An Intertemporal Risk Factor Model

Fousseni Chabi-Yo* Andrei S. Gonçalves† Johnathan Loudis‡

This Version: September 2020§

Link to Most Recent Version

Abstract

Current factor models do not identify risks that matter to investors. To address this issue, we provide a factor model implementation of the ICAPM, which captures market risk and intertemporal risk (i.e., changes in long-term expected returns and volatility). We build our intertemporal risk factors as mimicking portfolios for changes in dividend yield and realized volatility and demonstrate that, ex-post, they capture news to long-term expected returns and volatility. Our estimated risk price signs are in line with the ICAPM and their magnitudes imply an average risk aversion around five. Moreover, the ICAPM performs comparably with (and mostly better than) previous factor models in terms of its maximum (out-of-sample and cost-adjusted) sharpe ratio as well as its pricing of the testing assets Lewellen, Nagel, and Shanken (2010) recommend: single stocks, industry portfolios, correlation-clustered portfolios, and bond portfolios.

JEL Classification: G10; G11; G12.

Keywords: ICAPM; Intertemporal Risk; Long-term Investors; Factor Models.

*Isenberg School of Management, University of Massachusetts, Amherst, MA 01003. E-mail: fch-abiyo@isenberg.umass.edu.

†Kenan-Flagler Business School, University of North Carolina, Chapel Hill, NC 27599. E-mail: Andrei_Goncalves@kenan-flagler.unc.edu.

‡Mendoza College of Business, University of Notre Dame, Notre Dame, IN 46556. E-mail: jloudis@nd.edu.

§We thank Andrew Chen for providing us with data on anomaly decile portfolios as well as on stock-level trading costs. We are also grateful for the very helpful comments from Ravi Bansal, Kewei Hou, Tim Loughran, Ivan Shaliastovich, René Stulz, and Lu Zhang as well as from seminar participants at The Ohio State University, University of Notre Dame, and the Triangle Macro-Finance Workshop. All remaining errors are our own.

Introduction

Factor models are ubiquitous in the asset pricing literature as they allow us to reduce the cross-section of average returns into a small set of factors. However, current implementations of these models form factors based on signals that are not directly related to risk. This disconnect is present regardless of whether the factors are motivated empirically (Fama and French (1993) and Carhart (1997)), from a valuation identity (Fama and French (2015)), from firms' optimality conditions (Hou, Xue, and Zhang (2015) and Hou et al. (2020)), or from behavioral arguments (Stambaugh and Yuan (2017) and Daniel, Hirshleifer, and Sun (2020)). Consequently, we do not know whether the empirical factors in these models reflect relevant risks to the marginal investor and, if so, through what economic mechanisms.

The fundamental issue is that, under the law of one price, a factor model that mimics the tangency portfolio and prices all assets can always be constructed regardless of the underlying economic environment (Roll (1977)). As Cochrane (2008) puts, "*The only content to empirical work in asset pricing is what constraints the author puts on his fishing expedition to avoid rediscovering Roll's theorem…The main fishing constraint one can imagine is that the factor portfolios are in fact mimicking portfolios for some well-understood macroeconomic risk*".

The factor model literature has traditionally avoided constraining factors to reflect theory-based risks under the premise that the large comovement among stocks with similar factor exposures reflects risk. However, Kozak, Nagel, and Santosh (2018) show that even models in which the cross-section of returns is fully driven by sentiment imply returns have a factor structure as long as sharpe ratios are bounded, and thus the only way to learn about investors' motives from asset pricing data is to "*…develop and test structural models with explicit assumptions about beliefs and preferences*". Alas, usual tests of structural models are plagued by sensitivity issues due to non-tradable risk factors (Lewellen, Nagel, and Shanken (2010)).

To address these issues, in this paper we attempt to identify risks that matter to investors by providing a factor model implementation of the Intertemporal CAPM (ICAPM) of Campbell et al. (2018). Our main result is that the risk price signs estimated from our

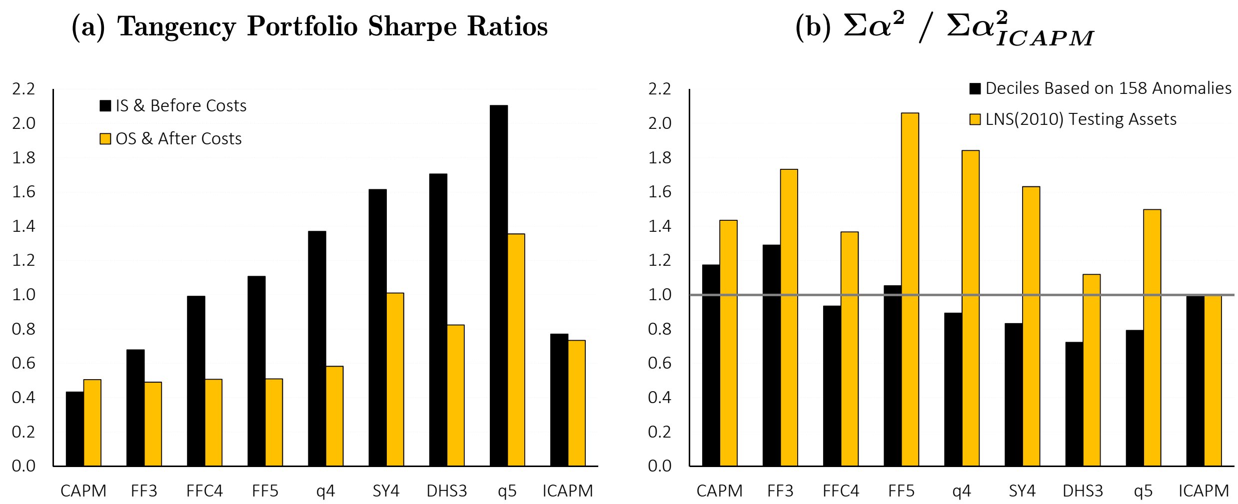


Figure 1

Main Results from Comparing Factor Models

These graphs summarize the main results from our factor model comparison. Panel (a) displays (in- and out-of-sample) maximum (gross and net of trading costs) sharpe ratios constructed using the ICAPM factors or using factors from other prominent factor models, which are described at the beginning of Section 3. Panel (b) displays averages of relative pricing errors, $\Sigma\alpha^2/\Sigma\alpha^2_{ICAPM}$, across deciles formed on 158 anomalies (from Chen and Zimmermann (2020)) and across the different groups of testing assets recommended in Lewellen, Nagel, and Shanken (2010) (single stocks, industry portfolios, correlation-clustered portfolios, and bond portfolios). Further details are provided in Sections 3 and 4.

implementation are consistent with the underlying ICAPM and their magnitudes imply the marginal investor has reasonable risk aversion (around five). Moreover, the ICAPM performs well relative to previous factor models. Specifically, while the (in-sample) ICAPM tangency portfolio sharpe ratio is relatively low, it is higher than the tangency sharpe ratio of most factor models when estimated out-of-sample and after adjusting for trading costs (see Figure 1(a)). Moreover, while some factor models perform better than the ICAPM in pricing anomaly deciles, Figure 1(b) shows the ICAPM performs better than all models we explore in pricing the testing assets recommended in Lewellen, Nagel, and Shanken (2010) (single stocks, industry portfolios, correlation-clustered portfolios, and bond portfolios), which are less subject to publication bias (see Lo and MacKinlay (1990) and Harvey (2017)).

The structural ICAPM underlying our factor model implementation features a long-term investor with Epstein-Zin preferences (Epstein and Zin (1989) and Epstein and Zin (1991)). Given her long-horizon, the investor cares about fluctuations in her current wealth (i.e., market risk) as well as in the investment opportunities she faces (i.e., intertemporal risk). The former concern induces the investor to price market risk with a positive sign. The latter leads her to price reinvestment risk (i.e., long-term expected return news, $N_{\mathbb{E}}$), and volatility risk (i.e., long-term volatility news, $N_{\mathbb{V}}$), with positive and negative signs, respectively. That is, holding current wealth fixed, declines in expected returns and increases in expected volatility are associated with high marginal utility because they imply worse prospects for long-term investing.

To translate the structural ICAPM into an intertemporal risk factor model, we must construct mimicking portfolios for $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$. The ideal procedure for constructing such mimicking portfolios involves three steps. First, we need to estimate $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$, which can be done through a Vector Autoregressive (VAR) system. Second, we need to sort stocks based on their $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$ exposures to construct base portfolios. Third, we need to project $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$ onto their respective base portfolio returns to recover their mimicking portfolios.

Our key insight is that it is possible to obtain tradable risk factors that proxy for the $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$ mimicking portfolios through an alternative procedure that does not require the estimation of $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$ in real time, which would be strongly affected by estimation error. Specifically, we sort stocks based on their past exposures to changes in the market dividend yield, Δdp, and realized (log) variance, $\Delta\sigma^2$, and then create our intertemporal risk factors, $r_{\mathbb{E}}$ and $r_{\mathbb{V}}$, as long-short portfolios in which the long (short) positions are composed of the stocks with the highest (lowest) exposures to Δdp and $\Delta\sigma^2$, respectively. As Figures 2(a) and 2(b) show, Δdp and $\Delta\sigma^2$ are highly correlated with (ex-post estimated) $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$, which leads our sorts to generate large cross-sectional variation in $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$ exposures (see Figure 3). Moreover, as Figures 2(c) and 2(d) demonstrate, our $r_{\mathbb{E}}$ and $r_{\mathbb{V}}$ risk factors are highly correlated with the "ideal factors" we obtain from projecting the (ex-post estimated) $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$ onto portfolios sorted on exposures to $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$, respectively. As such, $r_{\mathbb{E}}$ and

$r_{\mathbb{V}}$ can be used as real time proxies for the $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$ mimicking portfolios.

In our first empirical exercise, we estimate our intertemporal factor model by projecting the Stochastic Discount Factor (SDF) onto our (tradable) market and intertemporal risk factors (r_m, $r_{\mathbb{E}}$, and $r_{\mathbb{V}}$), which is equivalent to a Generalized Method of Moments (GMM) estimation that treats the factors as testing assets. We find that the average risk aversion is 4.9 in our long sample (1928-2019) and 5.4 in our modern sample (1973-2019), very close to the 5.1 (7.2) estimate from the early period (modern period) ICAPM structural estimation in Campbell et al. (2018). Moreover, consistent with the structural ICAPM, r_m and $r_{\mathbb{E}}$ have positive risk prices while $r_{\mathbb{V}}$ has a negative risk price.

While typical factor models are built in a way that the empirical factors are (roughly) uncorrelated and deliver significant average returns and sharpe ratios, this is not true of our intertemporal factor model. The ICAPM implies the factors should be correlated because long-term expected returns and volatility move together, and increases in long-term expected returns induce declines in market prices, leading to negative realized returns. Moreover, the ICAPM implies $r_{\mathbb{E}}$ ($r_{\mathbb{V}}$) has a positive (negative) risk price but such risk price must only translate into positive (negative) expected returns after controlling for r_m and $r_{\mathbb{V}}$ ($r_{\mathbb{E}}$).

Consistent with the correlation prediction, we find that our risk factors are strongly correlated, with $Cor(r_m, r_{\mathbb{E}}) = -0.77$, $Cor(r_m, r_{\mathbb{V}}) = -0.64$, and $Cor(r_{\mathbb{E}}, r_{\mathbb{V}}) = 0.83$.[1] In line with the risk price prediction, we show how to construct a strategy that exposes investors only to $r_{\mathbb{E}}$ ($r_{\mathbb{V}}$), and find that such strategy delivers a strong and significantly positive (negative) average return in-sample and out-of-sample.[2]

[1] The $Cor(r_{\mathbb{E}}, r_{\mathbb{V}}) = 0.83$ result may seem surprising given the mixed evidence in the literature on whether conditional volatility predicts short-term future returns. However, our correlation evidence is in line the literature as Bandi et al. (2019) show that current volatility strongly predicts long-term returns even though it has almost no predictive power for short-term returns. Moreover, Scruggs (1998) shows that the weak connection between volatility and short-term expected returns is in line with the ICAPM and happens because of time-variation in the ICAPM hedging component.

[2] Despite the strongly negative (and statistically significant) risk premium on a strategy that exposes investors to $r_{\mathbb{V}}$ (but not to r_m and $r_{\mathbb{E}}$), we find that our $r_{\mathbb{V}}$ risk factor has insignificant average returns over our modern sample in the absence of controls. This result helps to reconcile the apparent conflict between the negative risk price on $N_{\mathbb{V}}$ estimated in Campbell et al. (2018) and the (close to) zero risk premia on strategies that hedge against increases in expected volatility (Dew-Becker et al. (2017) and Dew-Becker,

In our second empirical exercise, we explore the relation between our intertemporal factor model and several prominent factor models proposed in the literature (Sharpe (1964), Fama and French (1993), Fama and French (2015), Carhart (1997), Hou, Xue, and Zhang (2015), Hou et al. (2020), Stambaugh and Yuan (2017), and Daniel, Hirshleifer, and Sun (2020)). We show that the risk prices on the ICAPM factors remain economically and statistically significant regardless of which factors are controlled for in SDF projections. We also estimate the correlations between the SDFs of the different models and find that the ICAPM's SDF is more correlated with the CAPM's SDF than with the SDF of any other factor model we explore, a result that only holds for the ICAPM. These findings indicate the ICAPM differs drastically from previous factor models and has a deep relation to the CAPM, which is in line with theory as the ICAPM only deviates from the CAPM due to the marginal investor's long horizon.

Finally, in our third empirical exercise, we compare the pricing ability of our intertemporal factor model to the factor models introduced above. We start by following the Barillas and Shanken (2017) argument that contrasting the tangency portfolio sharpe ratios (i.e., the "maximum sharpe ratios") of different factor models is sufficient for comparing them. We find that the ICAPM's maximum sharpe ratio is higher than the maximum sharpe ratio of the CAPM and the Fama and French (1993) 3-Factor model, but lower than the maximum sharpe ratios of all other models we explore. However, after partially adjusting for overfitting (through the out-of-sample analysis proposed by Kan, Wang, and Zheng (2019)) and for trading costs (as in Detzel, Novy-Marx, and Velikov (2020)), the ICAPM's maximum sharpe ratio is also higher than the Carhart (1997) 4-Factor model, the Fama and French (2015) 5-Factor model, and the Hou, Xue, and Zhang (2015) q4 model, remaining lower than only three models (the Hou et al. (2020) q5 model and the behavioral models proposed by Stambaugh and Yuan (2017) and Daniel, Hirshleifer, and Sun (2020)). As such, the ICAPM has a strong maximum sharpe ratio despite its factors being constrained to reflect theory-based risks.

Giglio, and Kelly (2020)). Specifically, the risk premium on expected volatility is only significantly negative after controlling for the market and reinvestment risk premia, which is in line with the ICAPM.

While focusing on maximum sharpe ratios is sufficient for comparing factor models in a world without publication bias, directly studying testing assets can be important when the publication prospects of proposed factor models correlate with the sharpe ratios on the proposed factors, which is likely to be the case in the asset pricing literature. As such, we also study the performance of the different factor models in pricing the testing assets recommended by Lewellen, Nagel, and Shanken (2010): single stocks, industry portfolios, correlation-clustered portfolios (Ahn, Conrad, and Dittmar (2009)), and bond portfolios. Tests based on these assets are likely to be subject to less publication bias (than sharpe ratio tests and tests based on the pricing of anomaly portfolios) as previous factor models were not tested against such assets when originally proposed. We find that the ICAPM is always among the models with the lowest pricing errors and is the only model to consistently do so across all four types of testings assets. In fact, after we rank models for each set of testing assets, we find that the ICAPM has the best average rank among all models we explore.

In summary, we propose an intertemporal factor model that reflects the structural ICAPM of Campbell et al. (2018), but with tradable risk factors. We show how such a factor model can be implemented empirically despite the limitations of estimating the structural ICAPM risk factors in real time. We then project the SDF onto the ICAPM risk factors and find that the risk price signs are consistent with theory and their magnitudes imply a reasonable average risk aversion. Finally, we demonstrate that the intertemporal factor model largely differs from previously proposed factor models as its SDF is more correlated with the CAPM's SDF than with the SDF of any other factor model we explore. Moreover, the intertemporal factor model performs well relative to other factor models in terms of its maximum sharpe ratio and its pricing of relevant testing assets.

Our main contribution is to connect the structural ICAPM literature (e.g., Campbell (1993), Campbell (1996), Brennan, Wang, and Xia (2004), Campbell and Vuolteenaho (2004), Maio (2013), Campbell et al. (2018), Cederburg (2019), Gonçalves (2020a), and Gonçalves (2020b)) to the reduced-form factor model literature (e.g., Fama and French (1993), Fama and French (2015), Carhart (1997), Hou, Xue, and Zhang (2015), Hou et al. (2020), Stambaugh and Yuan (2017), and Daniel, Hirshleifer, and Sun (2020)). While structural ICAPM tests provide a direct map from the risk considerations affecting investors' demand to empirically estimated risk prices, they require auxiliary assumptions (e.g., a state vector to estimate $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$) that make the overall results more subject to specification issues (see Chen and Zhao (2009) and Engsted, Pedersen, and Tanggaard (2012) for a debate on this matter). In contrast, reduced-form factor models are fairly robust to mispecification since they are based on portfolio sorts, but do not have a risk-based interpretation, and thus are silent on the fundamental risks the marginal investor cares about (Kozak, Nagel, and Santosh (2018)). Our work attempts to retain (at least part of) the theoretical link between risk prices and investors' demand while building on important implementation insights from the reduced-form factor model literature. The result is an intertemporal factor model that does not require the estimation of $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$, performs well empirically, and indicates the marginal investor has moderate risk aversion and a long investment horizon, which translates into the positive pricing of market risk (r_m) and reinvestment risk ($r_{\mathbb{E}}$), and the negative pricing of volatility risk ($r_{\mathbb{V}}$).

A recent literature has attempted to provide an ex-post ICAPM interpretation for factors in prominent factor models by linking them to variation in state variables that should capture investment opportunities (e.g., Maio and Santa-Clara (2012), Boons (2016), Cooper and Maio (2019), and Barroso, Boons, and Karehnke (2020)). Our work is different from (and complementary to) this literature as we construct tradable factors that are consistent with the ICAPM *ex-ante* as oppose to providing an *ex-post* analysis of whether well-known empirical factors are consistent with the ICAPM. Moreover, we focus on the structural ICAPM of Campbell et al. (2018), which imposes stronger restrictions than the consistency tests in this literature as they rely on the overall logic of Merton (1973) that tradable factors should be related (in some broad sense) to state variables that capture investment opportunities (also defined broadly). The result is that our analysis allows us to map each empirical factor we construct to a specific source of risk and also to infer the marginal investor's average risk aversion, tasks that are difficult to accomplish without a fully specified ICAPM framework.

Finally, our work provides a natural response to several papers that reveal important issues in asset pricing tests that raise skepticism about the ability of our current models to explain the cross-section of returns. First, we address criticisms related to the measurement of non-tradable risk factors and estimation of their risk prices by relying on tradable factors and requiring the model to price them, which makes our analysis immune to issues in the estimation of risk prices and relatively robust to mispecification in the factors' construction.[3] Second, we deal with problems arising from the testing assets used to evaluate models by relying on the testing assets recommended in Lewellen, Nagel, and Shanken (2010), which are reasonably immune to the core issues raised in the literature.[4] And third, we deal with the lack of economic interpretability of factor models (e.g., Ferson, Sarkissian, and Simin (1999) and Kozak, Nagel, and Santosh (2018)) by carefully constructing our risk factors in a way that they reasonably reflect the underlying ICAPM risks we would like to study.[5,6]

The rest of this paper is organized as follows. Section 1 introduces the structural ICAPM we rely on and details its factor model implementation. Then, Section 2 validates our intertemporal factor model and studies its risk prices as well as the risk premia associated with exposures to market and intertemporal risk. In turn, Section 3 compares our intertemporal

[3]Parker and Julliard (2005), Jagannathan and Wang (2007), Savov (2011), and Kroencke (2017) outline some important challenges in measuring consumption growth (needed to test consumption CAPMs) and Chen and Zhao (2009) focus on sensitivity issues in the measurement of news to long-term expected returns (necessary in structural tests of the ICAPM). Relatedly, Lewellen, Nagel, and Shanken (2010), Kan, Robotti, and Shanken (2013), and Laurinaityte et al. (2020) detail several limitations in the estimation of risk prices in models with non-tradable factors.

[4]See, for example, Lo and MacKinlay (1990), Berk (2000), Grauer and Janmaat (2004), Ahn, Conrad, and Dittmar (2009), Lewellen, Nagel, and Shanken (2010), Nagel and Singleton (2011), Cederburg and O'Doherty (2015), and Ang, Liu, and Schwarz (2020) for the limitations of asset pricing tests based on typical testing assets used in the literature (i.e., anomaly portfolios).

[5]Kozak, Nagel, and Santosh (2018) argue that their criticism also applies, albeit to a lesser degree, to the ICAPM when investment opportunities vary exogenously because they may be driven by sentiment. This is a limitation of our work as we do not take a stand on what drives variation in expected returns and volatility. However, even if sentiment fully drives investment opportunities and the strong correlation between $r_{\mathbb{E}}$ and $r_{\mathbb{V}}$, our results still shed light on the demand of rational investors as we uncover the risks that prevent arbitrageurs from fully exploiting the opportunities created by sentiment-driven investors.

[6]Models with ad-hoc macroeconomic variables face a similar economic interpretability issue in that average returns can be "explained" by macroeconomic variables even when they do not identify risks that matter to investors (Nawalkha (1997), Shanken (1992), and Reisman (1992)).

factor model with previous models in the literature and Section 4 explores anomalies. Finally, Section 5 concludes by summarizing our results and outlining our view on their implications. The Internet Appendix contains empirical details and supplementary empirical results.

1 The ICAPM and its Factor Model Implementation

This section introduces the ICAPM and details our factor model implementation, but only the latter is part of our core contribution since the structural ICAPM we rely on has been explored previously in the literature (e.g., Campbell et al. (2018) and Gonçalves (2020a)). Subsection 1.1 outlines the structural ICAPM, Subsection 1.2 builds the map from the structural ICAPM to an intertemporal risk factor model, and Subsection 1.3 details how we build our intertemporal risk factor model empirically.

1.1 The ICAPM

A long-term (i.e., infinitely lived) investor has Epstein-Zin recursive preferences (Epstein and Zin (1989) and Epstein and Zin (1991)) with time discount factor δ, intertemporal elasticity of substitution $\psi = 1$, and relative risk aversion γ.[7] She can invest in a set of assets delivering real gross returns summarized in the vector R_t to form her wealth portfolio with return $R_{w,t}$. She chooses consumption and portfolio allocation to maximize lifetime utility subject to the usual budget constraint. The Stochastic Discount Factor (SDF) derived from the investor's optimality conditions with respect to consumption and portfolio allocation is given by:[8]

$$logSDF_t \ = \kappa \ - \ \gamma \cdot logR_{w,t} \ - \ (\gamma - 1) \cdot \widetilde{vw}_t \qquad (1)$$

where $\widetilde{vw}_t = vw_t - \mathbb{E}_{t-1}[vw]$, with $vw_t = log(V_t/W_t)$ reflecting the investor's log value-wealth ratio (i.e., the ratio of her value function to her current wealth).

[7] ψ plays no role in our empirical analysis. We fix $\psi = 1$ in the exposition because in this case the ICAPM implications do not require any log-linear approximation. All ICAPM implications exposed follow (as approximations) if $\psi \neq 1$.

[8] To simplify the exposition, we assume $\mathbb{E}_t[vw] \approx vw_t/\delta$. Otherwise, the intercept has a small time varying component, $\kappa_{t-1} = (\gamma-1)\cdot(vw_{t-1}/\delta - \mathbb{E}_{t-1}[vw] + f(\psi, \delta))$. Such time varying component has no implications for risk premia (only for interest rate variation), and thus does not play a role in our analysis.

There are two fundamental sources of risk for a long-term investor in the ICAPM SDF. The first is the usual variation in current wealth ($logR_w$), which we refer to as market risk since it depends on realized returns in financial markets. The second is the variation in the investor's value function relative to her current wealth (vw), which we refer to as intertemporal risk since it reflects variation in investment opportunities.[9]

For our factor model implementation, we decompose $\widetilde{vw}$ into the two components that capture intertemporal risk in the ICAPM: news to long-term expected returns and volatility (i.e., reinvestment risk and volatility risk). Specifically, we assume $logSDF_t$ and $logR_{w,t}$ are joint normal so that we have (ignoring constants):

$$\begin{aligned} vw_t &= \mathbb{E}_t\left[\sum_{h=1}^{\infty}\delta^h \cdot logR_{w,t+h}\right] - \frac{(\gamma-1)}{2}\cdot\mathbb{E}_t\left[\sum_{h=1}^{\infty}\delta^h\cdot\mathbb{V}_{t+h-1}\left[v_{t+h}\right]\right] \\ &= \mathbb{E}_t\left[\sum_{h=1}^{\infty}\delta^h \cdot logR_{w,t+h}\right] - \eta\cdot\frac{(\gamma-1)}{2}\cdot\mathbb{E}_t\left[\sum_{h=1}^{\infty}\delta^h\cdot\mathbb{V}_{t+h-1}\left[logR_{w,t+h}\right]\right] \\ &= \mathbb{ER}_t - \eta\cdot\frac{(\gamma-1)}{2}\cdot\mathbb{VR}_t \\ &\Downarrow \\ \widetilde{vw}_t &= N_{\mathbb{E},t} \;-\; \eta\cdot\frac{(\gamma-1)}{2}\cdot N_{\mathbb{V},t} \end{aligned} \tag{2}$$

where the first equality is derived in Gonçalves (2020a), the second equality follows from the assumptions that $\mathbb{V}_t\left[\widetilde{vw}\right] \propto \mathbb{V}_t\left[logR_w\right]$ and $Cor_t(\widetilde{vw}, logR_w) = Cor(\widetilde{vw}, logR_w)$, and the third and fourth equalities define long-term expected returns and variance, $\mathbb{ER}_t$ and $\mathbb{VR}_t$, as well as their respective news $N_{\mathbb{E},t} = \mathbb{ER}_t - \mathbb{E}_{t-1}[\mathbb{ER}]$ and $N_{\mathbb{V},t} = \mathbb{VR}_t - \mathbb{E}_{t-1}[\mathbb{VR}]$.[10] $\eta \geq 0$ is a constant that is irrelevant for the purpose of this paper.[11]

[9] Note that intertemporal risk is only relevant to investors for which long-term prospects are important. Specifically, if the investor had a one period horizon, then the value function would be zero at the end of the period so that vw would not enter the SDF. Similarly, if investment opportunities did not vary over time, then $\widetilde{vw}$ would always be zero. Finally, if $\gamma = 1$, then the investor acts myopically and ignores vw.

[10] Our assumptions are a generalization of the assumptions in Campbell et al. (2018) since their VAR system satisfies $\mathbb{V}_t\left[vw\right] \propto \mathbb{V}_t\left[logR_w\right]$ and $Cor_t(vw, logR_w) = Cor(vw, logR_w)$.

[11] Specifically, $\eta = 1 + x + 2\cdot Cor(\widetilde{vw}, logR_w)\cdot\sqrt{x}$, where $x = \mathbb{V}_t[\widetilde{vw}]/\mathbb{V}_t[logR] \geq 0$. Setting

Increases in expected returns increase the value function relative to current wealth as wealth is expected to grow at a higher rate, delivering a more valuable future consumption stream. In contrast, higher future volatility decreases the value function relative to current wealth since achieving the given wealth growth requires facing more risk going forward.

Substituting Equation 2 into the log SDF in Equation 1 yields:[12]

$$logSDF_t \ = \kappa \ - \ \gamma \cdot logR_{w,t} \ - \ (\gamma - 1) \cdot N_{\mathbb{E},t} \ + \ \eta \cdot \frac{(\gamma-1)^2}{2} \cdot N_{\mathbb{V},t} \tag{3}$$

which shows that, with $\gamma > 1$, declines in expected returns and increases in expected volatility (holding current wealth fixed) represent bad news to the long-term investor.[13]

In summary, the ICAPM SDF reflects market risk (r_m), reinvestment risk ($N_{\mathbb{E}}$), and volatility risk ($N_{\mathbb{V}}$). Moreover, the last two terms are responsible for the intertemporal risk component in the ICAPM SDF, with the ICAPM reducing to the CAPM in the absence of these intertemporal risk terms, which occurs if long-term investing prospects are not important for the marginal investor (see Footnote 9).

1.2 The Intertemporal Risk Factor Model

This subsection translates from the ICAPM log SDF in Equation 3 to our intertemporal risk factor model (i.e., an SDF that is linear in a set of tradable long-short portfolios).

$Cor(\widetilde{vw}, logR_w) = -1$ provides the lower bound function $\eta_{\text{LB}} = 1 + x - 2 \cdot \sqrt{x}$, which reaches its minimum, $\eta^*_{\text{LB}} = 0$, at $x = 1$, implying $\eta \geq 0$.

[12] Equation 3 reflects the SDF in the ICAPM of Campbell et al. (2018) and resembles the SDF in the Long-run Risks (LRR) model of Bansal and Yaron (2004) and Bansal et al. (2014). Gonçalves (2020a) demonstrates that the ICAPM and the LRR model are deeply connected in that their SDFs can be written in identical forms. However, he also shows that the two models can yield substantially different asset pricing implications because they differ in the implicit marginal investor they consider. While the LRR model requires taking a stand on the consumption growth dynamics of the marginal investor (and is often implemented using aggregate consumption growth), the ICAPM requires taking a stand on the wealth portfolio dynamics of the marginal investor (and is often implemented using an equity index as the wealth portfolio).

[13] The $\gamma > 1$ condition for the positive $N_{\mathbb{E}}$ risk price is a consequence of two offsetting effects. An asset that comoves positively with expected returns is desirable since it provides more capital to be invested when expected returns are high, allowing the investor to take advantage of the better investment opportunities. However, the asset also exposes investors to reinvestment risk. When $\gamma > 1$, the latter effect dominates so that the $N_{\mathbb{E}}$ risk price is positive.

First, we apply a first order Taylor expansion to Equation 3 to obtain:

$$SDF_t \approx \mathbb{E}[SDF] \cdot \left(\kappa \; - \; \gamma \cdot logR_{w,t} \; - \; (\gamma - 1) \cdot N_{\mathbb{E}} \; + \; \eta \cdot \frac{(\gamma - 1)^2}{2} \cdot N_{\mathbb{V}} \right) \tag{4}$$

Now, project the risk factors onto a vector containing all excess returns (i.e., long-short portfolios) available to the representative investor, r_t, to get:

$$\begin{cases} logR_{w,t} & = & \phi_{0,m} \; + \; \phi_m \cdot r_{m,t} \; + \; \epsilon_{m,t} \\ N_{\mathbb{E}} & = & \phi_{0,\mathbb{E}} \; + \; \phi_{\mathbb{E}} \cdot r_{\mathbb{E},t} \; + \; \epsilon_{\mathbb{E},t} \\ N_{\mathbb{V}} & = & \phi_{0,\mathbb{V}} \; + \; \phi_{\mathbb{V}} \cdot r_{\mathbb{V},t} \; + \; \epsilon_{\mathbb{V},t} \end{cases} \tag{5}$$

where $r_{m,t} = \pi_m^{'} r_t$, $r_{\mathbb{E},t} = \pi_{\mathbb{E}}^{'} r_t$, $r_{\mathbb{V},t} = \pi_{\mathbb{V}}^{'} r_t$, and the orthogonality conditions $\mathbb{E}[\epsilon_w \cdot r_j] = \mathbb{E}[\epsilon_{\mathbb{E}} \cdot r_j] = \mathbb{E}[\epsilon_{\mathbb{V}} \cdot r_j] = 0$ hold by construction for any excess return, $r_{j,t}$. The weights are normalized so that $\Sigma_j \pi_{k,j} = 1$ and $\phi_k \geq 0$ for $k = m$, $\mathbb{E}$, $\mathbb{V}$.[14]

Substituting these projections into the linearized SDF in Equation 4 yields:

$$SDF_t = \underbrace{a \; - \; b_m \cdot r_{m,t} \; - \; b_{\mathbb{E}} \cdot r_{\mathbb{E},t} \; - \; b_{\mathbb{V}} \cdot r_{\mathbb{V},t}}_{M_t} \; + \; \epsilon_t \tag{6}$$

where M_t is the SDF_t projection onto the factors and $\mathbb{E}[\epsilon] = \mathbb{E}[\epsilon \cdot r_j] = 0$ holds by construction since ϵ is a linear combination of $(\epsilon_m, \epsilon_{\mathbb{E}}, \epsilon_{\mathbb{V}})$. Moreover, this intertemporal factor model implies the restrictions $b_m \geq 0$, $b_{\mathbb{V}} \leq 0$, and $\text{sign}(b_{\mathbb{E}}) = \text{sign}(\gamma - 1)$ (so $b_{\mathbb{E}} \geq 0$ as long as $\gamma \geq 1$), which we explore in our empirical analysis.[15]

In turn, the Euler condition $\mathbb{E}[M \cdot r_j] = 0$ implies $\mathbb{E}[r_j] = -Cov(r_j, M)/\mathbb{E}[M]$ so that substituting the ICAPM SDF in Equation 6 yields a factor model (with r_j representing an arbitrary excess return):

[14] This normalization is without loss of generality. To see why, note that, if $\phi_k < 0$, we can multiply ϕ_k and $r_{k,t}$ by -1 since $-r_{k,t}$ is in the space of excess returns whenever $r_{k,t}$ is. Similarly, any $\Sigma_j \pi_{k,j} \neq 1$ can be normalized by multiplying ϕ_k by $\Sigma_j \pi_{k,j}$ and dividing $\pi_{k,j}$ by $\Sigma_j \pi_{k,j}$. The $\Sigma_j \pi_{k,j} = 0$ case is not a concern because we can redefine one of the excess returns by multiplying it by -1 to return to the $\Sigma_j \pi_{k,j} \neq 0$ case.

