Assignment 4: Answers

(April 28, 1998)

1. An application of the Ho and Lee model.

   (a) The short rate tree is

		9.00
	7.00	
5.00		5.00
	3.00	
		1.00

   (b) The price path for the state-contingent claim is

		0.0000
	.4831	
.4760		1.0000
	.4926	
		0.0000

   The last column is the pure state-contingent claim [1 in state (1,2), zero in the others]. The earlier columns give us the value of this claim at prior dates.

   (c) Duffie's formula puts the current value of one dollar in state (1,2) in the (1,2) node of the tree, and tells us what current prices of other states are, too. The complete set of state prices is

		.2357
	.4878	
1.000		.4760
	.4878	
		.2403

   (d) Duffie's formula is helpful in generating discount factors (sums of state prices down columns) and spot rates:

Maturity (Half Years)	Discount Factor	Spot Rate
1	0.9756	5.000
2	0.9519	4.990
3	0.9290	4.974

2. An application of the Ho and Lee model in which we choose drift parameters to fit the current spot rate curve.

   (a) The drift parameters (expressed as percentages) are -0.979 and 0.037. The short rate tree is

		9.058
	7.021	
6.000		5.058
	3.021	
		1.058

(b,c) Here's the price path for a 3-period zero (which has a maturity of 2 in one period):

		95.67
	93.32	
92.41		97.53
	97.04	
		99.47

The possible prices of the two period zero in one period are 93.32 in the up state and 97.04 in the down state, implying two-period spot rates of 7.03 and 3.03. Thus the spot curve (y_1, y_2) changes from (6.00,5.50) in the first period to either (7.12,7.03) (in the up state) or (3.12,3.03) (in the down state). Ie, the curve shifts up or down, and flattens out in either case.

3. This example illustrates that we can value instruments with unusual cash flows. In this case, the cash flows exhibit extreme sensitivity to high interest rates.

(a) The rate increases sharply with r for $r \geq 5$. The tree for the coupon rate is

		17.00
	11.00	
5.00		5.00
	3.00	
		1.00

(b) The cash flows are half these rates, plus principal. Since rates are quoted six months prior to payment, it's convenient to move all of these cash flows back one period and discount them, which fits them nicely into the tree. The resulting cash flows are

		103.83
	5.31	
2.44		100.00
	1.48	
		100.00

Example: $5.31 = 0.5 * 11/(1 + .07/2)$.

(c) The price path of the note is

		103.83
	103.78	
101.84		100.00
	100.00	
		100.00

(d) The note's prices throughout the tree reflect the sensitivity to interest rates of the coupon rate. In this case, a coupon of exactly LIBOR would produce prices of 100.00 in all states: no sensitivity! Differences from 100.00 reflect the extra sensitivity (and value) for rates above 5. The initial value (101.84) reminds us that sensitivity of future payments can show up now.