The Allocation of Socially Responsible Capital

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Abstract

A rapidly increasing share of asset allocation decisions incorporate social values in addition to financial considerations. We argue that the most common strategies for socially motivated investing, which only consider the social value of the firms in an investors' portfolio, are misguided. We develop a tractable framework in which commercial and social investors compete, and identify alternative strategies for social investors that result in higher social welfare and deliver higher financial returns. We discuss several normative implications for socially-motivated investors. From the enterprise perspective, we demonstrate that a focus on increasing profitability can have a greater social impact than a focus on direct social value creation.

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1 Introduction

The last several decades have seen an invigoration of investing in companies that rank favorably on metrics of social value, such as environmental stewardship, social responsibility, and good governance practices (collectively referred to as ESG). For instance, one quarter of assets under management by professional investors–$30 trillion– are now allocated with such considerations (US SIF Foundation, 2018). This large shift in investment strategies has the potential to dramatically alter the allocation of capital in the economy. In fact, many argue the entire purpose of this movement is to help reallocate resources to socially beneficial uses and away from socially harmful ones. Thus, it is centrally important to understand whether and how this style of investing generates impact. This paper develops a general framework to explore how investing with social convictions results in the creation of social value and which investment strategies generate social value most efficiently.

We focus on "passive" strategies based entirely on selecting which assets to hold or avoid. By far the most common of such strategies in practice are constructed with attention to the financial returns and the social value of the companies included in an investor's portfolio. For example, ESG index funds attempt to track the returns of a benchmark index while maximizing some composite measure of the social good of the companies in the portfolio. Proponents of such "values-aligned" investing claim that they increase the valuation of (or equivalently decreases the cost of capital for) economic endeavors that contribute the most positively to society. This in turn shifts the set of projects that markets will finance towards those that create social value and away from those that destroy it.

We argue that the folk wisdom justifying "values-aligned" investing is misguided, and such investment strategies are an inefficient way to use asset allocation decisions to influence social value creation. Our framework builds on the insight that an investor's true contribution to social value is not reflected in the social value of the companies in their portfolio, but rather by the additional social value created relative to if the investor did not exist at all (e.g. Brest et al., 2016). The distinction between these perspectives is driven by the fact that many companies that have high social value could attract investors with a purely financial objective. Therefore, socially motivated investors who finance these companies may not be contributing to social value creation. In fact, their behavior could even result in social value destruction if it displaces investors unconstrained by social considerations into financing socially harmful projects. We formalize this critique in an equilibrium model of capital allocation and characterize its implications for the behavior of social investors. We further identify an alternative to "values-aligned" investment strategies that can generate more impact *and* achieve higher financial returns.

To understand the basic logic of why social investors who attend only to the social value of their portfolio companies might achieve sub-optimal outcomes, consider the following stylized example.

Suppose there are two investors each of whom holds one unit of capital. One *commercial investor* cares only about financial returns and the other *social investor* cares about both financial returns and social value. Suppose further that there are three enterprises, each of whom needs one unit of capital to operate:

- Firm A generates a 10% profit and 10 units of social value.
- Firm B generates a 8% profit and 5 units of social value.
- Firm C generates a 9% profit and 0 units of social value.

Investors finance companies at terms that ensure them a financial return no greater than the firm's total return on investment. A social investor who makes investment decisions based only on their own returns and the social value of the companies they finance would want to invest in Firm A. Such an investor would be willing to pay more for this opportunity than the commercial investor, leaving the commercial investor to finance Firm C and earn a 9% return. As a result, the social investor finances Firm A, enjoys a financial return of 9%, and 10 units of social value are created. If instead the social investor took a holistic view, they would appreciate that Firm A is profitable enough to attract the support of the commercial investor. The social investor might then want to invest in firm B. In this case the social investor would receive a financial return of 8% and 15 units of social value would be created.

This example highlights that social investors narrowly focused on the social value *attributable* to companies in their own portfolio are not effective at *generating* social value through their investment decisions.[1] We develop a framework to embed this logic in a competitive financial market, in which many commercial investors and social investors coexist and firms' costs of capital are determined in equilibrium. To highlight the nuances arising from the two approaches to social investing in the previous example, we introduce two types of social investors. *Values-aligned social investors* form portfolios based on the financial returns and social welfare generated by the companies they support. *Impact-aligned social investors* are similar, but consider implications of their investment decisions for total social welfare, not just the social value of firms in their portfolio.

Beyond admitting a tractable analysis of equilibrium behavior, our model yields several normative implications for social investors and entrepreneurs. First we demonstrate that capital held by different classes of investors has different social values, reflecting the investors' contribution to social welfare. We define the social value of capital for a particular class of investors as the increase in total social welfare associated with marginally expanding the pool of capital held by those investors. Both types of social investors have socially valuable capital. However, values-aligned investors have a lower social

[1] In fact, notice in this example that the social investor's choice of Firm A results in *zero* additional social value creation relative to an economy in which neither investor had preferences for social value.

value of their capital. Relative to impact-aligned investors, values-aligned investors over-prioritize companies that have both high social impact and high profitability. These companies could attract commercial investment independent of whether social investors support them, which limits the impact of values-aligned social investors. Impact-aligned social investors, by definition, internalize this effect of their investment behavior and do not support projects that could have been financed by commercial investors in their absence.

An important implication of this result is that there are improvements to the investment strategy of values-aligned social investors. Any capital that values-aligned investors deploy to fund profitable but also impactful projects that could have been commercially financed should be redeployed to projects with lower profitability (and potentially lower social value). Perhaps surprisingly, this can increase both the social impact *and* financial return to this capital. Why? As the result of values-aligned investors competing with each other to own shares in certain projects, they are both displacing commercial investors *and* transferring excessive value to entrepreneurs (or a firm's existing owners). Any financial concession made to own a firm that could have attracted commercial investment does not serve to expand the set of socially valuable projects that are economically viable. This generates scope to put a financial concession to more effective use. Instead investing directly in projects that are less profitable but more impactful than what a displaced commercial investor would have chosen can thus result in higher social value creation *and* higher financial returns to the social investor.

Our baseline model considers an environment where all firms have a fixed scale. In this case, impact-aligned social investors support firms that could not attract any commercial financing. This is not a realistic option for small socially conscious investors, who are typically limited to making investments through established capital markets. We extend our model to investigate the implications of socially responsible investing when firms have an intensive margin of scale and show that in this case there is scope for creating impact by investing in commercially viable firms. We illuminate the role of "blended" finance–issuing claims at different prices for different investors—in maximizing a social investor's impact. While values-aligned social investors never co-invest with commercial investors, blended finance allows impact-aligned social investors to provide low-cost financing for socially valuable projects without displacing a firm's existing commercial investors.

Our framework also has implications for evaluating the social impact of a firm, sometimes called its *enterprise impact* (Brest et al., 2016). Enterprise impact depends not only on the amount of capital used by the enterprise, but also on the *type* of capital used by the enterprise. All else equal, enterprises that attract the capital of socially minded investors have lower contribution to social welfare than those that attract the capital of purely commercial investors. Holding fixed the social value created by a firm, it can raise its enterprise impact by reducing its dependence on social capital, freeing social capital to fund another enterprise that is unable to obtain commercial financing. The more profitable is a firm,

the less likely it is to rely on scarce, socially valuable capital. Our framework thus provides a new connection between the profitability of an enterprise and its contribution to social welfare.

Finally we consider an extension in which social investors exhibit varying degrees of altruism. When social investors are values-aligned we identify a familiar result: social investors and entrepreneurs exhibit positive assortative matching, in that investors who care more about social welfare match to entrepreneurs that create more social value. However when social investors are impact-aligned, this result reverses. Holding fixed the level of an entrepreneur's profitability, social investors who value social welfare more highly match to entrepreneurs who create less social value. This result arises from the fact that social investors have interdependent utility in that they internalize the social value created by all firms that receive financing.

The investment strategies adopted by values-aligned investors in our model strongly mirror those of real-world socially conscious investors. There are at least two ways to understand this. First, investors may not care about social welfare *creation*, but instead derive utility from *association* with social value. They may not want to be associated with companies that are heavy polluters and would be willing to forgo higher returns from investing in those companies, for example, even if over-weighting low-polluting companies in their portfolio does little to reduce aggregate pollution. According to this interpretation of the motives of social investors, our analysis should be understood as exploring the positive implications of values aligned versus impact-aligned social investment strategies on aggregate resource allocation. Alternatively, real-world socially conscious investors could be motivated by making the world a better place, and therefore would be making a mistake in adopting values-aligned investment strategies. If they knew better, they might prefer to invest in a way that had a bigger impact on social value and generated higher returns. If investors are indeed making a mistake, our model delivers normative suggestions that can help investors improve asset allocation decisions. We return to this discussion in the conclusion.

This paper contributes to the literature on investing with social preferences. A number of papers study financing environments where social and commercial investors coexist, and ask how social investors should behave to maximize their impact. Oehmke and Opp (2019) and Landier and Lovo (2020) both study activist social investors who aim to resolve a moral hazard problem amongst entrepreneurs. The two papers investigate how the amount of social and commercial capital influences the bargaining power of social investors and their resulting social impact. Thakor and Quinn (2020) also studies how the presence of social and commercial investors interacts with the firm's moral hazard problem. Broccardo et al. (2020) highlights that if activist investors all vote as if they are pivotal, well-diversified investors with even a small concern for social welfare will vote in line with the social planner. Like in our analysis, these papers consider social investors who care about social value independently of if it was created by the companies they support. In contrast to these papers, we study

passive investors in a complete information environment, whose goal is to enable new projects by offering cheaper capital to firms with socially valuable projects.

Pedersen et al. (2019) and Pastor et al. (2020) study social and commercial investors within a Markowitz framework, derive the optimal portfolios for each class of investors and shows an ESG-adjusted CAPM emerges. The social investors in their framework maximize a combination of financial return and the impact of firms in their portfolio, and hence correspond to our values-aligned social investors.[2] Heinkel et al. (2001) demonstrates that when socially motivated investors divest from socially unproductive firms their stock price declines, as the remaining investors hold more concentrated portfolios. In contrast to these papers, we study a model without uncertainty, and focus our analysis on the behavior of impact-aligned social investors who aim to maximize social welfare rather than the impact of their own portfolio. Moisson (2020) has a related ambition, contrasting the behavior of consequentialist social investors with that of investors who hold other moral criteria.

Several papers analyze the behavior of individual firms and their prosocial investors. Focusing on the single investor case, Chowdhry et al. (2019) and Roth (2020) analyze when a socially minded investor can have more impact through an investment in a social enterprise than they can through a grant. Hart and Zingales (2017) fleshes out several cases for a stakeholder view of the firm, in which management's objective encompasses more than profits alone, and considers arrangements by which activist shareholders can induce the firm to take socially efficient actions. Morgan and Tumlinson (2019) argue that corporate social responsibility might be a vehicle to overcome collective action concerns amongst a firm's prosocial shareholders, and argues that corporate social responsibility might be an efficient channel for prosocial actions when the firm's production imposes societal externalities. This latter point is also highlighted in Nilsson and Robinson (2017). Dewatripont and Tirole (2020) studies how competition affects the degree to which firms' behaviors reflect the ethical concerns of their stakeholders.

