( d^{**}(p) \). We then define \( K^{**} \) from \( U_V (d^{**}(p)) = 0 \). Again, \( K^{**} \) is the highest level of \( K \) for which \( V \) can make zero profits under \( E - control \). It is immediate that this critical level is lower if \( E \) rather than \( V \) has the bargaining power in case of renegotiation.

For \( K > K^{**} \), \( V \) control is necessary, and we find some inefficient \( A \) decisions. The maximal level of \( K \), given by \( \overline{K} \), however does not depend on whether \( E \) or \( V \) has bargaining power. This is because for \( K = \overline{K} \), the optimal contract allocates the control rights and maximal cash flow rights to \( V \), so that no renegotiation ever occurs.

For part (iii) we examine the case where \( E \) also derives some private benefits from managing the company. Suppose that if \( I \) is chosen, \( E \) now also enjoys a private,
non-monetary benefit \( B > 0 \). For simplicity we assume that \( B \) is a constant, although this assumption is not necessary. The model with \( B > 0 \) is slightly different from proposition 2 in that control rights always matter, even for low values of \( K \). To see this, suppose \( V \) holds a fraction \( \bar{\phi} \) of straight equity, then he will prefer \( A \) over \( I \) whenever \( \bar{\phi}p > \bar{\phi}\pi \), i.e., whenever \( p > \pi \). \( E \) prefers \( A \) over \( I \) for \((1-\bar{\phi})p > (1-\bar{\phi})\pi + B\), i.e., whenever \( p > \pi + \frac{B}{1-\bar{\phi}} \).

We solve the model by first considering \( E - control \). We will see that this is always optimal for low values of \( K \). Since \( V \) is not wealth constrained, an efficient exit decision is always taken, possibly after renegotiation. The efficient exit decisions are now that \( A \) is chosen for \( p > \pi + B \), and \( I \) for \( p < \pi + B \). We assume again that \( V \) has all the bargaining power in case of renegotiation. The most favorable contract for \( V \) under \( E \) control is \( \phi(\pi) = \alpha\pi \) and \( f(p) = p \). \( E - control \) is feasible for \( K < K^{**} \) where

\[
K^{**} = \int_{p,\pi} \left((p - \alpha\pi - B) \cdot (p - B) + \alpha\pi \cdot (p - B)\right) d\Omega
\]

Note that this is a decreasing function of \( B \). The larger \( B \), the more reluctant is \( E \) to accept an \( A \) decision. As a consequence, the range where \( V \) requires control rights is larger. The maximal value \( \overline{K} \), however, is independent of \( B \), and for \( K > K^{**} \) the model is very similar than before. This is because \( V \) does not have to consider \( B \) if he has control. The only difference is that the optimal \( d(P) \) now maximizes
\[ U_E + U_V = \int_{p,\pi} \left[ p \mathbb{I}(p + \frac{d(p)}{1-\alpha}) + (\pi + B) \mathbb{I}(p + \frac{d(p)}{1-\alpha}) \right] d\Omega - K \]

This changes the optimal shape of \( d(p) \), but does not change the basic structure of the optimal contract.

### 6.9 Proof of Proposition 9

The joint value for \( E \) and \( V \) of pursuing \( I \) is now given by

\[ \pi_m \equiv m\mu_{\text{new}}\pi + (1 - m)\pi \]

Consider first giving \( \bar{\alpha}\% \) of straight equity to \( V \). And suppose that in case of replacement, the new manager is allocated new equity. In this case control doesn’t matter, since both \( E \) and \( V \) would both choose \( A \) for \( p > \pi_m \) and \( I \) otherwise. Using \( p > \pi_m \Leftrightarrow p \frac{\pi}{\pi_m} > \pi \), it follows that

\[ K^* = \int_{p,\pi} \left[ \mu_{\text{new}} \mathbb{I}(p \frac{\pi}{\pi_m}) + \bar{\alpha}\pi_m \mathbb{I}(p \frac{\pi}{\pi_m}) \right] d\Omega \]

For \( K > K^* \) we want to allocate additional cash flows to \( V \). In case of replacement, it is easy to allocate all the available returns \( \mu_{\text{new}}\pi \) to \( V \) rather than to \( E \). This essentially means that \( E \)’s equity has not yet vested at the time of replacement (see Hellmann (1998) for a more extensive discussion on vesting). Moreover, \( V \) may receive CPE. The maximal value that \( V \) can extract under \( I \) is
\[ \tilde{\pi}_m \equiv m \mu \tilde{\alpha}_{new} \pi + (1 - m) \pi \mu \]