[15] Specifically, the risk prices are given by $b_m = \phi_m \cdot \mathbb{E}[M] \cdot \gamma$, $b_{\mathbb{E}} = \phi_{\mathbb{E}} \cdot \mathbb{E}[M] \cdot (\gamma - 1)$, and $b_{\mathbb{V}} = -\phi_{\mathbb{V}} \cdot \mathbb{E}[M] \cdot \eta \cdot \frac{(\gamma-1)^2}{2}$. The ICAPM restrictions then follow from $\phi_k \geq 0$ for $k = m$, $\mathbb{E}$, $\mathbb{V}$.

$$
\begin{aligned}
\mathbb{E}[r_j] &= Cov(r_j, r_m) \cdot \frac{b_m}{\mathbb{E}[M]} \;+\; Cov(r_j, r_{\mathbb{E}}) \cdot \frac{b_{\mathbb{E}}}{\mathbb{E}[M]} \;+\; Cov(r_j, r_{\mathbb{V}}) \cdot \frac{b_{\mathbb{V}}}{\mathbb{E}[M]} \\
&= \frac{Cov(r_j, r_m)}{\sigma_m^2} \cdot \frac{\sigma_m^2 \cdot b_m}{\mathbb{E}[M]} \;+\; \frac{Cov(r_j, r_{\mathbb{E}})}{\sigma_m \cdot \sigma_{\mathbb{E}}} \cdot \frac{\sigma_m \cdot \sigma_{\mathbb{E}} \cdot b_{\mathbb{E}}}{\mathbb{E}[M]} \;+\; \frac{Cov(r_j, r_{\mathbb{V}})}{\sigma_m \cdot \sigma_{\mathbb{V}}} \cdot \frac{\sigma_m \cdot \sigma_{\mathbb{V}} \cdot b_{\mathbb{V}}}{\mathbb{E}[M]} \\
&= \beta_{m,j} \cdot \lambda_m \;+\; \beta_{\mathbb{E},j} \cdot \lambda_{\mathbb{E}} \;+\; \beta_{\mathbb{V},j} \cdot \lambda_{\mathbb{V}} \qquad (7)
\end{aligned}
$$

where the last equality defines the risk exposures ($\beta_{m,j}$, $\beta_{\mathbb{E},j}$, $\beta_{\mathbb{V},j}$) and the ICAPM factor risk premia (λ_m, $\lambda_{\mathbb{E}}$, $\lambda_{\mathbb{V}}$). For instance, $\lambda_{\mathbb{E}}$ equals the expected return of a portfolio with $\beta_{\mathbb{E}} = 1$ and $\beta_m = \beta_{\mathbb{V}} = 0$.

Note that we define risk exposures as univariate βs and normalize them to be in market beta units. For instance, $\beta_{\mathbb{E},j} = Cov(r_j, r_{\mathbb{E}})/(\sigma_m \cdot \sigma_{\mathbb{E}}) = Cov(r_j, \frac{\sigma_m}{\sigma_{\mathbb{E}}} \cdot r_{\mathbb{E}})/\sigma_m^2$, which reflects the univariate beta of excess return j with respect to $r_{\mathbb{E}}$ after we normalize $r_{\mathbb{E}}$ to have the same volatility as the market factor.[16]

Finally, letting $b = [b_m \quad b_{\mathbb{E}} \quad b_{\mathbb{V}}]'$, $\lambda = [\lambda_m \quad \lambda_{\mathbb{E}} \quad \lambda_{\mathbb{V}}]'$, and $f = [r_m \quad r_{\mathbb{E}} \quad r_{\mathbb{V}}]'$, the Euler condition $\mathbb{E}[M \cdot f] = 0$ implies:

$$
b = \begin{pmatrix} \mathbb{E}[M] \\ \mathbb{E}[M] \\ \mathbb{E}[M] \end{pmatrix} \times \Sigma_f^{-1} \mathbb{E}[f] \qquad \text{and} \qquad \lambda = \begin{pmatrix} \sigma_m^2 \\ \sigma_m \cdot \sigma_{\mathbb{E}} \\ \sigma_m \cdot \sigma_{\mathbb{V}} \end{pmatrix} \times \Sigma_f^{-1} \mathbb{E}[f] \qquad (8)
$$

where Σ_f is the f covariance matrix and $\times$ represents element-by-element multiplication

Since our risk factors are excess returns, the Euler condition $\mathbb{E}[M \cdot f] = 0$ does not identify a. As such, we normalize a by imposing $\mathbb{E}[M] = \mathbb{E}[1/R_f^n]$ where R_f^n is the nominal gross risk-

[16] Our choice of normalization attempts to make the units of the betas more comparable across factors by keeping factor volatility fixed. Similarly, we focus on univariate betas because the λs have more direct economic interpretations than the expected returns on the factors, which would be the λs if we focused on multivariate betas. The crux of the matter is that our risk factors are not orthogonal (nor they should be according to the ICAPM), which makes the distinction between univariate and multivariate betas important for the interpretation of the estimates we report in Section 2. However, neither the beta normalization nor the choice between univariate and multivariate betas influences the asset pricing implications of the model since alphas are the same regardless of how we define the βs as long as the λs are defined in accordance to the βs as in Equation 7.

free rate. This normalization does not affect any of the asset pricing implications for excess returns we study in this paper, as can be seeing in the λ expression above (see Chapter 13.2 in Cochrane (2005) for more details on this issue).

1.3 Constructing the ICAPM Risk Factors

The key to a valid factor model implementation of the ICAPM is to build r_m, $r_{\mathbb{E}}$, and $r_{\mathbb{V}}$ that represent mimicking portfolios (i.e., satisfy the orthogonality conditions implied by the Equations in 5) for the ICAPM risk factors capturing market risk ($logR_w$), reinvestment risk ($N_{\mathbb{E}}$), and volatility risk ($N_{\mathbb{V}}$). This subsection details how we approach this crucial task.

(a) The Market Risk Factor

Since $R_{w,t}$ are real gross returns on the representative investor's wealth portfolio, we have $logR_{w,t} = logR^n_{w,t} - log(I_t/I_{t-1}) \approx R^n_{w,t} - I_t/I_{t-1}$, where $R^n_{w,t}$ are nominal gross returns on the wealth portfolio and I_t/I_{t-1} is the gross inflation rate. If the equity market reflects the wealth portfolio so that $R^n_{w,t} = R^n_{m,t}$, then:

$$\begin{aligned} logR_{w,t} &\approx R^n_{w,t} - I_t/I_{t-1} \\ &= R^n_{m,t} - R^n_{f,t} \; + \; (R^n_{f,t} - I_t/I_{t-1}) \\ &= r_{m,t} \; + \; \epsilon_{m,t} \end{aligned} \tag{9}$$

where $r_{m,t} = R^n_{m,t} - R^n_{f,t}$ are returns on the equity market in excess of the risk-free rate and $\epsilon_{m,t} = R^n_{f,t} - I_t/I_{t-1}$ (roughly) reflects unexpected inflation, which we assume does not price excess returns since both legs of any long-short strategy have an inflation component.

The argument above motivates us to use $r_{m,t} = R^n_{m,t} - R^n_{f,t}$ as our market risk factor and implies $\gamma = b_m/(\phi_m \cdot \mathbb{E}[M]) = \lambda_m/\sigma^2_m$, which allows us to infer the marginal investor's relative risk aversion from our Σ_f and $\mathbb{E}[f]$ estimates.[17] Setting $r_m = R^n_{m,t} - R^n_{f,t}$ is also empirically convenient given that most CAPM tests use the same variable as the market

[17] We measure r_m using the market risk factor available in Kenneth French's data library (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html).

factor and effectively all factor models proposed in the literature have $r_{m,t} = R^n_{m,t} - R^n_{f,t}$ as one of their risk factors.

(b) The Intertemporal Risk Factors

Theoretically, $r_{\mathbb{E}}$ and $r_{\mathbb{V}}$ represent the projections of $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$ onto the space of excess returns. Given we do not observe $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$ and would not be able to project them onto the entire excess return space even if they were available, a feasible procedure to obtain $r_{\mathbb{E}}$ and $r_{\mathbb{V}}$ is to:

1. Estimate $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$ under auxiliary econometric assumptions

2. Sort stocks based on their exposures to $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$ in order to construct base portfolios

3. Project the estimated $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$ onto the base portfolios to recover $r_{\mathbb{E}}$ and $r_{\mathbb{V}}$

While the procedure above is (close to) theoretically ideal, Steps 1 and 3 would likely be plagued by estimation error. As such, we rely on an alternative procedure that follows as closely as possible the above steps while avoiding any estimation.

We start by obtaining easily measurable proxies for $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$. We proxy for $N_{\mathbb{E}}$ using monthly changes in the log dividend yield of the aggregate equity market, $\Delta dp_t = log(D_t/P_t) - log(D_{t-1}/P_{t-1})$, and for $N_{\mathbb{V}}$ using changes in monthly (log) realized variance in the aggregate equity market, $\Delta\sigma^2_t = log(\sigma^2_t) - log(\sigma^2_{t-1})$, where $\sigma^2_t = \frac{252}{N_t} \cdot \Sigma^{N_t}_{i=1}(logR_{w,t,i})^2$ and $logR_{w,t,i}$ are aggregate equity returns for day i of month t.[18]

The motivation for using Δdp_t and $\Delta\sigma^2_t$ as proxies for $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$ is as follows. In the Δdp_t case, it is standard practice to predict aggregate returns using the forecasting regression $logR_{w,t+h} = a^{(h)} + b^{(h)}dp_t + \varepsilon_{t+h}$ and such specification implies $\mathbb{ER}_t \propto \mathbb{E}_t[logR_{w,t+h}] \propto dp_t$, and thus $N_{\mathbb{E}} \approx \Delta\mathbb{ER}_t \propto \Delta dp_t$. Consequently, ranking stocks based on $N_{\mathbb{E}}$ exposures is similar

[18] Our dividend yield variable reflects all common stocks in the CRSP dataset, is based on the sum of annual dividends with no compounding (as suggested by Binsbergen and Koijen (2010)), and includes M&A paid in cash (as suggested by Allen and Michaely (2003)). The details about the dividend yield construction can be found in Gonçalves (2020a) as our measure is identical to his. Similarly, our $logR_{w,t,i}$ is based on daily log returns on the CRSP value-weighted index.

to ranking stocks based on Δdp_t exposures under this univariate return forecasting framework, which motivates our empirical decision to proxy for $N_{\mathbb{E}}$ with Δdp when creating $r_{\mathbb{E}}$. In the $\Delta\sigma_t^2$ case, if realized variance follows an AR(1) process, $\sigma_t^2 = a + b \cdot \sigma_{t-1}^2 + \varepsilon_t$, then $\mathbb{VR}_t \propto \mathbb{E}_t[\sigma_{t+h}^2] \propto \sigma_t^2$, and thus $N_{\mathbb{V}} \approx \Delta\mathbb{VR}_t \propto \sigma_t^2 - \sigma_{t-1}^2 \approx \Delta\sigma_t^2$.[19] Consequently, ranking stocks based on $N_{\mathbb{V}}$ exposures is similar to ranking stocks based on $\Delta\sigma_t^2$ exposures under an AR(1) model for σ_t^2, which motivates our empirical decision to proxy for $N_{\mathbb{V}}$ with $\Delta\sigma^2$ when creating $r_{\mathbb{V}}$.[20]

After measuring Δdp_t and $\Delta\sigma_t^2$, at each month t (with t from December/1927 to November/2019), we measure β_{dp} and β_{σ^2} for stocks in the CRSP dataset based on univariate betas of monthly returns on Δdp_t and $\Delta\sigma_t^2$ respectively.[21] We use a 5-year rolling window to estimate betas and require stocks to have the full five years of data available to be included in the analysis.[22] For instance, the betas as of December/2000 rely monthly returns from January/1996 to December/2000, and thus require stocks to have returns available over this entire five year period. Internet Appendix C.3 shows that the main results are very similar with a 3-year rolling window.

[19] We use $\Delta\sigma_t^2 = log\left(\sigma_t^2\right) - log\left(\sigma_{t-1}^2\right)$ instead $\Delta\sigma_t^2 = \sigma_t^2 - \sigma_{t-1}^2$ because realized variance is noisy and very skewed, which creates some extreme observations in the $\Delta\sigma_t^2$ distribution in the absence of the log transformation. Internet Appendix C shows that sorting on $\Delta\sigma_t^2 = log\left(\sigma_t^2\right) - log\left(\sigma_{t-1}^2\right)$ produces better ex-post sorts on the $N_{\mathbb{V}}$ exposures (than sorting on $\Delta\sigma_t^2 = \sigma_t^2 - \sigma_{t-1}^2$) despite the fact that our ex-post estimated $N_{\mathbb{V}}$ is based on a VAR for σ_t^2, not $log\left(\sigma_t^2\right)$.

[20] Papers that explore return and variance predictability using multivariate models indicate the univariate specifications we use to motivate our proxys are reasonable. For instance, Gonçalves (2020a) shows that shocks to dp_t have a 84% correlation with his reinvestment risk factor that effectively captures $N_{\mathbb{E}}$ (since his ICAPM is homoskedastic). Similarly, Campbell et al. (2018) rely on a VAR(1) and show that realized and expected variance have a 61% correlation.

[21] As it is standard in the literature, we restrict our analysis to common stocks of firms incorporated in the United States (shrcd = 10 or 11) that trade on NYSE, AMEX, or NASDAQ (exchcd = 1,2 or 3).

[22] The only exception to the 5-year rolling window rule is in the beginning of our sample period, before we have five years of data available to estimate betas. Our first observation for Δdp_t is on January/1927 and for $\Delta\sigma_t^2$ is on February/1926. As such, at the end of December/1927, we use stock returns going back to January/1927 for β_{dp} (i.e., 12 months of data) and February/1926 for β_{σ^2} (i.e., 23 months of data). For the subsequent months, we expand the window for each factor (keeping the January/1927 and February/1926 starting points) until we have five years of data to estimate the respective beta. Once we have five years of data to estimate beta (December/1931 for β_{dp} and January/1931 for β_{σ^2}) we start applying the five year rolling window procedure. To assure the universe of stocks used to construct the factors are always the same, we impose that stocks need to have returns available for the entire β_{σ^2} measurement window (which starts earlier than the β_{dp} measurement window before December/1931).

After estimating betas, we form four value-weighted portfolios ($R^n_{L\mathbb{E}}$, $R^n_{H\mathbb{E}}$, $R^n_{L\mathbb{V}}$, $R^n_{H\mathbb{V}}$) by sorting stocks based on their β_{dp} and β_{σ^2} each month. The $R^n_{L\mathbb{E}}$ ($R^n_{L\mathbb{V}}$) portfolio contains stocks that are below the 30% NYSE breakpoint for β_{dp} (β_{σ^2}) while the $R^n_{H\mathbb{E}}$ ($R^n_{H\mathbb{V}}$) portfolio contains stocks that are above the 70% NYSE breakpoint for β_{dp} (β_{σ^2}). Finally, our intertemporal risk factors (i.e., mimicking portfolios for $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$) are constructed as $r_{\mathbb{E}} = R^n_{H\mathbb{E}} - R^n_{L\mathbb{E}}$ and $r_{\mathbb{V}} = R^n_{H\mathbb{V}} - R^n_{L\mathbb{V}}$.[23] Since our betas are measured from December/1927 to November/2019, our $r_{\mathbb{E}}$ and $r_{\mathbb{V}}$ are available from January/1928 to December/2019.

(c) The $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$ (in-sample) Estimation and their Mimicking Portfolios

While our $r_{\mathbb{E}}$ and $r_{\mathbb{V}}$ construction relies on Δdp_t and $\Delta\sigma^2_t$ as simple proxies for $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$, we also explore (in-sample) estimates of $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$ in parts of our analysis (mainly to validate our $r_{\mathbb{E}}$ and $r_{\mathbb{V}}$ construction). These estimates largely follow previous implementations of the structural ICAPM (e.g., Campbell and Vuolteenaho (2004), Campbell et al. (2018), and Gonçalves (2020a)).

Let $z_t = [logR_{w,t},\, \sigma^2_t,\, s_t]$ be a set of state variables that follow a Vector Autoregression (VAR) of order one:

$$z_{t+1} - \overline{z} \;= B(z_t - \overline{z}) \;+\; u_{t+1} \tag{10}$$

where $u_t \sim IID\,(0, \Sigma)$.

Defining $B_\infty = \delta \cdot B\,(I - \delta \cdot B)^{-1}$, $1'_r z_t = logR_{w,t}$, and $1'_{\sigma^2} z_t = \sigma^2_t$, we have that news to long-term expected returns and volatility are given by $N_{\mathbb{E},t} = 1'_r B_\infty u_t$ and $N_{\mathbb{V},t} = 1'_{\sigma^2} B_\infty u_t$. We estimate B in-sample (using OLS equation-by-equation) and treat these $N_{\mathbb{E},t}$ and $N_{\mathbb{V},t}$ estimates as the intertemporal risk factors we intend to capture with $r_{\mathbb{E}}$ and $r_{\mathbb{V}}$.[24]

[23] Our factor construction is analogous to Fama and French (1993), except that we do not orthogonalize the factors through double sortings. The reason is that the ICAPM does not imply that factors are orthogonal. In fact, the ICAPM implies a tight link between $\mathbb{ER}_t$ and $\mathbb{VR}_t$ because expected returns go up in equilibrium as expected volatility increases. We verify this positive link in our empirical analysis.

[24] s_t is composed of five variables (all measured in natural log units): dividend yield, one year Treasury yield, term spread, credit spread, and value spread. These are the same variables used in Gonçalves (2020a), with all measurement details provided there. Also following Gonçalves (2020a), we set to zero the first column of B before its estimation because $logR_{w,t}$ is not a good predictor of future returns or variance, but the $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$ constructed without this step are very similar.

We also obtain (in-sample estimated) mimicking portfolios for $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$. Specifically, we form value-weighted decile portfolios by sorting stocks based on their exposures to $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$ ($\beta_{N\mathbb{E}}$ and $\beta_{N\mathbb{V}}$) calculated over the same 5-year rolling window used to obtain stock-level β_{dp} and β_{σ^2} (Internet Appendix show .[25] Then, we project our estimated $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$ onto their respective deciles (while requiring weights to add to zero) to obtain the mimicking portfolios, $r_{N\mathbb{E}}$ and $r_{N\mathbb{V}}$, as linear combinations of the returns on the $\beta_{N\mathbb{E}}$ and $\beta_{N\mathbb{V}}$ decile portfolios.

While $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$ represent the relevant risks in the ICAPM so that $r_{N\mathbb{E}}$ and $r_{N\mathbb{V}}$ are the "ideal factors" for the intertemporal risk factor model, $r_{N\mathbb{E}}$ and $r_{N\mathbb{V}}$ are not constructed in real time given the limitations in estimating $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$. As such, $r_{N\mathbb{E}}$ and $r_{N\mathbb{V}}$ are only used to verify (ex-post) that our tradable factors, $r_{\mathbb{E}}$ and $r_{\mathbb{V}}$, reasonably proxy for $r_{N\mathbb{E}}$ and $r_{N\mathbb{V}}$.

2 The Intertemporal Risk Factor Model: Main Results

This section presents the main empirical results from our intertemporal factor model. Subsection 2.1 demonstrates the intertemporal risk factors capture exposure to long-term expected returns and volatility, Subsection 2.2 presents the estimated risk prices, and Subsection 2.3 shows that investors can extract large risk premia in real time by constructing positions that expose them to the ICAPM risk factors.

2.1 Validating the Intertemporal Risk Factors

This subsection shows that sorting stocks on exposures to Δdp and $\Delta\sigma^2$ (which is how we construct $r_{\mathbb{E}}$ and $r_{\mathbb{V}}$) produces portfolios that display a strong sort not only on exposures to Δdp, $\Delta\sigma^2$, $r_{\mathbb{E}}$, and $r_{\mathbb{V}}$, but also on exposures to (in-sample estimated) $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$ as well as their respective (in-sample estimated) mimicking portfolios, $r_{N\mathbb{E}}$, and $r_{N\mathbb{V}}$.

[25] The only exception to the 5-year rolling window is that, for the first five years in each sample period, we use the $\beta_{N\mathbb{E}}$ and $\beta_{N\mathbb{V}}$ estimated over the first five years to create the deciles. This approach allows us to obtain our (in-sample) mimicking portfolios for $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$ over the same period we have our tradable risk factors, $r_{\mathbb{E}}$ and $r_{\mathbb{V}}$.

To start, we apply a filtering process to Δdp, $\Delta\sigma^2$, $N_{\mathbb{E}}$, and $N_{\mathbb{V}}$ so that their correlations can be easily visualized.[26] Figures 2(a) and 2(b) plot the resulting filtered processes to demonstrate that Δdp and $\Delta\sigma^2$ are strongly correlated with $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$, respectively. The results indicate that Δdp and $\Delta\sigma^2$ are reasonable proxies for $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$ for the objective of sorting stocks on their risk exposures. Figures 2(c) and 2(d) further verify that $r_{\mathbb{E}}$ and $r_{\mathbb{V}}$ proxy for the $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$ mimicking portfolios by plotting $r_{\mathbb{E}}$, $r_{\mathbb{V}}$, $r_{N\mathbb{E}}$, and $r_{N\mathbb{V}}$ (using the same filtering process described in Footnote 26). The results are similar, with a strong correlation between $r_{\mathbb{E}}$ and $r_{N\mathbb{E}}$ and between $r_{\mathbb{V}}$ and $r_{N\mathbb{V}}$.[27]

While Figure 2 visually demonstrates the connection between our tradable factors and the underlying risk factors they intend to capture, a formal analysis requires exploring risk exposures. Tables 1 and 2 provide such an exercise with betas normalized to market beta units (as described below Equation 7).[28] Table 1 focuses on β_{dp} sorted portfolios while Table 2 focuses on β_{σ^2} sorted portfolios. The Panel A of each table provides results over our long sample (1928-2019) while Panel B repeats the analysis over our modern sample (1973-2019). The long sample covers the entire period we are able to produce our factors while the modern

[26]Following Campbell et al. (2018) and Gonçalves (2020a), the filtering process for a generic risk factor x is based on an exponentially weighted moving average of normalized shocks, $fx_t = (1-\pi)\cdot(x_t - \overline{x})/\sigma_x + \pi \cdot fx_{t-1}$, with a half-life of two years ($\pi = 0.5^{1/24} \approx 0.97$). In the case of Δdp and $\Delta\sigma^2$ (which are not returns or news), we calculate the residuals of a moving average of order six and only then apply the filtering process. This last step is in principle important since Δdp and $\Delta\sigma^2$ are themselves autocorrelated, but the empirical effect is small, with results being similar whether or not we apply this step.

[27]One exception is the mismatch between $r_{\mathbb{V}}$ and $r_{N\mathbb{V}}$ in the late 1990s and beginning of 2000s. We verify (in untabulated results) that this mismatch does not affect our core results. Specifically, we estimate risk prices that are very close to our reported risk prices on a sample that excludes the 1996-2005 period.

[28]For each statistic, we report the values for each decile as well as the spread between deciles 10 and 1 and the slopes. The slopes are obtained from a panel regression with portfolio returns (or realized alphas) as the response variable. In the case of the β with respect to a generic factor x (already normalized to have the same volatility as the market), the regression is specified as $r_{p,t} = a_p + (a + b \cdot \text{Decile}_p)\cdot x_t + \epsilon_{p,t}$. In the case of average returns, the regression is specified as $r_{p,t} = a + b \cdot \text{Decile}_p + \epsilon_{p,t}$. Finally, in the case of αs, the regression is specified as $\widehat{\alpha}_p + \varepsilon_{p,t} = a + b \cdot \text{Decile}_p + \epsilon_{p,t}$, with $\varepsilon_{p,t}$ reflecting time-series residuals from the respective factor model. In all cases, we report $9 \cdot b$ so that the slopes are in the same units as the spread between deciles 10 and 1. The t-statistics are always obtained from the method in Driscoll and Kraay (1998), which is a generalization of Newey and West (1987) to panel data and accounts for autocorrelation as well as correlations across portfolios. We use the procedure in Newey and West (1994) to select the number of lags to account for.

sample focuses on the second half of our long sample, which has better data coverage and is more comparable to previous studies in the factor model literature. In fact, when comparing factor models in Section 3, we focus on our modern sample because it represents the longest period for which data is available to construct all factors we study.[29]

The first half of Table 1 reports betas of β_{dp} deciles relative to Δdp, $N_{\mathbb{E}}$, $r_{\mathbb{E}}$, and $r_{N\mathbb{E}}$ (labeled β_{dp}, $\beta_{N\mathbb{E}}$, $\beta_{\mathbb{E}}$, and $\beta_{N\mathbb{E}}^{proj}$). As it is clear from the table, higher deciles have higher (less negative) betas on all four measures. Moreover, the beta spreads between deciles 10 and 1 are all strongly significant as well as the beta slopes. Similarly, the first half of Table 2 reports betas of β_{σ^2} deciles relative to $\Delta\sigma^2$, $N_{\mathbb{V}}$, $r_{\mathbb{V}}$, and $r_{N\mathbb{V}}$ (labeled β_{σ^2}, $\beta_{N\mathbb{V}}$, $\beta_{\mathbb{V}}$, and $\beta_{N\mathbb{V}}^{proj}$) and shows that higher deciles have higher (less negative) betas on all four measures, with beta spreads between deciles 10 and 1 being strongly significant as well as the beta slopes. All these β results hold over the long sample as well as over the modern sample, with Figure 3 summarizing these results visually.

Interestingly, Tables 1 and 2 show that portfolios sorted on β_{dp} do not provide a meaningful spread in average returns and portfolios sorted on β_{σ^2} provide a statistically weak spread in average returns. This result is drastically different from what has been reported previously in the factor model literature as previous factors are created from signals that strongly predict returns going forward.

While unusual in the factor model literature, the above result is not puzzling because the ICAPM does not imply sorting on β_{dp} or β_{σ^2} should provide a spread in average returns. It implies that each β sort should provide a spread in alphas constructed from a model that accounts for all ICAPM risk factors except the one related to the β used in the given sort (we label such alphas $\alpha_{m,\mathbb{V}}$ and $\alpha_{m,\mathbb{E}}$ where the subscript shows which factors are included in the model). Moreover, sorting on β_{dp} or β_{σ^2} should not induce a sort on alphas constructed using the full intertemporal factor model (which we label $\alpha_{m,\mathbb{E},\mathbb{V}}$). The results in Tables 1 and 2 confirm these predictions over the long and modern samples. Sorting on β_{dp} provides a

[29]Strictly speaking, we could construct all factors up to July/1972, but we decide to start our modern sample in January/1973 because then the modern sample represents exactly half of our long sample period. Results are not sensitive to this choice.

positive sort on $\alpha_{m,\mathbb{V}}$ while sorting on β_{σ^2} provides a negative sort on $\alpha_{m,\mathbb{E}}$. Moreover, sorting on either β_{dp} or β_{σ^2} induces no spread in $\alpha_{m,\mathbb{E},\mathbb{V}}$. For completeness, we also report CAPM alphas (labeled α_m) even though the ICAPM does not have a clear prediction about them.

The overall results indicate our intertemporal risk factors, $r_{\mathbb{E}}$ and $r_{\mathbb{V}}$, in fact capture the intertemporal risks they are design to reflect. Specifically, $r_{\mathbb{E}}$ captures reinvestment risk (i.e., variation in long-term expected returns, $N_{\mathbb{E}}$) while $r_{\mathbb{V}}$ reflects volatility risk (i.e., variation in long-term volatility, $N_{\mathbb{V}}$).

2.2 Estimating the ICAPM Risk Prices

This subsection shows that, for many alternative sample periods, the risk prices for r_m and $r_{\mathbb{E}}$ are positive while the risk price for $r_{\mathbb{V}}$ is negative, results that are in line with the ICAPM predictions. It also demonstrates that the risk aversion implied by the ICAPM, $\gamma = \lambda_m/\sigma_m^2$, is generally reasonable (between 4 and 7 across the different sample periods we explore).

Equation 8 demonstrates that b and λ are functions of Σ_f and $\mathbb{E}[f]$. As such, we estimate them by plugging in the sample analogues of these two quantities.[30] Table 3 reports the estimated CAPM and ICAPM risk prices, b, their annualized risk premia, λ, and their t-stats. Since the bs are not easily comparable, we report $\sigma_x \cdot b_x$ for each factor x_t so that the reported values can be interpreted as the change in M_t induced by a one standard deviation change in the respective x_t (holding other factors fixed). We estimate the models over our long sample period (1928-2019), a postwar sample (1946-2019), a sample that matches many previous asset pricing studies (1963-2019), and our modern sample (1973-2019), which is the second half of our long sample and reflects the period over which we can construct all factors from the models we explore in the next section. We also estimate the models for sample

[30] In Internet Appendix A, we show that such procedure is equivalent to a just-identified GMM estimation in which the risk prices are obtained by requiring the model to price the factors themselves (and demonstrate how to obtain standard errors from this equivalence). Moreover, we show that such estimator can be motivated from efficiency and/or robustness arguments. In terms of efficiency, adding other testing assets to the GMM estimation leaves the estimator unaffected as long as we rely on the efficient GMM weighting matrix. In terms of robustness, we show that our b estimate converges in probability to the projection of the SDF onto f even if the $M = a + b^{'} f$ model is mispecified, a result that does not hold for other b estimators.

periods that start at the same years, but end in 2006 (i.e., before the great recession).

The results in Table 3 indicate that market and reinvestment risk (r_m and $r_{\mathbb{E}}$) are always positively priced while volatility risk ($r_{\mathbb{V}}$) is always negatively priced. That is, holding current wealth fixed, declines in expected returns and increases in expected volatility are associated with high marginal utility because they imply worse prospects for long-term investing. Moreover, the magnitudes are economically large. For instance, over our long sample, a one standard deviation movement in $r_{\mathbb{E}}$ induces a 0.34 change in M_t, which is substantial if we consider the $\mathbb{E}[M] = \mathbb{E}[1/R_f^n] \approx 1$. Such effect translates into a risk premium of $\lambda_{\mathbb{E}} = 21.8\%$ for a portfolio that has $\beta_{\mathbb{E}} = 1$ and $\beta_m = \beta_{\mathbb{V}} = 0$. As we show in the next subsection, such unit β portfolios have substantial volatility, but the risk premia remain high even if we require the different portfolios to have the same volatility as the market.