Our study also relates to the broader economic literature on altruistic motives. Andreoni (1990) highlights the distinction between "pure altruists," who derive utility from social welfare, and "impure altruists" who derive utility, or "warm glow" from having directly improved social welfare. In this light, our impact-aligned social investors can be understood as pure altruists, and our values-aligned social investors can be understood as impure altruists.

Our analysis bears a technical resemblance to assignment matching models, commonly employed

[2]Pastor et al. (2020) considers an extension in which investors incorporate the social value of all firms into their objective function, but the analysis assumes that individual investors cannot meaningfully influence the economy and hence their preferences for financial return and the social value of firms within their own portfolio fully determine their investment decisions. In contrast our impact-aligned social investors value their own financial return on the same order as their contribution to total social value, and hence the fact that they partially internalize the value created by all firms does influence their decisions. We expand on this discussion in Section **??**.

in trade and labor economics (e.g. Roy, 1951, Becker, 1973, Sattinger, 1979, Costinot and Vogel, 2010). We contribute to this literature by providing a model in which agents sort along two dimensions of heterogeneity, as in Gola (2020), and by studying an environment where one side of the market has interdependent utility in the sense that they care not only about their own match but also the matches of others. As discussed in Section 5, we show that this latter feature can partially reverse the classic result of positive assortative matching.

The rest of the paper proceeds as follows. In Section 2 we outline the model for the case where entrepreneurs have binary projects and in Section 3 we analyze it. In Section 4 we outline and analyze the model for the case where entrepreneurs have a non-trivial intensive margin of scale. In Section 5 we consider investors with varying degrees of concern for social welfare. Section 6 concludes.

2 Baseline Model

Players, Technology, and Contracts

There is a finite set E of entrepreneurs. Each one is endowed with a project that requires one unit of capital. If entrepreneur i receives the requisite capital, their project returns $\pi_i \in [0, \hat{\pi}]$ profit and $w_i \in [-\hat{w}, \hat{w}]$ "social value," where π_i and w_i represent the private and social return of the project respectively.[3] We assume that the features of each project are perfectly observable to all players.

There is a finite set S of social investors, each of whom allocates one unit of capital. There is also a market for commercial capital that inelastically supplies financing to all firms at required rate of return of r^C.[4]

A contract between some investor and an entrepreneur i specifies the transfer of one unit of capital from the investor to the entrepreneur in exchange for financial return r_i on their invested capital. The entrepreneur receives a share of profits $\pi_i - r_i$, and w_i social value is created. We will sometimes refer to r_i as the price or cost of capital offered to an entrepreneur. Because we are studying a complete information environment without contracting frictions, this contract can be understood as either debt or equity. In addition to being able to finance the entrepreneurs in E, social investors can also allocate their capital to a "social value-neutral" asset with financial return r^C profit and 0 social value.[5]

[3] We assume that w_i encompasses the full social return of the project, including the private return π_i, as well as any consumer and employee surplus and externalities arising from the project.

[4] We assume that the investors and firms we study are a small part of the global market and hence we take r^C to be exogenous.

[5] The results would be nearly unchanged if social investors did not have access to such a value-neutral asset.

Preferences

We index investors and entrepreneurs such that investor i matches with entrepreneur i. Each entrepreneur's utility is their share of the profit, $(\pi_i - r_i)$. We will separately examine two classes of social investors.

Values-aligned social investors make investment decisions based on financial returns they receive and the social value created by the entrepreneur they've financed. That is, their utility is

$$(r_i + \theta w_i),$$

where θ represents the strength of investor i's social preference.

Impact-aligned social investors receive utility from their income and from the total social value created by all entrepreneurs who receive financing. That is, their utility is

$$r_i + \theta \sum_{j \in \bar{E}} w_j = (r_i + \theta w_i) + \theta \sum_{j \in \bar{E} \setminus i} w_j,$$

where $\bar{E}$ is the set of entrepreneurs who receive financing. We can observe that the difference between the utility functions of values-aligned and impact-aligned social investors is that values-aligned investors do not derive utility from the social output generated by entrepreneurs they do not finance, while impact-aligned social investors derive utility equally from all social output regardless of who financed it.

Values-aligned investors do not fully internalize the implications of their investment decision on social welfare. This does not imply that the preferences of values-aligned investors are incorrect. As discussed in the introduction, values-aligned investors may derive intrinsic utility from owning firms that create social value, similar to the conception of warm-glow altruists in Andreoni (1990). In such a case, the analysis to follow should be understood as exploring the positive implications of these two modes of investment behavior. Alternatively values-aligned preferences may represent the *behavior* of socially conscious investors, while impact-aligned social preferences may more faithfully represent the *intentions* of socially-conscious investors to affect social change. Under this interpretation our analysis of the behavior of impact-aligned social investors offers normative guidance to real-world investors with social preferences.

Timing of Actions

The model proceeds in two stages. In stage 1 each social investor offers a contract to an entrepreneur. Simultaneously, all firms receive an offer for commercial financing at rate r^C. In stage 2 entrepreneurs then choose at most one contract to accept, and payoffs are realized.

Equilibrium

The solution concept is Subgame Perfect Equilibrium. All investors choose contracts that are mutual best responses. Among other things this implies that no entrepreneur ever receives offers from more than one social investor. Therefore we maintain the convention that entrepreneur i matches with investor i. Further, we adopt the convention that an entrepreneur i who accepts no offers for financing has a cost of capital $r_i = \pi_i$.

Social Welfare

Our measure of social welfare is $W = \sum_{i \in \bar{E}} w_i$, where $\bar{E}$ is the set of entrepreneurs that receive financing. Our interpretation is that w_i is the total social value created by firm i if it receives financing, including the value to the firm's owners.[6] Impact-aligned social investors can therefore be understood to be maximizing a modified variant of social welfare that increases the weight placed on their own consumption.

Note that we do not adopt a social welfare function that sums over the utility of entrepreneurs and investors. Doing so would induce a standard analysis of public good provision, wherein none of the investors we consider invest sufficiently in businesses that produce high w_i because they do not internalize the benefit accruing to other investors. Instead, we adopt the convention that the social planner cares about value creation w_i, but does not care about the intrinsic "altruistic" utility that social investors derive from the creation of w_i. In this sense we follow Hart and Zingales (2017) and Broccardo et al. (2020).

[6] Under this interpretation, the value accruing to the firm's owners is determined independently of how ownership is divided, i.e. the welfare weights placed on entrepreneurs and investors are the same.

3 Analysis of Baseline Model

To understand the behavior of values-aligned and impact-aligned social investors we first characterize the equilibrium of the model in which all investors are either values-aligned or impact-aligned. In Section 3.3 we present our main results in the model in which both types of social investors coexist.

3.1 Values-Aligned Social Investors

We begin by characterizing the equilibrium of the model where all social investors are values-aligned. For any two entrepreneurs i and j who are both supported by a social investor, their costs of capital satisfy $r_i + \theta w_i = r_j + \theta w_j$.[7] And for an entrepreneur i supported by a social investor and an entrepreneur k who is not, prices of capital must satisfy $r_i + \theta w_i \geq r_k + \theta w_k$. In equilibrium there exists a cutoff $\bar{w}$ such that commercial investors only support projects with social value below some $\bar{w}$ and profits above r^C. Therefore, $r^C + \theta\bar{w}$ is the social investor's effective outside option utility. Prices of capital in equilibrium are set such that social investors achieve this outside option utility. Social investors receive financial return of $r^C - \theta(w_i - \bar{w})$. Thus, they are willing to pay (in terms of reduced financial return) for projects that generate high social value.

The equilibrium investment allocation is depicted in Figure 1. A formal characterization of equilibrium investment allocations is relegated to the Appendix Section A.1.

3.2 Impact-aligned Social Investors

Next we analyze the equilibrium allocation of capital when social investors are all impact-aligned. The equilibrium allocation is depicted in Figure 2.

In equilibrium, social investors expect that entrepreneurs with profits above r^C can receive financing from a commercial investor. Therefore, impact-aligned social investors recognize that by supporting an entrepreneur i with profits $\pi_i \geq r^C$, their marginal contribution to social welfare is not w_i, but rather 0, as the firm would receive financing independently of the social investor's offer.[8]

Lemma 1. *For any entrepreneur i supported by a social investor, $r_i = r^C$ if $\pi_i \geq r^C$ and $r_i = \pi_i$ else.*

[7] The preceding equality holds so long as costs of capital are positive. In equilibrium a social investor may provide funding in exchange for zero share of the proceeds ($r = 0$) if the project has sufficiently high social impact (akin to philanthropy). In such a case, the above equality need not hold.

[8] In a full general-equilibrium model, the social investor's contribution to social welfare from supporting a commercially viable firm would be the social value of marginally expanding the set of commercially supported firms. We normalize this value to 0.