Suppose that \( E \) is in control, then the exit decision is always efficient. We have

\[ K^{**} = \int_{p,\pi} \left[ (p - (1 - m) \alpha \pi) \mathbb{I}(p \frac{\pi}{\pi_m}) + \tilde{\pi}_m \mathbb{I}(p \frac{\pi}{\pi_m}) \right] d\Omega \]

For \( K > K^{**} \), \( V - \) control is necessary. Inefficient exit decisions may be taken, and we have

\[ \overline{K} = \int_{p,\pi} \left[ p \mathbb{I}(p \frac{\pi}{\pi_m}) + \tilde{\pi}_m \mathbb{I}(p \frac{\pi}{\pi_m}) \right] d\Omega \]

Note that for \( m < 1 \), we have \( K^* < K^{**} < \overline{K} \), so that proposition 2 continues to hold. Moreover, \( K^* \), \( K^{**} \) and \( \overline{K} \) are all increasing in \( m \) and \( \mu \), so that having the option of replacing \( E \) increases the range of efficient and feasible contracting.

### 6.10 Proof of Proposition 10

Consider giving \( \overline{\alpha} \% \) of straight equity to \( V \), where \( \overline{\alpha} = Min[\alpha_A, \overline{\alpha}] \). In this case control doesn’t matter, since both \( E \) and \( V \) would choose \( A \) for \( p > \pi \) and \( I \) otherwise.

It follows that \( K^* \) is given by

\[ K^* = \int_{p,\pi} \left[ \overline{\alpha} p \mathbb{I}(p) + \overline{\alpha} \pi \mathbb{I}(p) \right] d\Omega \]

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For $K > K^*$ we want to allocate additional cash flows to $V$. The maximal value that $V$ can extract is $\bar{\alpha} \pi$ under $I$ and $\bar{\alpha} A \pi$ under $A$. Suppose that $E$ is in control, then the exit decision is always efficient. We have

$$K^{**} = \int_{p, \pi} [(p - \text{Max}[\alpha A p, \alpha \pi]) \bar{I}(p) + \bar{\alpha} \pi \bar{I}(p)] d\Omega$$

For $K > K^{**}$, $V - control$ is necessary, inefficient exit decisions may be taken, and we have

$$\overline{K} = \int_{p, \pi} [\alpha A p \bar{I}(p) \frac{1 - \alpha A}{1 - \alpha}) + \bar{\alpha} \pi \bar{I}(p) \frac{1 - \alpha A}{1 - \alpha})] d\Omega$$

It is easy to verify that $K^* < K^{**} < \overline{K}$ if and only if $\alpha A < \alpha$. In this case, proposition 2 continues to hold. Moreover, $K^{**}$ and $\overline{K}$ are both decreasing in $\alpha A$.

For part (ii) simply note that for a given distribution of $p$, $\beta$ does not enter anywhere in the optimal contract. For a given distribution of $P$, however, $\beta$ will shift down the distribution $p$.

### 6.11 A numerical example

To provide some further intuition for the main results of this section, we develop a series of simple numerical example. In particular, we assume that the venture is a failure with probability $q = \frac{1}{3}$ (Sahlman (1990) reports that approximately 1 in 3 venture capital investments fails). In this case it generates a liquidation value $L = 0.2$. With probability $(1 - q)$, it is not a failure. The expected return in case if $I$ is given by
a random variable $\pi$ that is distributed uniformly over the interval $[0.5, 1.5]$, so that the expected value is 1. In case of $A$, we assume that the expected value is simply the constant $p = 1$. For the rent capture coefficient, we use $\alpha = 0.1$. Obviously, the numerical examples are not meant to provide a realistic description of actual contracts, but merely seek to clarify the intuition for our results. All calculations were performed in Excel, and are available by the author upon request.