Beyond risk prices, the model allows us to infer the implied risk aversion, $\gamma = \lambda_m/\sigma_m^2$, which depend on the full Σ_f and $\mathbb{E}[f]$, not only on the r_m information. The highest risk aversion estimate is 6.9 (estimated from 1946 to 2019) and the lowest is 4.4 (estimate from 1928 to 2006). These estimates are reasonably close to the $\overline{\gamma} = 5$ benchmark used in the structural ICAPM of Gonçalves (2020a) and the 5.1 (7.2) estimate from the early period (modern period) ICAPM structural estimation in Campbell et al. (2018).

It is also interesting to note that the CAPM implies even lower risk aversion, with estimates ranging from 2.1 to 3.5. The reason is that accounting for variation in investment opportunities tend to lower the risk of the market portfolio (i.e., equities are safer in the long-run), and thus a higher risk aversion is required in the ICAPM (relative to the CAPM) to justify the equity premium we observe in the data.

2.3 Extracting the ICAPM Factor Risk Premia

The λs in the previous subsection reflect the risk premia on portfolios that are exposed to a single risk factor each. This subsection shows how to use this concept to create trading strategies that expose investors to a single risk factor each while maintaining reasonable volatility (targeted to the equity market volatility). It also demonstrates that such strategies deliver substantial risk premia whether constructed ex-post or in real time.

While typical factor models are built in a way that the empirical factors are (roughly) uncorrelated and deliver significant average returns and sharpe ratios, this is not true of our intertemporal factor model. The ICAPM implies the factors should be correlated because long-term expected returns and volatility move together, and increases in long-term expected returns induce declines in market prices, leading to negative realized returns.

The first part of each Panel in Table 4 provides correlations across the ICAPM factors as well as their (annualized) average returns, volatilities, and sharpe ratios. Consistent with the correlation prediction, we find that our risk factors are strongly correlated, with (over the long sample) $Cor(r_m, r_{\mathbb{E}}) = -0.77$, $Cor(r_m, r_{\mathbb{V}}) = -0.64$, and $Cor(r_{\mathbb{E}}, r_{\mathbb{V}}) = 0.83$. However, we also find that $r_{\mathbb{E}}$ and $r_{\mathbb{V}}$ deliver insignificant average returns (with the $r_{\mathbb{V}}$ average return being significant over our long sample). While unusual in the factor model literature, this result is not surprising because the ICAPM implies $r_{\mathbb{E}}$ ($r_{\mathbb{V}}$) has a positive (negative) risk price but such risk price must only translate into positive (negative) expected returns after controlling for r_m and $r_{\mathbb{V}}$ ($r_{\mathbb{E}}$).

To verify that controlling for other factors results in significant risk premia for the ICAPM factors, we extract the ICAPM factor risk premia. That is, we compute expected returns for strategies that have unit exposure to each ICAPM factor with no exposure to the other two. In the case of factor $f_{i,t}$, we construct a portfolio, $r^*_{i,t} = w'_{\lambda,i} f_t$, with $\beta_i = 1$ and other betas equal zero so that $\mathbb{E}[r^*_i] = \lambda_i$. The weights for such portfolio are given by $w'_{\lambda,i} = \sigma_m \cdot \sigma_i \cdot \Sigma_f^{-1}$ (see Equation 8). We found that this approach produces returns that are much more volatile than market returns. As such, and to make these risk premia more comparable to that on the market factor, we scale the weights to match a given target volatility (σ_m in our empirical exercise). Specifically, we first find $w_{\lambda,i}$ and then obtain $w_i = \left(\sigma_m / \sqrt{w'_{\lambda,i} \Sigma_f w_{\lambda,i}}\right) \cdot w_{\lambda,i}$ for each factor $f_{i,t}$ so that $\sqrt{w'_i \Sigma_f w_i} = \sigma_m$ by construction.

The results from this ICAPM factor risk premia extraction are provided in the second part of each Panel in Table 4. The resulting strategies (r^*_m, $r^*_{\mathbb{E}}$, $r^*_{\mathbb{V}}$) are by construction strongly correlated with their respective factors, uncorrelated with the other factors, and as volatile as r_m. Moreover, they deliver substantial (and statistically significant) risk premia, with $\mathbb{E}[r_m^*] = 10.7\%$, $\mathbb{E}[r_{\mathbb{E}}^*] = 10.1\%$, and $\mathbb{E}[r_{\mathbb{V}}^*] = -6.7\%$ over the long sample.

However, the weights required to construct r_m^*, $r_{\mathbb{E}}^*$, and $r_{\mathbb{V}}^*$ (which rely on Σ_f only) were obtained ex-post using the full sample estimate for Σ_f. In the third part of each Panel in Table 4, we repeat our factor risk premia extraction exercise, but constructing the weights using a rolling window of 10 years (other rolling windows deliver similar results). Such "real time factor extraction" still results in strong (and statistically significant) risk premia, with $\mathbb{E}[r_m^*] = 10.0\%$, $\mathbb{E}[r_{\mathbb{E}}^*] = 10.5\%$, and $\mathbb{E}[r_{\mathbb{V}}^*] = -7.5\%$ over the long sample.

3 Comparing the ICAPM with Other Factor Models

The previous section shows that the ICAPM risk factors properly capture market and reinvestment risk and are priced consistently with the ICAPM predictions. In this section, we compare the ICAPM with eight prominent factor models proposed in the literature. Specifically, we consider (ordered by publication year): the Sharpe (1964) CAPM, the Fama and French (1993) 3-Factor model (FF3), the Carhart (1997) 4-Factor model (FFC4), the Fama and French (2015) 5-Factor model (FF5), the Hou, Xue, and Zhang (2015) 4-Factor model (q4), the Stambaugh and Yuan (2017) 4-Factor model (SY4), the Daniel, Hirshleifer, and Sun (2020) 3-Factor Model (DHS3), and the Hou et al. (2020) 5-Factor model (q5).[31]

The rest of this section is organized as follows. Subsection 3.1 estimates the ICAPM risk prices after controlling for other factors, Subsection 3.2 compares the ICAPM with other factor models based on their implied tangency portfolio sharpe ratios (i.e., their "maximum sharpe ratios"), and Subsection 3.3 extends the comparison to pricing errors on the testing assets recommended by Lewellen, Nagel, and Shanken (2010): single stocks, industry portfolios,

[31] For the construction of the factors, we rely on the data provided by the original authors whenever possible and on our replication of their factors whenever needed. For instance, the data provided by Daniel, Hirshleifer, and Sun (2020) on their DHS3 factors ends in December/2018, and thus we use our replication of their factors to extend the data up to December/2019, which is the end of our sample period. A detailed description of the measurement of the factors is provided in Internet Appendix B. Moreover, a detailed description of the econometric procedures used during our analyses (e.g., our bootstrap simulations) is provided in Internet Appendix A

correlation-clustered portfolios (Ahn, Conrad, and Dittmar (2009)), and bond portfolios.

3.1 ICAPM Risk Prices Controlling for Other Factors

This subsection estimates the ICAPM risk prices controlling for other factors. Specifically, we estimate the SDF projection $M_t = a - b' f_t - b_x' x_t$ with x_t reflecting the factors in each of the factor models mentioned above, with the exception that we do not add the market factor to x_t since it is already included in f_t. These SDF projections are analogous to the typical factor spanning tests in the literature, with the added advantage that SDF projections control not only for x_t but also for other f_t factors when testing each f_t factor, which is important in the context of the ICAPM.[32]

For the modern sample (1973-2019), we can construct all factors, and thus our estimation is analogous to the one used in Table 3. For the long sample (1928-2019), we can only construct the ICAPM risk factors and the factors in FFC4, which are known as SMB, HML, and MOM. However, since the risk price estimates depend only on estimates for Σ_f and $\mathbb{E}[f]$, we are still able to estimate risk prices using the method in Stambaugh (1997), which allows for factors with different time-series lengths in the estimation process.[33]

The ICAPM normalized risk prices, annualized risk premia, and t-statistics after controlling for other factors are reported in Table 5. The ICAPM risk prices remain strong and statistically significant regardless of which factors we control for. One exception is that, over the modern sample, the economically large $\lambda_{\mathbb{E}} \geq 7.0\%$ premium becomes statistically insignificant when controlling for the DHS3 and q5 factors. However, this result seems to be driven by sample size since $\lambda_{\mathbb{E}}$ remains strong and statistically significant in the long sample

[32] See Internet Appendix A.2 for a formalization of the statement that SDF projections capture the same economic information as factor spanning tests.

[33] In all cases over the long sample, we extend the factors to their first date available and then apply the Stambaugh (1997)'s estimation procedure. For instance, for the q5 model, we use data on the q5 factors until 1967 and then apply the estimation in Stambaugh (1997) to get the long sample estimates for b and λ, treating the 1928-2019 period as the "long history" and the 1967-2019 period as the "short history" as per Stambaugh (1997)'s terminology. We always treat the ICAPM and FFC4 factors as available over the "long history" regardless of which SDF projection we are estimating. As such, in the case of the comparison between the ICAPM and the FFC4 model, the procedure is equivalent to the analysis performed over our modern sample. We use bootstrap standard errors to obtain t-statistics for our long sample estimates.

even after controlling for the DHS3 and q5 factors.

Given that some of the models we control for capture a very large number of anomalies in the literature (see Hou et al. (2020)), it is surprising that they do not subsume the ICAPM factors. This fact raises the question of whether previous factor models can be interpreted as capturing state variables in the economy or need to be interpreted through the lens of the Arbitrage Pricing Theory (APT) of Ross (1976), which is substantially less informative about investor's demand beyond non-arbitrage pricing restrictions (see Kozak, Nagel, and Santosh (2018)).

Table 6 sheds light on this issue by reporting correlations between the SDFs implied by the different factor models (over our modern sample). The average pairwise correlation between the ICAPM and the different factor models (excluding the CAPM) is only 0.30. Moreover, the ICAPM has a higher correlation with the CAPM ($Cor(M_{ICAPM}, M_{CAPM}) = 0.56$) than with any of the factor models we explore ($Max\ Cor(M_{ICAPM}, M_{FacModel}) = 0.40$). These results indicate that if these factor models reflect state variables that are relevant to the marginal investor, then such state variables must capture either higher order moments of the distribution of future market returns or other components of wealth (e.g., labor wealth). Otherwise, the models largely reflect comovement in the APT spirit without reflecting macroeconomic risks in the ICAPM spirit.

3.2 ICAPM vs Other Factor Models: Maximum Sharpe Ratios

In this subsection, we compare the tangency portfolio sharpe ratios (i.e., the "maximum sharpe ratios") of different factor models. Based on Barillas and Shanken (2017), such analysis is sufficient for comparing the pricing ability of different factor models.[34]

[34] A simplified version of the Barillas and Shanken (2017) argument can be seen directly from the Gibbons, Ross, and Shanken (1989) (GRS) test statistic. Specifically, the GRS statistic is given by $SR^2(f,R) - SR^2(f) = \alpha^{'}\Sigma\alpha$, where $SR^2(f)$ is the sharpe ratio of the tangency portfolio formed with f. If we include all existing assets in the set of testing assets, R, then $SR^2(f,R) = \overline{SR}^2$ is the same for any set of factors we include in f. Therefore, the factor model with the highest $SR^2(f)$ is also the model with the best pricing ability according to the GRS statistic. As such, it is sufficient to compare models based on their maximum sharpe ratios (i.e., to compare $SR(f)$ for different f) in order to rank their pricing abilities.

Table 7 Panel A shows the (annualized) maximum sharpe ratios, SR_{max}, of the different factor models, with parentheses reporting the percent of bootstrap simulations over which the ICAPM has higher SR_{max} than the given model. The first row shows in-sample SR_{max} over the entire modern sample with models ordered by publication year. Interestingly, SR_{max} increases monotonically with the publication year, with the q5 model displaying the highest (and impressive) in-sample $SR_{max} = 2.10$. In contrast, the ICAPM's SR_{max} is higher than the SR_{max} of the CAPM and FF3, but lower than the SR_{max} of all other factor models we explore.

However, as pointed out by Fama and French (2018) and Kan, Wang, and Zheng (2019), a model's in-sample SR_{max} has an overfitting problem in that the weights of the tangency portfolio are chosen over the same period that the sharpe ratio is calculated. To explore this issue, we follow Kan, Wang, and Zheng (2019) and break the modern sample into two periods, with the 1st half representing the period from January/1973 to June/1995 and the 2nd half the period from July/1996 to December/2019 (the respective in-sample sharpe ratios are also provided in the table). We then estimate the SR_{max} weights over the 1st half and apply them to the second half to get our out-of-sample SR_{max}. The results indicate the ICAPM's SR_{max} is higher than the SR_{max} of the CAPM, FF3, and FFC4 and effectively the same as the SR_{max} of the FF5 and q4 (remaining lower than the SR_{max} of the SY4, DHS3, and q5).

The out-of-sample analysis above restricts the weights to be estimated over the first half of the modern sample. However, some factor models (such as the ICAPM) have data going back further, which allows investors to obtain weights in real time using a much longer sample. As such, we also provide the out-of-sample SR_{max} of each model after allowing the weights to be estimated from all data available up to June/1995. The results indicate the ICAPM's SR_{max} is higher than the SR_{max} of the CAPM, FF3, FFC4, and FF5 and effectively the same as the SR_{max} of the q4 (remaining lower than the SR_{max} of the SY4, DHS3, and q5).

While the analysis above considers the SR_{max} overfitting problem, there is another issue related to the comparability of the factors themselves. Specifically, each factor is constructed following a different procedure. For instance, the FF3 factors are based on 30% and 70%

breakpoints while the DHS3 factors rely on 20% and 80% breakpoints. In the absence of theory, there is no reason to restrict a given factor to a given set of construction rules, but the implementation differences can generate substantial differences in trading costs, which can have a strong effect on sharpe ratios (a point highlighted by Detzel, Novy-Marx, and Velikov (2020)).

To address this issue, we follow Detzel, Novy-Marx, and Velikov (2020) and also report, in Table 7 Panel B, a SR_{max} analysis that relies on net (of trading costs) factor returns.[35] The key difference relative to the results in Panel A is that the ICAPM compares more favorably relative to other factor models, indicating that the trading costs in implementing the ICAPM are smaller than for other models. In particular, the out-of-sample SR_{max} analysis (whether we restrict weights to be estimated from data starting in January/1973 or not) indicates the ICAPM has higher sharpe ratios than all models except for the SY4, DHS3, and q5.

The core results from Table 7 are repeated visually in Figure 4. In summary, the results indicate the ICAPM performs well relative to other factor models in terms of its maximum sharpe ratio despite its factors being constrained to reflect theory-based risks. In fact, only three out of the eight factor models studied have SR_{max} that are higher than the ICAPM SR_{max} once we account for overfitting and trading costs.

3.3 ICAPM vs Other Factor Models: Pricing Errors

While focusing on SR_{max} is sufficient for comparing factor models in a world without publication bias, there are important limitations of a SR_{max} analysis when we consider that the publication prospects of proposed factor models likely correlate with the sharpe ratios of the proposed factors. This issue can be seen directly from Table 7 as SR_{max} increases

[35] We follow the same procedure as Detzel, Novy-Marx, and Velikov (2020) with the exception that we use the trading cost measure of Chen and Velikov (2020) for the trading cost adjustment (and thank Andrew Chen for sharing the data). As Chen and Velikov (2020) demonstrate, their high frequency trading cost measure more accurately reflects trading costs and implies, on average, lower trading costs than the main alternative measures used in the literature. This choice is conservative from our perspective because, as we show, the ICAPM is less affected by trading costs than the other factor models we consider. This result is in line with the fact that our factor construction relies on single sorts whereas other models use double or triple sorts, which overweight small stocks.

monotonically with the model's publication year. The out-of-sample analysis in the previous section deals with overfitting but not with publication bias because the factors are still based on a publication process that relied on sharpe ratios over the period we treat as "out-of-sample". In some sense, the only solution is to wait another twenty years to perform a truly out-of-sample SR_{max} analysis.

This subsection considers an imperfect, but still useful, alternative solution. Namely, we compare the ICAPM to previous factor models based on the pricing of testing assets that the original studies did not consider. For this task, we focus on the testing assets recommended by Lewellen, Nagel, and Shanken (2010): single stocks, industry portfolios, correlation-clustered portfolios (Ahn, Conrad, and Dittmar (2009)), and bond portfolios. This choice alleviates concerns associated with publication bias in testing assets (Lo and MacKinlay (1990) and Harvey (2017)) and with testing assets that are formed from signals closely related to the factors themselves (e.g., Ferson, Sarkissian, and Simin (1999)), which would be a problem if we compared models based on, for example, well-known anomalies or the twenty decile portfolios we study in Tables 1 and 2 (but we also provide results for these testing assets in Section 4). When analysing testing assets, we always report results for the modern sample as it reflects the longest period for which we can construct factor returns for all factors we explore.

(a) Single Stocks

At each month t, we select all CRSP common stocks of firms incorporated in the United States (shrcd = 10 or 11) that trade on NYSE, AMEX, or NASDAQ (exchcd = 1,2 or 3) and have all returns available over the last τ months including t (i.e., from $t-\tau+1$ to t). We then estimate their pricing errors (αs) based on the usual factor regressions and calculate the squared and absolute sum of pricing errors, $\Sigma\alpha^2$ and $\Sigma|\alpha|$.[36]

Table 8 reports, for each factor model, the time-series averages of $\Sigma\alpha^2$ and $\Sigma|\alpha|$ normalized

[36] Note that the pricing error obtained from Equation 7 as $\alpha = \mathbb{E}[r_j] - (\beta_{m,j} \cdot \lambda_m + \beta_{\mathbb{E},j} \cdot \lambda_{\mathbb{E}} + \beta_{\mathbb{V},j} \cdot \lambda_{\mathbb{V}})$ is numerically identical to the α obtained from a factor regression.

by the respective sums of pricing errors computed under risk-neutral pricing ($\alpha_{RN} = \mathbb{E}[r]$), the CAPM ($\alpha_{CAPM}$), and the ICAPM ($\alpha_{ICAPM}$). We also report the percent of months for which the respective sum is lower under the ICAPM than under the given factor model (e.g., $\%(\Sigma\alpha^2_{ICAPM}<\Sigma\alpha^2)$), which incorporates sampling variability in a manner similar to Fama and MacBeth (1973) regressions. Panel A uses $\tau = 60$ (i.e., five years) and Panel B uses $\tau = 120$ (i.e., ten years).

Figure 5 provides a visual summary of the main findings. The results based on $\Sigma\alpha^2$ unambiguously indicate the ICAPM produces the smallest pricing errors and the differences are particularly pronouced when comparing the ICAPM with models that were published more recently, such as SY4, DHS3 and q5. The results based on $\Sigma|\alpha|$ are similar when comparing the ICAPM with recent models, but differ for early models as the CAPM, FF3, and FFC4 have slightly lower $\Sigma|\alpha|$ than the ICAPM when $\tau = 120$.

(b) Industry Portfolios

For this analysis, we rely on the Fama and French (1997)'s 30 and 48 industry portfolios. We choose these two sets of portfolios because Lewellen, Nagel, and Shanken (2010) also use 30 industries and 48 is the maximum number of industries based on Fama and French (1997)'s classification. In contrast to single stocks, we have a balanced panel with industry portfolios, and thus calculate one α per portfolio, reporting the same $\Sigma\alpha^2$ and $\Sigma|\alpha|$ (with the same normalizations) as we do with single stocks, but with no time-series average being required. The percent of samples for which the ICAPM has lower pricing errors than each model come from bootstrap simulations in this case.

The results (provided in Table 9 and summarized in Figures 6(a) and 6(b)) unambiguously indicate the ICAPM produces the smallest pricing errors and the differences are very high, with the ICAPM always outperforming other models in more than 75% of the bootstrap samples. Such findings hold whether we focus on $\Sigma\alpha^2$ or $\Sigma|\alpha|$.

Figures 6(c) and 6(d) plot average returns against ICAPM αs for each set of industry portfolios, which is possible since we have a balanced panel.[37] The ICAPM performs well not only relative to other models, but also in absolute terms since it reduces the relatively large variation in average returns to a relatively small variation in (statistically insignificant) αs. That said, the ICAPM has a small tendency to produce positive αs, which indicates the model is not perfect and opens the door for future research on how to improve our intertemporal risk factor model.

(c) Correlation-Clustered Portfolios

We follow Ahn, Conrad, and Dittmar (2009) in constructing two sets of correlation-clustered portfolios (one set with 10 portfolios and another set with 25 portfolios). Specifically, at each time t we calculate return correlations (on a five year rolling window) between all pairs of stocks used in our analysis of single stocks. We then obtain the distance between stocks i and j as $d_{i,j} = \sqrt{2 \cdot (1 - Cor_{i,j})}$ and use these distance measures in a hierarchical clustering analysis, applying the Ward's minimum variance method to identity groups. Finally, we combine clusters formed at different points in time by assuring that, across adjacent months, each cluster portfolio has the most consistent firm membership possible (i.e., we combine clusters each month based on the firms that belong to the different clusters in the previous month). As in Ahn, Conrad, and Dittmar (2009), our correlation-clustered portfolios provide a large spread in average returns (as in typical anomaly sorts) and low correlation across portfolios (in contrast to typical anomaly sorts). Further details are provided in Ahn, Conrad, and Dittmar (2009).

[37] Figures 6(c) and 6(d) follow the bootstrap simulations in Fama and French (2010) to correct for the bias associated with the fact that ex-post αs tend to display a larger cross-sectional distribution than ex-ante αs due to sampling variation (in the context of Fama and French (2010), this bias reflects "luck vs skill" in mutual fund performance). We also apply this bias correction when analysing correlation-clustered portfolios in Figures 7(c) and 7(d), but not when analysing Treasury bond portfolios in Figures 8(c) and 8(d) because Treasury bond portfolios (similar to anomaly portfolios) have an ex-ante rank based on duration, and thus there is no ex-post α bias. We provide details on the adjustment and on why it is not needed for Treasury bonds in Internet Appendix A.4. For simplicity, the model comparison results in the main text do not adjust for any ex-post α bias given that all models are affected by this issue, and thus the problem is less relevant in model comparison. However, we provide (qualitatively similar) model comparison results after bias adjustment for ex-post αs in Internet Appendix C.4.

Table 10 reports the results for these correlation-clustered portfolios in the same format as Table 9 and Figures 7(a) and 7(b) summarize the core findings. The two best performing factor models in this analysis are the FF3 and FFC4. The ICAPM comes next and performs substantially better than all other factor models. For instance, the next best performing model is the DHS and the ICAPM always performs better than it in more than 75% of the bootstrap samples regardless of whether we focus on $\Sigma\alpha^2$ or $\Sigma|\alpha|$.

Figures 7(c) and 7(d) plot average returns against ICAPM αs for each set of correlation-clustered portfolios. Similar to the results for industry portfolios, The ICAPM performs well not only relative to other models, but also in absolute terms since it reduces the relatively large variation in average returns to a relatively small variation in (statistically insignificant) αs. However, the model still displays the small tendency to produce positive αs, which is incompatible with the hypothesis that the model fully explains the cross-section of returns.

(d) Treasury Bond Portfolios

For this analysis, we rely on the two sets of Treasury bond portfolios available in CRSP. The first set is composed of the Fama bond portfolios and reflects Treasury bond portfolios with bond maturities up to $h = 1, 2, 3, 4, 5, 10, 30$ years. The second set is composed of the CRSP US Treasury Indexes and reflects Treasury bond portfolios with bond maturities up to $h = 1, 2, 5, 7, 10, 20, 30$ years.

Table 11 reports the results for these Treasury bond portfolios in the same format as the previous two tables and Figures 8(a) and 8(b) provide a visual summary of the main results. The best performing model is unambiguously the DHS3. The q5 and the ICAPM are pretty much tied as the second/third best performing model. In contrast, all other models perform substantially worse than the ICAPM, with the next best performing model being the q4, which performs worse than the ICAPM in at least 68% of the bootstrap samples regardless of whether we focus on $\Sigma\alpha^2$ or $\Sigma|\alpha|$.

Figures 8(c) and 8(d) plot average returns against ICAPM αs for each set of Treasury bond portfolios. In contrast to the results for industry and correlation-clustered portfolios, The ICAPM performs well relative to other models, but not in absolute terms since it explains only about a third of the variation in average returns observed across Treasury bond portfolios. One possibility for improving the model is to consider that the wealth portfolio of the marginal investor may also include Treasury bonds (which are in zero net supply at the aggregate, but can still have positive weights on the wealth portfolio of the marginal investor). We leave this type of exploration for future research since perfectly identifying the wealth portfolio of the marginal investor is an exercise worth an entire paper.

(e) Summarizing α Results

In summary, we find that the ICAPM is always among the best performing models (in terms of lowest pricing errors) and is the only factor model to consistently do so across all four types of testings assets. Table 12 formalizes this statement by ranking models based on normalized $\Sigma\alpha^2$ or $\Sigma|\alpha|$ for each set of testing assets we consider. Regardless of whether we focus on $\Sigma\alpha^2$ or $\Sigma|\alpha|$, the ICAPM is the model with the best average rank among all models we consider. Figure 1(b) in the introduction provides similar conclusions from a different perspective by averaging the eight $\Sigma\alpha^2/\Sigma\alpha^2_{ICAPM}$ values reported in Tables 8 to 11 for each factor model.

4 Anomalies

For completeness, this section provides a comparison of the different factor models we study using anomaly deciles as testing assets.

Table 13, Panel A, provides model comparison results using 158 anomaly decile portfolios (that are value-weighted and based on NYSE breakpoints) from Chen and Zimmermann (2020), which gives us a total of 1,580 portfolios.[38] The ICAPM performs better than the CAPM, FF3, and FF5 models in pricing anomalies, but worse than the other models. The

[38] We begin with the 180 "clear predictors" from Chen and Zimmermann (2020), which reflect anomalies that they classify as being "clearly significant in the original papers". From these 180 significant anomalies, we remove anomalies that do not have return records for all 10 decile portfolios for at least half of our 1973-2019 sample. This procedure yields the 158 anomalies (and the corresponding 1,580 decile portfolios) we explore in Table 13.

DHS3 and the q5 models are the two best performing ones, with the ICAPM performing better than these models in only 8% of the simulations based on $\Sigma\alpha^2$ and in no more than 45% of the simulations based on $\Sigma|\alpha|$.

As pointed out in the previous section, model comparison based on anomalies has an inherent publication bias (see Lo and MacKinlay (1990) and Harvey (2017)). In addition, the signals used in the construction of previous factor models are, by design, strongly connected (and sometimes identical) to the signals used in the construction of anomaly portfolios, which creates another set of important issues to consider when studying factor models (Ferson, Sarkissian, and Simin (1999)). Table 13, Panel B, gives a rough indication of the problem. It replaces the deciles formed on the 158 anomalies in Panel A with the deciles formed on β_{dp} and β_{σ^2} (studied in Tables 1 and 2). The results indicate the ICAPM is by far the best model in pricing these testing assets. Of course, it would be misleading to conclude the ICAPM is the best model based on such an analysis because the ICAPM factors are constructed using β_{dp} and β_{σ^2} as signals. The same logic (but in reverse) plagues the analysis in Panel A.

5 Conclusion

In this paper, we propose an intertemporal risk factor model that reflects the structural ICAPM of Campbell et al. (2018), but with tradable risk factors. We show how such an intertemporal factor model can be implemented empirically despite the limitations of estimating the structural ICAPM risk factors in real time. We then project the SDF onto the ICAPM risk factors and find that the risk price signs are consistent with theory and their magnitudes imply a reasonable average risk aversion. Finally, we demonstrate that the intertemporal factor model largely differs from previously proposed factor models and performs well relative to them in terms of its (out-of-sample and cost-adjusted) maximum sharpe ratio and its pricing of relevant testing assets.

Our results provide a direct indication that it can be fruitful to build factor models in which the factors are truly risk factors (i.e., are constructed to properly reflect the risks of some underlying theory). Such risk factor models are more informative than traditional factor models, which are not directly linked to investors' preferences and/or beliefs, and thus do not help us learn about investors' demand (Kozak, Nagel, and Santosh (2018)). At the same time, risk factor models allow for the identification of risk factors that matter to investors without being subject to the important sensitivity issues associated with estimating the risk prices of non-tradable factors (Lewellen, Nagel, and Shanken (2010)).

We hope the future literature continues to tackle the hard task of building risk factor models (or sentiment factor models closely guided by theory). This can be done by extending the ICAPM to incorporate issues we abstract from (e.g., including other assets in the wealth portfolios and tackling the non-normality of returns) or by bringing other frameworks (such as intermediary-based asset pricing) to the factor model literature. The important unifying theme is that future work in this literature should always demonstrate that the proposed factors capture the underlying risks they intend to proxy for.

References

Ahn, Dong-Hyun, Jennifer Conrad, and Robert F. Dittmar (2009). "Basis Assets". In: *Review of Financial Studies* 22.12, pp. 5133–5174.

Allen, Franklin and Roni Michaely (2003). "Payout Policy". In: *Handbook of the Economics of Finance*. Ed. by George M. Constantinides, Milton Harris, and René M. Stulz. Vol. 1. A. Elsevier Science. Chap. 7, pp. 337–429.

Ang, Andrew, Jun Liu, and Krista Schwarz (2020). "Using Stocks or Portfolios in Tests of Factor Models". In: *Journal of Financial and Quantitative Analysis* 55.3, pp. 709–750.

Bandi, Federico et al. (2019). "The Scale of Predictability". In: *Journal of Econometrics* 208.1, pp. 120–140.

Bansal, Ravi and Amir Yaron (2004). "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles". In: *Journal of Finance* 59.4, pp. 1481–1509.

Bansal, Ravi et al. (2014). "Volatility, the Macroeconomy, and Asset Prices". In: *Journal of Finance* 69.6, pp. 2471–2511.

Barillas, Francisco and Jay Shanken (2017). "Which Alpha?" In: *Review of Financial Studies* 30.4, pp. 1316–1338.

Barroso, Pedro, Martijn Boons, and Paul Karehnke (2020). "Time-varying state variable risk premia in the ICAPM". In: *Journal of Financial Economics* Forthcoming.

Berk, Jonathan B. (2000). "Sorting out sorts". In: *Journal of Finance* 55.1, pp. 407–427.

Binsbergen, Jules H. van and Ralph S. J. Koijen (2010). "Predictive Regressions: A Present-Value Approach". In: *Journal of Finance* 65.4.

Boons, Martijn (2016). "State variables, macroeconomic activity, and the cross section of individual stocks". In: *Journal of Financial Economics* 119, pp. 489–511.

Brennan, Michael J., Ashley W. Wang, and Yihong Xia (2004). "Estimation and Test of a Simple Model of Intertemporal Capital Asset Pricing". In: *Journal of Finance* 59.4, pp. 1743–1775.

Campbell, John Y. (1993). "Intertemporal Asset Pricing without Comsumption Data". In: *American Economic Review* 83.3, pp. 487–512.

– (1996). "Understanding Risk and Return". In: *Journal of Political Economy* 104.2, pp. 298–345.

Campbell, John Y. and Tuomo Vuolteenaho (2004). "Bad Beta, Good Beta". In: *American Economic Review* 94.5, pp. 1249–1275.

Campbell, John Y. et al. (2018). "An Intertemporal CAPM with Stochastic Volatility". In: *Journal of Financial Economics* 128.2, pp. 207–233.

Carhart, Mark M. (1997). "On Persistence in Mutual Fund Performance". In: *Journal of Finance* 52.1, pp. 57–82.

Cederburg, Scott (2019). "Pricing intertemporal risk when investment opportunities are unobservable". In: *Journal of Financial and Quantitative Analysis* 54.4, pp. 1759–1789.

Cederburg, Scott and Michael S. O'Doherty (2015). "Asset-pricing anomalies at the firm level". In: *Journal of Econometrics* 186, pp. 113–128.

Chen, Andrew Y. and Mihail Velikov (2020). "Zeroing in on the Expected Returns of Anomalies". Working Paper.