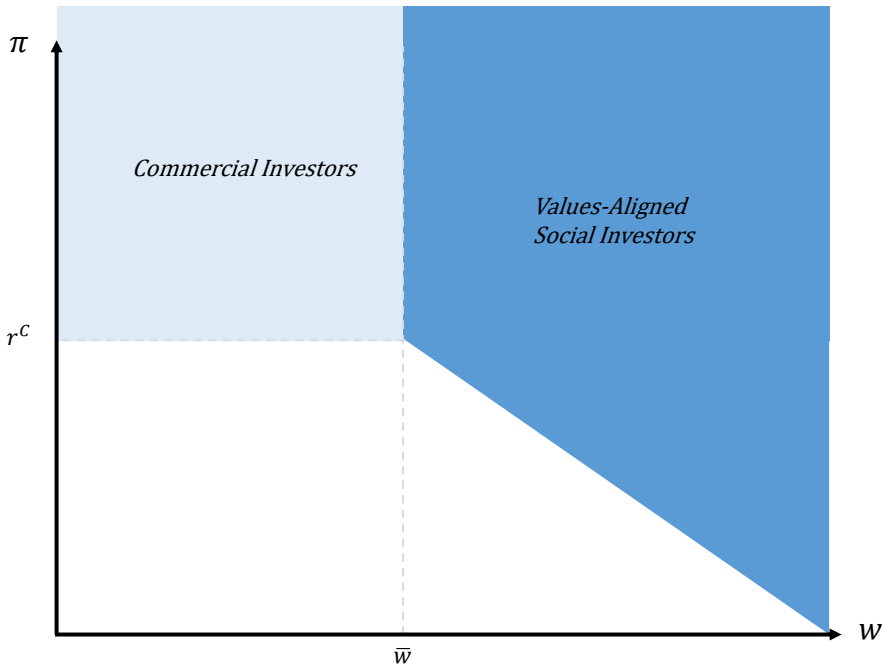

Figure 1: Equilibrium investment with values-aligned social investors

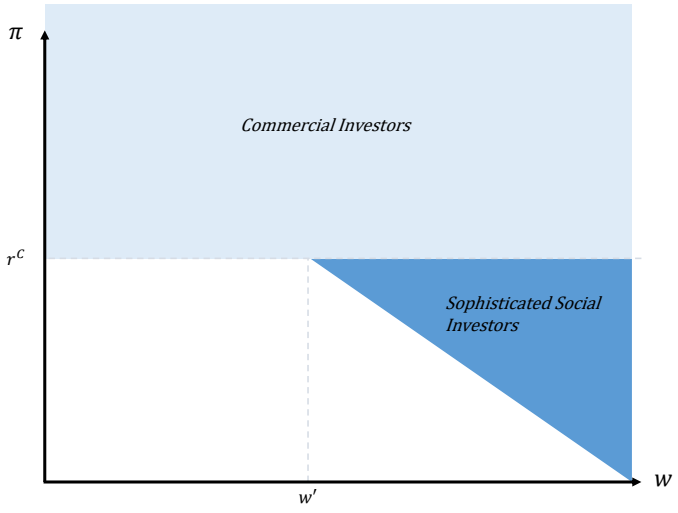

Figure 2: Equilibrium investment with impact-aligned social investors

Lemma 1 dictates that if a social investor were to support an entrepreneur with profit $\pi_i \geq r^C$, they must offer contracts that allow the entrepreneur to retain $\pi_i - r^C$ of their profit, as this is the profit the entrepreneur could achieve from a commercial investment. In contrast, social investors who support entrepreneurs with $\pi_i < r^C$ can extract the entrepreneur's full profits. These entrepreneurs cannot attract commercial financing. Further, because impact-aligned social investors recognize that they cannot contribute to social welfare by undercutting another social investor, no social investor will undercut another who is supporting an entrepreneur with profits $\pi_i < r^C$, as they would always prefer to instead invest in a commercially viable firm and earn return r^C.

In equilibrium, impact-aligned social investors support the firms i who have the highest $\pi_i + \theta w_i$, amongst those that generate profits $\pi_i < r^C$ (so that they would not receive commercial financing). This is depicted in Figure 2. In Appendix Section A.2, we formally characterize the allocation of impact-aligned social investor's capital.

Comparison of the Two Equilibria

Before analyzing the model where impact-aligned and values-aligned social investors coexist, we compare social welfare across the regimes with only one type of social investor. Recall, our measure of social welfare is $W = \sum_{i \in \bar{E}} w_i$, where $\bar{E}$ is the set of entrepreneurs that receive financing in equilibrium.

Proposition 1. *Social welfare is higher when all social investors are impact-aligned than when all social investors are values-aligned.*

Though all social investors place weight θ on social value, impact-aligned social investors create more social value in equilibrium. This is because values-aligned social investors prioritize investment in entrepreneurs that have high social value w_i even if they could also attract investment in the commercial market. Relative to values-aligned social investors, impact-aligned social investors may support entrepreneurs with lower contributions to social welfare. But the impact-aligned investor's contribution to social welfare is higher, as they are not displacing a commercial investor who would have supported the firm in their absence.

3.3 Main Results

In this section we discuss a number of normative results about social investing, in a market in which both impact-aligned and values-aligned social investors coexist. In particular, we demonstrate that there are investment strategies that deliver higher financial returns and create more social value than

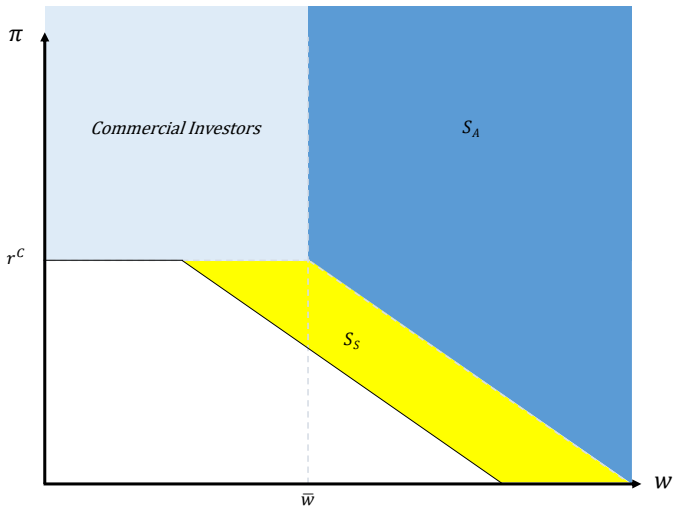

Figure 3: Equilibrium investment with both types of social investors

those employed by values-aligned social investors. And we draw a new link between the profitability of a firm and the firm's social value.

First we characterize the equilibrium in the market with both types of social investors.

3.3.1 Equilibrium Structure with Both Types of Social Investors

Let there be a mass S_A of values-aligned social investors, and S_S impact-aligned social investors.

There is no longer a unique equilibrium in this market. In Appendix Section A.3 we characterize the full set of equilibria. In this section we focus on the investor-optimal equilibrium, which is also the welfare-optimal equilibrium, depicted in Figure 3.[9]

As in Section 3.1 there is some $\bar{w}$ such that values-aligned social investors only support entrepreneurs with social value greater than $\bar{w}$ and commercial investors only support entrepreneurs with social value less than $\bar{w}$. As in Section 3.1, entrepreneurs with profits higher than r^C and social value less than $\bar{w}$ are supported by commercial investors.

To characterize the behavior of impact-aligned social investors, the following analogue of Lemma 1 holds.

Lemma 2. *For any entrepreneur i supported by an impact-aligned social investor,*

if $\pi_i \geq r^C$ *and* $w_i \leq \bar{w}$, $r_i = r^C$,

[9]This is the equilibrium we would expect to emerge if investors were allowed to make side transfers to one another.

if $\min\{\pi_i, r^C\} + \theta w_i \geq r^C + \theta\bar{w}$ *then* r_i *solves* $r + \theta w_i = r^C + \theta\bar{w}$ *if such an* r *exists and* $r_i = 0$ *if no such* r *exists*

and $r_i = \pi_i$ *else.*

The return required by a social investor is disciplined by the commercial market if the entrepreneur could have attracted commercial financing, and by the values-aligned social investors if the entrepreneur could have attracted financing from values-aligned social investors. Else, an impact-aligned social investor can demand an entrepreneur's entire profit, as in Section 3.2.

In the investor-optimal equilibrium impact-aligned social investors do not compete with either commercial investors or values-aligned social investors, and instead support the set of entrepreneurs i who maximize $\pi_i + \theta w_i$ amongst those who could not attract financing from other investors. We defer formal characterization of this equilibrium to Appendix A.3. For the remainder of our analysis we focus on the invesstor-optimal equilibrium to illuminate the model's comparative statics.

3.3.2 Reallocating Values-Aligned Social Capital to Improve Social Welfare and Financial Returns

In this section we consider two thought exercises. First, holding fixed the equilibrium behavior of all other investors, we consider the possibility of reallocating the investment of a single values-aligned social investor. We demonstrate that any values-aligned social investor who supports a firm with $\pi_i \geq r^C$ and who earns a financial return of $r_i < r^C$ could reallocate their capital to increase total social welfare and increase their financial return. In this sense values-aligned investors leave both money and impact on the table. We then consider the possibility of converting a values-aligned social investor into an impact-aligned social investor, and show that this always leads to an increase in social welfare and sometimes (but not always) leads to an increase in the investor's financial return.

The logic underlying both results is as follows. Both types of social investors make a financial concession, relative to commercial investors, in order to finance socially valuable firms. Impact-aligned social investors only make concessions to support socially valuable firms that would not be commercially viable. In contrast, because values-aligned investors compete with one another to own socially-valuable firms, some of the concession made by values-aligned social investors provides rents for entrepreneurs *that would have been able to attract commercial capital*. Therefore there is scope to reallocate values-aligned capital in a way that reduces their financial concession and puts it to more effective use (from the perspective of increasing total social welfare).

Proposition 2. *Consider any values-aligned social investor* i *that supports a firm with* $\pi_i > r^C$ *and earns a return* $r_i < r^C$ *in equilibrium. If the distribution of firms is sufficiently dense, there exists an*

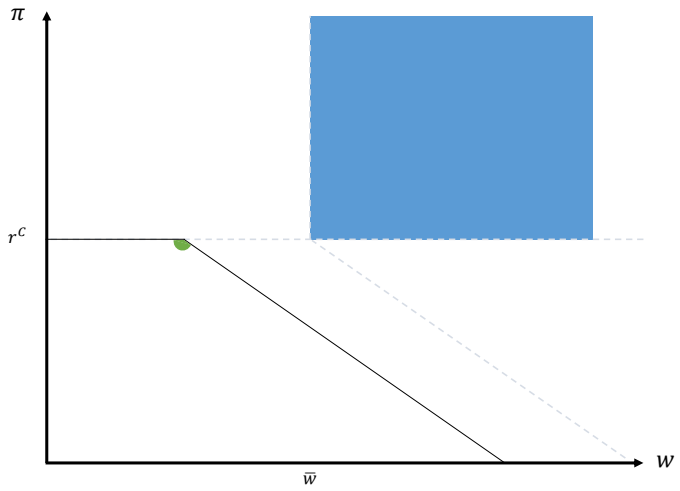

Figure 4: Reallocating Values-aligned Social Capital Out of Equilibrium

unfinanced firm j with profits $\pi_j > r_i$, such that if the values-aligned social investor i were to deviate and offer firm j financing at cost π_j, total social welfare would increase as would investor i's financial return.

In the proof of Proposition 2 we formalize the notion that the distribution of entrepreneurs is *sufficiently dense*. Intuitively, it guarantees that for any combination of π and w, there is an entrepreneur with profits π_i near π and social value w_i near w. Proposition 2 can be understood with reference to Figure 4. Fix any values-aligned social investor i that supports a firm i with profits $\pi_i \geq r^C$ and who earns financial return $r_i < r^C$ (generically this holds for all values-aligned investors who support firms with $\pi_i \geq r^C$). These investors support the firms highlighted in blue. And consider among the set of unfinanced firms some firm j with profits $\pi_j > r_i$ and with social value $w_j > 0$.[10] This firm is guaranteed to exist by the assumption that the distribution of firms has full support, and one such firm is highlighted in green.

The contribution to social welfare of the equilibrium investment for investor i is 0, regardless of the social value w_i of firm i, as investor i is merely displacing commercial investment. Firm j creates less social value than any firm in the blue region of the diagram, but by reallocating investor i's capital to firm j social welfare increases, as firm j was previously unfinanced.

Further, by offering firm j a cost of capital $r_j = \pi_j$, investor i can earn higher financial return as well. As with social value, firm j earns lower profits than any firm in the blue region of the diagram. But social investor i can demand the full profit of firm j in exchange for financing, whereas the price

[10] Or, more generally, we identify a firm j with profits $\pi_j > r_i$ and with social value w_j greater than the social value of the marginal firm receiving commercial support.

of capital that values-aligned investor i offered to firm i was disciplined by competition with other values-aligned social investors. Values-aligned social investors compete down the price of capital of firms with large contributions to social value even when these firms could have attracted commercial financing. The financial compromise made by values-aligned investors to support such firms results in a transfer of wealth to the entrepreneur rather than expanding the pool of socially valuable firms. In contrast, the financial compromise made to support a firm that could not attract commercial financing goes entirely toward expanding the pool of socially valuable firms rather than transferring rents to entrepreneurs whose projects would anyway have been feasible.