We first illustrate ex-post bargaining over exit decisions. Consider a simple CPE contract where $f(p) = \text{Max}[\text{Min}[p, D], \pi p]$ and $\phi(\pi) = \pi p$. Consider $D = 0.92$. In case of $A$, $V$ does not want to convert since $\text{Min}[p, D] = \text{Min}[1, 0.92] = 0.92 > 0.9 = \pi p$. Consider the case where the venture has not failed and an acquisition offer $p = 1$ is on the table. Suppose first that $\pi = 0.98$, so that choosing $A$ is efficient. $V$ gets $D = 0.92$ in case of $A$ and $\pi \pi = 0.882$ in case of $I$. $E$ gets $p - D = 0.08$ in case of $A$ and $\alpha \pi = 0.098$ in case of $I$. If $V$ controls the exit decision, he simply chooses $A$. If $E$ controls the exit decision, she would want to choose $I$ prior to renegotiation. However, $V$ can simply offer to reduce $D = 0.92$ to $D' = 0.902$, so that $p - D' = 0.098$. $E$ is now willing to choose $A$ over $I$, and an efficient exit decision is made. (In proposition 4 we also show how this renegotiation can be avoided if $C$ folds this renegotiation directly into the original acquisition offer.) Suppose next that $\pi = 1.02$, so that $I$ is more efficient. If $A$ is chosen, $E$ receives $p - D = 0.08$ in case of $A$ and $\alpha \pi = 0.102$ in case of $I$, so that under $E - control$ $I$ is chosen over $A$. But if $V$ controls the exit decision, he gets 0.92 in case of $A$ and $\pi \pi = 0.918$ in case of $I$. He thus prefers $A$ over $I$, even though $A$ is inefficient. No renegotiation is possible. This shows the
main intuition for proposition 1.

Consider now the optimal ex-ante contract of proposition 2 and 3. A straight equity contact with $\tilde{\phi} = 0.8$, for example would yield the following expected utilities.

$$U_V = \frac{1}{3} \cdot 0.8 \cdot 0.2 + \frac{2}{3} \left( \int_{\pi=0.5}^{\pi=1} 0.8 + \int_{\pi=1}^{\pi=1.5} 0.8\pi \right) - K = 0.6533 - K$$

$$U_E = \frac{1}{3} \cdot 0.2 \cdot 0.2 + \frac{2}{3} \left( \int_{\pi=0.5}^{\pi=1} 0.2 + \int_{\pi=1}^{\pi=1.5} 0.2\pi \right) = 0.1633$$

Straight equity is feasible for $K \leq K^*$ where

$$K^* = \frac{1}{3} \cdot 0.9 \cdot 0.2 + \frac{2}{3} \left( \int_{\pi=0.5}^{\pi=1} 0.9 + \int_{\pi=1}^{\pi=1.5} 0.9\pi \right) = 0.735$$

For $K > 0.735$, however, $V$ requires additional cash flows. CPE allows $V$ to access these cash flows. We will focus here on simple CPE, although other types of CPE could also be used. Suppose first that $E$ retains control. Setting $D = 0.92$, for example, is optimal at $K = 0.747$. To see this note that under $E-control$ renegotiation is necessary for $p > \pi$ but $p - D = 0.08 < \alpha\pi = 0.1 \cdot \pi \leftrightarrow \pi > 0.8$. In this case, $V$ reduces $D$ to $D' = p - \alpha\pi$. The expected utilities are thus given by (the subscript indicates the recipient agent, the superscript indicates the controlling agent)

$$U^E_V = \frac{1}{3} \cdot 0.2 + \frac{2}{3} \left( \int_{\pi=0.5}^{\pi=0.8} 0.92 + \int_{\pi=0.8}^{\pi=1} 1 - 0.1\pi + \int_{\pi=1}^{\pi=1.5} 0.9\pi \right) - K = 0.747 - K$$

$$U^E_E = \frac{1}{3} \cdot 0 + \frac{2}{3} \left( \int_{\pi=0.5}^{\pi=0.8} 0.08 + \int_{\pi=0.8}^{\pi=1} 0.1\pi + \int_{\pi=1}^{\pi=1.5} 0.1\pi \right) = 0.0697$$
$E - control$ is feasible for $K \leq K^{**}$, where $K^{**}$ given by

$$K^{**} = \frac{1}{3} \times 0.2 + \frac{2}{3} \left( \int_{\pi=0.5}^{\pi=1} 1 - 0.1\pi + \int_{\pi=1}^{\pi=1.5} 0.9\pi \right) = 0.75$$