Chen, Andrew Y. and Tom Zimmermann (2020). "Open Source Cross-Sectional Asset Pricing". Working Paper.

Chen, Long and Xinlei Zhao (2009). "Return Decomposition". In: *Review of Financial Studies* 22.12, pp. 5213–5249.

Cochrane, John H (2005). *Asset Pricing*. Revised Edition. Princeton University Press.

– (2008). "Financial Markets and the Real Economy". In: *Handbook of the Equity Premium*. Ed. by Rajnish Mehra. 1st ed. Elsevier Science. Chap. 7, pp. 237–330.

Cooper, Ilan and Paulo Maio (2019). "New Evidence on Conditional Factor Models". In: *Journal of Financial and Quantitative Analysis* 54.5, pp. 1975–2016.

Daniel, Kent, David Hirshleifer, and Lin Sun (2020). "Short- and Long-Horizon Behavioral Factors". In: *Review of Financial Studies* 4, pp. 1673–1736.

Detzel, Andrew, Robert Novy-Marx, and Mihail Velikov (2020). "Model Selection with Transaction Costs". Working Paper.

Dew-Becker, Ian, Stefano Giglio, and Bryan Kelly (2020). "Hedging Macroeconomic and Financial Uncertainty and Volatility". Working Paper.

Dew-Becker, Ian et al. (2017). "The price of variance risk". In: *Journal of Financial Economics* 123, pp. 225–250.

Driscoll, John C. and Aart C. Kraay (1998). "Consistent Covariance Matrix Estimation with Spatially Dependent Panel Data". In: *Review of Economics and Statistics* 80.4.

Engsted, Tom, Thomas Q. Pedersen, and Carsten Tanggaard (2012). "Pitfalls in VAR based return decomposition: A clarification". In: *Journal of Banking and Finance* 36.5, pp. 1255–1265.

Epstein, Larry G. and Stanley E. Zin (1989). "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework". In: *Econometrica* 57.4, pp. 937–969.

Epstein, Larry G. and Stanley E. Zin (1991). "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis". In: *Journal of Political Economy* 99.2, pp. 263–286.

Fama, Eugene F. and Kenneth R. French (1993). "Common Risk Factors in the Returns on Stocks and Bonds". In: *Journal of Financial Economics* 33, pp. 3–56.

– (1997). "Industry Costs of Equity". In: *Journal of Financial Economics* 43.2, pp. 153–193.

– (2010). "Luck versus Skill in the Cross-Section of Mutual Fund Returns". In: *Journal of Finance* 65.5, pp. 1915–1947.

– (2015). "A five-factor asset pricing model". In: *Journal of Financial Economics* 116, pp. 1–22.

– (2018). "Choosing Factors". In: *Journal of Financial Economics* 128, pp. 234–252.

Fama, Eugene F. and James D. MacBeth (1973). "Risk, Return and Equilibrium: Empirical Tests". In: *Journal of Political Economy* 81.3, pp. 607–636.

Ferson, Wayne E., Sergei Sarkissian, and Timothy Simin (1999). "The alpha factor asset pricing model: A parable". In: *Journal of Financial Markets* 2.1, pp. 49–68.

Gibbons, Michael R., Stephen A. Ross, and Jay Shanken (1989). "A Test of the Efficiency of a Given Portfolio". In: *Econometrica* 57.5, pp. 1121–1152.

Gonçalves, Andrei S. (2020a). "Reinvestment Risk and the Equity Term Structure". In: *Journal of Finance* Forthcoming.

– (2020b). "The Short Duration Premium". In: *Journal of Financial Economics* Forthcoming.

Grauer, Robert R. and Johannus A. Janmaat (2004). "The unintended consequences of grouping in tests of asset pricing models". In: *Journal of Banking and Finance* 28, pp. 2889–2914.

Harvey, Campbell R. (2017). "Presidential Address: The Scientific Outlook in Financial Economics". In: *Journal of Finance* 72.4, pp. 1399–1440.

Hou, Kewei, Chen Xue, and Lu Zhang (2015). "Digesting Anomalies: An Investment Approach". In: *Review of Financial Studies* 28.3, pp. 650–705.

Hou, Kewei et al. (2020). "An Augmented q-Factor Model with Expected Growth". In: *Review of Finance* Forthcoming.

Jagannathan, Ravi and Yong Wang (2007). "Lazy Investors, Discretionary Consumption, and the Cross-Section of Stock Returns". In: *Journal of Finance* 62.4, pp. 1623–1661.

Kan, Raymond, Cesare Robotti, and Jay Shanken (2013). "Pricing Model Performance and the Two-Pass Cross-Sectional Regression Methodology". In: *Journal of Finance* 68.6, pp. 2617–2649.

Kan, Raymond, Xiaolu Wang, and Xinghua Zheng (2019). "In-sample and Out-of-sample Sharpe Ratios of Multi-factor Asset Pricing Models". Working Paper.

Kozak, Serhiy, Stefan Nagel, and Shrihari Santosh (2018). "Interpreting Factor Models". In: *Journal of Finance* 73.3, pp. 1183–1223.

Kroencke, Tim A. (2017). "Asset pricing without garbage". In: *The Journal of Finance* 72.1, pp. 47–98.

Laurinaityte et al. (2020). "GMM Weighting Matrices in Cross-Sectional Asset Pricing Tests". Working Paper.

Lewellen, Jonathan, Stefan Nagel, and Jay Shanken (2010). "A Skeptical Appraisal of Asset Pricing Tests". In: *Journal of Financial Economics* 96.2, pp. 175–194.

Lo, Andrew W. and A. Craig MacKinlay (1990). "Data-Snooping Biases in Tests of Financial Asset Pricing Models". In: *Review of Financial Studies* 3.3, pp. 431–467.

Maio, Paulo (2013). "Intertemporal CAPM with Conditioning Variables". In: *Management Science* 59.1, pp. 122–141.

Maio, Paulo and Pedro Santa-Clara (2012). "Multifactor models and their consistency with the ICAPM". In: *Journal of Financial Economics* 106, pp. 586–613.

Merton, Robert C. (1973). "An Intertemporal Capital Asset Pricing Model". In: *Econometrica* 41.5, pp. 867–887.

Nagel, Stefan and Kenneth J. Singleton (2011). "Estimation and Evaluation of Conditional Asset Pricing Models". In: *Journal of Finance* 66.3, pp. 873–909.

Nawalkha (1997). "A multibeta representation theorem for linear asset pricing theories". In: *Journal of Financial Economics* 46.3, p. 1997.

Newey, Whitney K. and Kenneth D. West (1987). "A Simple, Positive-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix". In: *Econometrica* 55.3, pp. 703–708.

– (1994). "Automatic Lag Selection in Covariance Matrix Estimation". In: *Review of Economic Studies* 61.4, pp. 631–653.

Parker, Jonathan A. and Christian Julliard (2005). "Consumption risk and the cross section of expected returns". In: *Journal of Political Economy* 113.1, pp. 185–222.

Reisman, Haim (1992). "Reference Variables, Factor Structure, and the Approximate Multibeta Representation". In: *Journal of Finance* 47.4, pp. 1303–1314.

Roll, Richard (1977). "A Critique of the Asset Pricing Theory's Tests". In: *Journal of Financial Economics* 4, pp. 129–176.

Ross, Stephen A. (1976). "The Arbitrage Theory of Capital Asset Pricing". In: *Journal of Economic Theory* 13.3, pp. 341–360.

Savov, Alexi (2011). "Asset Pricing with Garbage". In: *Journal of Finance* 66.1, pp. 177–201.

Scruggs, John T. (1998). "Resolving the Puzzling Intertemporal Relation between the Market Risk Premium and Conditional Market Variance: A Two-Factor Approach". In: *Journal of Finance* 53.2, pp. 575–603.

Shanken, Jay (1992). "The Current State of the Arbitrage Pricing Theory". In: *Journal of Finance* 47.4, pp. 1569–1574.

Sharpe, William F. (1964). "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk". In: *Journal of Finance* 19.3, pp. 425–442.

Stambaugh, Robert F. (1997). "Analyzing investments whose histories differ in length". In: *Journal of Financial Economics* 45, pp. 285–331.

Stambaugh, Robert F. and Yu Yuan (2017). "Mispricing Factors". In: *Review of Financial Studies* 4, pp. 1270–1315.

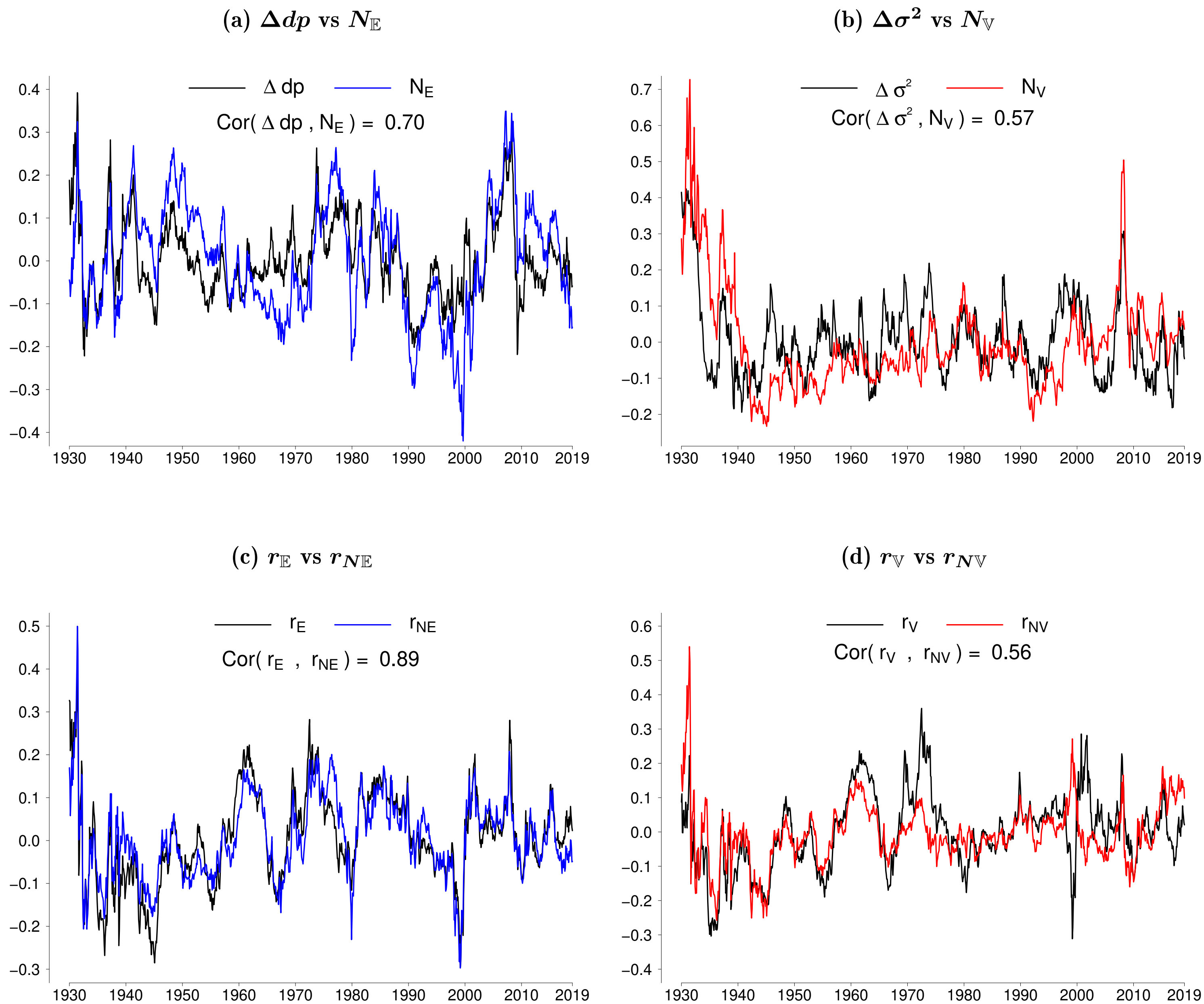


Figure 2

Filtered Intertemporal Risk Factors

These graphs compare filtered versions of different proxies for reinvestment risk and volatility risk. Panels (a) and (b) compare long-term expected return and volatility news to our news proxies. Panels (c) and (d) compare our tradable factors to mimicking factors constructed by projecting each news term onto tradable assets. In particular, Panel (a) compares the proxy for long-term expected return news (Δdp) to the news itself ($N_{\mathbb{E}}$). Panel (b) compares the proxy for long-term volatility news ($\Delta\sigma^2$) to the news itself ($N_{\mathbb{V}}$). Expected return news ($N_{\mathbb{E}}$) and volatility news ($N_{\mathbb{V}}$) are estimated in-sample using the VAR approach described in Subsection 1.3. Details on the construction of Δdp and $\Delta\sigma^2$ can be found in Footnote 18. Panels (c) and (d) compare our reinvestment risk factor ($r_{\mathbb{E}}$) and our volatility risk factor ($r_{\mathbb{V}}$) to the mimicking portfolios for $N_{\mathbb{E}}$ and $N_{\mathbb{V}}$ ($r_{N\mathbb{E}}$ and $r_{N\mathbb{V}}$). To construct $r_{N\mathbb{E}}$ ($r_{N\mathbb{V}}$), we project $N_{\mathbb{E}}$ ($N_{\mathbb{V}}$) onto returns from decile portfolios constructed by sorting stocks based on their exposure to $N_{\mathbb{E}}$ ($N_{\mathbb{V}}$) and imposing that portfolio weights sum to zero (i.e., the mimicking portfolios are zero-net-cost portfolios). The filtering procedure applied to each of these time series is the same as in Campbell et al. (2018) and is described in more detail in Subsection 2.1.

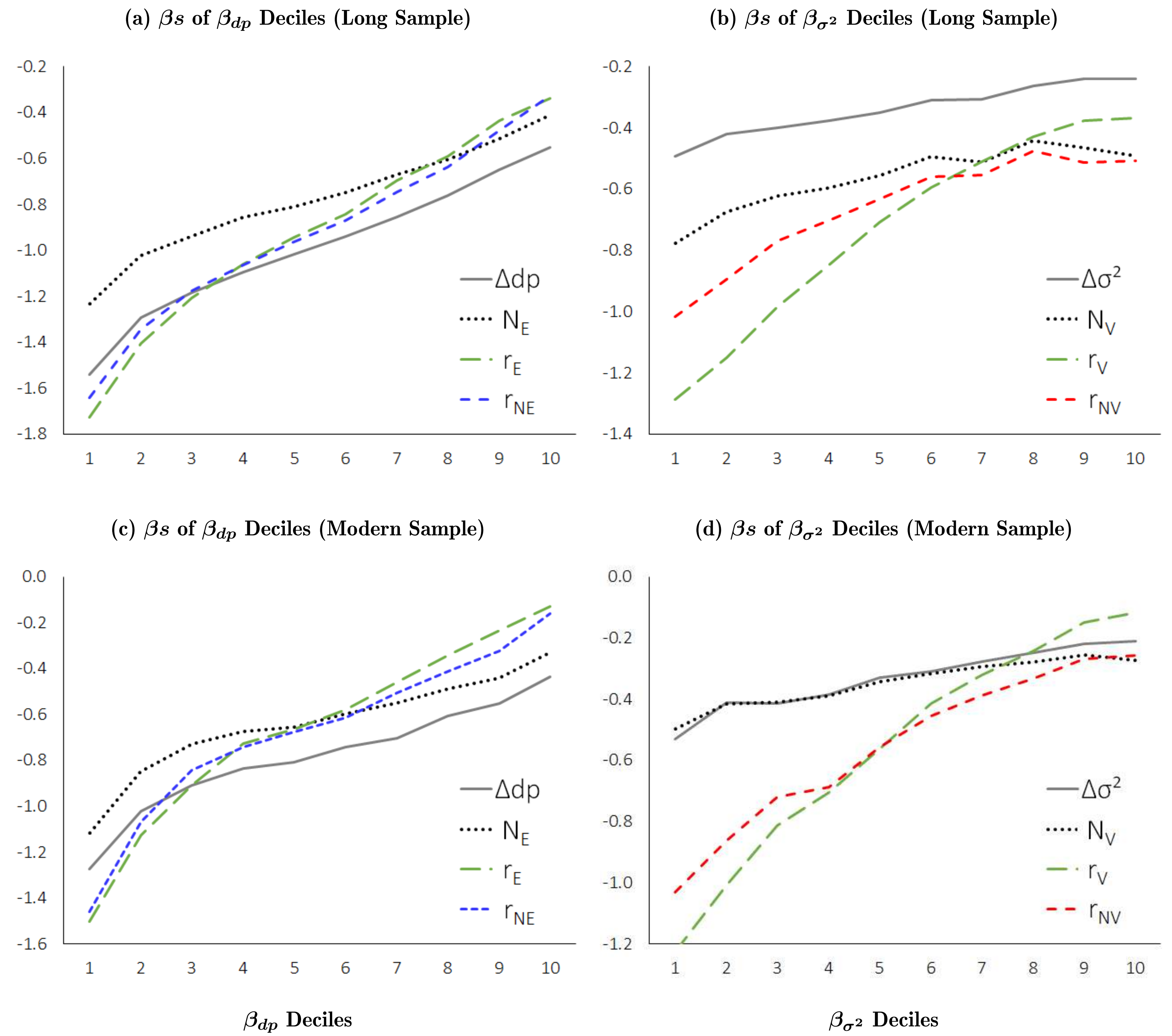


Figure 3
βs of β_{dp} and β_{σ^2} Decile Portfolios

These graphs show ex post decile portfolio betas with various measures of reinvestment risk (Δdp, $N_{\mathbb{E}}$, $r_{\mathbb{E}}$, and $r_{N\mathbb{E}}$) and volatility risk ($\Delta\sigma^2$, $N_{\mathbb{V}}$, $r_{\mathbb{V}}$, and $r_{N\mathbb{V}}$). Panels (a) and (b) use data from our Long Sample (1928-2019). Panels (c) and (d) use data from our Modern Sample (1973-2019). Panels (a) and (c) use portfolios sorted on ex ante exposure to Δdp. Panels (b) and (d) use portfolios sorted on ex ante exposure to $\Delta\sigma^2$. See Subsection 1.3 for details related to the measurement of risk factors and decile portfolio construction.

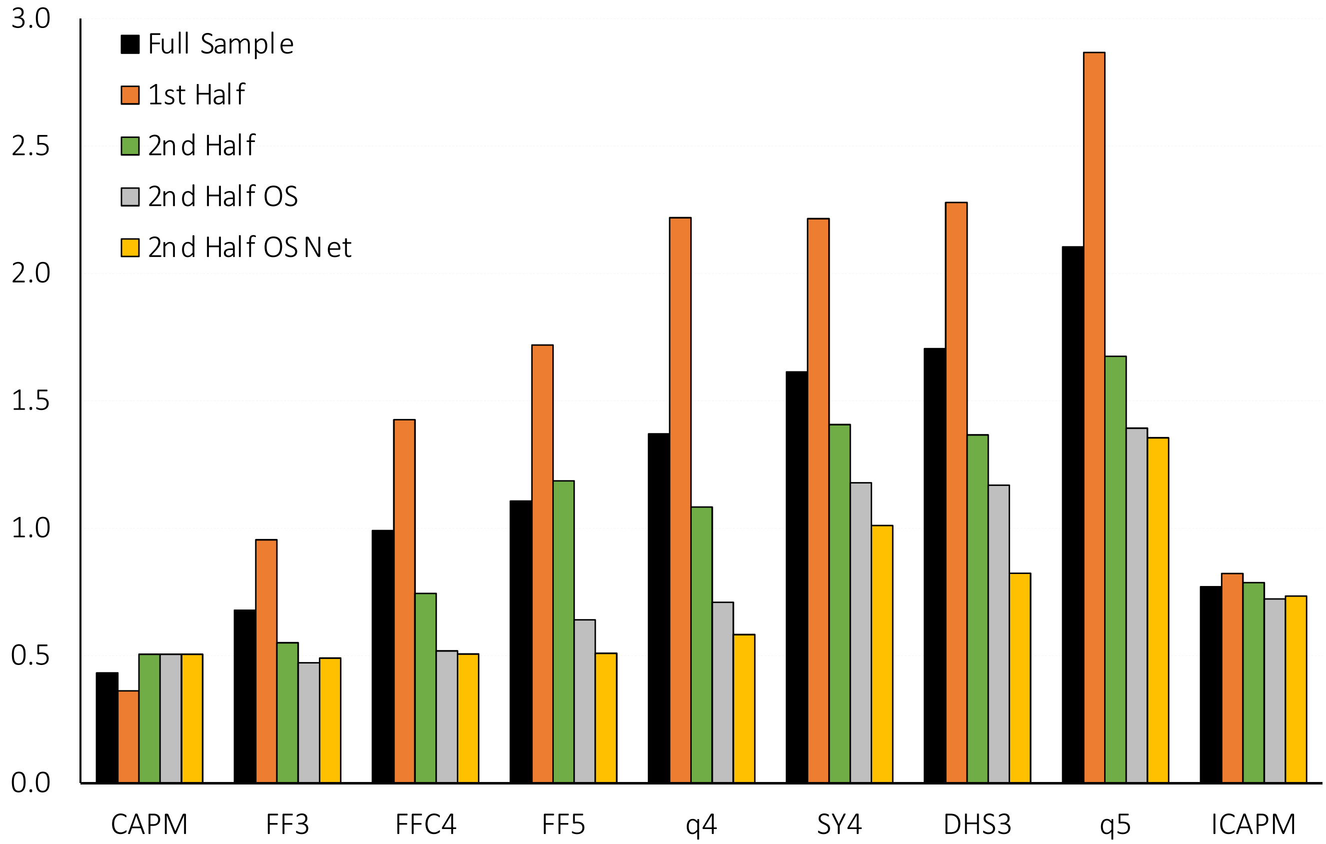
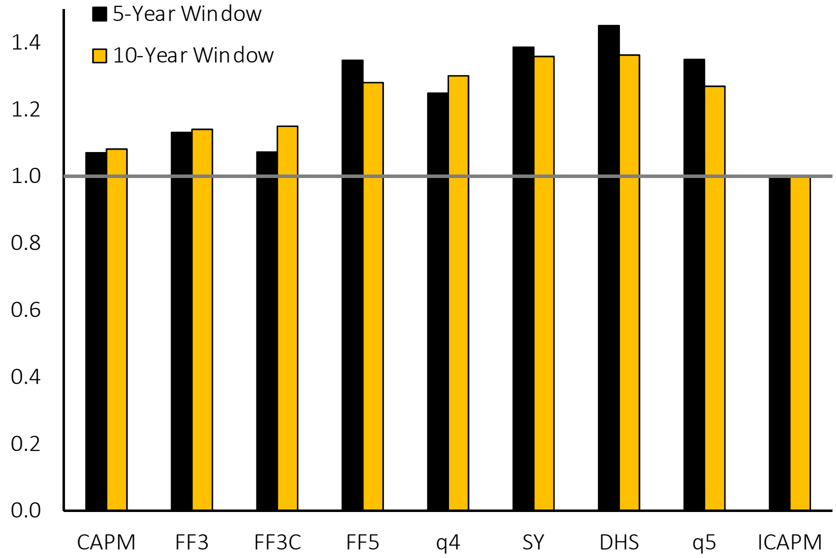


Figure 4
ICAPM vs Other Factor Models: Maximum Sharpe Ratios

This graph shows (in- and out-of-sample) maximum (gross and net-of-trading-cost) sharpe ratios constructed using the ICAPM factors or using factors from other prominent factor models, which are described at the beginning of Section 3. Results are provided for three different periods: "Full Data" (1973-2019), "1st Half" (1973-1995), and "2nd Half" (1995-2019). For each model, the first three bars display maximum sharpe ratios constructed using tangency portfolio weights estimated in-sample and applied to factors within each of these three periods. The fourth bar displays the sharpe ratio that results from applying tangency portfolio weights estimated during the "1st Half" period to construct a portfolio of factors during the "2nd Half" period. The last bar displays the analogous out-of-sample sharpe ratio when net-of-cost factors are used to both estimate weights and form the factor portfolio (see Footnote 35 for a description of the net-of-cost factor construction).

(a) $\Sigma\alpha^2$ / $\Sigma\alpha^2_{ICAPM}$



(b) $\Sigma|\alpha|$ / $\Sigma|\alpha_{ICAPM}|$

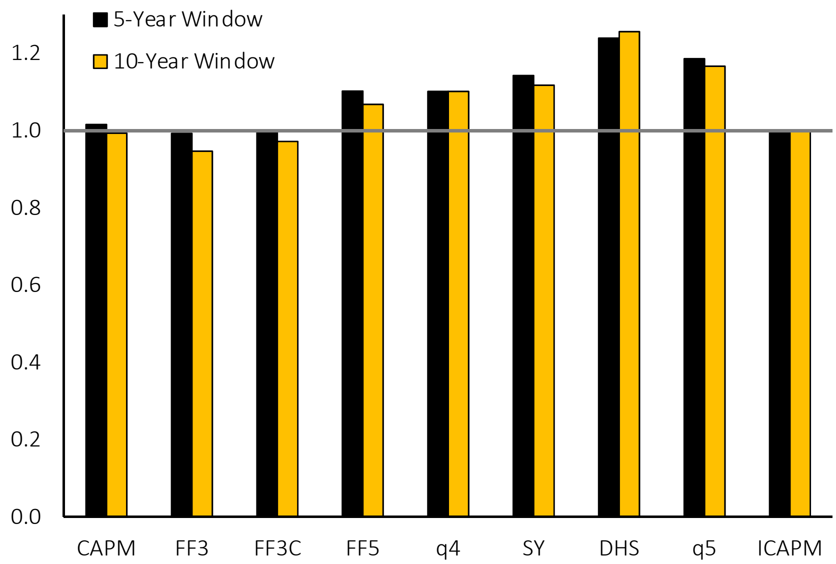


Figure 5

ICAPM vs Other Factor Models: αs of Single Stocks

These figures show pricing error metrics for the ICAPM and other prominent factor models (described at the beginning of Section 3) when we use single stocks as test assets. Panel (a) shows the sum of squared pricing errors from various models relative to that from the ICAPM ($\Sigma\alpha^2/\Sigma\alpha^2_{ICAPM}$). Panel (b) shows the sum of absolute pricing errors from various models relative to that from the ICAPM ($\Sigma|\alpha|/\Sigma|\alpha_{ICAPM}|$). αs for each stock are computed on either a 5- or 10-year rolling basis and recorded each month. Stocks are required to have full return data in each rolling period in order to be included in a particular month. Sum-squared or sum-absolute pricing errors are then computed each month by summing across the pricing errors for each valid stock, and the relevant ratios are computed each month for each model. The plotted ratios are time-series averages of the monthly ratios, similar to a Fama and MacBeth (1973) estimator. All data are from our Modern Sample (1973-2019).

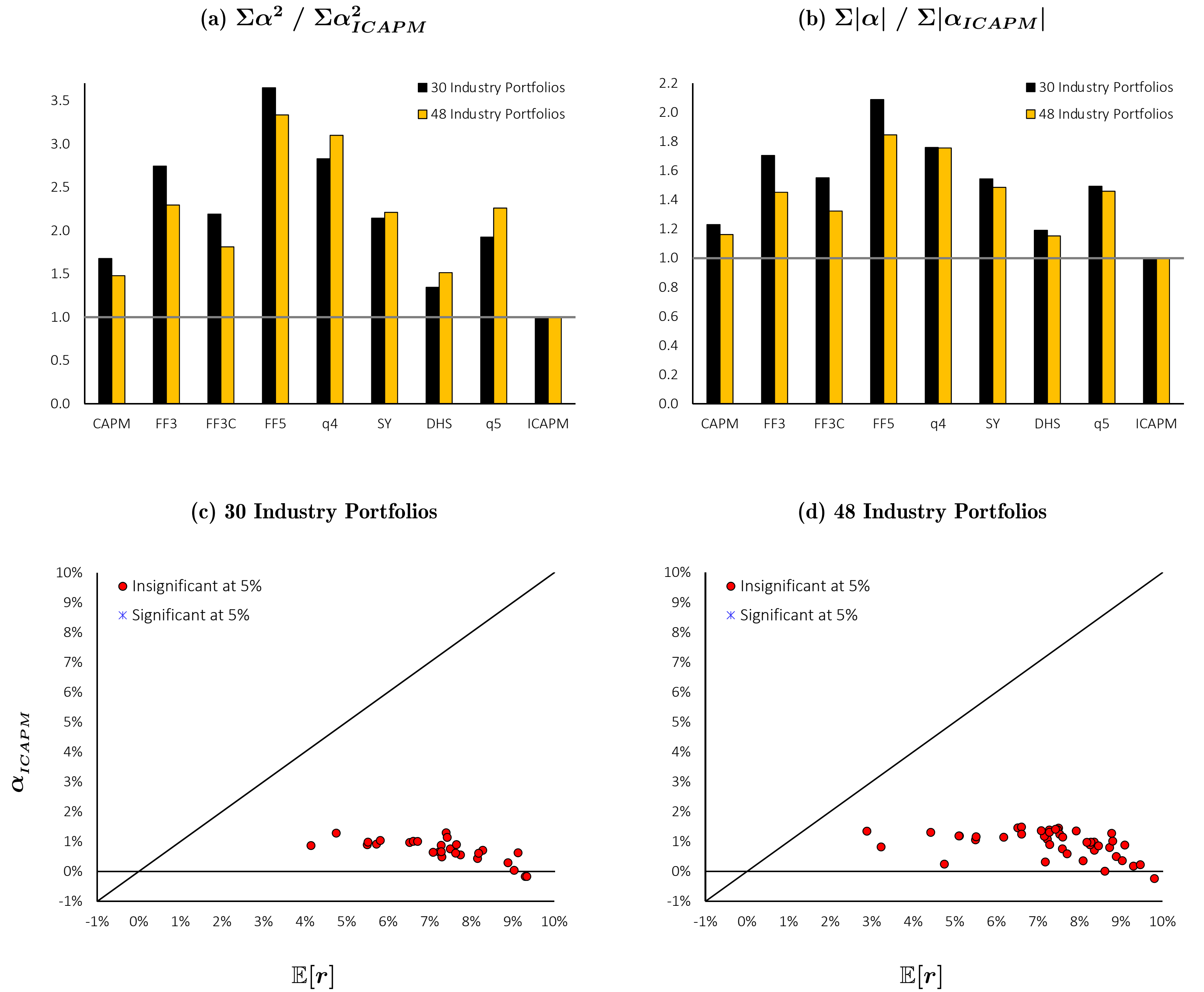


Figure 6

ICAPM vs Other Factor Models: αs of Industry Portfolios

These figures show pricing error metrics for the ICAPM and other prominent factor models (described at the beginning of Section 3) when we use Fama and French (1997)'s 30 or 48 industry portfolios as test assets. Panel (a) shows the sum of squared pricing errors from various models relative to that from the ICAPM ($\Sigma\alpha^2/\Sigma\alpha^2_{ICAPM}$) for both sets of test assets. Panel (b) shows the sum of absolute pricing errors from various models relative to that from the ICAPM ($\Sigma|\alpha|/\Sigma|\alpha_{ICAPM}|$) for both sets of test assets. Panels (c) and (d) show debiased pricing errors from the ICAPM plotted against average portfolio returns when the 30 or 48 industry portfolios are used as test assets, respectively (see Internet Appendix A.4 for a description of the debiasing procedure). All data are from our Modern Sample (1973-2019).