Proposition 2 demonstrates that values-aligned social investors leave both impact and money on the table in the sense that, relative to the firms these investors support, there are unfinanced firms that could deliver higher financial returns and increase social welfare. However, while it is straightforward to show that converting values-aligned social investors to impact-aligned social investors would result in higher total social welfare, in general we cannot guarantee that this conversion would lead investors to earn higher financial returns. The simple reason is that once values-aligned investors have been converted to impact-aligned social investors, while they would prefer to finance firms in the green region of Figure 4 relative to any firm in the blue region, there may be yet another firm they prefer to firms in the green region that contributes more to social welfare but has lower financial return. Nevertheless, we can demonstrate the following result.

Proposition 3. *In the investor-optimal equilibrium, there may exist a set of values-aligned social investors such that were they to be converted to impact-aligned social investors they would earn higher returns. Moreover total social welfare would increase.*

The logic of Proposition 3 is depicted in Figure 5. We identify a mass of values-aligned social investors, shaded in blue, who are supporting entrepreneurs that satisfy two properties.

1. These entrepreneurs could attract commercial capital at market rates, i.e. $\pi_i \geq r^C$,
2. These entrepreneurs have very high contribution to social value w_i, so that values-aligned social investors make a large financial compromise to support them.

By converting these values-aligned social investors to impact-aligned social investors, they instead support the firms shaded in yellow. As above, this results in an increase in social welfare, as the firms in yellow could not have attracted commercial capital, while those in blue could. Moreover, by identifying values-aligned social investors who were making a sufficiently large financial compromise to support firms i with high w_i, we can guarantee that the newly converted impact-aligned social investors earn higher profits, as these impact-aligned social investors can demand the full profits of the firms they support.

Finally, we note that while converting these values-aligned social investors to impact-aligned so-

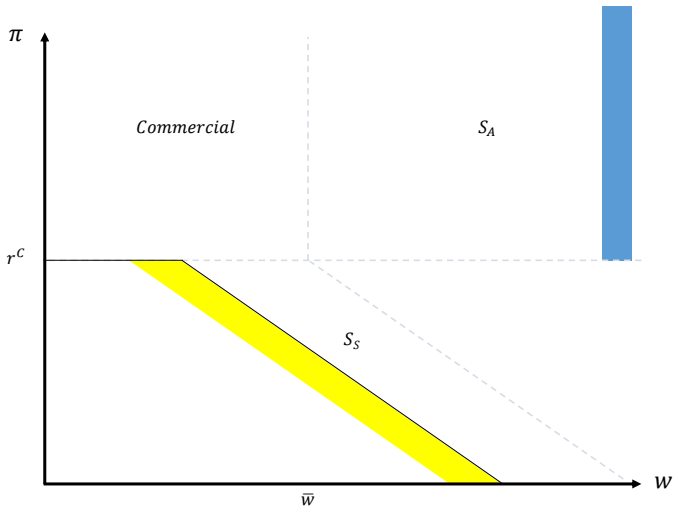

Figure 5: Converting Values-aligned to Impact-aligned Investors

cial investors increases their profits and total social welfare, it does not increase their utility judged according to the utility function of values-aligned social investors. Nevertheless, Proposition 3 offers encouraging news about the prospect of converting values-aligned social investors to impact-aligned social investors in practice. A substantial amount of effort has gone into investigating the hypothesis that ESG investing can increase impact and financial returns (e.g. Eccles et al., 2014), suggesting investors are sensitive to the financial implications of values investing. Our model demonstrates that relative to conventional ESG strategies there is room for improvement in this dimension.

3.3.3 Enterprise Impact

How should one judge the contribution to social welfare of a particular entrepreneur, sometimes referred to as *enterprise impact* (e.g. Brest et al., 2016)? On first pass it might seem natural for w_i to be the measure of enterprise impact. However, we argue that a firm's enterprise impact should also account for the social value of the capital it employs.

Let $W(S_A, S_S)$ be the total social value created in the investor-optimal equilibrium given masses of investors S_A, and S_S. Define v_{S_A} to be the increase in social value corresponding to adding one additional values-aligned social investor, and v_{S_S} to be the increase in social value corresponding to adding one additional impact-aligned social investor. The social value of commercial capital, v_C is normalized to 0. It is straightforward to show that $0 < v_{S_A} < v_{S_S}$.

We define the enterprise impact of firm i to be $e_i \equiv w_i - v_i$ where v_i is the social value of capital attributable to the investor who supports entrepreneur i in the investor-optimal equilibrium. We define

the enterprise impact to be 0 for firms that do not receive financing.

This definition of enterprise impact might have practical value for socially motivated investors aiming to quantify the social value of a particular enterprise. Frontier efforts in the impact investing industry often attempt to account for the social value created by the enterprise and the amount of capital employed by the enterprise, such as in the *impact multiple of money* method (Addy et al., 2019). Our analysis highlights that it is also critical to account for the composition of social capital versus commercial capital raised by an enterprise in judging its impact.

This definition of enterprise impact also highlights an alignment between the enterprise impact and profitability of a firm. Firms can increase their enterprise impact by increasing their profitability *even holding fixed their social value* w_i. Increasing the profitability of the firm makes it more likely to attract commercial capital, freeing up capital that is willing to accept lower returns to fund higher social value endeavors. In particular, we have the following result.

Proposition 4. *Suppose firm i attracts financing from an impact-aligned social investor in equilibrium. Increasing its profits π_i while holding fixed its social value w_i increases its enterprise impact e_i and total social value created in equilibrium.*

Proposition 4 states that making a firm more profitable increases its enterprise impact *even holding its social value w_i fixed.* Importantly, this result is not driven by an assumption that a firm's profitability and its social value are correlated. Instead, this result is driven by the observation that once a firm becomes profitable enough to attract commercial financing, impact-aligned social investors will step aside, freeing up their capital to support another socially valuable firm. Therefore, more profitable firms use less socially valuable capital, and have higher enterprise impact. Note that this phenomenon does not hold for firms supported by values-aligned social investors, as values-aligned social investors pay no regard to whether a firm could attract commercial capital in their absence.

4 Impact on the Intensive Margin and Blended Finance

So far we have assumed that every firm has a single project, which is completed if and only if it raises a unit of capital. Within this setting, we demonstrated that the impact-aligned investing strategy can outperform the values-aligned investment strategy on both total social value creation and financial returns. However, the impact-aligned approach required that investors allocate their capital to firms that are not commercially viable. In reality, this would effectively relegate impact-aligned social investors to private capital markets, which is likely infeasible for small investors. In this section we consider a variant of the model in which firms have a non-trivial intensive margin of scale and demonstrate that impact-aligned social investors *can* have impact by inducing commercially viable

firms to change their scale of operation. Therefore, there may be room for impact-aligned social investors to induce change in public markets.

In what follows, we demonstrate that our results from Section 3 have natural analogues when firms have continuous, concave production functions. We also identify several new considerations. Most importantly, we observe a role for blended finance—where both social and commercial investors support the same firm at different terms— when social investors are impact-aligned, though not when they are values-aligned, and we draw out implications for security design.

4.1 Model

Agents, Technology, and Contracts

The model in Section 2 is now modified in the following ways. First, rather than assuming that firms have projects of binary scale, we now assume that each firm i can absorb any positive mass of capital k. Firm i then produces $\pi_i(k)$ profit and $w_i(k)$ social value. Both functions are increasing, concave, and continuously differentiable, although the case where $w(\cdot)$ is at some points decreasing could be easily accommodated. We maintain the assumption that there is a finite set of social investors denoted by S each of whom owns one unit of capital, however now capital is divisible so that in principle one investor could support several firms. There is a market for commercial capital that inelastically supplies financing to all firms at rate r^C. Social investors can also allocate their capital to a "social value-neutral" asset with financial return r^C profit and 0 social value.[11]

A contract between an investor and entrepreneur now specifies not only the transfer of capital from the investor to the entrepreneur at a cost of $r \geq 0$, but also a minimum-scale contingency k, discussed below. A contract is therefore represented by $\langle r, k \rangle$, where r represents the cost of 1 unit of capital. The minimum-scale contingency can also be left unspecified, represented by $\emptyset$.

Preferences

Let x_i^j be the amount of capital that investor i allocates to firm j in equilibrium, r_i^j be the cost of capital that investor i charges firm j and $\bar{k}_j$ be the mass of capital raised by firm j. A values-aligned social investor i's utility is represented by

$$\sum_j x_j^i \left(r_j^i + \theta \frac{w_j\left(\bar{k}_j\right)}{\bar{k}_j} \right).$$

[11] The results would be nearly unchanged if social investors did not have access to a social value-neutral asset.

That is, values-aligned social investors care about their total financial return and social value of the firm they support, weighted by their ownership share.

In contrast, an impact-aligned social investor i's utility is represented by

$$\sum_j x_j^i r_j^i + \theta \sum_j w_j \left(\bar{k}_j\right)$$

That is, impact-aligned social investors care about their financial return and the total social value created by all firms, regardless of who supports them.

Entrepreneur i has preferences represented by $\pi_i\left(\bar{k}_i\right) - r_i$, where $r_i \equiv \sum r_j$, and the sum is taken over all contracts the entrepreneur accepts from investors j. That is, entrepreneurs maximize firm profit net of the cost of external capital.

Timing

The timing unfolds in two stages. First, all social investors offer a contract to at most one entrepreneur. Simultaneously, entrepreneurs receive offers from the commercial market for an arbitrary amount of capital at rate r^C. Second, entrepreneurs accept any number of such contracts and operate at scale $\bar{k}_i$, where $\bar{k}_i$ represents the mass of capital they have accepted. Entrepreneurs may only accept a contract that specifies a minimum-scale contingency k if they operate at a scale $\bar{k}_i \geq k$.

Finally, we maintain the solution concept is pure-strategy Subgame Perfect Equilibrium.

Discussion of Minimum-Scale Contingency

The minimum-scale contingency gives investors the ability to influence a firm's scale on the margin. Without it, if an investor unilaterally offered a firm cheaper capital, the firm may merely accept that capital as a substitute for more expensive capital, without changing its scale. As we demonstrate below, this will be especially relevant for impact-aligned social investors, who always utilize minimum-scale contingencies in equilibrium. We discuss implications for security design in Section 4.2.2.

4.2 Equilibrium Structure with Just One Type of Social Investor

For simplicity we focus separately on the case when all social investors are values-aligned and when all social investors are impact-aligned.

4.2.1 Values-aligned Social Investors

The equilibrium when social investors are values-aligned works in much the same way as the binary-project case. We separately examine equilibrium implications for cost of capital and firm scales.