Note that it would never be efficient to have $V - control$ when $E - control$ is still feasible (and $K > K^*$). As an example, consider once more $K = 0.747$. Under $V - control$, $V$ makes zero profits at $D = 0.9157$. To see this, we first derive the range of inefficient $A$ decisions. They occur whenever $p < \pi$ but $f(p) = D = 0.9157 > \pi\pi = 0.9\pi \Leftrightarrow \pi < 1.0147$. We then have

$$U^V_N = \frac{1}{3} \times 0.2 + \frac{2}{3} \left( \int_{\pi=0.5}^{\pi=1.0147} 0.9157 + \int_{\pi=1.0147}^{\pi=1.5} 0.9\pi \right) - K = 0.747 - K$$

$$U^V_E = \frac{1}{3} \times 0 + \frac{2}{3} \left( \int_{\pi=0.5}^{\pi=1.0147} 0.0843 + \int_{\pi=1.0147}^{\pi=1.5} 0.1\pi \right) = 0.0696$$

While it is possible to find an equivalent contract with $V - control$, it is inefficient to do so. We recognize this in two ways. $E$’s utility is lower under $V - control$ than under $E - control$. And the exit decision is inefficient within the interval $[1, 1.0147]$. Using the properties of the uniform distribution, this implies a $1.47\%$ probability of inefficient acquisitions.

For $K > K^{**}$, $V - control$ becomes unavoidable. For example, at $K = 0.77$, $V$ needs $D = 0.9782$, $E$ gets $U^V_E = 0.0441$ and inefficient exit decisions occur with a probability of $8.69\%$ (the structure of these calculations is analogous to the ones we
did for $K = 0.747$ and $D = 0.9157$). Using $\frac{D}{\pi} = 1.1111$, the highest $K$ is given by

$$K = \frac{1}{3} \times 0.2 + \frac{2}{3} \left( \int_{\pi=0.5}^{\pi=1.1111} 1 + \int_{\pi=1.1111}^{\pi=1.5} 0.9\pi \right) = 0.7787$$

In this case inefficient exit decisions occur with probability 11.11%.

Consider also the case of contingent contracting. We revisit the case of $K = 0.77$, but now examine contingent control. For $D = 0.985$ and $\lambda = 1$ we get

$$U_V^\lambda = \frac{1}{3} \times 0.2 + \frac{2}{3} \left( \int_{\pi=0.5}^{\pi=1} 0.985 + \int_{\pi=1}^{\pi=1.5} 0.9\pi \right) - K = 0.77 - K$$

$$U_V^\lambda = \frac{1}{3} \times 0.2 + \frac{2}{3} \left( \int_{\pi=0.5}^{\pi=1} 0.015 + \int_{\pi=1}^{\pi=1.5} 0.1\pi \right) = 0.0467$$

From $U_E^\lambda = 0.0467 > 0.0441 = U_E^V$ we see that contingent contracting is more efficient. $\lambda$ equals 1 and no inefficient A decisions occur for $K < K^{***}$, where $K^{***}$ given by

$$K^{***} = \frac{1}{3} \times 0.2 + \frac{2}{3} \left( \int_{\pi=0.5}^{\pi=1} 1 + \int_{\pi=1}^{\pi=1.5} 0.9\pi \right) = 0.775$$

But at $K = 0.7776$, for example, we get $\lambda = 1.05$, implying a 5% probability of an inefficient A decision.
7 References


Min\([V, D]\) converting to \(\phi V\)

\(V = \text{value of the firm}\)
\(D = \text{face value of preferred dividends (or debt)}\)
\(\phi = \text{percentage equity after conversion}\)
Participating Preferred Convertible Equity

\[ V = \text{value of the firm} \]
\[ D = \text{face value of preferred dividends (or debt)} \]
\[ \phi = \text{percentage equity after conversion} \]
\[ V = V(f(V)) \]

Figure 3

Generalized Convertible Preferred Equity

- \( f(V) \) represents the face value of cash flows before conversion.
- \( \phi V \) represents the percentage equity after conversion.

\[ V = \text{value of the firm} \]
\[ \phi = \text{percentage equity after conversion} \]
\[ f(V) = \text{face value of cash flows before conversion} \]
Figure 4
Exit Decisions with a Tight Finance Constraint

Efficient "A" Decision
Inefficient "A" Decision
Efficient "I" Decision
Figure 5
Exit Decisions Under Contingent Control

- Efficient "A" Decision
- Inefficient "A" Decision
- Efficient "I" Decision
- V-Control
- E-Control