

Assignment 3: Answers

(October 21, 1998)

1. DM swaps.

- (a) Standard swap rate calculation (par yield formula). Use the spot rates to compute discount factors: $d_n = 1/(1 + y/2)^n$, with n the maturity in half-years. For the 2-year swap, the rate is computed from

$$\begin{aligned}\text{Swap Rate} &= \frac{1 - d_4}{d_1 + d_2 + d_3 + d_4} \times 200 \\ &= 4.653.\end{aligned}$$

For 3 years, a similar calculation gives us 5.056.

- (b) The fixed rate leg has coupons of 5.056/2 (see above) for three years plus a principal of 100 at the end. The duration is computed by the same method we use for bonds:

$$D = (1 + .05056/2)^{-1} (w_1 \times 0.5 + w_2 \times 1 + \cdots + w_6 \times 3),$$

with

$$\begin{aligned}w_j &= \frac{(5.056/2)/(1 + .05056/2)^j}{100} \quad \text{for } j = 1, 2, 3, 4, 5 \\ w_6 &= \frac{(100 + 5.056/2)/(1 + .05056/2)^6}{100}.\end{aligned}$$

100 in the denominator is the value of the fixed leg, conventionally set equal to par at the trade date. As a result, the coupon rate is the yield expressed as a percent. The answer is $D = 2.751$.

- (c) Spot rates have fallen, esp at the long end, leading to a flatter curve. (These are real numbers, by the way, for January and July of 1998.)
- (d) Although the swap rate is set to make the initial value of the fixed rate leg 100, the value can change over the life of the swap. At a future date, we value the fixed rate payments and coupon the same way we value a coupon bond:

$$\begin{aligned}\text{Value} &= d_1 \times 5.056/2 + d_2 \times 5.056/2 + d_3 \times 5.056/2 + d_4 \times 5.056/2 \\ &\quad + d_5 \times (100 + 5.056/2) \\ &= 102.638.\end{aligned}$$

The value of the swap is therefore $102.638 - 100 = 2.638$. Your profit is the change in value plus the initial net interest payment:

$$\begin{aligned}\text{Profit} &= (2.638 - 0) + (5.056/2 - 3.90/2) \\ &= 3.216.\end{aligned}$$

- (e) Duration can be used to estimate the change in price of the fixed leg, which is the same as the change in the value of the swap:

$$p - p_0 = -D \times p_0 \times \Delta y.$$

Here the initial price $p_0 = 100$, $D = 2.75$, and $\Delta y = .03938 - 0.05056 = -0.0112$. We would estimate the price p , then, as 103.077, which is a little higher than what happened (102.638). This is yet another example of duration giving only an approximate guide to the change in price.

2. A yield spread trade.

- (a) The DV01's are 0.0189 (the 2-year) and 0.0795 (10-year). Along the way you need the yields, which are 4.838 and 6.410.
- (b) You want to be long at short maturities to take advantage of the fall in yields, and possibly long at long maturities, too, if you expect long rates to fall.
- (c) If you want to protect yourself against general movements in yields but profit from steepening, you should hold short bonds and short enough long bonds to make the overall interest sensitivity zero. The change in value if yields change by the same amount at all maturities is approximately

$$\Delta v = -(x_2 \times DV01_2 + x_{10} \times DV01_{10}) \times \Delta y \text{ in basis points .}$$

For this to be zero, you need to hold quantities of the two bonds in this ratio:

$$\frac{x_2}{x_{10}} = -\frac{DV01_{10}}{DV01_2} = -4.206.$$

Intuitively, you need to hold more 2's than you're short 10's, because the 2's are less sensitive to yield changes.

- (d) I've made this optional.
- (i) 100 in the 2 is $x = 100.54/100 = 0.995$ units. The standard deviation is

$$\sigma = x \times DV01 \times 1140 = \$21.4$$

Note that I multiplied by 10000, since the standard deviation of the yield was a percentage, not basis points. (We need bp's, since DV01 is the response to a one bp change.)

- (ii) 100 in the 10 is $x = 100.54/114.34 = 0.875$ units. The standard deviation is

$$\sigma = x \times DV01 \times 660 = \$45.9$$

The higher number is the result of greater interest sensitivity (the DV01), offset partly by a smaller yield standard deviation.

- (iii) First we need to solve these two equations to find the positions:

$$\begin{aligned} x_2 \times 100.54 + x_{10} \times 114.34 &= 100 \\ x_2 \times 100.54 + x_{10} \times 114.34 &= 0 \end{aligned}$$

which keep the value at 100 and eliminate general interest sensitivity. The answer is $x_2 = 1.363$ and $x_{10} = -0.324$ (short position). With these positions, the risk in the portfolio is

$$\begin{aligned}\sigma^2 &= (1.363 \times 0.0189 \times 1140)^2 + (-0.324 \times 0.0795 \times 660)^2 \\ &\quad + 2 \times 0.531 (-0.324 \times 1.363 \times 0.0189 \times 0.0795 \times 1140 \times 660) \\ &= (\$24.9)^2.\end{aligned}$$

In short, this is only slightly more risky (by this measure) than the 2-years alone. One of the reasons is that the two positions aren't that highly correlated (we get credit for some diversification).

Comment: these numbers are too high. I got them from the RiskMetrics home page, but I must have messed up somewhere.)