Cost of Capital. First, we analyze the model fixing investors' expectations about firm scales $\{\bar{k}_i\}$. Competition between social investors guarantees that for each firm i and j that receive financing from social investors we have

$$r_i + \theta \frac{w_i(\bar{k}_i)}{\bar{k}_i} = r_j + \theta \frac{w_j(\bar{k}_j)}{\bar{k}_j}. \tag{1}$$

That is, the financial return plus the social value derived by owning a share of firm must be equalized across all firms that receive social investment. Moreover, competition from commercial investors guarantees that no firm i accepts financing at cost $r_i > r^C$.

Together these pin down the cost of financing for all firms that receive social investment in equilibrium. Social investors support the firms with highest $\frac{w_i(\bar{k}_i)}{\bar{k}_i}$ up to the point where the sum of their invested capital equals $|S|$.[12] Of all firms that receive social investment, the firm i with the lowest average social value $\bar{w} \equiv \frac{w_i(\bar{k}_i)}{\bar{k}_i}$ receives financing at rate $r_i = r^C$, and for all others, the cost of financing is pinned down by Equation 1.

Moreover, in equilibrium all firms with social value less than $\frac{w_i(\bar{k}_i)}{\bar{k}_i} < \bar{w}$, but with average profit $\frac{\pi_i(\bar{k}_i)}{\bar{k}_i} \geq r^C$ receive commercial financing at rate $r_i = r^C$, and all other firms remain unfinanced. This determines the cost of capital r_i for all firms in equilibrium, as a function of their expected scales $\{\bar{k}_i\}$.

Firm Scales. On the firm's side, firms choose their scale in one of two ways. Firms that are unconstrained by a minimum-scale contingency choose their scale $\bar{k}_i$ to maximize $\pi_i(k) - r_i k$, so that $\bar{k}_i$ solves

$$\pi_i'(\bar{k}_i) = r_i. \tag{2}$$

Firms that are constrained by a minimum-scale contingency need not set their marginal profit equal to their cost of capital. Specifically, let a firm's commercial scale k_i^C solves $\pi_i'(k_i^C) = r^C$, and define its commercial share of profits to be $q_i^C \equiv \pi_i(k_i^C) - r^C k_i^C$. A firm constrained by a minimum-scale

[12] Generically, the firm among these with the lowest $\frac{w_i(\bar{k}_i)}{\bar{k}_i}$ will only be partially financed by social investors, with the remainder being financed on the commercial market.

contingency $\bar{k}_i$ need only satisfy

$$\pi_i\left(\bar{k}_i\right) - r_i\bar{k}_i \geq q_i^C \tag{3}$$

Equilibrium is determined by a set of costs of financing $\{r_i\}$ that satisfy the above equations, and a set of firm scales $\left\{\bar{k}_i\right\}$ each of which either satisfies Equation 2 or 3. We note that there exists an equilibrium where no investor utilizes scale-contingencies, and Equation 2 determines the scale of all firms.

No Role For Blended Finance. Next, we note that across all equilibria, there is no co-investment between commercial investors and social investors within any firm that receives a subsidy relative to the commercial cost of capital.

Lemma 3. *Firms that receive a cost of capital $r_i < r^C$ from any social investor are financed wholly by social investors.*

Social and commercial investors disagree on the relative value of companies with the same profits but different contributions to social value, so there is no equilibrium price at which both sets of investors would be happy to finance the same investment.[13] While this extreme separation would not arise in a model with, for example, diversification motives, it illustrates an important point. Disagreement about the value of a company among investors implies that to change the scale of the company requires displacing some of its existing investors. This idea is closely related to the observations of Heinkel et al. (2001) and Broccardo et al. (2020), that commercial investors will partially "undo" the actions of social investors, insofar as social investors may partially crowd out commercial investors in the firms they support. We will see in the following section that impact-aligned social investors do not displace commercial investors, and co-investment does occur in equilibrium.

4.2.2 Impact-aligned Social Investors

Next we analyze how the allocation of capital differs when social investors are impact-aligned. As there are multiple equilibria, we focus on the one that maximizes the sum of investors' utilities.[14]

Blended Finance. We first observe that unlike in the case of values-aligned social investors, there is a role for blended finance; impact-aligned social investors co-invest with commercial investors.

Lemma 4. *In equilibrium, all firms raise at least k_i^C capital from commercial investors.*

If an impact-aligned social investor were to marginally undercut a commercial investor, they would earn a return of r^C, and create 0 additional social value. In equilibrium, there is always a more

[13] The one exception is the firm for which $\frac{w_i\left(\bar{k}_i\right)}{\bar{k}_i} = \bar{w}$. This firm may be financed by both commercial and social investors at cost of capital r^C.

[14] This is the equilibrium we would expect to emerge if investors were allowed to make side transfers to one another.

attractive use of their capital. Therefore, firms that receive social investment raise at least k_i^C capital from commercial markets, and the remaining capital from social investors. In fact, as we demonstrate in the proof of Lemma 4, in equilibrium firms supported by social investors may raise more than k_i^C commercial capital. In such settings impact-aligned social investors subsidize the entry of commercial investors to increase the scale of firms with high marginal social value.

Prices. As in the case with binary projects, because impact-aligned social investors care about total social value creation rather than the social value of the firm they support, they do not compete with one another or with commercial investors. Rather than being determined by competitive forces, equilibrium prices of capital are determined by a no-rents condition.

Lemma 5. *In equilibrium, all entrepreneurs earn a payoff of* q_i^C.

Impact-aligned social investors demand a return r_i^S that solves $\pi_i\left(\bar{k}_i\right) - r^C\bar{k}_i^C - r_i\left(\bar{k}_i - \bar{k}_i^C\right) = q_i^C$, where $\bar{k}_i^C$ is the amount of commercial capital raised by firm i in equilibrium. If social investors demanded a higher return, firm owners would prefer to invest at their commercial scale k_i^C and to rely exclusively on commercial capital. And because impact-aligned social investors recognize that by undercutting one another they are not contributing to total social value creation, required returns are set so as to make entrepreneurs indifferent between accepting social capital versus relying exclusively on commercial financing.

Scale Contingencies. We now turn to minimum-scale contingencies.

Lemma 6. *Impact-aligned social investors utilize minimum-scale contingencies in equilibrium.*

Because impact-aligned social investors set prices so as to leave entrepreneurs with their commercial payoff, the firms they support are always faced with a marginal cost of capital that is above their marginal return on investment at $\bar{k}_i$. If they were free to choose their own scale, they would accept the subsidized social capital in lieu of commercial capital, and still choose a smaller scale than social investors desired. Therefore, unlike in the case with values-aligned investors, impact-aligned social investors always utilize scale-contingent contracts.

Firm Scales. Finally, we turn to equilibrium capital allocation.

Lemma 7. *In equilibrium, for any two firms* i *and* j *that receive capital from social investors, we have*

$$\pi_i'\left(\bar{k}_i\right) + \theta w_i'\left(\bar{k}_i\right) = \pi_j'\left(\bar{k}_j\right) + \theta w_j'\left(\bar{k}_j\right).$$

Impact-aligned social investors allocate their capital so as to equalize the marginal profits plus the marginal social value of all firms that receive a subsidy.

Security Design

We close this section with a discussion of security design. In equilibrium, blended financing occurs in that impact-aligned social investors co-invest with commercial investors but utilize different terms. Impact-aligned social investors offer cheaper financing, and include a scale contingency requiring the firm to operate at a certain scale if it is to accept their capital. This contingent financing scheme resembles the idea of green bonds, in that they are a means to offer subsidized capital to firms, earmarked for projects with high social or environmental value, without displacing commercial investors. The primary distinction is that social investors in our model support levels of output that would not have been financed by commercial investors. Therefore, a practical implementation of this market would verify that each project has negative net present value at its commercial cost of capital in addition to verifying that the project has high social or environmental value.

4.3 Analogues of Main Results from Section 3.3

In this section we demonstrate that natural analogues of Propositions 2 and 4 extend to this setting. Namely, we demonstrate that in equilibrium values-aligned investors leave both money and impact on the table, and that increasing a firm's profitability may also increase its enterprise impact.

First we consider our analogue to Proposition 2.

Proposition 5. *In the equilibrium there may exist a deviation for a values-aligned social investor that would result in higher financial return and increase total welfare.*

When there is a non-trivial intensive margin of scale, both values-aligned and impact-aligned social investors bid up the value of commercially viable firms and increase their impact. Therefore, both values-aligned and impact-aligned social investors make a financial concession. However, as in Section 3 from the perspective of increasing total social welfare, values-aligned investors spend their concessions inefficiently. The fact that values-aligned social investors place intrinsic value on being the owner of socially valuable firms has two important implications. First, they allocate their capital to firms that could have attracted commercial support. Second, they compete with one another to be the owners of socially valuable firms, and bid up the prices of these firms. This crowds out commercial capital and it transfers rents to the entrepreneur that do not result in additional social value creation. The fact that values-aligned investors crowd out commercial capital creates scope to reallocate their capital that does not meet commercial returns, thereby increasing total social impact. And in doing so, if they can find a socially valuable project that not only cannot attract commercial capital, but also requires a smaller subsidy than the transfer they are already making to the entrepreneur they support, then they can both increase total social welfare and their financial return. The reason that this is stated

as a possibility result, rather than as an opportunity that exists across all equilibria as in Section 3, is that we no longer have an analogue of the sufficient density assumption about the distribution of firms. Therefore, while values-aligned investors spend their concessions inefficiently from a social welfare perspective, there may not exist an appropriate firm with which they can unilaterally change their investment behavior to increase their impact and financial return.

Next we consider an analogue to Proposition 4. We extend the definition of enterprise impact to account for the possibility that firm i attract capital from more than one type of investor. We define the enterprise impact of firm i to be $e_i(k) \equiv w_i(k) - \nu_i k$, where ν_i is now the *average* social value of capital utilized by entrepreneur i. We have the following result.

Proposition 6. *Suppose firm i attracts financing from an impact-aligned social investor in equilibrium. Increasing its profitability while holding fixed its social value $w_i(\cdot)$ increases its enterprise impact e_i and total social value created in equilibrium.*

The logic proposition is exactly parallel to that of Proposition 6. Take any entrepreneur supported by impact-aligned social investors in equilibrium. As its profit function is scaled up, commercial investors will finance a larger fraction of its output, which frees impact-aligned social capital to invest elsewhere and increase total social value.

5 Extension: Heterogeneous Investor Altruism θ

In this section we consider an extension of the model in which we allow the altruism parameter θ to vary across investors. Our aim is to explore the model's implications with regards to assortative matching. A classic exercise in the assignment matching literature is to identify conditions under which agents exhibit positive assortative matching (e.g. Roy, 1951, Becker, 1973, Sattinger, 1979, Costinot and Vogel, 2010, Gola, 2020) -– i.e. when do agents with higher "types" match with one another? We demonstrate when social investors in our model are values-aligned, investors with higher altruism match with entrepreneurs with higher social value for familiar reasons. In contrast, when investors are impact-aligned, they exhibit a variant of *negative* assortative matching. This latter result arises from the fact that impact-aligned social investors have interdependent utility; their utility depends not only on the terms of their own match but also on the matches of other investors.