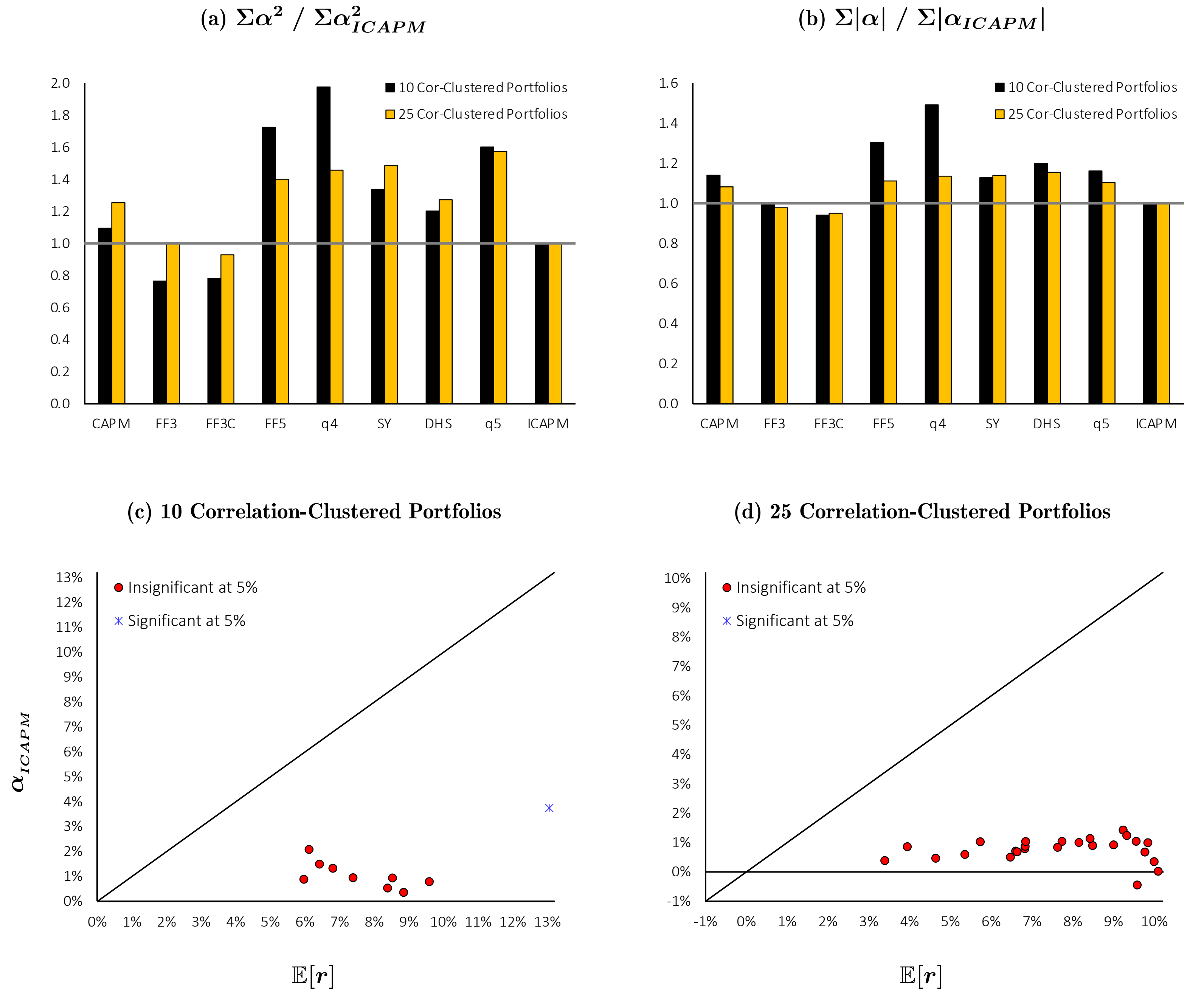


Figure 7

ICAPM vs Other Factor Models: αs of Correlation-clustered Portfolios

These figures show pricing error metrics for the ICAPM and other prominent factor models (described at the beginning of Section 3) when we use either 10 or 25 correlation-clustered portfolios as test assets. Correlation-clustered portfolios are constructed using the method from Ahn, Conrad, and Dittmar (2009) (see Subsection 3.3 for more details). Panel (a) shows the sum of squared pricing errors from various models relative to that from the ICAPM ($\Sigma\alpha^2/\Sigma\alpha^2_{ICAPM}$) for both sets of test assets. Panel (b) shows the sum of absolute pricing errors from various models relative to that from the ICAPM ($\Sigma|\alpha|/\Sigma|\alpha_{ICAPM}|$) for both sets of test assets. Panels (c) and (d) show debiased pricing errors from the ICAPM plotted against average portfolio returns when the 10 or 25 correlation-clustered portfolios are used as test assets, respectively (see Internet Appendix A.4 for a description of the debiasing procedure). All data are from our Modern Sample (1973-2019).

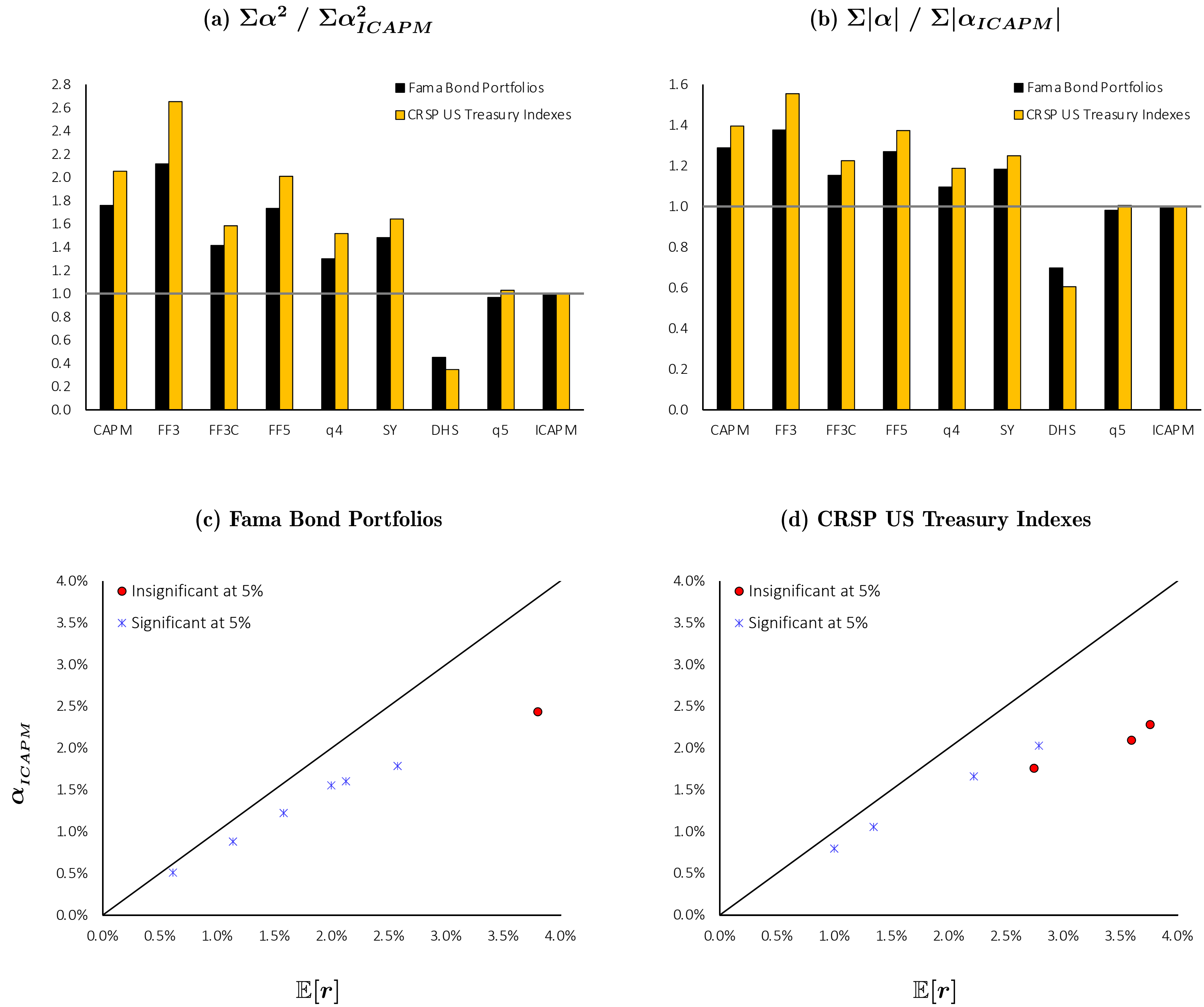


Figure 8

ICAPM vs Other Factor Models: αs of Treasury Bond Portfolios

These figures show pricing error metrics for the ICAPM and other prominent factor models (described at the beginning of Section 3) when we use either the Fama bond portfolios or the CRSP US Treasury Indexes as test assets. Panel (a) shows the sum of squared pricing errors from various models relative to that from the ICAPM ($\Sigma\alpha^2/\Sigma\alpha^2_{ICAPM}$) for both sets of test assets. Panel (b) shows the sum of absolute pricing errors from various models relative to that from the ICAPM ($\Sigma|\alpha|/\Sigma|\alpha_{ICAPM}|$) for both sets of test assets. Panels (c) and (d) show pricing errors from the ICAPM plotted against average portfolio returns when the Fama bond portfolios or the CRSP US Treasury Indexes are used as test assets, respectively. All data are from our Modern Sample (1973-2019).

Table 1

Decile Portfolios Sorted on β_{dp}

This table reports statistics related to monthly returns on 10 β_{dp}-sorted portfolios. Panels A and B provide results from our Long (1928-2019) and Modern (1973-2019) samples, respectively. In the top portion of each panel, we report portfolio return exposures to our expected return news proxy (Δdp), the in-sample expected return news measure ($N_{\mathbb{E}}$), our reinvestment risk factor ($r_{\mathbb{E}}$), and the expected return news mimicking portfolio ($r_{N\mathbb{E}}$). Portfolio return exposures to each of these time series are denoted by β_{dp}, $\beta_{N\mathbb{E}}$, $\beta_{\mathbb{E}}$, and $\beta_{N\mathbb{E}}^{proj}$, respectively, and are normalized to be in market beta units as described in Subsection 2.1. In the bottom portion of each panel, we report portfolio average returns ($\mathbb{E}[r]$) and αs when computed with respect to the CAPM (α_m), the ICAPM excluding $r_{\mathbb{E}}$ ($\alpha_{m,\mathbb{V}}$), and the full ICAPM ($\alpha_{m,\mathbb{E},\mathbb{V}}$). All returns are in percent and annualized (approximately) by multiplying monthly returns by 12. The "Slope" statistic is a measure of the slope of the 10 related portfolio statistics with respect to portfolio decile (see Footnote 28). Portfolios are rebalanced monthly based on individual stock exposures to Δdp with further details provided in Subsection 1.3. The 10-1 portfolio t-statistics are computed according to Newey and West (1987) and Newey and West (1994). The Slope t-statistics are computed according to the method in Driscoll and Kraay (1998) using the procedure in Newey and West (1994) to select the number of lags.

PANEL A: Long Sample (1928-2019)

Dec =	1	2	3	4	5	6	7	8	9	10	10-1	(t_{10-1})	Slope	(t_{Slope})
β_{dp}	-1.54	-1.29	-1.18	-1.09	-1.01	-0.94	-0.85	-0.76	-0.65	-0.55	0.99	(11.5)	0.89	(8.46)
$\beta_{N\mathbb{E}}$	-1.23	-1.02	-0.94	-0.86	-0.81	-0.75	-0.67	-0.60	-0.52	-0.41	0.82	(12.4)	0.72	(8.82)
$\beta_{\mathbb{E}}$	-1.73	-1.40	-1.21	-1.06	-0.94	-0.84	-0.69	-0.59	-0.43	-0.34	1.39	(57.0)	1.28	(54.3)
$\beta_{N\mathbb{E}}^{proj}$	-1.64	-1.34	-1.18	-1.06	-0.96	-0.87	-0.75	-0.64	-0.48	-0.33	1.32	(34.5)	1.18	(23.9)
$\mathbb{E}[r]$	9.7%	8.8%	9.2%	9.2%	9.0%	9.5%	8.8%	8.1%	8.3%	6.7%	-3.0%	(-1.11)	-2.0%	(-0.57)
α_m	-3.9%	-2.6%	-1.4%	-0.6%	-0.1%	1.2%	1.2%	1.3%	2.5%	1.9%	5.8%	(2.87)	5.9%	(2.54)
$\alpha_{m,\mathbb{V}}$	-3.8%	-2.6%	-1.3%	-0.6%	-0.1%	1.2%	1.2%	1.2%	2.4%	1.8%	5.6%	(3.70)	5.7%	(3.28)
$\alpha_{m,\mathbb{E},\mathbb{V}}$	0.8%	1.1%	1.1%	0.8%	0.7%	1.8%	0.9%	0.5%	1.1%	0.5%	-0.2%	(-0.25)	-0.2%	(-0.32)

PANEL B: Modern Sample (1973-2019)

Dec =	1	2	3	4	5	6	7	8	9	10	10-1	(t_{10-1})	Slope	(t_{Slope})
β_{dp}	-1.27	-1.02	-0.91	-0.84	-0.81	-0.74	-0.70	-0.61	-0.55	-0.44	0.84	(10.4)	0.70	(7.69)
$\beta_{N\mathbb{E}}$	-1.12	-0.85	-0.73	-0.67	-0.66	-0.60	-0.55	-0.49	-0.44	-0.33	0.79	(11.0)	0.63	(8.00)
$\beta_{\mathbb{E}}$	-1.50	-1.13	-0.91	-0.72	-0.66	-0.58	-0.46	-0.34	-0.23	-0.13	1.37	(42.2)	1.22	(40.6)
$\beta_{N\mathbb{E}}^{proj}$	-1.46	-1.07	-0.84	-0.74	-0.68	-0.61	-0.51	-0.41	-0.32	-0.16	1.30	(19.8)	1.08	(13.4)
$\mathbb{E}[r]$	7.5%	6.7%	7.0%	7.1%	7.9%	8.0%	7.2%	7.2%	8.2%	6.4%	-1.1%	(-0.31)	0.1%	(0.02)
α_m	-3.3%	-2.2%	-1.0%	-0.3%	0.9%	1.5%	1.0%	1.8%	3.3%	2.6%	5.9%	(2.30)	6.0%	(2.08)
$\alpha_{m,\mathbb{V}}$	-3.3%	-2.2%	-1.0%	-0.3%	0.9%	1.5%	1.0%	1.7%	3.2%	2.6%	5.8%	(3.15)	5.9%	(2.84)
$\alpha_{m,\mathbb{E},\mathbb{V}}$	0.2%	0.9%	1.1%	0.4%	1.5%	2.0%	0.2%	0.4%	1.2%	0.3%	0.1%	(0.08)	0.0%	(-0.04)

Table 2
Decile Portfolios Sorted on β_{σ^2}

This table reports statistics related to monthly returns on 10 β_{σ^2}-sorted portfolios. Panels A and B provide results from our Long (1928-2019) and Modern (1973-2019) samples, respectively. In the top portion of each panel, we report portfolio return exposures to our volatility news proxy ($\Delta\sigma^2$), the in-sample volatility news measure ($N_{\mathbb{V}}$), our volatility risk factor ($r_{\mathbb{V}}$), and the volatility news mimicking portfolio ($r_{N\mathbb{V}}$). Portfolio return exposures to each of these time series are denoted by β_{σ^2}, $\beta_{N\mathbb{V}}$, $\beta_{\mathbb{V}}$, and $\beta_{N\mathbb{V}}^{proj}$, respectively, and are normalized to be in market beta units as described in Subsection 2.1. In the bottom portion of each panel, we report portfolio average returns ($\mathbb{E}[r]$) and αs when computed with respect to the CAPM (α_m), the ICAPM excluding $r_{\mathbb{V}}$ ($\alpha_{m,\mathbb{E}}$), and the full ICAPM ($\alpha_{m,\mathbb{E},\mathbb{V}}$). All returns are in percent and annualized (approximately) by multiplying monthly returns by 12. The "Slope" statistic is a measure of the slope of the 10 related portfolio statistics with respect to portfolio decile (see Footnote 28). Portfolios are rebalanced monthly based on individual stock exposures to $\Delta\sigma^2$ with further details provided in Subsection 1.3. The 10-1 portfolio t-statistics are computed according to Newey and West (1987) and Newey and West (1994). The Slope t-statistics are computed according to the method in Driscoll and Kraay (1998) using the procedure in Newey and West (1994) to select the number of lags.

PANEL A: Long Sample (1928-2019)

Dec =	1	2	3	4	5	6	7	8	9	10	10-1	(t_{10-1})	Slope	(t_{Slope})
β_{σ^2}	-0.49	-0.42	-0.40	-0.38	-0.35	-0.31	-0.31	-0.26	-0.24	-0.24	0.25	(6.77)	0.24	(5.00)
$\beta_{N\mathbb{V}}$	-0.78	-0.67	-0.62	-0.60	-0.55	-0.49	-0.51	-0.44	-0.46	-0.49	0.29	(4.93)	0.29	(3.70)
$\beta_{\mathbb{V}}$	-1.29	-1.15	-0.98	-0.85	-0.71	-0.59	-0.51	-0.43	-0.38	-0.37	0.92	(33.3)	0.96	(59.4)
$\beta_{N\mathbb{V}}^{proj}$	-1.02	-0.89	-0.77	-0.70	-0.63	-0.56	-0.55	-0.48	-0.51	-0.51	0.51	(7.02)	0.50	(5.83)
$\mathbb{E}[r]$	9.8%	10.2%	10.9%	9.3%	9.1%	8.2%	8.0%	7.4%	7.5%	5.8%	-4.0%	(-1.98)	-4.2%	(-1.55)
α_m	-2.0%	-0.3%	1.4%	0.4%	0.8%	0.8%	0.8%	0.8%	0.9%	-1.0%	0.9%	(0.53)	0.8%	(0.38)
$\alpha_{m,\mathbb{E}}$	1.0%	1.9%	2.7%	1.1%	1.0%	0.4%	0.2%	-0.1%	0.0%	-1.8%	-2.8%	(-2.09)	-3.0%	(-2.00)
$\alpha_{m,\mathbb{E},\mathbb{V}}$	-0.3%	0.3%	1.4%	0.3%	0.9%	0.2%	0.8%	0.6%	1.2%	0.0%	0.3%	(0.27)	0.3%	(0.54)

PANEL B: Modern Sample (1973-2019)

Dec =	1	2	3	4	5	6	7	8	9	10	10-1	(t_{10-1})	Slope	(t_{Slope})
β_{σ^2}	-0.53	-0.41	-0.41	-0.39	-0.33	-0.31	-0.28	-0.25	-0.22	-0.21	0.32	(4.94)	0.29	(3.64)
$\beta_{N\mathbb{V}}$	-0.50	-0.42	-0.41	-0.39	-0.34	-0.32	-0.29	-0.28	-0.26	-0.27	0.22	(2.32)	0.22	(1.73)
$\beta_{\mathbb{V}}$	-1.22	-1.01	-0.81	-0.70	-0.56	-0.41	-0.32	-0.24	-0.15	-0.12	1.11	(33.1)	1.10	(59.9)
$\beta_{N\mathbb{V}}^{proj}$	-1.03	-0.86	-0.72	-0.69	-0.56	-0.45	-0.39	-0.33	-0.27	-0.26	0.77	(11.3)	0.77	(10.8)
$\mathbb{E}[r]$	8.8%	9.1%	9.5%	7.4%	9.1%	7.8%	7.6%	6.7%	6.6%	4.7%	-4.1%	(-1.47)	-3.8%	(-1.03)
α_m	-0.9%	0.6%	1.8%	-0.1%	2.3%	1.5%	1.6%	0.9%	1.2%	-1.1%	-0.1%	(-0.05)	0.2%	(0.06)
$\alpha_{m,\mathbb{E}}$	2.3%	2.5%	2.9%	0.7%	2.5%	0.9%	0.7%	-0.5%	-0.4%	-2.7%	-5.0%	(-2.99)	-4.6%	(-2.23)
$\alpha_{m,\mathbb{E},\mathbb{V}}$	0.1%	0.3%	1.5%	0.1%	2.3%	1.0%	1.5%	0.6%	1.1%	-0.4%	-0.6%	(-0.53)	-0.1%	(-0.10)

Table 3

ICAPM Risk Prices

This table reports estimated CAPM (top panel) and ICAPM (bottom panel) risk prices (b) and their associated risk premia (λ) according to Equation 8 over various sub-periods. Since bs are not easily comparable, we report $\sigma_x \cdot b_x$ for each factor x_t so that the reported values can be interpreted as the change in M_t induced by a one standard deviation change in the respective x_t (holding other factors fixed). λs are annualized (approximately) by multiplying by 12. The implied relative risk aversion is estimated as $\gamma = \lambda_m/\sigma_m^2$. The estimator for b is a Generalized Method of Moments (GMM) estimator, and thus t-statistics are computed according to GMM asymptotic theory (see Internet Appendix A).

	$T_0 =$ / $T_1 =$	1928 / 2019	1946 / 2019	1963 / 2019	1973 / 2019	1928 / 2006	1946 / 2006	1963 / 2006	1973 / 2006
CAPM									
r_m	b	0.12	0.15	0.13	0.12	0.11	0.14	0.11	0.11
	[λ]	[7.8%]	[7.6%]	[6.6%]	[6.8%]	[7.6%]	[7.3%]	[5.9%]	[6.0%]
	(t_{stat})	(3.83)	(3.99)	(3.00)	(2.70)	(3.41)	(3.54)	(2.40)	(2.03)
Implied γ		2.3	3.5	2.9	2.8	2.1	3.4	2.6	2.4
ICAPM									
r_m	b	0.26	0.29	0.24	0.24	0.24	0.28	0.21	0.21
	[λ]	[16.7%]	[14.9%]	[12.6%]	[13.3%]	[16.1%]	[14.2%]	[11.1%]	[11.7%]
	(t_{stat})	(4.92)	(5.77)	(4.33)	(4.00)	(4.33)	(5.24)	(3.56)	(3.17)
$r_{\mathbb{E}}$	b	0.34	0.39	0.35	0.36	0.32	0.39	0.33	0.35
	[λ]	[21.8%]	[20.1%]	[18.3%]	[19.8%]	[21.3%]	[19.8%]	[17.3%]	[19.1%]
	(t_{stat})	(4.86)	(5.36)	(4.20)	(4.06)	(4.32)	(5.01)	(3.63)	(3.47)
$r_{\mathbb{V}}$	b	-0.19	-0.22	-0.22	-0.23	-0.19	-0.22	-0.22	-0.24
	[λ]	[-12.2%]	[-11.0%]	[-11.6%]	[-12.7%]	[-12.3%]	[-11.2%]	[-11.7%]	[-13.2%]
	(t_{stat})	(-3.29)	(-3.33)	(-3.04)	(-3.01)	(-3.02)	(-3.16)	(-2.76)	(-2.74)
Implied γ		4.9	6.9	5.5	5.4	4.4	6.6	4.8	4.6

Table 4
Extracting ICAPM Factor Risk Premia

This table reports ICAPM risk factor correlations, expected returns, standard deviations, and sharpe ratios over our Long (1928-2019) and Modern (1973-2019) samples (Panels A and B, respectively). Statistics are provided for the raw factors (r) as well as for factors that are orthogonalized in-sample ($IS\ r^*$) according to the weighting procedure outlined in Subsection 2.3. Out-of-sample versions of the orthogonalized factors ($OS\ r^*$) are constructed using weights estimated using this same procedure but applied to 10-year rolling windows preceding each month in which the weights are applied. $\mathbb{E}[r]$ and $\sigma[r]$ are the annualized average returns and return standard deviations of each of each factor. The t-statistics are computed according to Newey and West (1987) and Newey and West (1994).

PANEL A: Long Sample (1928-2019)

		$Cor(r, r_m)$	$Cor(r, r_{\mathbb{E}})$	$Cor(r, r_{\mathbb{V}})$	$\mathbb{E}[r]$	$(t_{\mathbb{E}[r]})$	$\sigma[r]$	$\mathbb{E}[r]/\sigma[r]$
r	m	1.00	-0.77	-0.64	7.8%	(3.74)	18.5%	0.42
	$\mathbb{E}$	-0.77	1.00	0.83	-1.1%	(-0.62)	17.1%	-0.06
	$\mathbb{V}$	-0.64	0.83	1.00	-3.4%	(-2.45)	13.1%	-0.26
$IS\ r^*$	m	0.64	0.00	0.00	10.7%	(4.67)	18.5%	0.58
	$\mathbb{E}$	0.00	0.46	0.00	10.1%	(5.33)	18.5%	0.54
	$\mathbb{V}$	0.00	0.00	0.55	-6.7%	(-3.84)	18.5%	-0.36
$OS\ r^*$	m	0.57	0.13	0.12	10.0%	(4.84)	18.0%	0.56
	$\mathbb{E}$	-0.01	0.49	0.05	10.5%	(5.91)	17.4%	0.61
	$\mathbb{V}$	-0.05	0.12	0.58	-7.5%	(-3.75)	17.9%	-0.42

PANEL B: Modern Sample (1973-2019)

		$Cor(r, r_m)$	$Cor(r, r_{\mathbb{E}})$	$Cor(r, r_{\mathbb{V}})$	$\mathbb{E}[r]$	$(t_{\mathbb{E}[r]})$	$\sigma[r]$	$\mathbb{E}[r]/\sigma[r]$
r	m	1.00	-0.67	-0.52	6.8%	(2.83)	15.6%	0.43
	$\mathbb{E}$	-0.67	1.00	0.82	0.5%	(0.25)	14.2%	0.04
	$\mathbb{V}$	-0.52	0.82	1.00	-2.8%	(-1.49)	12.5%	-0.22
$IS\ r^*$	m	0.75	0.00	0.00	9.9%	(4.21)	15.6%	0.63
	$\mathbb{E}$	0.00	0.50	0.00	10.0%	(4.43)	15.6%	0.64
	$\mathbb{V}$	0.00	0.00	0.57	-7.3%	(-3.37)	15.6%	-0.47
$OS\ r^*$	m	0.75	-0.03	-0.02	9.2%	(3.51)	16.4%	0.56
	$\mathbb{E}$	-0.03	0.47	-0.03	12.5%	(4.93)	17.5%	0.72
	$\mathbb{V}$	0.05	-0.02	0.51	-9.9%	(-3.64)	18.2%	-0.55

Table 5
ICAPM Risk Prices Controlling for Factors in Other Factor Models

This table reports estimated risk prices (b) and their associated risk premia (λ) for ICAPM risk factors (f_t) according to Equation 8 when controlling for factors from other prominent factor models (x_t), which are described at the beginning of Section 3. Panels A and B cover our Long (1928-2019) and Modern (1973-2019) samples, respectively. In the case of the Long Sample, we include the earliest factor data available for each model (with the starting year listed in parentheses below each model) and use the Stambaugh (1997) procedure to estimate b and λ over the entire Long Sample (see Subsection 3.1 for more details). Since bs are not easily comparable, we report $\sigma_f \cdot b$ so that the reported values can be interpreted as the change in M_t induced by a one standard deviation change in the respective factor (holding other factors fixed). λs are annualized (approximately) by multiplying by 12. The estimator for b is a Generalized Method of Moments (GMM) estimator and the t-statistics are computed using a bootstrap exercise in Panel A and GMM asymptotic theory in Panel B (see Internet Appendix A).

PANEL A: Long Sample (1928-2019)

$$M_t = a - b' f_t - b_x' x_t$$

	$x =$	None (1928)	FF3 (1928)	FFC4 (1928)	FF5 (1963)	q4 (1963)	SY4 (1967)	DHS3 (1972)	q5 (1967)
	b	0.26	0.27	0.27	0.30	0.28	0.44	0.35	0.41
r_m	[λ]	[16.7%]	[17.1%]	[17.4%]	[19.5%]	[17.9%]	[28.5%]	[22.5%]	[26.4%]
	(t_{stat})	(4.92)	(4.98)	(4.81)	(5.04)	(4.49)	(6.50)	(5.38)	(5.41)
	b	0.34	0.47	0.39	0.27	0.33	0.36	0.25	0.21
$r_{\mathbb{E}}$	[λ]	[21.8%]	[30.2%]	[24.8%]	[17.2%]	[21.5%]	[22.9%]	[15.8%]	[13.4%]
	(t_{stat})	(4.86)	(6.21)	(4.98)	(3.31)	(3.72)	(3.92)	(2.66)	(2.01)
	b	-0.19	-0.25	-0.25	-0.33	-0.40	-0.41	-0.37	-0.38
$r_{\mathbb{V}}$	[λ]	[-12.2%]	[-15.8%]	[-15.9%]	[-21.5%]	[-25.5%]	[-26.6%]	[-23.9%]	[-24.3%]
	(t_{stat})	(-3.29)	(-4.11)	(-3.83)	(-4.71)	(-5.38)	(-5.33)	(-4.63)	(-4.37)

PANEL B: Modern Sample (1973-2019)

$$M_t = a - b' f_t - b_x' x_t$$

	$x =$	None	FF3	FFC4	FF5	q4	SY4	DHS3	q5
	b	0.24	0.27	0.26	0.28	0.25	0.40	0.33	0.35
r_m	[λ]	[13.3%]	[14.5%]	[14.0%]	[15.3%]	[13.8%]	[21.9%]	[17.8%]	[19.0%]
	(t_{stat})	(4.00)	(4.06)	(3.69)	(4.21)	(3.72)	(5.09)	(4.49)	(4.45)
	b	0.36	0.36	0.27	0.28	0.25	0.20	0.16	0.13
$r_{\mathbb{E}}$	[λ]	[19.8%]	[19.5%]	[14.5%]	[15.1%]	[13.8%]	[10.8%]	[8.6%]	[7.0%]
	(t_{stat})	(4.06)	(3.97)	(2.80)	(2.93)	(2.53)	(1.93)	(1.50)	(1.19)
	b	-0.23	-0.23	-0.24	-0.36	-0.38	-0.38	-0.37	-0.34
$r_{\mathbb{V}}$	[λ]	[-12.7%]	[-12.4%]	[-13.0%]	[-19.5%]	[-20.7%]	[-20.6%]	[-20.4%]	[-18.8%]
	(t_{stat})	(-3.01)	(-2.76)	(-2.68)	(-3.95)	(-4.13)	(-3.94)	(-3.94)	(-3.38)

Table 6

Correlations Between SDFs Implied by Different Factor Models

This table reports correlations between the SDFs (M_t) implied by each factor model we investigate, which are described at the beginning of Section 3. Results use data from the Modern Sample (1973-2019).

	CAPM	**FF3**	**FFC4**	**FF5**	**q4**	**SY4**	**DHS3**	**q5**	**ICAPM**
CAPM	1	0.64	0.44	0.39	0.32	0.27	0.25	0.21	0.56
FF3	0.64	1	0.68	0.60	0.45	0.46	0.29	0.30	0.39
FFC4	0.44	0.68	1	0.51	0.61	0.68	0.53	0.48	0.40
FF5	0.39	0.60	0.51	1	0.76	0.70	0.40	0.53	0.31
q4	0.32	0.45	0.61	0.76	1	0.66	0.47	0.65	0.27
SY4	0.27	0.46	0.68	0.70	0.66	1	0.57	0.65	0.27
DHS3	0.25	0.29	0.53	0.40	0.47	0.57	1	0.48	0.29
q5	0.21	0.30	0.48	0.53	0.65	0.65	0.48	1	0.25
ICAPM	0.56	0.39	0.40	0.31	0.27	0.27	0.29	0.25	1
Av Correlation (Excluding CAPM)	0.37	0.42	0.56	0.55	0.59	0.60	0.47	0.51	0.30

Table 7
ICAPM vs Other Factor Models: Maximum Sharpe Ratios

This table reports the maximum sharpe ratios constructed (in- and out-of-sample) using the ICAPM factors or using factors from other prominent factor models, which are described at the beginning of Section 3. We also simulate factor data and report the percent of times the ICAPM maximum sharpe ratio is higher than each alternative model in parenthesis. The simulation results are based on a bootstrap analysis that samples the data 100,000 times with replacement, then recomputes each model's maximum sharpe ratio within each simulation (see Internet Appendix A). Panel A presents results using the gross (of trading cost) factors and Panel B presents results using net-of-trading-cost factors (described in Footnote 35). Results reported for the "Modern Sample", "1st Half", and "2nd Half" are based on portfolio weights (w) estimated in-sample portfolio. Results reported for "2nd Half OS (w from 1973-1995)" use factor data from 1973 through the first half of 1995 to estimate w. Results reported in "2nd Half OS (w from 1928-1995)" use factor data from 1928 through the first half of 1995 (or the earliest factor data available for each factor model, which is summarized in Table 5, Panel A) to estimate w.

PANEL A: Gross (of Trading Costs) Factor Returns

	CAPM	**FF3**	**FFC4**	**FF5**	**q4**	**SY4**	**DHS3**	**q5**	**ICAPM**
Modern Sample	0.43	0.68	0.99	1.11	1.37	1.61	1.70	2.10	0.77
(1973-2019)	(100%)	(70%)	(9%)	(2%)	(0%)	(0%)	(0%)	(0%)	-
1st Half	0.36	0.96	1.43	1.72	2.22	2.21	2.28	2.87	0.82
(1973-1995)	(100%)	(25%)	(1%)	(0%)	(0%)	(0%)	(0%)	(0%)	-
2nd Half	0.51	0.55	0.74	1.19	1.08	1.41	1.37	1.67	0.79
(1995-2019)	(100%)	(88%)	(52%)	(2%)	(8%)	(0%)	(1%)	(0%)	-
2nd Half OS	0.51	0.43	0.56	0.66	0.66	1.16	1.17	1.32	0.65
(w from 1973-1995)	(69%)	(73%)	(59%)	(45%)	(45%)	(4%)	(4%)	(2%)	-
2nd Half OS	0.51	0.47	0.52	0.64	0.71	1.18	1.17	1.39	0.72
(w from 1928-1995)	(84%)	(84%)	(73%)	(57%)	(49%)	(6%)	(8%)	(2%)	-

PANEL B: Net (of Trading Costs) Factor Returns

	CAPM	**FF3**	**FFC4**	**FF5**	**q4**	**SY4**	**DHS3**	**q5**	**ICAPM**
Modern Sample	0.43	0.62	0.75	0.87	0.90	1.19	1.23	1.70	0.64
(1973-2019)	(100%)	(57%)	(22%)	(6%)	(5%)	(0%)	(0%)	(0%)	-
1st Half	0.36	0.85	0.99	1.17	1.18	1.36	1.50	2.06	0.58
(1973-1995)	(100%)	(9%)	(2%)	(0%)	(0%)	(0%)	(0%)	(0%)	-
2nd Half	0.51	0.53	0.63	1.06	0.90	1.30	1.20	1.50	0.74
(1995-2019)	(100%)	(88%)	(66%)	(3%)	(19%)	(0%)	(2%)	(0%)	-
2nd Half OS	0.51	0.39	0.48	0.53	0.54	0.95	0.82	1.30	0.66
(w from 1973-1995)	(63%)	(73%)	(67%)	(54%)	(55%)	(13%)	(22%)	(2%)	-
2nd Half OS	0.51	0.49	0.51	0.51	0.58	1.01	0.82	1.35	0.73
(w from 1928-1995)	(84%)	(86%)	(77%)	(70%)	(63%)	(14%)	(31%)	(2%)	-

Table 8

ICAPM vs Other Factor Models: αs of Single Stocks

This table reports time-series averages of $\Sigma\alpha^2$ and $\Sigma|\alpha|$ normalized by the respective sums of pricing errors computed under risk-neutral pricing ($\alpha_{RN} = \mathbb{E}[r]$), the CAPM ($\alpha_{CAPM}$), and the ICAPM ($\alpha_{ICAPM}$) for each factor model (described at the beginning of Section 3) applied to single stocks. αs for each stock are computed on either a 5- (Panel A) or 10-year (Panel B) rolling basis and recorded each month. Stocks are required to have full return data in each rolling period in order to be included in a particular month. Sum-squared or sum-absolute pricing errors are then computed each month by summing across the pricing errors for each valid stock, and the relevant ratios are computed each month. Ratios reported in the table are time-series averages of the monthly ratios, similar to a Fama and MacBeth (1973) estimator. We also report the percent of months in which $\Sigma\alpha^2/\Sigma\mathbb{E}[r]^2$ or $\Sigma|a|/\Sigma|\mathbb{E}[r]|$ generated by the ICAPM is below that generated by the alternate model.

PANEL A: 5-Year Rolling Window

	CAPM	FF3	FFC4	FF5	q4	SY4	DHS3	q5	ICAPM
$\Sigma\alpha^2$ / $\Sigma\mathbb{E}[r]^2$	0.60	0.62	0.60	0.74	0.69	0.76	0.82	0.75	0.57
$\Sigma\alpha^2$ / $\Sigma\alpha^2_{CAPM}$	1.00	1.07	1.02	1.27	1.19	1.32	1.38	1.29	0.95
$\Sigma\alpha^2$ / $\Sigma\alpha^2_{ICAPM}$	1.07	1.13	1.07	1.35	1.25	1.39	1.45	1.35	1.00
$\%(\Sigma\alpha^2_{ICAPM} < \Sigma\alpha^2)$	(62%)	(74%)	(59%)	(90%)	(87%)	(89%)	(98%)	(90%)	-
$\Sigma|\alpha|$ / $\Sigma|\mathbb{E}[r]|$	0.84	0.83	0.84	0.93	0.93	0.96	1.05	1.00	0.84
$\Sigma|\alpha|$ / $\Sigma|\alpha_{CAPM}|$	0.99	0.98	0.99	1.09	1.09	1.13	1.22	1.17	0.99
$\Sigma|\alpha|$ / $\Sigma|\alpha_{ICAPM}|$	1.00	0.99	1.00	1.10	1.10	1.14	1.24	1.19	1.00
$\%(\Sigma|\alpha_{ICAPM}| < \Sigma|\alpha|)$	(54%)	(41%)	(40%)	(84%)	(79%)	(78%)	(90%)	(86%)	-

PANEL B: 10-Year Rolling Window

	CAPM	FF3	FFC4	FF5	q4	SY4	DHS3	q5	ICAPM
$\Sigma\alpha^2$ / $\Sigma\mathbb{E}[r]^2$	0.47	0.48	0.49	0.54	0.55	0.56	0.58	0.53	0.44
$\Sigma\alpha^2$ / $\Sigma\alpha^2_{CAPM}$	1.00	1.06	1.08	1.20	1.22	1.27	1.28	1.19	0.93
$\Sigma\alpha^2$ / $\Sigma\alpha^2_{ICAPM}$	1.08	1.14	1.15	1.28	1.30	1.36	1.36	1.27	1.00
$\%(\Sigma\alpha^2_{ICAPM} < \Sigma\alpha^2)$	(71%)	(75%)	(73%)	(93%)	(90%)	(89%)	(100%)	(93%)	-
$\Sigma|\alpha|$ / $\Sigma|\mathbb{E}[r]|$	0.70	0.66	0.68	0.75	0.77	0.78	0.88	0.82	0.71
$\Sigma|\alpha|$ / $\Sigma|\alpha_{CAPM}|$	1.00	0.96	0.98	1.08	1.11	1.13	1.27	1.18	1.01
$\Sigma|\alpha|$ / $\Sigma|\alpha_{ICAPM}|$	0.99	0.95	0.97	1.07	1.10	1.12	1.26	1.17	1.00
$\%(\Sigma|\alpha_{ICAPM}| < \Sigma|\alpha|)$	(31%)	(27%)	(33%)	(74%)	(70%)	(82%)	(88%)	(84%)	-

Table 9
ICAPM vs Other Factor Models: αs of Industry Portfolios

This table reports $\Sigma\alpha^2$ and $\Sigma|\alpha|$ normalized by the respective sums of pricing errors computed under risk-neutral pricing ($\alpha_{RN} = \mathbb{E}[r]$), the CAPM ($\alpha_{CAPM}$), and the ICAPM ($\alpha_{ICAPM}$) for each factor model (described at the beginning of Section 3) applied to either the 30 (Panel A) or 48 (Panel B) Fama-French industry portfolios. αs for each set of portfolios are estimated once using the full data from our Modern Sample (1973-2019) for each model. We also simulate industry portfolio return data and report the percent of times in which $\Sigma\alpha^2/\Sigma\mathbb{E}[r]^2$ or $\Sigma|a|/\Sigma|\mathbb{E}[r]|$ generated by the ICAPM is below that generated by the alternate model. The simulation results are based on a bootstrap analysis that samples the data with replacement, then recomputes each model's α ratio metric within each simulation (see Internet Appendix A).

PANEL A: 30 Fama-French Industries

	CAPM	FF3	FFC4	FF5	q4	SY4	DHS3	q5	ICAPM
$\Sigma\alpha^2$ / $\Sigma\mathbb{E}[r]^2$	0.09	0.15	0.12	0.20	0.16	0.12	0.07	0.11	0.06
$\Sigma\alpha^2$ / $\Sigma\alpha^2_{CAPM}$	1.00	1.64	1.31	2.18	1.69	1.28	0.80	1.15	0.60
$\Sigma\alpha^2$ / $\Sigma\alpha^2_{ICAPM}$	1.68	2.75	2.19	3.65	2.83	2.14	1.35	1.93	1.00
$\%(\Sigma\alpha^2_{ICAPM} < \Sigma\alpha^2)$	(88%)	(96%)	(94%)	(97%)	(93%)	(86%)	(76%)	(83%)	-
$\Sigma|\alpha|$ / $\Sigma|\mathbb{E}[r]|$	0.23	0.32	0.29	0.39	0.33	0.29	0.22	0.28	0.19
$\Sigma|\alpha|$ / $\Sigma|\alpha_{CAPM}|$	1.00	1.39	1.26	1.70	1.43	1.26	0.97	1.21	0.81
$\Sigma|\alpha|$ / $\Sigma|\alpha_{ICAPM}|$	1.23	1.70	1.55	2.09	1.76	1.54	1.19	1.49	1.00
$\%(\Sigma|\alpha_{ICAPM}| < \Sigma|\alpha|)$	(86%)	(95%)	(94%)	(97%)	(93%)	(88%)	(80%)	(86%)	-

PANEL B: 48 Fama-French Industries

	CAPM	FF3	FFC4	FF5	q4	SY4	DHS3	q5	ICAPM
$\Sigma\alpha^2$ / $\Sigma\mathbb{E}[r]^2$	0.11	0.16	0.13	0.24	0.22	0.16	0.11	0.16	0.07
$\Sigma\alpha^2$ / $\Sigma\alpha^2_{CAPM}$	1.00	1.55	1.22	2.25	2.10	1.49	1.02	1.53	0.68
$\Sigma\alpha^2$ / $\Sigma\alpha^2_{ICAPM}$	1.48	2.29	1.81	3.34	3.10	2.21	1.51	2.26	1.00
$\%(\Sigma\alpha^2_{ICAPM} < \Sigma\alpha^2)$	(88%)	(96%)	(93%)	(98%)	(97%)	(92%)	(87%)	(92%)	-
$\Sigma|\alpha|$ / $\Sigma|\mathbb{E}[r]|$	0.26	0.32	0.29	0.41	0.39	0.33	0.25	0.32	0.22
$\Sigma|\alpha|$ / $\Sigma|\alpha_{CAPM}|$	1.00	1.25	1.14	1.59	1.51	1.28	0.99	1.26	0.86
$\Sigma|\alpha|$ / $\Sigma|\alpha_{ICAPM}|$	1.16	1.45	1.32	1.85	1.76	1.48	1.15	1.46	1.00
$\%(\Sigma|\alpha_{ICAPM}| < \Sigma|\alpha|)$	(85%)	(94%)	(92%)	(98%)	(96%)	(91%)	(86%)	(92%)	-

Table 10
ICAPM vs Other Factor Models: αs of Correlation-clustered Portfolios

This table reports $\Sigma\alpha^2$ and $\Sigma|\alpha|$ normalized by the respective sums of pricing errors computed under risk-neutral pricing ($\alpha_{RN} = \mathbb{E}[r]$), the CAPM ($\alpha_{CAPM}$), and the ICAPM ($\alpha_{ICAPM}$) for each factor model (described at the beginning of Section 3) applied to either the 10 (Panel A) or 25 (Panel B) correlation-clustered portfolios constructed using the method from Ahn, Conrad, and Dittmar (2009) (see Subsection 3.3 for more details). αs for each set of portfolios are estimated once using the full data from our Modern Sample (1973-2019) for each model. We also simulate correlation-clustered portfolio return data and report the percent of times in which $\Sigma\alpha^2/\Sigma\mathbb{E}[r]^2$ or $\Sigma|a|/\Sigma|\mathbb{E}[r]|$ generated by the ICAPM is below that generated by the alternate model. The simulation results are based on a bootstrap analysis that samples the data with replacement, then recomputes each model's α ratio metric within each simulation (see Internet Appendix A).

PANEL A: 10 Cluster Portfolios

	CAPM	FF3	FFC4	FF5	q4	SY4	DHS3	q5	ICAPM
$\Sigma\alpha^2$ / $\Sigma\mathbb{E}[r]^2$	0.09	0.06	0.06	0.14	0.16	0.11	0.10	0.13	0.08
$\Sigma\alpha^2$ / $\Sigma\alpha^2_{CAPM}$	1.00	0.70	0.71	1.58	1.80	1.22	1.10	1.46	0.91
$\Sigma\alpha^2$ / $\Sigma\alpha^2_{ICAPM}$	1.10	0.77	0.78	1.73	1.98	1.34	1.20	1.60	1.00
$\%(\Sigma\alpha^2_{ICAPM} < \Sigma\alpha^2)$	(62%)	(18%)	(26%)	(93%)	(97%)	(81%)	(76%)	(87%)	-
$\Sigma\|\alpha\|$ / $\Sigma\|\mathbb{E}[r]\|$	0.23	0.20	0.19	0.26	0.30	0.22	0.24	0.23	0.20
$\Sigma\|\alpha\|$ / $\Sigma\|\alpha_{CAPM}\|$	1.00	0.87	0.83	1.14	1.31	0.99	1.05	1.02	0.88
$\Sigma\|\alpha\|$ / $\Sigma\|\alpha_{ICAPM}\|$	1.14	0.99	0.94	1.30	1.49	1.13	1.20	1.16	1.00
$\%(\Sigma\|\alpha_{ICAPM}\| < \Sigma\|\alpha\|)$	(76%)	(39%)	(41%)	(84%)	(92%)	(77%)	(83%)	(84%)	-

PANEL B: 25 Cluster Portfolios

	CAPM	FF3	FFC4	FF5	q4	SY4	DHS3	q5	ICAPM
$\Sigma\alpha^2$ / $\Sigma\mathbb{E}[r]^2$	0.08	0.07	0.06	0.09	0.10	0.10	0.09	0.11	0.07
$\Sigma\alpha^2$ / $\Sigma\alpha^2_{CAPM}$	1.00	0.80	0.74	1.12	1.16	1.18	1.01	1.26	0.80
$\Sigma\alpha^2$ / $\Sigma\alpha^2_{ICAPM}$	1.26	1.01	0.93	1.40	1.46	1.49	1.27	1.58	1.00
$\%(\Sigma\alpha^2_{ICAPM} < \Sigma\alpha^2)$	(86%)	(46%)	(44%)	(78%)	(86%)	(87%)	(86%)	(89%)	-
$\Sigma\|\alpha\|$ / $\Sigma\|\mathbb{E}[r]\|$	0.25	0.22	0.22	0.26	0.26	0.26	0.26	0.25	0.23
$\Sigma\|\alpha\|$ / $\Sigma\|\alpha_{CAPM}\|$	1.00	0.90	0.88	1.03	1.05	1.05	1.07	1.02	0.92
$\Sigma\|\alpha\|$ / $\Sigma\|\alpha_{ICAPM}\|$	1.08	0.98	0.95	1.11	1.14	1.14	1.16	1.10	1.00
$\%(\Sigma\|\alpha_{ICAPM}\| < \Sigma\|\alpha\|)$	(80%)	(44%)	(44%)	(74%)	(82%)	(84%)	(83%)	(87%)	-

Table 11
ICAPM vs Other Factor Models: αs of Treasury Bond Portfolios

This table reports $\Sigma\alpha^2$ and $\Sigma|\alpha|$ normalized by the respective sums of pricing errors computed under risk-neutral pricing ($\alpha_{RN} = \mathbb{E}[r]$), the CAPM ($\alpha_{CAPM}$), and the ICAPM ($\alpha_{ICAPM}$) for each factor model (described at the beginning of Section 3) applied to either the Fama bond portfolios (Panel A) or the CRSP US Treasury Indexes (Panel B). αs for each set of portfolios are estimated once using the full data from our Modern Sample (1973-2019) for each model. We also simulate bond portfolio return data and report the percent of times in which $\Sigma\alpha^2/\Sigma\mathbb{E}\left[r\right]^2$ or $\Sigma|a|/\Sigma|\mathbb{E}[r]|$ generated by the ICAPM is below that generated by the alternate model. The simulation results are based on a bootstrap analysis that samples the data with replacement, then recomputes each model's α ratio metric within each simulation (see Internet Appendix A).

PANEL A: Fama Bond Portfolios ($h = 1, 2, 3, 4, 5, 10, 30$ years)

	CAPM	FF3	FFC4	FF5	q4	SY4	DHS3	q5	ICAPM
$\Sigma\alpha^2 / \Sigma\mathbb{E}[r]^2$	0.87	1.04	0.70	0.85	0.64	0.73	0.22	0.48	0.49
$\Sigma\alpha^2 / \Sigma\alpha^2_{CAPM}$	1.00	1.20	0.80	0.99	0.74	0.84	0.26	0.55	0.57
$\Sigma\alpha^2 / \Sigma\alpha^2_{ICAPM}$	1.76	2.12	1.42	1.74	1.30	1.48	0.45	0.97	1.00
$\%(\Sigma\alpha^2_{ICAPM} < \Sigma\alpha^2)$	(98%)	(99%)	(83%)	(94%)	(72%)	(82%)	(17%)	(51%)	-
$\Sigma|\alpha| / \Sigma|\mathbb{E}[r]|$	0.93	0.99	0.83	0.92	0.79	0.85	0.50	0.71	0.72
$\Sigma|\alpha| / \Sigma|\alpha_{CAPM}|$	1.00	1.07	0.90	0.99	0.85	0.92	0.54	0.76	0.78
$\Sigma|\alpha| / \Sigma|\alpha_{ICAPM}|$	1.29	1.38	1.15	1.27	1.10	1.18	0.70	0.98	1.00
$\%(\Sigma|\alpha_{ICAPM}| < \Sigma|\alpha|)$	(98%)	(99%)	(81%)	(93%)	(68%)	(80%)	(16%)	(50%)	-

PANEL B: CRSP US Treasury Indexes ($h = 1, 2, 5, 7, 10, 20, 30$ years)

	CAPM	FF3	FFC4	FF5	q4	SY4	DHS3	q5	ICAPM
$\Sigma\alpha^2 / \Sigma\mathbb{E}[r]^2$	0.87	1.13	0.67	0.86	0.65	0.70	0.15	0.44	0.43
$\Sigma\alpha^2 / \Sigma\alpha^2_{CAPM}$	1.00	1.29	0.77	0.98	0.74	0.80	0.17	0.50	0.49
$\Sigma\alpha^2 / \Sigma\alpha^2_{ICAPM}$	2.06	2.65	1.59	2.01	1.52	1.64	0.35	1.03	1.00
$\%(\Sigma\alpha^2_{ICAPM} < \Sigma\alpha^2)$	(96%)	(97%)	(82%)	(92%)	(75%)	(81%)	(24%)	(56%)	-
$\Sigma|\alpha| / \Sigma|\mathbb{E}[r]|$	0.93	1.04	0.82	0.92	0.79	0.84	0.41	0.67	0.67
$\Sigma|\alpha| / \Sigma|\alpha_{CAPM}|$	1.00	1.11	0.88	0.98	0.85	0.90	0.43	0.72	0.72
$\Sigma|\alpha| / \Sigma|\alpha_{ICAPM}|$	1.40	1.55	1.22	1.37	1.19	1.25	0.61	1.01	1.00
$\%(\Sigma|\alpha_{ICAPM}| < \Sigma|\alpha|)$	(96%)	(98%)	(81%)	(92%)	(73%)	(80%)	(22%)	(55%)	-

Table 12
ICAPM vs Other Factor Models: Summarizing α Results

This table reports $\Sigma\alpha^2/\Sigma\mathbb{E}[r]^2$ (Panel A) and $\Sigma|a|/\Sigma|\mathbb{E}[r]|$ (Panel B) ranks across each model within each test asset group we investigate. Lower ranks correspond to lower ratio values. The last row in each panel reports the average rank of the ICAPM and each other factor model (described at the beginning of Section 3) across all test asset groups.

PANEL A: Squared Pricing Errors ($\Sigma\alpha^2/\Sigma\mathbb{E}[r]^2$) Ranks

		CAPM	FF3	FFC4	FF5	q4	SY4	DHS3	q5	ICAPM
Single Stocks	5-Year Window	2	4	3	6	5	8	9	7	1
	10-Year Window	2	3	4	6	7	8	9	5	1
Industry Port	30 Portfolios	3	7	6	9	8	5	2	4	1
	48 Portfolios	2	7	4	9	8	5	3	6	1
Cor Clust Port	10 Portfolios	4	1	2	8	9	6	5	7	3
	25 Portfolios	4	3	1	6	7	8	5	9	2
Bond Portfolios	Fama Bond Port	8	9	5	7	4	6	1	2	3
	CRSP Bond Port	8	9	5	7	4	6	1	3	2
	Average Rank =	4.1	5.4	3.8	7.3	6.5	6.5	4.4	5.4	1.8

PANEL B: Absolute Pricing Errors ($\Sigma|\alpha|/\Sigma|\mathbb{E}[r]|$) Ranks

		CAPM	FF3	FFC4	FF5	q4	SY4	DHS3	q5	ICAPM
Single Stocks	5-Year Window	4	1	2	5	6	7	9	8	3
	10-Year Window	3	1	2	5	6	7	9	8	4
Industry Port	30 Portfolios	3	7	6	9	8	5	2	4	1
	48 Portfolios	3	5	4	9	8	7	2	6	1
Cor Clust Port	10 Portfolios	5	2	1	8	9	4	7	6	3
	25 Portfolios	4	2	1	6	7	8	9	5	3
Bond Portfolios	Fama Bond Port	8	9	5	7	4	6	1	2	3
	CRSP Bond Port	8	9	5	7	4	6	1	3	2
	Average Rank =	4.8	4.5	3.3	7.0	6.5	6.3	5.0	5.3	2.5

Table 13

ICAPM vs Other Factor Models: Anomaly Deciles

This table reports $\Sigma\alpha^2$ and $\Sigma|\alpha|$ normalized by the respective sums of pricing errors computed under risk-neutral pricing ($\alpha_{RN} = \mathbb{E}[r]$), the CAPM ($\alpha_{CAPM}$), and the ICAPM ($\alpha_{ICAPM}$) for each factor model (described at the beginning of Section 3) applied to either deciles formed on 158 anomalies from Chen and Zimmermann (2020) (Panel A) or the 20 deciles formed on β_{dp} and β_{σ^2} sorts (Panel B). αs for each set of portfolios are estimated once using the full data from our Modern Sample (1973-2019) for each model. We also simulate bond portfolio return data and report the percent of times in which $\Sigma\alpha^2/\Sigma\mathbb{E}[r]^2$ or $\Sigma|a|/\Sigma|\mathbb{E}[r]|$ generated by the ICAPM is below that generated by the alternate model. The simulation results are based on a bootstrap analysis that samples the data with replacement, then recomputes each model's α ratio metric within each simulation (see Internet Appendix A).

PANEL A: Deciles Based on 158 Anomalies

	CAPM	FF3	FFC4	FF5	q4	SY4	DHS3	q5	ICAPM
$\Sigma\alpha^2$ / $\Sigma\mathbb{E}[r]^2$	0.29	0.32	0.23	0.26	0.22	0.20	0.18	0.19	0.25
$\Sigma\alpha^2$ / $\Sigma\alpha^2_{CAPM}$	1.00	1.10	0.80	0.90	0.76	0.71	0.62	0.67	0.85
$\Sigma\alpha^2$ / $\Sigma\alpha^2_{ICAPM}$	1.17	1.29	0.93	1.05	0.89	0.83	0.72	0.79	1.00
$\%(\Sigma\alpha^2_{ICAPM} < \Sigma\alpha^2)$	(90%)	(97%)	(28%)	(61%)	(24%)	(10%)	(8%)	(8%)	-
$\Sigma|\alpha|$ / $\Sigma|\mathbb{E}[r]|$	0.30	0.30	0.25	0.28	0.27	0.25	0.26	0.26	0.27
$\Sigma|\alpha|$ / $\Sigma|\alpha_{CAPM}|$	1.00	0.98	0.83	0.93	0.88	0.83	0.87	0.85	0.90
$\Sigma|\alpha|$ / $\Sigma|\alpha_{ICAPM}|$	1.11	1.09	0.92	1.03	0.98	0.92	0.97	0.94	1.00
$\%(\Sigma|\alpha_{ICAPM}| < \Sigma|\alpha|)$	(94%)	(87%)	(10%)	(59%)	(45%)	(22%)	(45%)	(32%)	-

PANEL B: 20 Deciles Based on β_{dp} and β_{σ^2} Sorts

	CAPM	FF3	FFC4	FF5	q4	SY4	DHS3	q5	ICAPM
$\Sigma\alpha^2$ / $\Sigma\mathbb{E}[r]^2$	0.05	0.04	0.03	0.06	0.07	0.06	0.09	0.08	0.02
$\Sigma\alpha^2$ / $\Sigma\alpha^2_{CAPM}$	1.00	0.82	0.50	1.07	1.27	1.10	1.65	1.53	0.39
$\Sigma\alpha^2$ / $\Sigma\alpha^2_{ICAPM}$	2.54	2.07	1.26	2.72	3.23	2.80	4.19	3.88	1.00
$\%(\Sigma\alpha^2_{ICAPM} < \Sigma\alpha^2)$	(99%)	(88%)	(78%)	(94%)	(95%)	(92%)	(98%)	(98%)	-
$\Sigma|\alpha|$ / $\Sigma|\mathbb{E}[r]|$	0.20	0.17	0.14	0.20	0.21	0.21	0.23	0.24	0.12
$\Sigma|\alpha|$ / $\Sigma|\alpha_{CAPM}|$	1.00	0.85	0.68	1.02	1.07	1.05	1.14	1.19	0.58
$\Sigma|\alpha|$ / $\Sigma|\alpha_{ICAPM}|$	1.71	1.46	1.17	1.76	1.83	1.81	1.96	2.04	1.00
$\%(\Sigma|\alpha_{ICAPM}| < \Sigma|\alpha|)$	(98%)	(87%)	(74%)	(93%)	(94%)	(91%)	(96%)	(96%)	-

Internet Appendix

"An Intertemporal Risk Factor Model"

By Fousseni Chabi-Yo, Andrei S. Gonçalves, and Johnathan Loudis

This Internet Appendix is organized as follows. Section A provides some econometric details, Section B describes data sources and measurement for the analysis, and Section C provides some further results that supplement the main findings in the paper.

A Econometric Details

This section provides econometric details related to some of the results described in the main text. Subsection A.1 focuses on our estimation of risk prices, Subsection A.3 describes our bootstrap simulations, and Subsection A.4 outlines the bias adjustment for pricing errors we apply in some of our graphs.

A.1 Estimating Risk Prices

Equation 8 shows that $b = \mathbb{E}\left[1/R_f^n\right] \cdot \Sigma_f^{-1} \mathbb{E}\left[f\right]$ and we estimate b from the sample analogue $\widehat{b} = \widehat{\mathbb{E}}\left[1/R_f^n\right] \cdot \widehat{\Sigma}_f^{-1} \widehat{\mathbb{E}}\left[f\right]$. Despite its simplicity, our estimation approach has an interesting economic justification (i.e., it represents the projection of the true SDF onto the factors) and can be motivated by efficiency and/or robustness arguments. This subsection elaborates on these aspects and shows how to compute standard errors for $\widehat{b}$.

We begin by motivating our risk price estimation using a linear regression framework. Next, we show how this maps directly into a GMM estimate of the same risk prices and use GMM theory to develop asymptotic standard errors. Finally, we show that this methodology can be motivated by efficiency and robustness arguments. In terms of efficiency, if we were to add other testing assets to the analogous GMM framework, this would leave our estimator unaffected as long as we rely on the efficient GMM weighting matrix. In terms of robustness, we show that our b estimate using the linear regression framework converges in probability to the projection of the SDF onto f even if the $M = a + b^{'} f$ model is mispecified, a result that does not hold for other b estimators.

We consider an environment that is more general than our intertemporal factor model so that we can understand what our estimation procedure uncovers if the intertemporal factor model is false. As such, for the rest of this section, M_t represents the true SDF and $\dddot{M}_t$ reflects a given model for M_t. Obviously, any model's implication is that $M_t = \dddot{M}_t$, but the implication might not hold in reality (the model might be misspecified), in which case it is important to consider the distinction between M_t and $\dddot{M}_t$. We consider models of the form

$\dddot{M}_t = a - b' f_t$, where f_t are excess returns on traded factors and a risk-free asset exists, since this is by far the most common case in the empirical asset pricing literature (and nests our ICAPM risk factor model).

A.1.1 Linear Regression Framework

Since any model of interest is likely misspecified, we want a procedure that is consistent for a and b if $M_t = \dddot{M}_t = a - b' f_t$ holds, but that is "reasonable" (i.e., delivers $\dddot{M}_t = a - b' f_t$ with some useful properties) if $M_t \neq \dddot{M}_t$ holds instead.

To find such a procedure, start by projecting M_t onto f_t, which can always be done whether $\dddot{M}_t$ is well specified or not. Then, we have $M_t = a - b' f_t - \epsilon_t$ where $\mathbb{E}[\epsilon \cdot f] = 0$ and $\mathbb{E}[\epsilon] = 0$. Thus, M_t can always be understood as a linear factor model with $k + 1$ factors in which the last factor, ϵ_t, is orthogonal to the first k. This means that for any given set of factors, the linearity assumption can always be justified as long as we recognize that we might have a missing factor. In this case, the original misspecification is incorporated into this unobservable factor, making the linearity of M_t on f_t well specified. If $\dddot{M}_t$ is well specified, then we have $M_t = \dddot{M}_t = a - b' f_t$ and $\epsilon_t = 0 \,\forall\, t$. However, if it is misspecified, we have $M_t \neq \dddot{M}_t = a - b' f_t$ and $\dddot{M}_t$ recovers the best linear predictor of M_t given f_t or $\dddot{M}_t = Proj\,(M_t | f_t)$. Moreover, ϵ_t does not help pricing f_t or the risk-free rate, $R^n_{f,t}$.

If we demean M_t, we have $M_t - \mathbb{E}[M] = -b' (f_t - \mathbb{E}[f])$. Then, a projection of $M_t - \mathbb{E}[M]$ onto $f_t - \mathbb{E}[f]$ yields $b = -\Sigma_f^{-1} \mathbb{E}[(f_t - \mathbb{E}[f])(M_t - \mathbb{E}[M])]$ which, after some algebra, becomes:

$$b = \mathbb{E}\left[1/R^n_f\right] \cdot \Sigma_f^{-1} \mathbb{E}[f] \tag{IA.1}$$

where Σ_f is the covariance matrix of the factors.

Note that Equation IA.1 is equivalent to the expression for b in Equation 8, and that a can be easily recovered from $a = \mathbb{E}\left[1/R^n_f\right] + b' \mathbb{E}[f]$. Simply plugging in sample analogues to the moments in these expressions gives a consistent estimator:

$$\begin{cases} \widehat{a} = \widehat{\mathbb{E}}\left[1/R_f^n\right] + \widehat{b}'\widehat{\mathbb{E}}\left[f\right] \\ \widehat{b} = \widehat{\mathbb{E}}\left[1/R_f^n\right] \cdot \widehat{\Sigma}_f^{-1}\widehat{\mathbb{E}}\left[f\right] \end{cases} \tag{IA.2}$$

where $\widehat{\Sigma}_f = \widehat{\mathbb{E}}\left[\left(f - \widehat{\mathbb{E}}\left[f\right]\right)\left(f - \widehat{\mathbb{E}}\left[f\right]\right)'\right]$ represents the sample covariance matrix and $\widehat{\mathbb{E}}\left[\cdot\right]$ represents the sample average.