5.1 Model

The model is the same as in Section 2 with the exception that for the set of social investors we now index their altruism parameter θ_i by i, and let it vary across investors. Specifically we assume that

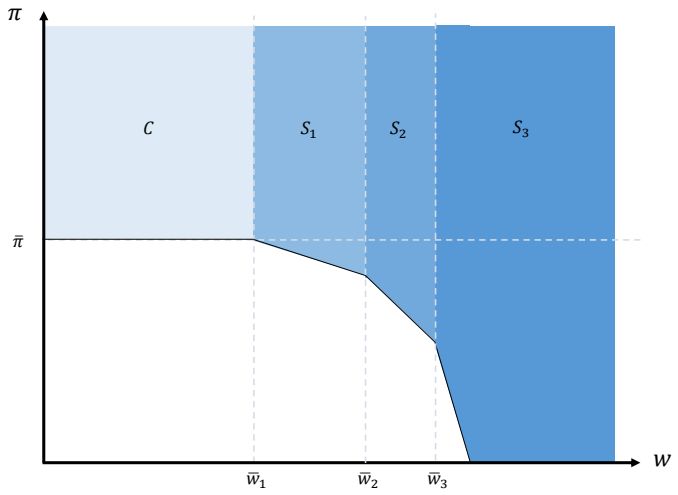

Figure 6: Equilibrium sorting with values-aligned social investors

there is a finite set $\Theta \equiv \{\theta^1, \ldots, \theta^n\}$ of potential levels of altruism, with $\theta^j < \theta^k$ for $1 \leq j < k \leq n$. We make no assumption about the distribution of θ_i. We now let S^l be the set of social investors with altruism parameter θ^l. We maintain all other assumptions of the model in Section 2.

5.2 Values-aligned Social Investors

We now characterize the equilibrium of the model where all social investors are values-aligned and demonstrate that social investors and entrepreneurs exhibit positive assortative matching on θ and w.

Prices of capital offered to any two entrepreneurs i and j who are both supported by a social investor with type θ^l satisfy $r_i + \theta^l w_i = r_j + \theta^l w_j$.[15] And for an entrepreneur i supported by a social investor of type θ^l and an entrepreneur k who is not, prices of capital must satisfy $r_i + \theta^l w_i \geq r_k + \theta^l w_k$.

With the above pricing equations we can characterize the set of entrepreneurs financed by each type of investor in equilibrium. The equilibrium investment allocation is depicted in Figure 6.

Relative to Section 3.1 the principle novelty is that we can now establish assortative matching in equilibrium. Namely, investors partition the set of entrepreneurs who receive financing such that investors with higher θ_i match with entrepreneurs who have higher w_i. This stems from the fact that the utility of investor i is super-modular in θ_i and w_i, and hence social investors with higher altruism have a higher willingness to pay for projects with high social value. This positive assortative matching

[15]The preceding equality holds so long as prices are finite. In equilibrium a social investor may provide funding in exchange for zero share of the proceeds ($r = 0$) if the project has sufficiently high social impact (akin to philanthropy). In such a case, the above equality need not hold.

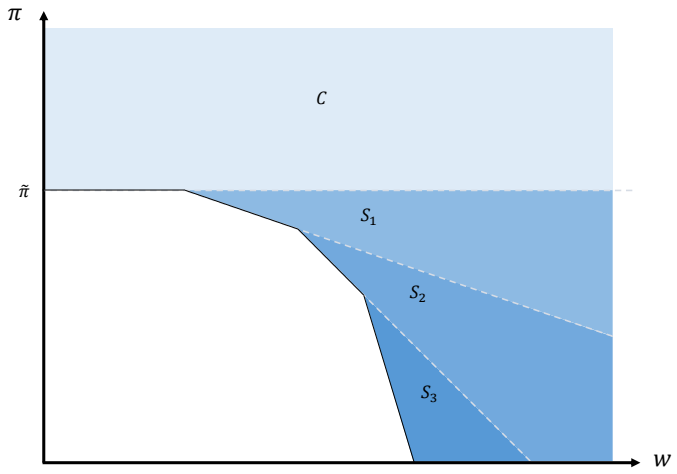

Figure 7: Equilibrium sorting with impact-aligned social investors

echoes many results in the assignment matching literature cited above. As we will see in the following section, this result breaks down, and partially reverses when social investors are impact-aligned.

5.3 Impact-aligned Social Investors

When social investors are impact-aligned there is a multiplicity of equilibria; Figure 7 depicts the investment allocation in the investor- and welfare-optimal equilibrium. Appendix Section A.5 offers a formal characterization of this equilibrium.

Relative to when social investors are values-aligned, the equilibrium allocation features two important differences. First, as in Section 3.2, so long as social capital is sufficiently scarce, impact-aligned social investors exclusively support firms that could not attract commercial financing. Second, and novel to this section, positive assortative matching breaks down, *even among the set of firms supported by social investors.* In fact, holding fixed a level of profits π, social investors exhibit negative assortative matching; the higher is the social investor's altruism parameter θ_i, the lower is the social value w_i of the firm they support. This negative assortative matching holds despite the fact that the utility of impact-aligned social investors is still super-modular in their altruism parameter θ_i and the social value w_i of the firm they support.[16]

In equilibrium, in order for an impact-aligned social investor i not to deviate and support a firm

[16] All equilibria with the same investment frontier depicted in Figure 7 are investor- and welfare-optimal. Therefore, formally, there exists an investor- and welfare-optimal equilibrium such that holding the level of entrepreneur profit fixed, social investors engage in negative assortative matching. But there may be other equilibria with equivalent allocations that do not feature negative assortative matching.

that could have attracted commercial investment it must be that

$$\pi_i + \theta_i w_i \geq r^C.$$

This incentive compatibility condition is easier to satisfy for social investors with higher altruism parameters. Therefore in the welfare optimal equilibrium, it is the impact-aligned social investors who care the least about social welfare that match to the most impactful entrepreneurs for a given level of profitability, as these are the entrepreneurs who are most able to entice social investors away from commercial markets. In contrast, impact-aligned social investors with higher altruism parameters are willing to forgo commercial returns to support entrepreneurs with lower contribution to social welfare for a given profit level. And because impact-aligned social investors derive utility from the social value created by all firms supported in equilibrium, social investors with high altruism parameters do not compete with social investors with low altruism parameters, as they recognize that doing so would not expand social value. Therefore, that positive assortative matching breaks down, and partially reverses when social investors are impact-aligned arises from the fact that impact-aligned social investors have interdependent utilities in the sense that their utility depends not only on the firm they match to and their financial return but also on the matching of other investors and entrepreneurs.

We close this section with one final remark. We view this result as an interesting contribution to the literature on assignment matching models but are cautious in interpreting it as a normative result for social investors, as it relies on a feature of the model that may be our biggest departure from the real world. Namely, while impact-aligned social investors recognize that firms they do not finance can still search for financing on the commercial market, impact-aligned social investors who support firms that are not eligible for commercial financing are all pivotal. Consider an equilibrium in which social investor i with low altruism parameter is assigned to support a high social value firm j that cannot attract commercial investment. Because impact-aligned social investors all choose their investment decisions at the same time, if social investor i deviated to invest in a higher profit firm, then firm j would go unfinanced. However, in a richer model in which social investors make their decisions dynamically, social investor i might expect that another social investor with higher altruism parameter would replace them if they were to deviate and leave firm j unfinanced. In such a case, this negative assortative matching result would break down.

6 Conclusion

This paper provides a new framework understand to how values-based investing generates impact. We consider a model in which investors influence social outcomes only through their asset allocation.

Investors' choices affect the set of companies or projects that are financed in equilibrium. Precisely how socially conscious investors think about social value when making investment decisions matters. If investors act as if they only care about the social value generated by companies in which they invest – what we term *values-aligned investing* –they have limited impact on total social value creation.

Investors following values-aligned investment strategies, which closely resemble the construction of conventional ESG and emissions reduction portfolios, have limited impact because they displace commercial investors who do not care about social value creation but would have supported some socially valuable companies anyway. The impact of socially conscious investors is therefore in part determined by the preferences of the investors displaced by the arrival of socially conscious capital. The mechanism for this displacement further constrains their impact. By competing with each other to invest in high social value companies, values-aligned investors are wasting a financial concession that could have been better spent investing in a socially valuable project that could not have attracted commercial capital. Because of this, we show it is possible for a values-based investor to alter their investment strategy in a way that generates more impact and delivers higher financial returns.

The idea of displacement also has efficiency considerations from a firm's perspective. By investing in their own profitability, companies are able to reduce their reliance on social capital, freeing it to invest in other endeavors and thus contributing to social value creation. This represents a new link between a firm's profitability and its social value.

And when firms have an intensive margin of scale, the displacement logic also gives rise to blended finance. Social investors can maximize their impact by co-investing with commercial investors, so as not to displace them. Our analysis yields several implications for security design.

An important question not addressed by our analysis regards the true preferences of socially aware investors. There has been an explosive growth in investment strategies that resemble the behavior values-aligned preferences in our model, and are thus consistent with preferences for *association* with social value rather than *creation* of social value. If these are indeed the true preferences of investors, our analysis delivers a positive prediction that preferences for social value association result in only limited social value creation.

On the other hand, it is not unreasonable to believe that investors have preferences for social value creation, yet, perhaps mistakenly, pursue strategies that are inefficient in generating social value. Investment advisers and mutual fund managers advertising ESG portfolios routinely allude to the positive impact of their investment strategies. For a few examples of many, Nuveen, an investment management firm with one trillion dollars in assets under management asserts that ESG investing is the approach "that is most likely to produce optimal financial and societal outcomes."[17] Candriam, a

[17] See: https://www.tiaa.org/public/pdf/how_nuveen_uses_responsible_investing_across_asset_classes.pdf Last Ac-

socially responsible investment fund with €130 billion under management asserts on its webpage that they "invest in the future, channeling capital for the common good."[18] While these quotes indicate that ESG investment funds aim to attract investors with a direct preference for the creation of social value, Calvert, another provider of ESG mutual funds with $23 billion in assets under management, goes one step further; their website allows investors to calculate the impact of their investment in a Calvert mutual fund across a variety of outcomes.[19]

To the extent that ESG funds are marketed to investors that have a preference for the creation of impact rather than merely being the owner of impactful firms, our analysis offers normative guidance. Namely, we offer investment strategies that dominate typical ESG approaches on both financial return and the creation of social impact.

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cessed: November 27, 2020

[18] See: https://www.candriam.com/en/private/about-us/ Last Accessed: November 27, 2020

[19] In fact, Calvert's calculation of the impact of an investment conflates measures of social value of portfolio companies with the impact of an investor in these companies. For example, it reports that a $10,000 investment in Calvert US Large-Cap Core Responsible Index Fund results in an annual reduction in emissions equivalent to burning 147 gallons of gasoline. This figure is based on the difference between the value-weighted emissions of constituents in the Calvert fund and the Russell 1000 Index. See: https://www.calvert.com/what-is-your-impact.php Last Accessed: November 27, 2020

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A Equilibrium Characterizations and Proofs

A.1 Characterization of the Equilibrium with Values-aligned Investors in Section 3.1

Lemma 8. *Projects financed by social investors have higher social value than projects financed by commercial investors. Formally, if entrepreneur i is financed by a social investor and entrepreneur j is financed by a commercial investor, $w_i \geq w_j$.*

Lemma 8 implies that there is some $\bar{w}$ such that social investors only support entrepreneurs with social impact greater than $\bar{w}$ and commercial investors only support entrepreneurs with social value less than $\bar{w}$. Commercial investors support entrepreneurs with profit higher than r^C and with social impact lower than $\bar{w}$. Entrepreneurs with profit lower than r^C cannot generate sufficient profits to attract commercial support, and entrepreneurs with social impact higher than $\bar{w}$ are priced by social investors and therefore yield returns too low to attract commercial support.