In summary, IA.2 delivers consistent estimates for a and b under the validity of $\dddot{M}_t$ and recovers the projection of M_t onto f_t when $\dddot{M}_t$ is misspecified, transforming the misspecification into an orthogonal missing factor, ϵ_t, such that $\mathbb{E}\left[\epsilon \cdot f\right] = 0$ and $\mathbb{E}\left[\epsilon\right] = 0$.

A.1.2 GMM Framework

We formalize the GMM procedure that leads to the $\widehat{a}$ and $\widehat{b}$ estimates in the previous subsection and provide standard errors for it. For the rest of this subsection, we stack the risk-free payoff (a constant 1) together with the factors. We do the same for prices and parameters. Thus, we define the new terms $F_t \equiv (1\ ,\ f_t')'$, $P_{Ft-1} \equiv (1/R_{f,t}^n\ ,\ 0')'$, and $\theta \equiv (a\ ,\ -b')'$.

We have $k+1$ Euler conditions summarized by $\mathbb{E}[M_t F_t] = \mathbb{E}[P_{Ft-1}]$. Then, substituting $M_t = \theta' F_t - \epsilon_t$ and $\mathbb{E}[\epsilon \cdot F] = 0$ into the Euler conditions yields $\mathbb{E}[\theta' F_t F_t] = \mathbb{E}[P_{Ft-1}]$. Given this just identified system, $\widehat{\theta}$ is the solution to $\widehat{\mathbb{E}}[F_t F_t' \widehat{\theta}] = \widehat{\mathbb{E}}[P_{Ft-1}]$, which is given by:[39]

$$\widehat{\theta} = \widehat{\mathbb{E}}\left[F_t F_t'\right]^{-1} \widehat{\mathbb{E}}\left[P_{Ft-1}\right]. \tag{IA.3}$$

We can use GMM to get the asymptotic covariance matrix for $\widehat{\theta}$ as follows. Let $u_t\left(\theta\right) = \theta' F_t F_t - P_{Ft-1}$ and $g\left(\theta\right) = \mathbb{E}\left[u_t\left(\theta\right)\right]$. Then, from general GMM theory (see Hansen (1982)), we have that $aVar(\widehat{\theta}) = \left(\frac{\partial g(\theta)}{\partial \theta}' S^{-1} \frac{\partial g(\theta)}{\partial \theta}\right)^{-1}$, where $S = \sum_{j=-\infty}^{\infty} \mathbb{E}\left[u_t\left(\theta\right) u_{t-j}\left(\theta\right)'\right]$.

[39]Note that $\widehat{\mathbb{E}}[F_t F_t' \widehat{\theta}] = \widehat{\mathbb{E}}[P_{Ft-1}]$ is equivalent to

$$\begin{cases} \widehat{\mathbb{E}}\left[\widehat{a} - \widehat{b}' f\right] = \widehat{\mathbb{E}}\left[1/R_f^n\right] \\ \widehat{\mathbb{E}}\left[\left(\widehat{a} - \widehat{b}' f\right) f\right] = 0. \end{cases}$$

which yields the same estimates as in Equation IA.2 once we solve for $\widehat{a}$ and $\widehat{b}$.

Thus, simply substituting terms in the expression for the asymptotic variance-covariance matrix gives:

$$aVar(\widehat{\theta}) = \frac{1}{T}\left\{\mathbb{E}\left[F_t F_t'\right]'\left(\sum_{j=-\infty}^{\infty}\mathbb{E}\left[u_t\left(\theta\right)u_{t-j}\left(\theta\right)'\right]\right)^{-1}\mathbb{E}\left[F_t F_t'\right]\right\}^{-1}. \tag{IA.4}$$

The asymptotic variance-covariance matrix can be estimated by plugging in $\widehat{\theta}$ and using some suitable estimator for the spectral density (i.e., the infinity sum), compatible with the assumptions one intends to impose on u_t. For instance, if we assume u_t is uncorrelated over time (which is valid under the null that $M_t = \ddot{M}_t$), but can have arbitrary covariance structure across equations (allowing for factor covariance and heteroskedasticity), then a "White-type" estimator is appropriate:

$$\widehat{aVar}(\widehat{\theta}) = \frac{1}{T}\left\{\widehat{\mathbb{E}}\left[F_t F_t'\right]'\left(\widehat{\mathbb{E}}\left[u_t(\widehat{\theta})u_t(\widehat{\theta})'\right]\right)^{-1}\widehat{\mathbb{E}}\left[F_t F_t'\right]\right\}^{-1} \tag{IA.5}$$

which is the estimator we use to construct standard errors for our risk price estimates.

A.1.3 Efficiency and Robustness

a) Efficiency

In general, $b = \mathbb{E}\left[1/R_f^n\right]\cdot\Sigma_f^{-1}\varphi$, where φ is the vector of risk premia for the factors (see Ludvigson (2013)). Thus, we effectively estimate b by the respective sample moments, $\widehat{b} = \widehat{\mathbb{E}}\left[1/R_f^n\right]\cdot\widehat{\Sigma}_f^{-1}\widehat{\mathbb{E}}\left[f\right]$. The first two moments are obviously fine, and thus we should question how efficient it is to estimate φ using $\widehat{\mathbb{E}}\left[f\right]$.

Suppose we try to estimate φ using a cross-sectional regression of betas on average returns of testing assets including not only the factors, but also all other assets in the economy. In this case, the *efficient* Generalized Least Squares (GLS) gives $\widehat{\varphi} = \mathbb{E}_T\left[f_t\right]$ and ignores the cross-section information on all other assets (see Cochrane (2005)). A similar result holds for the maximum likelihood estimator of φ in factor regressions (Gibbons, Ross, and Shanken (1989)) and on two-pass regressions (Shanken (1992)). Hence, $\widehat{\mathbb{E}}\left[f\right]$ is the efficient choice of estimator for φ.

This well-known result extends to our $\widehat{b}$. Suppose we try to estimate b directly using GMM, but relying on a set of testing assets that includes f_t as well as other assets. Then, the *efficient* GMM ignores (asymptotically) all testing assets other than f_t (see Section 3 in Nagel (2013)), which makes our $\widehat{b}$ asymptotically efficient.

b) Robustness

If we include other testing assets in the GMM, then, in finite samples, the estimates for a and b vary with the assets included. This is quite problematic as the estimates will depend not only on the model under analysis, but also on the testing assets used. The problem gets worse if the model is misspecified. In this case, not only the estimates are dependent on the cross-section of assets used, but also their probability limit (as T grows). This means that, if the model is mispecified, *what* we are estimating varies with the testing assets under GMM.

To see this point, consider a well specified model such that $M_t = \dddot{M}_t = a - b'f_t$. In this case, although $\widehat{a}$ and $\widehat{b}$ depend on the cross-section of asset used, GMM with any set of testing assets provides estimates that converge to the same (and correct) a and b. That is not the case if the model is misspecified. Under misspecification, the $\widehat{a}$ and $\widehat{b}$ converge to the a and b in the projection $M_t = a - b'f_t - \epsilon_t$ if and only if $\widehat{\theta}$ converge to $\mathbb{E}[F_tF_t']^{-1}\mathbb{E}[M_tF_t]$. This is the case with our estimator because $\mathbb{E}[P_{Ft-1}] = \mathbb{E}[M_tF_t]$, and thus $\widehat{\theta} = \widehat{\mathbb{E}}[F_tF_t']^{-1}\widehat{\mathbb{E}}[P_{Ft-1}] \xrightarrow{p} \mathbb{E}[F_tF_t']^{-1}\mathbb{E}[P_{Ft-1}] = \mathbb{E}[F_tF_t']^{-1}\mathbb{E}[M_tF_t]$.

Therefore, our $\widehat{a} - \widehat{b}'f_t$ has the robust interpretation that it always reflects the projection of M_t onto f_t regardless of whether $M_t = \dddot{M}_t$ or not. The same does not hold true for GMM estimators that rely on other testing assets and a non-optimal weighting matrix.

A.2 SDF Projections as Factor Spanning Tests

We would like to understand whether adding f_t to x_t in a factor model improves the pricing of testing assets (i.e., increases the sharpe ratio of the tangency portfolio). It is well-known in the literature that $f_{i,t}$ adds to x_t if and only if $\alpha_i \neq 0$ and that the sign of α_i tells us whether we would like to long or short f_i. Tests of α_i are called factor spanning tests. Below, we show that the b in the SDF $M_t = a - b' f_t - b_x' x_t$ provides the same economic information as factor spanning tests, with the added advantage that the SDF version of the test controls not only for x_t but also for other f_t factors when testing each f_t factor, which is important in the context of the ICAPM as discussed in Subsection 2.3.

To start, generalize the framework to allow for mispecification as in the previous subsection. That is, consider $M_t = \dddot{M}_t - \epsilon_t$ in which M_t is the true SDF and $\dddot{M}_t$ is a model under consideration. Moreover, note that the α of any asset r_j relative to any arbitrary model $\dddot{M}_t$ is given by $\alpha_j = -\mathbb{E}[\epsilon \cdot r_j]/\mathbb{E}[\dddot{M}]$.[40] Then, the proposition below assures the SDF projection $M_t = a - b' f_t - b_x' x_t - \epsilon_t$ contains the same economic information as a factor spanning test.

Proposition 1. *The SDF projection $M_t = a - b' f_t - b_x' x_t - \epsilon_t$ results in $b_i = 0$ if and only if $\alpha_i = 0$, where α_i represents the pricing error of $f_{i,t}$ relative to the model $\dddot{M}_t = \ddot{a} - \dddot{b}' f_t - \dddot{b}_x' x_t$, with $f_{-i,t}$ reflecting all factors in f_t except $f_{i,t}$.*

Proof. Suppose $b_i = 0$. In this case, estimating $M_t = \dddot{M}_t - \epsilon_t$ results in the same ϵ_t obtained from the SDF projection $M_t = a - b' f_t - b_x' x_t - \epsilon_t$. Then, we have $\alpha_i = -\mathbb{E}[\epsilon \cdot f_i]/\mathbb{E}[\dddot{M}] = 0$ since, by construction, the SDF projection implies $\mathbb{E}[\epsilon \cdot f_i] = 0$.

Alternatively, suppose $\alpha_i = 0$. Then, we must have $\mathbb{E}[\epsilon \cdot f_i] = 0$ since $\alpha_i = -\mathbb{E}[\epsilon \cdot f_i]/\mathbb{E}[\dddot{M}]$. As such, estimating the SDF projection $M_t = a - b' f_t - b_x' x_t - \epsilon_t$ yields $b_i = 0$ because the ϵ_t in $M_t = \ddot{a} - \dddot{b}' f_{-i,t} - \dddot{b}_x' x_t - \epsilon_t$ is already orthogonal to $f_{i,t}$. □

A.3 Details Related to Bootstrap Analyses

We use bootstrap simulations for two purposes. The first is to estimate t-statistics associated with the Long-Sample risk prices (and risk premia) estimated in Table 5 when applying the Stambaugh (1997) procedure. The second is to estimate sampling statistics related to Sharpe

[40] To see this result, note that $\dddot{M}$ implies $\dddot{\mathbb{E}}[r_j] = -Cov(\dddot{M}, r_j)/\mathbb{E}[\dddot{M}]$, and thus

$$\alpha_j = \mathbb{E}[r_j] - \dddot{\mathbb{E}}[r_j] = \mathbb{E}[r_j] - \frac{Cov(\dddot{M}, r_j)}{\mathbb{E}[\dddot{M}]} = \mathbb{E}[r_j] - \frac{Cov(M, r_j)}{\mathbb{E}[M]} - \frac{Cov(\epsilon, r_j)}{\mathbb{E}[\dddot{M}]} = -\frac{\mathbb{E}[\epsilon \cdot r_j]}{\mathbb{E}[\dddot{M}]}$$

ratios (Table 7) and sum-squared- or sum-absolute-alpha metrics (Tables 9-11 and 13) across different factor models. We describe each of the related bootstrapping methodologies below.

a) Long-Sample Risk Price t-Statistics

The Stambaugh (1997) procedure allows us to estimate both Σ_f and $\mathbb{E}[f]$ needed to compute risk prices according to Equation 8 for a set of factors over the Long Sample (1928-2019) when some of these factors are not available during the early period ("short factors" from the "short sample") where other factors are available ("long factors" from the "long sample"). For example, we have ICAPM risk factor data available from 1928-2019, but only have FF5 factor data available from 1963-2019. We use this procedure to estimate ICAPM risk prices over the Long Sample when controlling for factors from the following models: FF5, q4, SY4, DHS3, and q5.

We would like to estimate Σ_f and $\mathbb{E}[f]$ for all factors in both models over the long sample so that we can estimate the Long-Sample risk prices in Table 5. The Stambaugh (1997) procedure allows us to compute consistent estimates of these variables (loosely) by projecting short factors onto long factors that are available during both the long and short samples,[41] then using this projection to extrapolate over the long sample. The fact that both Σ_f and $\mathbb{E}[f]$ are estimated with error results in complications for the asymptotic theory needed to estimate standard errors on the resulting b estimates. As opposed to using something like the Delta Method to estimate asymptotic standard errors, we instead choose to estimate standard errors using simulation.

We run bootstrap simulations as follows. Let T be the total number of months in the long sample (i.e., the number of months from 1928-2019). For a given alternative factor model (such as the FF5 model), let T_S be the total number of months in the short sample over which data for the alternative model is available (e.g., in the case of the FF5 model, the number of months from 1963-2019). We randomly select T_S months of factor data (with replacement)

[41] We use the ICAPM and FFC4 factors, which are available during the entire Long Sample (1928-2019) as long factors in the Stambaugh (1997) procedure.

from the the short sample to estimate the projection of the short factors onto the long factors. Next, we randomly select $T - T_S$ months of data from the early period (1928-1962 in the example) with replacement and merge the data with the short-period simulated data to estimate Σ_f and $\mathbb{E}[f]$ following Stambaugh (1997). Finally, we calculate the resulting Long Sample b according to Equation 8. We repeat this procedure 100,000 times and record the estimated b in each simulation. We use the standard deviation from the resulting distribution of bs as an estimate of the standard error on our b estimated from the full data using the Stambaugh (1997) procedure.

b) Sharpe Ratio Statistics

We use bootstrap simulations to estimate the percent of times the ICAPM sharpe ratio is *higher* than that from alternative models under various IS/OS constructions given bootstrapped distributions. These values are reported in parentheses in Table 7

The simulation procedure is slightly different depending on whether we are concerned with IS sharpe ratios (i.e., those reported for the "Modern Sample", "1st Half", and "2nd Half"), OS sharpe ratios using weights estimated from 1973-1995 (i.e., those reported for "2nd Half OS (w from 1973-1995)"), or OS sharpe ratios using weight estimates from 1928-1995 (i.e., those reported for "2nd Half OS (w from 1928-1995)"). We will describe each of the three simulation procedures in turn below. Regardless of the procedure, we repeat it 100,000 and note the resulting sharpe ratios for each model in each simulation. We then compute the reported metric as the percent of simulations in which the ICAPM sharpe ratio was higher than the sharpe ratio for a particular alternative model. These simulations are motivated by a similar sampling procedure in Fama and French (2018) and similar data-splitting procedure in Kan, Wang, and Zheng (2019).

We simulate IS sharpe ratios for three different periods: (i) The "Modern Sample" from 1973-2019, (ii) The "1st Half" of the Modern Sample from 1973-1995, and (iii) The "2nd Half" of the Modern Sample from 1995-2019. Let the particular period of interest contain T months of factor data. Within each simulation, we randomly sample T months of data from the original sample period (with replacement) and then use these simulated data to compute the maximum sharpe ratios for the ICAPM and each alternative model. We record these sharpe ratios and repeat the process 100,000 times. Reported values are the percent of times the ICAPM sharpe ratio is greater than that for the alternative model across all simulations.

We simulate the "2nd Half OS (w from 1973-1995)" sharpe ratios as follows.[42] Let there be T months in the Modern Sample from 1973-2019. Within each simulation, we randomly select $T/2$ months of data from the first half of the Modern Sample (i.e., from 1973-1995) with replacement. For each of these randomly-selected months, t, we pair it with month $t + T/2$ from the second half of the Modern Sample (1995-2019). We use the simulated data from the first half of the sample to construct factor weights, w, that produce the maximum in-sample sharpe ratio for each model, then apply these weights to the paired simulated factor data from the second half of the sample and compute the resulting OS sharpe ratios for each model. We record these sharpe ratios and repeat the process 100,000 times. Reported values are the percent of times the ICAPM sharpe ratio is greater than that for the alternative model across all simulations.

The simulation methodology we use for "2nd Half OS (w from 1928-1995)" is similar to that used for "2nd Half OS (w from 1973-1995)", but allows us to use factor data from before 1973 (where available) to estimate in-sample weights, w. Within each simulation, we select IS and OS data from 1973-1995 and 1995-2019, respectively, in the same manner as for the "2nd Half OS (w from 1973-1995)" simulations. For a given model, let there be N months of data available before 1973. We also randomly sample N months from this data (with replacement) and combine it with the data sampled from 1973-1995. We the use this combined data to estimate factor weights that produce the maximum in-sample sharpe ratio, then apply these weights to the randomly-sampled data from 1995-2019 and compute the OS sharpe ratio from this data. In this way, we make use of data available before 1973 (when available), which helps improve the stability of the estimated weights, w. We record these sharpe ratios

[42] Recall that we use data from 1973-1995 to estimate weights, w, applied to the factors to construct maximum sharpe ratios in the 1973-1995 sample, and then apply these weights to factor data from 1995-2019 to construct the OS sharpe ratios.

and repeat the process 100,000 times. Reported values are the percent of times the ICAPM sharpe ratio is greater than that for the alternative model across all simulations.

c) Comparing $\Sigma\alpha^2/\Sigma\mathbb{E}[r]^2$ and $\Sigma|a|/\Sigma\mathbb{E}|r|$ across factor models

We use bootstrap simulations to estimate the percent of times the ICAPM $\Sigma\alpha^2/\Sigma\mathbb{E}[r]^2$ and $\Sigma|a|/\Sigma\mathbb{E}|r|$ ratios are *lower* than those from alternative models under the bootstrapped distribution when the models are applied to various testing assets (Tables 9-11) and 158 anomalies from Chen and Zimmermann (2020) (Table 13).

The simulation procedure is as follows. Let there be T months of data from 1973-2019. In each simulation, we randomly select T months of data (with replacement) from the 1973-2019 period and record the associated factor values and test asset returns. We then compute the $\Sigma\alpha^2/\Sigma\mathbb{E}[r]^2$ and $\Sigma|a|/\Sigma\mathbb{E}|r|$ for each model given the sampled data and record these values. We repeat the process 100,000 times for all results except for those reported in Table 13 related to anomalies. In that case, we repeat the process 10,000 times due to data processing limitations associated with simulating returns from 1,580 anomaly portfolios. In all cases, reported values are the percent of times the ICAPM $\Sigma\alpha^2/\Sigma\mathbb{E}[r]^2$ or $\Sigma|a|/\Sigma\mathbb{E}|r|$ metric is less than that for the alternative model across all simulations.

A.4 Bias Adjustment for Ex-post Alphas

In order to assess a given model's ability to price a set of testing assets, we would like to study ex-ante pricing errors (αs). However, in any cross-section of assets without an ex-ante ranking, we expect the highest ex-post α (i.e., the highest $\widehat{\alpha}$) to be higher than its ex-ante α and likewise for the lowest $\widehat{\alpha}$. This issue is well-known in the mutual fund literature (often referred to as "luck vs skill") and gives the false impression that pricing errors are larger (in absolute value) than they are in reality.

To overcome this issue, we follow the bootstrap simulation procedure in Fama and French (2010) to debias $\widehat{\alpha}$s before plotting Figures 6(c), 6(d), 7(c), and 7(d).[43] In particular, we

[43]No debiasing is applied to Figure 8 because Treasury bonds have an ex-ante ranking (on duration), and

debias $\widehat{\alpha}$s under the null hypothesis that the ICAPM is the correct model. Hence, we call this the "Model Null" procedure. The debiased $\widehat{\alpha}$ of a portfolio then reflects its $\widehat{\alpha}$ in excess of the $\widehat{\alpha}$ expected (given its ex-post $\widehat{\alpha}$ rank) if the ICAPM were true. In simple words, the debiased $\widehat{\alpha}$s measure pricing errors relative to what an econometrician would expect if the ICAPM were correct (and the econometrician should expect a non-zero distribution of $\widehat{\alpha}$s despite the ICAPM being valid).

We begin by estimating the ICAPM over the entire sample period of interest, T, and for all testing assets of interest. We then assign each portfolio an $\widehat{\alpha}$ rank, k, in ascending order and index $\widehat{\alpha}$ by j and k so that we have $\widehat{\alpha}_{j,k}$. We use $\mathring{r}_{j,t} = r_{j,t} - \widehat{\alpha}_{j,k}$ as data in the bootstrap simulations (with r reflecting excess returns). In this way, the simulations take the estimated ICAPM as the true model so that ex-ante αs are, by construction, zero. In each simulation, i, we randomly select T months from the full data (with replacement) and obtain the respective $\mathring{r}_{j,t}$ for each testing asset j. We then project $\mathring{R}_{j,t}$ onto f_t in the simulation to estimate $\widehat{\widehat{\alpha}}^{(i)}_{j,k}$, with k reflecting the $\widehat{\widehat{\alpha}}$ rank for portfolio j in simulation i. We repeat this process from $i = 1$ to $i = 100,000$. Finally, we obtain debiased $\widehat{\alpha}s$ by $\widehat{\alpha}^*_{j,k} = \widehat{\alpha}_{j,k} - \frac{1}{100,000}\Sigma_i \widehat{\widehat{\alpha}}^{(i)}_{j_i,k}$, where j_i varies across is while k if fixed across is. Results in Figures 6(c), 6(d), 7(c), and 7(d) report these $\widehat{\alpha}^*$s and whether they are statistically significant.[44]

While the "Model Null" procedure above is analogous to the mutual fund performance evaluation in Fama and French (2010) and is appropriate for studying how much the data deviates from the implications of a given model (in our case, the ICAPM), we cannot rely on this procedure to compare across models. The reason is that the simulation environment (i.e., the null hypothesis used for the simulation) would vary across models, which can distort model comparison. Since model comparison is not strongly affected by the $\widehat{\alpha}$ bias (because such a bias affects all models), we perform model comparison in the main text using the raw $\widehat{\alpha}$s. In Table IA.5, however, we summarize model comparison results with debiased $\widehat{\alpha}$s (and

---

thus there is no implicit ranking based on $\widehat{\alpha}$s. This is similar to the ranking of firms by book-to-market ratio, where no bias adjustment is needed.

[44]Note that the bias adjustment, $\frac{1}{100,000}\Sigma_i \widehat{\widehat{\alpha}}^{(i)}_{j_i,k}$, can be made arbitrarily precise with more simulations, and thus we use the same standard errors for $\widehat{\alpha}^*_{j,k}$ and $\widehat{\alpha}_{j,k}$.

the results are qualitatively similar to the ones reported in the main text).

The debiasing on IA.5 deviates from Fama and French (2010) as it is performed following a "Data Null" procedure. Specifically, we take the same steps as in our "Model Null" procedure except that we rely on $\mathring{r}_{j,t} = r_{j,t}$ and $\widehat{\alpha}^*_{j,k} = \widehat{\alpha}_{j,k} - \frac{1}{100{,}000}\Sigma_i \widehat{\widehat{\alpha}}^{(i)}_{j_i,k} - \widehat{\alpha}_{j_i}$. The first modification simply changes from a "Model Null" to a "Data Null" simulation. The second modification recognizes that ex-ante α is not necessarily zero in the "Data Null" simulation, and thus $\widehat{\widehat{\alpha}}^{(i)}_{j_i,k} - \widehat{\alpha}_{j_i}$ measures the bias of rank k in simulation i. As explained in Footnote 43, we do not apply any debiasing procedure to Treasury bonds. Moreover, we also do not apply any debiasing procedure to single stocks because of data processing limitations associated with running simulations on a rolling basis each month and for all stocks.

B Data Sources and Measurement

This section contains details on the data sources and measurement beyond the details provided in the main text. Subsection B.1 focuses on the factor models we study in Sections 3 and 4 (beyond the intertemporal factor model) while Subsection B.2 focuses on the anomaly deciles we explore in Section 4.

B.1 Replicating Factors from Prominent Factor Models

We obtain the factor data for all factor models from the original authors.[45] We also replicate factors from all the factor models we investigate for two reasons. First, replicating the factors allows us to extend factors from the SY4 and DHS3 models beyond their publicly-available end dates (2016 and 2018, respectively) to the end of our sample (2019). In these cases, we use our replicated versions of the factors from 2017-2019 and in 2019, respectively, in our main results. Second, we need to reconstruct the factors each month to create the net-trading-cost factors used to produce results in Figure 4 and in Table 7 (Panel B). We follow Detzel, Novy-Marx, and Velikov (2020) in the cost adjustment, with the necessary details provided in Footnote 35.

We describe how we obtain or construct signals used to create factors from each model we investigate below. Once the signals are constructed, we follow the sorting procedures described in the original papers to create the respective factors. For instance, to construct the FF3 SMB and HML factors, we use the 2x3 sorting procedure described in Fama and French (1993). Note that we need not account for trading costs associated with the market factor since it is a passive strategy that just holds the market portfolio.

[45] 1928-2019 data for the factors in FF3 and FFC4 as well as 1963-2019 data for the factors in FF5 are obtained from Kenneth French's data library (https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). 1967-2019 data for the factors in q4 and q5 are obtained from the global-q data library (http://global-q.org/index.html). 1963-2016 data for the factors in SY4 are obtained from Robert Stambaugh's webpage (http://finance.wharton.upenn.edu/~stambaug/). 1972-2018 data for the factors in DHS3 are obtained from Lin Sun's webpage (https://sites.google.com/view/linsunhome).

a) FF3 factors (SMB and HML)

SMB and HML are constructed annually at the end of June in each year t using the same procedure as in Fama and French (1993). We use market equity, computed by multiplying CRSP shares outstanding (shrout) with absolute price (prc) at the end of June in year t, as the sorting signal for size. The sorting signal for HML is book-to-market equity, where book equity is based on accounting data that is available at the end of December in year $t-1$ and market equity is from the same month. We construct book equity using COMPUSTAT data as described in Fama and French (1993) with two slight modifications. First, we exclude deferred taxes from the calculation for fiscal years starting in 1993 due to changes in tax treatments.[46] Second, we augment the COMPUSTAT book equity data with hand-collected book equity data from Moody's Manuals.[47] Finally, we use NYSE breakpoints for the size and book-to-market signals for sorting, which we obtain from Kenneth French's website. We reconstruct SMB and HML over our Long Sample (1928-2019) and achieve correlations of 99.8% and 99.6% with the original factors, respectively, over that period.

b) FFC4 factors (MOM)

MOM is constructed monthly using a stock's cumulative returns over the past 12 months and skipping the last month (i.e., the "12-2 return") as in Carhart (1997). The factor is then constructed using an independent double-sorting procedure that sorts on the momentum measure and market equity, computed by multiplying CRSP shares outstanding (shrout) with absolute price (prc), at the end of each month as in Carhart (1997). We reconstruct MOM over our Long Sample (1928-2019) and achieve a correlation of 99.9% with the original factor over that period.

[46]Note that this modification is also used to construct the publicly available SMB and HML factors. Further details can be found on Kenneth French's webpage.

[47]This data is available on Kenneth French's website and is based on Davis, Fama, and French (2000).

c) FF5 factors (SMB, CMA, and RMW)

The FF5 model uses the same HML factor as the FF3 model. However, its construction of SMB is slightly different than that in the FF3 model. The FF5 model also adds CMA and RMW factors. Therefore, we reconstruct the FF5 SMB, CMA, and RMW factors annually at the end of June in each year t using the same procedure as in Fama and French (2015). To construct the CMA and RMW, we use COMPUSTAT data to construct asset growth ("investment") and profitability ("operating profitability") signals as described in Fama and French (2015). We use NYSE breakpoints for the investment and operating profitability signals for sorting, which we obtain from Kenneth French's website. We reconstruct the FF5 SMB, CMA, and RMW factors from 1963-2019 and achieve correlations of 99.7%, 98.2%, and 98.9% with the original factors, respectively, during the period in which they are available (1963-2019).

d) q4 and q5 models (ME, IA, ROE, and EG)

We reconstruct the ME, IA, ROE, and EG factors using the same procedure described in Hou, Xue, and Zhang (2015) and Hou et al. (2020) using signals that were provided directly by the authors.[48] Our reconstructed ME, IA, ROE, and EG factors achieve correlations of 99.9%, 99.6%, 99.8%, and 99.7% with the original factors, respectively, during the period in which they are available (1967-2019).

e) SY4 model (SMB, MGMT, and PERF)

We reconstruct the SY4 SMB, MGMT, and PERF factors using the same procedure described in Stambaugh and Yuan (2017). For the SY4 SMB factor, the signal is the same as in the SMB factor of the FF3 model even though the factor construction differs from the approach in FF3. Therefore we rely on the same size signal we use in the FF3 SMB construction to obtain the SY4 size signal. The MGMT factor requires six different signals: net stock issues, composite equity issues, accruals, net operating assets, asset growth, and the investment-to-assets ratio.

[48]We thank the authors of these papers for sharing these data.

We obtain four of these signals (accruals, net operating assets, asset growth, and investment-to-asset ratio) from the signal data Chen and Zimmermann (2020) make publicly available (the respective labels are "Accruals", "NOA", "AG", and "InvestPPEInv").[49] We construct the other two signals (net stock issues and composite equity issues) directly from CRSP and COMPUSTAT data following Stambaugh and Yuan (2017) because Chen and Zimmermann (2020)'s construction of the two analogous signals differs slightly from the construction in Stambaugh and Yuan (2017).[50] The PERF factor requires five different signals: financial distress, O-score, 12-2 momentum, gross profitability, and quarterly return on assets. We use the same momentum signal as in our MOM factor. The remaining four signals (financial distress, O-score, gross profitability, and quarterly return on assets) are obtained from the signal data in Chen and Zimmermann (2020) (the respective labels are "FailureProbability", "OScore", "GP", and "roaq"). Given these signals, we construct SMB, MGMT, and PERF as in Stambaugh and Yuan (2017). Our reconstructed SMB, MGMT, and PERF factors achieve correlations of 95.7%, 96.7%, and 93.0% with the original factors, respectively, during the period in which they are available (1963-2016). Note that we reconstruct these factors over the 1963-2019 period and augment the original factors with our reconstructed factors only from 2017 to 2019 in our main results.

f) DHS3 model (FIN and PEAD)

We construct FIN and PEAD factors using the same procedure described in Daniel, Hirshleifer, and Sun (2020). The FIN factor requires two signals: net stock issues and composite equity issues. We construct net stock issues as in Daniel, Hirshleifer, and Sun (2020) directly using COMPUSTAT data.[51] We use the "CompEqIss" signal from Chen and Zimmermann (2020) for composite equity issues since it is constructed in the same way as the corresponding signal in Daniel, Hirshleifer, and Sun (2020). The PEAD factor uses the four-day cumula-

[49] See https://github.com/OpenSourceAP/CrossSection

[50] Note that the construction of net stock issues and composite equity issues in Stambaugh and Yuan (2017) differs from the construction used in the DHS3 FIN factor of Daniel, Hirshleifer, and Sun (2020).

[51] Note that the construction of net stock issues and composite equity issues in Daniel, Hirshleifer, and Sun (2020) differs from the construction used in the SY4 MGMT factor of Stambaugh and Yuan (2017).

tive returns around earnings announcements as a signal. We use the "AnnouncementReturn" signal from Chen and Zimmermann (2020) since it is constructed in the same way as the corresponding signal in Daniel, Hirshleifer, and Sun (2020). Our reconstructed FIN and PEAD factors achieve correlations of 97.3% and 97.2% with the original factors, respectively, during the period in which they are available (1972-2018). Note that we reconstruct these factors over the 1972-2019 period and augment the original factors with our reconstructed factors only for 2019 in our main results.