Lemma 9. *For entrepreneurs with $w_i \geq \bar{w}$, social investors support those with $\min\{\pi_i, r^C\} + \theta w_i \geq r^C + \theta\bar{w}$ and the price they pay is such that either $r_i + \theta w_i = r^C + \theta\bar{w}$ if such an $r_i \geq 0$ exists and $r_i = 0$ else.*

Because commercial investors support all entrepreneurs with profit higher than r^C and with social value lower than $\bar{w}$, $r^C + \theta\bar{w}$ is the social investor's effective outside option utility. Prices in equilibrium are set such that investors achieve this outside option utility.

Social investors receive financial return $r^C - \theta(w_i - \bar{w})$. Thus, they are willing to pay (in terms of reduced financial return) for projects that generate high social value. Commercial investors do not find it attractive to invest in companies with $\pi_i > r^C$ and $w_i > \bar{w}$ precisely because social investors are willing to invest in these companies at higher valuations.

Lemma 9 allows for the graphical characterization of the equilibrium, depicted in Figure 1. That $\bar{w}$ is uniquely pinned down is implied by the resource constraints, i.e. that social investors have mass S.

A.2 Characterization of the Equilibrium with impact-aligned Investors in Section 3.2

As impact-aligned social investors care about total social welfare rather than the social value of the entrepreneur they support, their expectations about which entrepreneurs will be financed in the commercial market will determine whom they finance. In equilibrium, social investors and entrepreneurs

expect that those entrepreneurs with profits above r^C can receive commercial financing and earn $\pi_i - r^C$.

Using the equilibrium prices from Lemma 1, we can characterize the set of entrepreneurs supported by social investors.

Define $s_w \equiv \left\{ i \in E : \pi_i \leq r^C, \pi_i + \theta w_i \geq r^C + \theta w \right\}$. If there exists a $\tilde{w} > 0$ such that $|s_{\tilde{w}}| = |S|$, then in equilibrium the social investors support entrepreneurs in $s_{\tilde{w}}$ and have a strict preference for these firms over supporting any firm with $\pi_i > r^C$. Prices for firms in $s_{\tilde{w}}$ are set such that $p_i = \pi_i$.

To see that this behavior can be supported in equilibrium, there are three deviations we must consider for impact-aligned social investors – supporting an entrepreneur j with profit $\pi_j > r^C$, who will get commercial financing, supporting an entrepreneur $k \in s_{\tilde{w}}$ who already receives support from another social investor, or supporting an entrepreneur l who does not receive any investment in equilibrium.

In the first case, the social investor cannot earn more than r^C profit, as this is what the commercial investor earns. Moreover, she recognizes that by undercutting a commercial investor, she is not contributing to total social value, and so by definition of S_w this is not a profitable deviation.

In the second case entrepreneur k produces $\pi_k \leq r^C$ profit and the social investor recognizes that by undercutting another social investor she is not contributing to the total social welfare created because entrepreneur k will be financed independently of her actions, and the displaced social investor cannot reallocate her capital. Therefore by definition of S_w these are not profitable deviations. In the final case, the most attractive entrepreneur to support is the one with profit $\pi_l = r^C$ and social impact $w_l = \tilde{w}$, but once again by definition of S_w this is not a profitable deviation.

If there is no $\tilde{w} > 0$ such that $|s_{\tilde{w}}| = |S|$, the in equilibrium impact-aligned social investors support all firms in s_0, and allocate the remainder of their capital either to firms with profits $\pi_i > r^C$ or to the outside option asset.

Proof of Proposition 1

First, we observe that no social investor ever provides financing to a firm with $w_i < 0$, as the value-neutral outside option asset is always preferable. Moreover, it is straightforward to show that the set of firms that receive financing when social investors are impact-aligned contains the set of firms that receive financing when social investors are values aligned, as impact-aligned social investors do not compete with commercial investors. Therefore total social welfare is higher when social investors are impact-aligned.

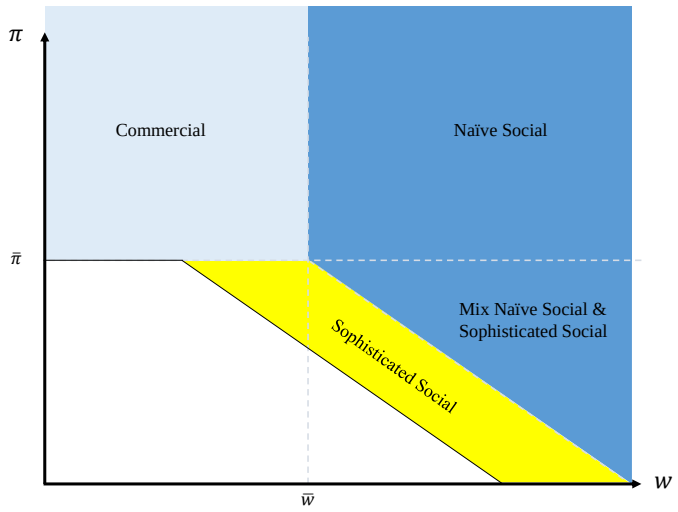

Figure 8: General Equilibrium With Both Types of Social Investors

A.3 Characterization of the Equilibrium with Both Types of Social Investors in Section 3.3

Figure 8 depicts the equilibrium structure. To characterize this structure we use the following three lemmas.

Analogous to Section 3.1 there exists a $\bar{w}$ such that

Lemma 10. *Commercial investors support the entrepreneurs with* $\pi_i \geq r^C$ *and with* $w_i \leq \bar{w}$.

Analogous to Lemma 9 we have

Lemma 11. *Values-aligned investors support entrepreneurs with* $\min\left\{\pi_i, r^C\right\} + \theta w_i \geq r^C + \theta \bar{w}$.

Finally, analogous to the equilibrium characterized in Section A.2, there exists a $\tilde{w} \geq 0$ such that

Lemma 12. *Impact-aligned social investors support entrepreneurs with profits* $\pi_i < r^C$ *and with* $\pi_i + \theta w_i > r^C + \tilde{w}$. *If* $\tilde{w} > 0$ *then the former inequalities completely characterize the set of entrepreneurs supported by impact-aligned social investors. If* $\tilde{w} = 0$ *then some impact-aligned social investors utilize the outside option asset.*

A few features of the equilibrium are of note. First, both impact-aligned and values-aligned investors may support firms in the region below the horizontal line $\pi = r^C$ and to the right of the diagonal line $\pi_i + \theta w_i \geq r^C + \theta \bar{w}$. This is because in equilibrium, impact-aligned investors know they are pivotal for the firms they support in this region, since no values-aligned social investor would support them if the impact-aligned investor were to deviate.

Second, only values-aligned social investors support firms in the region above $\pi = r^C$ and to the

right of $w = \bar{w}$ because these are firms that could attract commercial support if only a values-aligned investor did not support them, and so by the same logic as in Section 3.2 these firms cannot attract the support of impact-aligned social investors.

Next we argue that the equilibrium depicted in Figure 3 is welfare-optimal. This is the equilibrium in which only values-aligned social investors support firms in the region below the horizontal line $\pi = r^C$ and to the right of the diagonal line $\pi_i + \theta w_i \geq r^C + \theta \bar{w}$. It is straightforward to show that the set of firms supported in this equilibrium contains the set of firms supported in all other equilibria. By the same logic as in the proof of Proposition 1, the equilibrium in which this line is the highest has the highest social welfare.

Next we argue that the equilibrium depicted in Figure 3 is investor-optimal. This is the equilibrium in which only values-aligned social investors support firms in the region below the horizontal line $\pi = r^C$ and to the right of the diagonal line $\pi_i + \theta w_i \geq r^C + \theta \bar{w}$. It is straightforward to show that this is the equilibrium where $\bar{w}$ is the highest. To see that the sum of the financial returns is highest in this equilibrium, note that only values-aligned social investors place competitive pressure on prices, and prices for all firms to the right of the diagonal line $\pi_i + \theta w_i \geq r^C + \theta \bar{w}$ satisfy $r_i + \theta w_i = r^C + \theta \bar{w}$. Because $\bar{w}$ is the highest, prices are also the highest in equilibrium. To see that social welfare is the highest in this equilibrium, note that he set of firms supported in this equilibrium contains the set of firms supported in all other equilibria. Because this equilibrium has features both the highest total social investor returns, and the highest social welfare, it is the equilibrium in which the sum of social investor welfare is maximized (and commercial investors welfare is constant across all equilibria).

A.4 Omitted Proofs From Sections 3.3 and 4

Proof of Propositions 2 and 3

First, we provide a formal definition of "sufficient density" referenced in Proposition 2. We say that the distribution of firms is ε-*dense* if for every $\pi \in [0, \hat{\pi}]$ and for every $w \in [-\hat{w}, \hat{w}]$, there is a firm i with $|\pi_i - \pi| + |w_i - w| < \varepsilon$. Proposition 2 is formally stated as follows: *Consider any values-aligned social investor i that supports a firm with $\pi_i > r^C$ and earns a return $r_i < r^C$ in equilibrium. There exists an $\varepsilon > 0$ such that If the distribution of firms is ε-dense, there exists an unfinanced firm j with profits $\pi_j > r_i$, such that if the values-aligned social investor i were to deviate and offer firm j financing at cost π_j, total social welfare would increase as would investor i's financial return.*

We sketch the proof of Proposition 2 with reference to Figure4, where a values-aligned investor in the blue region is moved to instead support an entrepreneur in the green region. For any level of

financial compromise that a values-aligned social investor makes to support a firm that could have attracted commercial financing, ε-density guarantees that there exists a firm that is not supported by any investor but that could offer a higher return than the values-aligned social investor is earning, and such that if the values-aligned social investor were to reallocate her capital to the new firm total social welfare would increase. This new firm has lower w_i than the one that the values-aligned social investor supported, but the values-aligned social investor's contribution to social welfare is higher when supporting the new firm because it could not attract commercial financing.

We sketch this proof of Proposition 3 with reference to Figure 5. First we identify a set of values-aligned social investors who are earning 0 return in equilibrium. We convert these values-aligned social investors to impact-aligned social investors, noting that this strictly expands the set of firms supported by impact-aligned social investors. We make no assumption about which of the firms supported by impact-aligned social investors are supported by our "converted" investors. Instead we note that in equilibrium all impact-aligned social investors earn positive profits, as the price for each of these firms is $p_i = \pi_i$. Therefore our converted investors earn higher profits than prior to their conversion. That social welfare is higher is a straightforward consequence of Proposition 1.