B.2 Anomaly Portfolios

We obtain 158 anomaly decile portfolios (that are value-weighted and based on NYSE breakpoints) from Chen and Zimmermann (2020), which gives us a total of 1,580 portfolios. In particular, we begin with the 180 "clear predictors" from Chen and Zimmermann (2020), which reflect anomalies that they classify as being "clearly significant in the original papers". From these 180 significant anomalies, we remove anomalies that do not have return records for all 10 decile portfolios for at least half of our 1973-2019 sample. This procedure yields the 158 anomalies (and the corresponding 1,580 decile portfolios) we use as test assets in Section 4.

C Supplementary Empirical Results

C.1 Net Payout Yield as Proxy for $N_{\mathbb{E}}$

In this subsection, we consider using changes in the log payout yield (Δpoy) as a proxy for $N_{\mathbb{E}}$ instead of our main measure, changes in the log-dividend-price ratio including M&A (Δdp).[52]

We provide motivation for why Δdp is the preferable proxy. In particular, we show that this measure provides better ex post sorts on exposure to $N_{\mathbb{E}}$ than Δpoy in Table IA.2. Portfolios sorted on exposure to Δdp provide a better proxy for exposure to $N_{\mathbb{E}}$ than those sorted on exposure to Δpoy in the sense that they produce a larger 10-1 portfolio beta (0.82 versus 0.49) with a higher t-statistic (12.4 versus 6.26) in the Long Sample. These portfolios also yield a larger beta slope (0.72 versus 0.45) with a higher t-static (8.82 versus 4.91) in the Long Sample. Results are similar in the Modern Sample. Note that portfolios sorted on exposure to Δdp also produce better sorts on $r_{N\mathbb{V}}$ (as measured by $\beta^{proj}_{N\mathbb{E}}$) compared to portfolios sorted on exposure to Δpoy across both samples based on their respective 10-1 portfolio beta and beta slope measures.

C.2 Realized Volatility as Proxy for $N_{\mathbb{V}}$

In this subsection, we consider using changes in realized variance ($\Delta\sigma^2$) as a proxy for $N_{\mathbb{V}}$ instead of our main measure, changes in the log of realized variance ($\Delta\log(\sigma^2)$). Note that we change notation slightly from our main text in this subsection to make the distinction between these two variance proxies more clear. Namely, in the main text we refer to $\Delta\log(\sigma^2)$ as "$\Delta\sigma^2$" for simplicity, which we avoid here for clarity.

We provide motivation for why $\Delta\log(\sigma^2)$ is the preferable proxy. In particular, we show that $\Delta\log(\sigma^2)$ provides better ex post sorts on exposure to $N_{\mathbb{V}}$ than $\Delta\sigma^2$ in Table IA.2 even though $N_{\mathbb{V}}$ reflects news to long-term expected σ^2, not $\log(\sigma^2)$. Portfolios sorted on exposure to $\Delta\log(\sigma^2)$ provide a better proxy for exposure to $N_{\mathbb{V}}$ than those sorted on exposure to

[52] We construct poy exactly as in Gonçalves (2020). Note that poy includes both issuances and repurchases.

$\Delta\sigma^2$ in the sense that they produce a larger 10-1 portfolio beta (0.29 versus 0.22) with a higher t-statistic (4.93 versus 3.77) in the Long Sample. These portfolios also yield a larger beta slope (0.29 versus 0.20) with a higher t-statistic (3.70 versus 2.78) in the Long Sample. The dominance is weaker in the Modern Sample where the corresponding 10-1 betas are 0.22 versus 0.21 with t-statistics of 2.32 versus 1.97, respectively. The beta slopes are both 0.22 with t-statistics of 1.73 versus 1.55, respectively. Note that portfolios sorted on exposure to $\Delta\log(\sigma^2)$ also produce better sorts on $r_{N\mathbb{V}}$ (as measured by $\beta^{proj}_{N\mathbb{V}}$) compared to portfolios sorted on exposure to $\Delta\sigma^2$ in the Long Sample (1928-2019) and similar sorts in the Modern Sample (1973-2019) based on their respective 10-1 portfolio beta and beta slope measures.

C.3 Exposure to Δdp and $\Delta\sigma^2$ using Three-Year Rolling Windows

In this subsection, we construct portfolios and factors based on stock exposures to Δdp and $\Delta\sigma^2$ (β_{dp} and β_{σ^2}, respectively) using three-year rolling estimation windows rather than five-year windows as in our main results.

Table IA.3 reports analogous results to those in Tables 1 and 2 when stocks are sorted into decile portfolios based on either β_{dp} (Panel A) or β_{σ^2} (Panel B) estimated using the three-year rolling windows. As in our main results, these portfolios sort well on ex post exposure to the original expected return and variance news proxies (Δdp and $\Delta\sigma^2$) as well as the news components themselves ($N_{\mathbb{E}}$ and $N_{\mathbb{V}}$).[53]

Table IA.4 reports estimated ICAPM risk prices and risk premia when the ICAPM factors are constructed from stocks based on β_{dp} and β_{σ^2} estimated using the three-year rolling windows. These results are analogous to our main results in Table 3, which use five-year rolling windows to estimate β_{dp} and β_{σ^2}. Results are both qualitatively and quantitatively similar to our main results, implying that the consistency of ICAPM factor model risk prices with ICAPM theory is robust to modifying the β estimation window.

[53] We only provide results for our Long Sample (1928-2019), but results for our Modern Sample (1973-2019) are also similar to our main results for that period.

C.4 Model Rankings Using Bias-Adjusted Alphas

In this subsection, we show that our main ranking results reported in Table 12 are qualitatively unchanged when accounting for the ex post bias in αs described in Internet Appendix A.4. The analogous results when applying the bias adjustment to $\Sigma\alpha^2/\Sigma\mathbb{E}\left[r\right]^2$ and $\Sigma|a|/\Sigma\mathbb{E}|r|$ associated with industry and correlation-clustered portfolios are provided in Table IA.5, which shows that the ICAPM maintains the best average rank among all models we investigate after applying the α bias correction.

References for Internet Appendix

Carhart, Mark M. (1997). "On Persistence in Mutual Fund Performance". In: *Journal of Finance* 52.1, pp. 57–82.

Chen, Andrew Y. and Tom Zimmermann (2020). "Open Source Cross-Sectional Asset Pricing". Working Paper.

Cochrane, John H (2005). *Asset Pricing*. Revised Edition. Princeton University Press.

Daniel, Kent, David Hirshleifer, and Lin Sun (2020). "Short- and Long-Horizon Behavioral Factors". In: *Review of Financial Studies* 4, pp. 1673–1736.

Davis, James L., Eugene F. Fama, and Kenneth R. French (2000). "Characteristics, Covariances, and Average Returns: 1929-1997". In: *Journal of Finance* 55.1, pp. 389–406.

Detzel, Andrew, Robert Novy-Marx, and Mihail Velikov (2020). "Model Selection with Transaction Costs". Working Paper.

Driscoll, John C. and Aart C. Kraay (1998). "Consistent Covariance Matrix Estimation with Spatially Dependent Panel Data". In: *Review of Economics and Statistics* 80.4.

Fama, Eugene F. and Kenneth R. French (1993). "Common Risk Factors in the Returns on Stocks and Bonds". In: *Journal of Financial Economics* 33, pp. 3–56.

– (2010). "Luck versus Skill in the Cross-Section of Mutual Fund Returns". In: *Journal of Finance* 65.5, pp. 1915–1947.

– (2015). "A five-factor asset pricing model". In: *Journal of Financial Economics* 116, pp. 1–22.

– (2018). "Choosing Factors". In: *Journal of Financial Economics* 128, pp. 234–252.

Gibbons, Michael R., Stephen A. Ross, and Jay Shanken (1989). "A Test of the Efficiency of a Given Portfolio". In: *Econometrica* 57.5, pp. 1121–1152.

Gonçalves, Andrei S. (2020). "The Short Duration Premium". In: *Journal of Financial Economics* Forthcoming.

Hansen, Lars Peter (1982). "Large Sample Properties of Generalized Method of Moments Estimators". In: *Econometrica* 50.4, pp. 1029–1054.

Hou, Kewei, Chen Xue, and Lu Zhang (2015). "Digesting Anomalies: An Investment Approach". In: *Review of Financial Studies* 28.3, pp. 650–705.

Hou, Kewei et al. (2020). "An Augmented q-Factor Model with Expected Growth". In: *Review of Finance* Forthcoming.

Kan, Raymond, Xiaolu Wang, and Xinghua Zheng (2019). "In-sample and Out-of-sample Sharpe Ratios of Multi-factor Asset Pricing Models". Working Paper.

Ludvigson, Sydney C. (2013). "Advances in Consumption-Based Asset Pricing: Empirical Tests". In: *Handbook of the Economics of Finance*. Ed. by George M. Constantinides, Milton Harris, and Rene M. Stulz. 1st ed. Vol. 2. B. Elsevier Science. Chap. 12, pp. 799–906.

Nagel, Stefan (2013). "Empirical cross-sectional asset pricing". In: *Annual Review of Financial Economics* 5, pp. 167–199.

Newey, Whitney K. and Kenneth D. West (1987). "A Simple, Positive-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix". In: *Econometrica* 55.3, pp. 703–708.

– (1994). "Automatic Lag Selection in Covariance Matrix Estimation". In: *Review of Economic Studies* 61.4, pp. 631–653.

Shanken, Jay (1992). "On the Estimation of Beta-Pricing Models". In: *Review of Financial Studies* 5.1, pp. 1–33.

Stambaugh, Robert F. (1997). "Analyzing investments whose histories differ in length". In: *Journal of Financial Economics* 45, pp. 285–331.

Stambaugh, Robert F. and Yu Yuan (2017). "Mispricing Factors". In: *Review of Financial Studies* 4, pp. 1270–1315.

Table IA.1
Decile Portfolios Sorted on β_{dp} using dp versus poy

This table reports statistics related to monthly returns on 10 β_{dp}-sorted portfolios. Here "β_{dp}" is measured as either exposure to Δdp or Δpoy. Panels A and B provide results from our Long (1928-2019) and Modern (1973-2019) samples, respectively. The top portion of each panel reports metrics related to portfolios that are sorted on exposure to Δdp (our main proxy for $N_{\mathbb{E}}$). The bottom portion of each panel reports metrics related to portfolios that are sorted on exposure to Δpoy. In the top portion of each panel, we report portfolio return exposures to our expected return news proxy (Δdp or Δpoy), the in-sample expected return news measure ($N_{\mathbb{E}}$), reinvestment risk factors constructed using either Δdp (top portion of each panel) or Δpoy (bottom portion of each panel), and the expected return news mimicking portfolio based on portfolios sorted on exposure to either Δdp (top portion of each panel) or Δpoy (bottom portion of each panel). Portfolio return exposures to each of these time series are denoted by β_{dp}, $\beta_{N\mathbb{E}}$, $\beta_{\mathbb{E}}$, and $\beta_{N\mathbb{E}}^{proj}$, respectively, and are normalized to be in market beta units as described in Subsection 2.1. The "Slope" statistic is a measure of the slope of the 10 related portfolio statistics with respect to portfolio decile (see Footnote 28). Portfolios are rebalanced monthly based on individual stock exposures to Δdp or Δpoy with further details provided in Subsection 1.3. The 10-1 portfolio t-statistics are computed according to Newey and West (1987) and Newey and West (1994). The Slope t-statistics are computed according to the method in Driscoll and Kraay (1998) using the procedure in Newey and West (1994) to select the number of lags.

PANEL A: Long Sample (1928-2019)

Proxy	Dec	1	2	3	4	5	6	7	8	9	10	10-1	(t_{10-1})	Slope	(t_{Slope})
Δdp	β_{dp}	-1.54	-1.29	-1.18	-1.09	-1.01	-0.94	-0.85	-0.76	-0.65	-0.55	0.99	(11.5)	0.89	(8.46)
	$\beta_{N_{\mathbb{E}}}$	-1.23	-1.02	-0.94	-0.86	-0.81	-0.75	-0.67	-0.60	-0.52	-0.41	0.82	(12.4)	0.72	(8.82)
	$\beta_{\mathbb{E}}$	-1.73	-1.40	-1.21	-1.06	-0.94	-0.84	-0.69	-0.59	-0.43	-0.34	1.39	(57.0)	1.28	(54.3)
	$\beta_{N\mathbb{E}}^{proj}$	-1.64	-1.34	-1.18	-1.06	-0.96	-0.87	-0.75	-0.64	-0.48	-0.33	1.32	(34.5)	1.18	(23.9)
Δpoy	β_{poy}	-0.85	-0.77	-0.67	-0.62	-0.56	-0.55	-0.49	-0.44	-0.40	-0.38	0.47	(3.38)	0.46	(2.82)
	$\beta_{N_{\mathbb{E}}}$	-1.11	-0.95	-0.88	-0.82	-0.79	-0.77	-0.69	-0.66	-0.61	-0.62	0.49	(6.26)	0.45	(4.91)
	$\beta_{\mathbb{E}}$	-1.48	-1.29	-1.11	-0.92	-0.84	-0.77	-0.63	-0.49	-0.40	-0.30	1.18	(45.1)	1.14	(68.2)
	$\beta_{N\mathbb{E}}^{proj}$	-1.46	-1.25	-1.11	-0.97	-0.93	-0.88	-0.77	-0.68	-0.60	-0.58	0.88	(8.58)	0.84	(7.23)

PANEL B: Modern Sample (1973-2019)

Proxy	Dec	1	2	3	4	5	6	7	8	9	10	10-1	(t_{10-1})	Slope	(t_{Slope})
Δdp	β_{dp}	-1.27	-1.02	-0.91	-0.84	-0.81	-0.74	-0.70	-0.61	-0.55	-0.44	0.84	(10.4)	0.70	(7.69)
	$\beta_{N_{\mathbb{E}}}$	-1.12	-0.85	-0.73	-0.67	-0.66	-0.60	-0.55	-0.49	-0.44	-0.33	0.79	(11.0)	0.63	(8.00)
	$\beta_{\mathbb{E}}$	-1.50	-1.13	-0.91	-0.72	-0.66	-0.58	-0.46	-0.34	-0.23	-0.13	1.37	(42.2)	1.22	(40.6)
	$\beta_{N\mathbb{E}}^{proj}$	-1.46	-1.07	-0.84	-0.74	-0.68	-0.61	-0.51	-0.41	-0.32	-0.16	1.30	(19.8)	1.08	(13.4)
Δpoy	β_{poy}	-0.39	-0.39	-0.34	-0.34	-0.30	-0.30	-0.29	-0.31	-0.26	-0.32	0.07	(1.40)	0.10	(1.48)
	$\beta_{N_{\mathbb{E}}}$	-0.96	-0.79	-0.74	-0.68	-0.66	-0.62	-0.60	-0.58	-0.53	-0.59	0.37	(6.91)	0.34	(4.63)
	$\beta_{\mathbb{E}}$	-1.15	-0.97	-0.86	-0.67	-0.61	-0.57	-0.43	-0.36	-0.25	-0.25	0.90	(28.4)	0.89	(53.4)
	$\beta_{N\mathbb{E}}^{proj}$	-1.17	-0.93	-0.85	-0.71	-0.68	-0.63	-0.61	-0.56	-0.47	-0.59	0.57	(7.83)	0.55	(6.53)

Table IA.2
Decile Portfolios Sorted on β_{σ^2} using $\log(\sigma^2)$ versus σ^2

This table reports statistics related to monthly returns on two sets of 10 portfolios sorted on either exposure to $\Delta \log(\sigma^2)$ or $\Delta\sigma^2$. Panels A and B provide results from our Long (1928-2019) and Modern (1973-2019) samples, respectively. The top portion of each panel reports metrics related to portfolios that are sorted on exposure to $\Delta \log(\sigma^2)$ (our main proxy for $N_{\mathbb{V}}$). The bottom portion of each panel reports metrics related to portfolios that are sorted on exposure to $\Delta\sigma^2$. In each panel, we report portfolio return exposures to our volatility news proxy ($\Delta \log(\sigma^2)$ or $\Delta\sigma^2$), the in-sample volatility news measure ($N_{\mathbb{V}}$), volatility risk factors constructed using either $\Delta \log(\sigma^2)$ (top portion of each panel) or $\Delta\sigma^2$ (bottom portion of each panel), and the variance news mimicking portfolio based on portfolios sorted on exposure either $\Delta \log(\sigma^2)$ (top portion of each panel) or $\Delta\sigma^2$ (bottom portion of each panel). Portfolio return exposures to each of these time series are denoted by $\beta_{\log\sigma^2}/\beta_{\sigma^2}$, $\beta_{N\mathbb{V}}$, $\beta_{\mathbb{V}}$, and $\beta^{proj}_{N\mathbb{V}}$, respectively, and are normalized to be in market beta units as described in Subsection 2.1. The "Slope" statistic is a measure of the slope of the 10 related portfolio statistics with respect to portfolio decile (see Footnote 28). Portfolios are rebalanced monthly based on individual stock exposures to either $N_{\mathbb{V}}$ proxy with further details provided in Subsection 1.3. The 10-1 portfolio t-statistics are computed according to Newey and West (1987) and Newey and West (1994). The Slope t-statistics are computed according to the method in Driscoll and Kraay (1998) using the procedure in Newey and West (1994) to select the number of lags.

PANEL A: Long Sample (1928-2019)

Proxy	Dec	1	2	3	4	5	6	7	8	9	10	10-1	(t_{10-1})	Slope	(t_{Slope})
$\Delta\log(\sigma^2)$	$\beta_{\log\sigma^2}$	-0.49	-0.42	-0.40	-0.38	-0.35	-0.31	-0.31	-0.26	-0.24	-0.24	0.25	(6.77)	0.24	(5.00)
	$\beta_{N_{\mathbb{V}}}$	-0.78	-0.67	-0.62	-0.60	-0.55	-0.49	-0.51	-0.44	-0.46	-0.49	0.29	(4.93)	0.29	(3.70)
	$\beta_{\mathbb{V}}$	-1.29	-1.15	-0.98	-0.85	-0.71	-0.59	-0.51	-0.43	-0.38	-0.37	0.92	(33.3)	0.96	(59.4)
	$\beta^{proj}_{N\mathbb{V}}$	-1.02	-0.89	-0.77	-0.70	-0.63	-0.56	-0.55	-0.48	-0.51	-0.51	0.51	(7.02)	0.50	(5.83)
$\Delta\sigma^2$	β_{σ^2}	-0.38	-0.34	-0.33	-0.27	-0.28	-0.24	-0.24	-0.24	-0.21	-0.22	0.16	(1.91)	0.16	(1.53)
	$\beta_{N_{\mathbb{V}}}$	-0.76	-0.62	-0.58	-0.57	-0.52	-0.48	-0.48	-0.48	-0.50	-0.54	0.22	(3.77)	0.20	(2.78)
	$\beta_{\mathbb{V}}$	-1.18	-0.98	-0.83	-0.65	-0.60	-0.51	-0.40	-0.34	-0.27	-0.31	0.87	(29.4)	0.88	(45.7)
	$\beta^{proj}_{N\mathbb{V}}$	-0.98	-0.80	-0.71	-0.63	-0.60	-0.55	-0.53	-0.52	-0.55	-0.62	0.36	(5.21)	0.35	(4.17)

PANEL B: Modern Sample (1973-2019)

Proxy	Dec	1	2	3	4	5	6	7	8	9	10	10-1	(t_{10-1})	Slope	(t_{Slope})
$\Delta\log(\sigma^2)$	$\beta_{\log\sigma^2}$	-0.53	-0.41	-0.41	-0.39	-0.33	-0.31	-0.28	-0.25	-0.22	-0.21	0.32	(4.94)	0.29	(3.64)
	$\beta_{N_{\mathbb{V}}}$	-0.50	-0.42	-0.41	-0.39	-0.34	-0.32	-0.29	-0.28	-0.26	-0.27	0.22	(2.32)	0.22	(1.73)
	$\beta_{\mathbb{V}}$	-1.22	-1.01	-0.81	-0.70	-0.56	-0.41	-0.32	-0.24	-0.15	-0.12	1.11	(33.1)	1.10	(59.9)
	$\beta^{proj}_{N\mathbb{V}}$	-1.03	-0.86	-0.72	-0.69	-0.56	-0.45	-0.39	-0.33	-0.27	-0.26	0.77	(11.3)	0.77	(10.8)
$\Delta\sigma^2$	β_{σ^2}	-0.39	-0.33	-0.29	-0.27	-0.29	-0.23	-0.23	-0.23	-0.20	-0.23	0.16	(1.02)	0.15	(0.82)
	$\beta_{N_{\mathbb{V}}}$	-0.52	-0.43	-0.38	-0.38	-0.35	-0.31	-0.30	-0.28	-0.24	-0.31	0.21	(1.97)	0.22	(1.55)
	$\beta_{\mathbb{V}}$	-1.19	-0.96	-0.76	-0.59	-0.52	-0.41	-0.30	-0.18	-0.11	-0.13	1.06	(26.7)	1.05	(53.6)
	$\beta^{proj}_{N\mathbb{V}}$	-1.02	-0.85	-0.70	-0.63	-0.57	-0.49	-0.42	-0.30	-0.26	-0.28	0.75	(12.0)	0.74	(9.82)

Table IA.3

Decile Portfolios Sorted on Three-Year βs

This table reports statistics related to monthly returns on 10 β_{dp}-sorted portfolios (Panel A) and 10 β_{σ^2}-sorted portfolios (Panel B) when sorting on exposure to either Δdp or $\Delta\sigma^2$, respectively, estimated over three-year rolling windows. All data is from our Long Sample (1928-2019). The portion of Panel A reports ex post portfolio return exposures to Δdp, the in-sample expected return news measure ($N_{\mathbb{E}}$), a reinvestment risk factor ($r_{\mathbb{E}}$) constructed using the three-year exposures, and the expected return news mimicking portfolio ($r_{N\mathbb{E}}$) denoted by β_{dp}, $\beta_{N\mathbb{E}}$, $\beta_{\mathbb{E}}$, and $\beta_{N\mathbb{E}}^{proj}$, respectively. The top portion of Panel B reports portfolio return exposures to $\Delta\sigma^2$, the in-sample volatility news measure ($N_{\mathbb{V}}$), a volatility risk factor ($r_{\mathbb{V}}$) constructed using the three-year exposures, and the volatility news mimicking portfolio ($r_{N\mathbb{V}}$) denoted by β_{σ^2}, $\beta_{N\mathbb{V}}$, $\beta_{\mathbb{V}}$, and $\beta_{N\mathbb{V}}^{proj}$, respectively. All βs are normalized to be in market beta units as described in Subsection 2.1. In the bottom portion of each panel, we report portfolio average returns ($\mathbb{E}[r]$) and αs when computed with respect to the CAPM (α_m), the ICAPM excluding $r_{\mathbb{E}}$ ($\alpha_{m,\mathbb{V}}$) or $r_{\mathbb{V}}$ ($\alpha_{m,\mathbb{E}}$), and the full ICAPM ($\alpha_{m,\mathbb{E},\mathbb{V}}$). All returns are in percent and annualized (approximately) by multiplying monthly returns by 12. The "Slope" statistic is a measure of the slope of the 10 related portfolio statistics with respect to portfolio decile (see Footnote 28). Portfolios are rebalanced monthly based on individual stock exposures to Δdp or σ^2 with further details provided in Subsection 1.3. The 10-1 portfolio t-statistics are computed according to Newey and West (1987) and Newey and West (1994). The Slope t-statistics are computed according to the method in Driscoll and Kraay (1998) using the procedure in Newey and West (1994) to select the number of lags.

PANEL A: β_{dp}-Sorted Portfolios

Dec	1	2	3	4	5	6	7	8	9	10	10-1	(t_{10-1})	Slope	(t_{Slope})
β_{dp}	-1.52	-1.30	-1.18	-1.07	-1.00	-0.94	-0.83	-0.74	-0.67	-0.57	0.96	(9.93)	0.88	(7.98)
$\beta_{N\mathbb{E}}$	-1.23	-1.03	-0.95	-0.84	-0.79	-0.75	-0.66	-0.58	-0.52	-0.43	0.80	(11.07)	0.72	(8.70)
$\beta_{\mathbb{E}}$	-1.70	-1.41	-1.20	-1.04	-0.93	-0.84	-0.68	-0.55	-0.45	-0.34	1.36	(52.70)	1.27	(50.63)
$\beta_{N\mathbb{E}}^{proj}$	-1.64	-1.33	-1.14	-1.03	-0.92	-0.84	-0.70	-0.57	-0.45	-0.31	1.32	(27.17)	1.20	(23.65)
$\mathbb{E}[r]$	9.4%	8.7%	9.1%	9.3%	8.8%	9.3%	9.5%	8.2%	7.6%	6.8%	-2.7%	(-0.98)	-1.9%	(-0.54)
α_m	-4.1%	-2.9%	-1.4%	-0.3%	-0.2%	0.9%	2.1%	1.6%	1.6%	1.8%	5.9%	(2.98)	5.9%	(2.50)
$\alpha_{m,\mathbb{V}}$	-4.2%	-3.0%	-1.5%	-0.3%	-0.2%	1.0%	2.1%	1.6%	1.7%	1.8%	6.0%	(4.04)	6.1%	(3.34)
$\alpha_{m,\mathbb{E},\mathbb{V}}$	0.4%	0.5%	1.1%	1.3%	0.7%	1.4%	1.9%	0.6%	0.2%	0.0%	-0.4%	(-0.42)	-0.3%	(-0.47)

PANEL B: β_{σ^2}-Sorted Portfolios

Dec	1	2	3	4	5	6	7	8	9	10	10-1	(t_{10-1})	Slope	(t_{Slope})
β_{σ^2}	-0.50	-0.41	-0.38	-0.37	-0.33	-0.30	-0.29	-0.29	-0.26	-0.26	0.24	(6.55)	0.21	(5.02)
$\beta_{N\mathbb{V}}$	-0.74	-0.69	-0.63	-0.58	-0.51	-0.49	-0.48	-0.47	-0.46	-0.54	0.20	(4.05)	0.25	(3.29)
$\beta_{\mathbb{V}}$	-1.27	-1.14	-0.99	-0.83	-0.67	-0.61	-0.49	-0.43	-0.41	-0.43	0.84	(14.96)	0.90	(39.79)
$\beta_{N\mathbb{V}}^{proj}$	-0.97	-0.93	-0.82	-0.72	-0.61	-0.57	-0.54	-0.51	-0.51	-0.59	0.37	(5.10)	0.46	(6.16)
$\mathbb{E}[r]$	10.8%	10.1%	10.5%	9.6%	9.0%	8.7%	7.8%	7.6%	6.5%	6.2%	-4.5%	(-2.27)	-4.7%	(-1.82)
α_m	-0.6%	-0.4%	1.0%	0.8%	1.0%	1.3%	0.7%	0.8%	-0.3%	-1.1%	-0.5%	(-0.27)	-0.2%	(-0.10)
$\alpha_{m,\mathbb{E}}$	2.0%	1.6%	2.2%	1.5%	1.0%	1.1%	-0.1%	0.0%	-1.0%	-1.6%	-3.6%	(-2.65)	-3.6%	(-2.28)
$\alpha_{m,\mathbb{E},\mathbb{V}}$	0.1%	-0.1%	0.6%	0.8%	0.8%	0.9%	0.4%	0.9%	0.3%	0.0%	0.0%	(-0.04)	0.1%	(0.20)

Table IA.4
ICAPM Risk Prices with Factors Based on Three-Year βs

This table reports estimated ICAPM risk prices (b) and their associated risk premia (λ) according to Equation 8 over various sub-periods using ICAPM factors constructed from stocks sorted on exposure to Δdp and $\Delta\sigma^2$ using three-year rolling estimation windows. Since bs are not easily comparable, we report $\sigma_x \cdot b_x$ for each factor x_t so that the reported values can be interpreted as the change in M_t induced by a one standard deviation change in the respective x_t (holding other factors fixed). λs are annualized (approximately) by multiplying by 12. The implied relative risk aversion is estimated as $\gamma = \lambda_m/\sigma_m^2$. The estimator for b is a Generalized Method of Moments (GMM) estimator, and thus t-statistics are computed according to GMM asymptotic theory (see Internet Appendix A).

	$T_0 =$	**1928**	**1946**	**1963**	**1973**	**1928**	**1946**	**1963**	**1973**
	$T_1 =$	**2019**	**2019**	**2019**	**2019**	**2006**	**2006**	**2006**	**2006**
	b	0.26	0.30	0.25	0.25	0.24	0.29	0.22	0.22
r_m	$[\lambda]$	[16.6%]	[15.4%]	[13.3%]	[13.7%]	[15.8%]	[14.7%]	[11.8%]	[12.0%]
	(t_{stat})	(4.77)	(5.64)	(4.33)	(3.91)	(4.14)	(5.07)	(3.55)	(3.06)
	b	0.33	0.42	0.38	0.38	0.31	0.41	0.35	0.34
$r_{\mathbb{E}}$	$[\lambda]$	[21.6%]	[21.4%]	[19.8%]	[20.4%]	[20.7%]	[20.7%]	[18.6%]	[19.1%]
	(t_{stat})	(5.09)	(5.81)	(4.74)	(4.38)	(4.51)	(5.41)	(4.16)	(3.73)
	b	-0.19	-0.24	-0.23	-0.23	-0.18	-0.24	-0.23	-0.23
$r_{\mathbb{V}}$	$[\lambda]$	[-12.1%]	[-12.1%]	[-12.2%]	[-12.6%]	[-12.1%]	[-12.0%]	[-12.1%]	[-12.6%]
	(t_{stat})	(-3.52)	(-3.95)	(-3.50)	(-3.27)	(-3.23)	(-3.72)	(-3.18)	(-2.90)
Implied γ		4.8	7.1	5.8	5.6	4.4	6.9	5.1	4.7

Table IA.5

ICAPM vs Other Factor Models: Summarizing α Results with Bias Correction

This table reports $\Sigma\alpha^2/\Sigma\mathbb{E}[r]^2$ (Panel A) and $\Sigma|a|/\Sigma\mathbb{E}|r|$ (Panel B) ranks across each model within each test asset group we investigate when αs are corrected for the ex post measurement bias described in Internet Appendix A.4. Single Stock and Bond Portfolio measures are not bias-adjusted for reasons described in that subsection. Lower ranks correspond to lower ratio values. The last row in each panel reports the average rank of the ICAPM and each other factor model (described at the beginning of Section 3) across all test asset groups.

PANEL A: Squared Pricing Errors ($\Sigma\alpha^2/\Sigma\mathbb{E}[r]^2$) Ranks

		CAPM	FF3	FFC4	FF5	q4	SY4	DHS3	q5	ICAPM
Single Stocks	5-Year Window	2	4	3	6	5	8	9	7	1
	10-Year Window	2	3	4	6	7	8	9	5	1
Industry Port	30 Portfolios	4	7	6	9	8	5	1	3	2
	48 Portfolios	3	7	4	9	8	6	2	5	1
Cor Clust Port	10 Portfolios	3	1	2	8	9	6	4	7	5
	25 Portfolios	7	2	1	6	5	4	9	8	3
Bond Portfolios	Fama Bond Port	8	9	5	7	4	6	1	2	3
	CRSP Bond Port	8	9	5	7	4	6	1	3	2
	Average Rank =	4.6	5.3	3.8	7.3	6.3	6.1	4.5	5.0	2.3

PANEL B: Absolute Pricing Errors ($\Sigma|\alpha|/\Sigma|\mathbb{E}[r]|$) Ranks

		CAPM	FF3	FFC4	FF5	q4	SY4	DHS3	q5	ICAPM
Single Stocks	5-Year Window	4	1	2	6	5	7	9	8	3
	10-Year Window	3	1	2	5	6	7	9	8	4
Industry Port	30 Portfolios	3	7	6	9	8	5	1	4	2
	48 Portfolios	3	7	4	9	8	6	1	5	2
Cor Clust Port	10 Portfolios	6	1	2	4	8	3	9	7	5
	25 Portfolios	7	2	1	6	3	4	9	8	5
Bond Portfolios	Fama Bond Port	8	9	5	7	4	6	1	2	3
	CRSP Bond Port	8	9	5	7	4	6	1	3	2
	Average Rank =	5.3	4.6	3.4	6.6	5.8	5.5	5.0	5.6	3.3