Proof of Proposition 4

This is a straightforward implication of the equilibrium depicted in Figure 3. Fix any entrepreneur i who is being supported by an impact-aligned social investor. Raising his profit π_i to $\pi_j < r^C$ does not change the type of capital he attracts. But raising his profit to $\pi_j > r^C$ causes him to instead be supported by a commercial investor and his enterprise impact increases. Social welfare increases because the impact-aligned social investor can now support another entrepreneur.

Proof of Lemma 3

In any equilibrium all values-aligned social investors are indifferent between supporting any two firms with average social value exceeding $\bar{w}$. Suppose that for one such firm i, where a social investor j offers a cost of capital $r_i < r^C$ there was co-investment. That is, the firm receives some capital at $r_i < r^C$ and some capital at r^C. Then a social investor supporting another firm could deviate and offer firm i a unit of capital at $r_i' = r_i + \varepsilon < r^C$, thereby displacing one unit of commercial capital that firm i previously accepted. This social investor would have strictly higher utility than social investor j, violating the equilibrium indifference condition.

Proof of Lemmas 4, 5, 6, and 7

We directly construct the investor-optimal equilibrium. Let $\bar{k}_i$ be the equilibrium scale of firm i, representing the total social capital it is offered in equilibrium plus $\bar{k}_i^C$, which represents the amount of commercial capital firm i raises in equilibrium. Each social investor who offers firm i a contract specifies the minimum-scale contingency $\bar{k}_i$, and the required return r_i^S solving $\pi_i\left(\bar{k}_i\right) - r^C\bar{k}_i^C - r_i^S\left(\bar{k}_i - \bar{k}_i^C\right) = q_i^C$.

There is a unique allocation of capital such that

1. $\pi_i'\left(\bar{k}_i\right) + \theta w_i'\left(\bar{k}_i\right) = \pi_j'\left(\bar{k}_j\right) + \theta w_j'\left(\bar{k}_j\right)$ for all firms i and j that receive social capital
2. $\pi_i'\left(\bar{k}_i\right) + \theta w_i'\left(\bar{k}_i\right) \geq \pi_k'\left(\bar{k}_j\right) + \theta w_k'\left(\bar{k}_j\right)$ for firm i that receives social capital and firm k that does not
3. Either,
   (a) For all firms that receive social capital, $\pi_i'\left(\bar{k}_i\right) + \theta w_i'\left(\bar{k}_i\right) = r^C$, or
   (b) $\pi_i'\left(\bar{k}_i\right) + \theta w_i'\left(\bar{k}_i\right) > r^C$ and $r_i^S = 0$.

We first verify that this is an equilibrium and second verify that it is the investor-optimal equilibrium.

To see that it is an equilibrium, note that it is incentive compatible for firms to accept all contracts that they are offered, as they earn q_i^C regardless of whether they accept all of their social investment contracts and operate at $\bar{k}_i$ or whether they accept only the commercial capital and operate at k_i^C. To see that this is incentive compatible from the investor's perspective, note that every investor in a socially supported firm is pivotal, in the sense that if they were to rescind their offer the firm could no longer operate at $\bar{k}_i$ an still earn q_i^C. Therefore, it would revert to its commercial scale k_i^C. Finally, note that social investors cannot raise their required return r_i^S as they would violate the firm's incentive compatibility constraint.

Now, to see that this is the investor optimal equilibrium, we note that if, holding the allocation of commercial capital fixed, the capital allocation of social investors across firms was changed, this would by definition reduce the sum of total social investor returns plus total social welfare. Reducing the amount of commercial capital that any firm i raised would weakly reduce social investor welfare, and strictly so if social investors in this equilibrium all strictly prefer their allocation to the outside option asset, as either it would result in firm contraction, or it would need to be replaced by social capital previously allocated to another firm, both of which reduce the sum of investors' welfare by more than r^C by construction. Finally, if $r_i^S = 0$ then increasing the amount of commercial capital held by any firm i is not feasible, as it would result in a social investor earning a negative return. And

if $r_i^S > 0$ then by construction we have $\pi_i'\left(\bar{k}_i\right) + \theta w_i'\left(\bar{k}_i\right) = r^C$ for all firms that receive social capital, and therefore raising the level of commercial capital would not increase social investors' welfare.

Proof of Proposition 5

Consider the unique equilibrium of the model with values-aligned investors in which no investor utilizes a scale-contingency. In this setting, each firm supported by a social investor operates at the scale $\bar{k}_i$ such that $\pi_i'\left(\bar{k}_i\right) = r_i$. Now consider the case where there are two firms i and j in equilibrium that are both supported by social investors and for whom $\frac{w_i\left(\bar{k}_i\right)}{\bar{k}_i} > \frac{w_j\left(\bar{k}_j\right)}{\bar{k}_j}$ and $w_i'\left(\bar{k}_i\right) < w_j'\left(\bar{k}_j\right)$. The former inequality implies that the equilibrium cost of capital will be lower for firm i, i.e. $r_i < r_j$, and the latter implies that the marginal social value of expanding firm j is higher than for expanding firm i. Now, consider the an arbitrary social investor who supports firm i, and suppose that this investor were to deviate in the following way. Firm i is offered ε less capital, a scale-contingent contract of $\bar{k}_i - \varepsilon$, and charged $\pi_i\left(\bar{k}\right) - \pi_i\left(\bar{k} - \varepsilon\right)$ less. Firm j is offered ε more capital, a scale contingent contract of $\bar{k}_j + \varepsilon$, and charged $\pi_j\left(\bar{k} + \varepsilon\right) - \pi_j\left(\bar{k}\right)$ more. Both firms would accept these contracts, ash they do not change the entrepreneurs' share of the profits. And for sufficiently small ε, the deviation would result in higher social welfare as $w_i'\left(\bar{k}_i\right) < w_j'\left(\bar{k}_j\right)$, and increased profit for the investor as $r_i < r_j$.

Proof of Proposition 6

Consider the case where social investors are impact-aligned, and in equilibrium each investor has a strict preference to support some firm i rather than investing in the outside option asset. And now take one such firm i that receives social capital. Let $\bar{k}_i$ be its equilibrium scale. Modify firm i's profitability to a new function $\tilde{\pi}$ in the following way.

$$\tilde{\pi}'(k) = \begin{cases} \max\left\{\pi', r^C\right\} & \text{if } k \leq \bar{k}_i \\ \pi' & \text{else} \end{cases}$$

This modification increases the profitability of firm i so that it can attract commercial capital up to its former equilibrium scale $\bar{k}_i$, and holds fixed the marginal profitability of firm i at all higher scales. Impact-aligned social investors now recognize that their investment up to $\bar{k}_i$ in equilibrium has no impact. Thus the full mass of impact-aligned social investors who used to support firm i up to $\bar{k}_i$ are now free to allocate their capital elsewhere (potentially in part by increasing firm i's scale beyond $\bar{k}_i$),

increasing social welfare.

A.5 Section 5.3 and Proof Sketch of Negative Assortative Matching

The welfare-optimal equilibrium can be established following the construction in SectionA.2, where the process is first followed for social investors with $\theta_i = \theta^1$ and then is repeated for each group of social investors with progressively higher levels of altruism. However, to establish that the equilibrium depicted in Figure 7 is indeed welfare-optimal, it is instructive to outline a different method for constructing an equilibrium in this model.

Consider the following process.

Define $\sigma(i)$ to be any ordering over all impact-aligned social investors $i \in S^1 \cup \cdots \cup S^n$.

At step 1, social investor $\sigma^{-1}(1)$ is called to support an entrepreneur. If she chooses an entrepreneur i with profits $\pi_i \geq r^C$, or the outside option asset, assign her a price of $r_i = r^C$, and a social value of $w_i' = 0$. Else assign her a price of $r_i = \pi_i$ and a social value of $w_i' = w_i$. Social investor $\sigma^{-1}(1)$ chooses the entrepreneur i that maximizes $r_i + \theta_{\sigma^{-1}(1)} w_i'$.

At step k, social investor $\sigma^{-1}(k)$ is called on to support an entrepreneur. She chooses an entrepreneur to support according to the same process, excluding any entrepreneur that has been chosen in a previous step. It is straightforward to show that this process results in an equilibrium allocation.

Now, assign every social investor an index that is increasing in their altruism parameter θ_i, so that the least altruistic social investors have the lowest indices. Then the equilibrium depicted in Figure 7 corresponds to the ordering $\sigma(i) = i$. We will demonstrate that the equilibrium arising from any other ordering σ' that results in a different allocation produces lower social welfare than σ.

Take some σ'. Identify two social investors, j and k such that

- $\theta_j > \theta_k$
- $\sigma'(k) = \sigma'(j) + 1$

That is, j is more altruistic than k, but j chooses an investment one step before k in the ordering σ'. If no such pair can be found then $\sigma' = \sigma$. Now consider an alternative ordering σ'', which is the same as σ' except that $\sigma''(j) = \sigma'(k)$ and $\sigma''(k) = \sigma'(j)$ (i.e. j and k are reordered but everything else is preserved). Let a denote the entrepreneur chosen by k under σ' and b denote the entrepreneur chosen by j under σ'. If in the ordering σ'', k chooses a at step $\sigma''(k)$, then the two orderings result in the exact same allocation. Else, under the ordering σ'', at step $\sigma''(k)$, k chooses b. At step $\sigma''(j)$ under the ordering σ'', j chooses an entrepreneur c such that $w_c \geq w_a$, as j is more altruistic than k. In this

case, it is straight forward to show that σ'' results in an allocation with weakly higher welfare than does σ'.

Now take σ'' and repeat the above process (i.e. identify mis-ordered pairs of investors and re-order them). Continue to do so until σ'' results in the same allocation as σ. So long as the allocation arising from σ' and from σ are different, it is straightforward to show that at least one transformation resulted in a strict welfare improvement. Therefore σ induces the welfare-optimal equilibrium.

To see that σ is also the investor-optimal equilibrium, we need only demonstrate that the shift from σ' to σ'' described above also weakly improves aggregate investor welfare. If in σ'' k chooses a then the allocation and aggregate investor welfare are the same in σ' and σ'' . Otherwise, k chooses b. This improves k's welfare by revealed preference. If in σ'' j then chooses a, then aggregate investor welfare is unchanged from σ' to σ''.

The remaining case is that under σ'', k chooses b, and j chooses $c \neq a$ with $w_c \geq w_a$ and $\pi_c \leq \pi_a$. We complete the proof by demonstrating that this results in higher aggregate investor welfare than the case where j chose a under σ''. Relative to if j had chosen a under σ'', j's welfare is higher by revealed preference. And all investors who chose before j also have weakly higher welfare, because $w_c \geq w_a$. It remains to show that this also weakly improves the welfare of investors who choose after j under σ''. Relative to the case where j had chosen a, the set of entrepreneurs that these investors can choose from is fixed, except that now c is guaranteed financing, and a remains eligible for financing. Because c has higher social value and lower profits, this can only improve the aggregate welfare of the remaining investors.