

# Aggregate Issuance and Savings Waves \*

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## Abstract

We document the fact that at both the aggregate and the firm level, corporations tend to simultaneously raise external finance and accumulate liquid assets, and we use this fact to make inferences about the aggregate cost of external finance over time. For all but the very largest firms, the aggregate correlation between external finance raised and liquidity accumulation is 0.6, and the average firm level correlation is 0.2. Conditioning on firms that raise external finance, the aggregate correlation increases to 0.74. We also show that firms' decisions in the *cross-section* about their sources and uses of funds can be useful for identifying the *aggregate* level of the cost of external finance. Specifically, we measure the cross-sectional correlation between external finance and liquidity accumulation at each date, and show that the time series of this cross-sectional correlation is highly correlated with traditional measures of the cost of external finance. Accordingly, we use our dynamic model of firm financing and savings, along with cross-sectional moments describing firms' internal and external financing decisions to estimate a time series for the aggregate cost of external finance in the US time series 1980-2010.

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# I. Introduction

We document the fact that at both the aggregate and the firm level, corporations tend to simultaneously raise external finance and accumulate liquid assets. For all but the very largest firms, the aggregate correlation between external finance raised and liquidity accumulation is 0.6, and the average firm level correlation is 0.2. Conditioning on firms that raise external finance, the aggregate correlation increases to 0.74. These facts seem puzzling if internal and external finance are substitutes and external finance is costly. In fact, static pecking order intuition predicts that firms will first draw down liquid balances and only then issue costly external finance.<sup>1</sup> On the other hand, if the cost of external finance varies over time, then the fact that there appear to be aggregate waves of issuance and savings activity may not be surprising.

To see this, consider the fact that firms which raise external finance can invest their issuance proceeds in productive capital assets, or in liquid financial assets with a low physical rate of return. Thus, if firms raise costly external finance in order to save in liquid assets, either the cost of external finance is relatively low at that time, or the shadow return to liquidity is particularly high. In this paper, we exploit this intuition, and explore the relationship between firms' issuance and savings decisions and the aggregate cost of external finance. We also show that firms' decisions in the cross section about how they use the proceeds of the external funds they raise is informative about the aggregate cost of external finance. We then use the information in the cross section of Compustat data to infer the average cost of external finance at each point in time.

We begin by constructing a simple two period model which formalizes the intuition that firms are more likely to issue external finance and save the proceeds when the cost of external finance is low. We assume that physical capital is more productive than liquid assets, that there are decreasing returns to physical capital, and that there are constant returns to liquidity accumulation. Finally, we assume that the marginal cost of external finance is increasing in the amount of funds raised. In this model, firms invest in physical capital, and raise external finance as necessary, until the net marginal benefit of an additional unit of capital declines enough to equal the net return on liquidity accumulation. If external finance is cheap enough, the firm will raise additional funds in order to accumulate liquidity until the marginal cost of an additional dollar of external finance equals the net return on liquid assets. It is easy to show in this environment that, except for at corner solutions, external finance and liquidity accumulation will increase one for one together as the cost of external finance decreases. As a result, in the cross section, the correlation between external finance and liquidity accumulation

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<sup>1</sup>See Myers (1984) p. 581.

naturally increases as the cost of external finance decreases.

Empirically, there is a strong relationship between firms' issuance and savings decisions, and traditional proxies for the cost of external finance. We show that the cross sectional correlation between external finance raised and liquidity accumulated tends to be low when the default spread and the tightness of lending standards indicate that external finance is particularly costly, and vice versa. We compute the cross-section correlation between liquidity accumulation and external finance,  $xsrho_{i,e}$ , at each date, and show that the time series correlation between  $xsrho_{i,e}$  and the negative of the default spread is 0.64. The correlation between  $xsrho_{i,e}$  and the negative of the percent of banks tightening lending standards is 0.58. We argue, then, using the intuition from our model, and the empirical relationship between  $xsrho_{i,e}$  and traditional proxies for the cost of external finance, that firms' behavior in the cross section contains useful information about the the aggregate cost of external finance.

Based on the intuition from our simple model, and these empirical facts, we construct a dynamic, quantitative model of firms' issuance and savings decisions. Firms maximize the present value of their payouts by choosing investment and savings. They face three shocks, an aggregate and an idiosyncratic productivity shock, and an aggregate shock to the cost of external finance. Their investment and financing decisions are subject to both physical capital adjustment costs, and costly external finance. Our dynamic model extends the results from the two period framework in several ways. First, it endogenizes the shadow return on liquid assets by incorporating firms' dynamic motives to hedge both investment opportunities and the stochastic cost of external finance. Second, it incorporates the dynamic effects of the intertemporal tradeoffs inherent in the physical and financial adjustment costs on firms' investment and financing decisions. This is important, as we will show, because fixed costs of external finance, along with investment smoothing due to quadratic investment adjustment costs, can lead to waves of issuance and savings even if the cost of external finance does not vary. Finally, the full model allows us to show that such a model with stochastic costs of external finance can quantitatively match the empirical relationship between savings and external finance, which then enables us to use results from this model to make inferences about the aggregate cost of external finance in the data.

Using the calibrated version of the dynamic model, we simulate a panel of firms analogous to the Compustat panel we study empirically. The calibrated model replicates the aggregate correlation between liquidity accumulation and external finance of about 0.60, as well as the fact that the correlation between liquidity accumulation and external finance is decreasing in firm size. Moreover, the main intuition from the simple model regarding the relationship between firms' financing and savings decisions and the aggregate cost of external finance carries over to

the quantitative model. When the cost of external finance is low, firms are more likely to raise external finance and save the proceeds. In the model, the correlation between the time series of the cross-section correlation between liquidity accumulation and external finance,  $xsrho_{i,e}$ , and the cost of external finance is 0.86. A regression of the cost of external finance on  $xsrho_{i,e}$  yields an R-squared of 0.67. Thus, we argue that the dynamic, quantitative model, as well as the two-period model shows that this cross-section correlation contains useful information about the aggregate cost of external finance.

We then extend this idea, and use the model implied relationship between firms' behavior in the cross section and the aggregate cost of external finance to construct a time series for the aggregate cost of external finance in the US from 1980-2010. We show that results are similar using two methods. First, we use a regression methodology to construct an aggregate index of the cost of external finance. We use this index, along with Compustat data, to construct a time series of the predicted aggregate cost of external finance for the US. The weights in our cost of external finance index are determined by the coefficients in a regression using data from our quantitative model of the average cost of external finance at any given date on the cross sectional moments describing firms' issuance and savings decisions on that date. We use the coefficients in our cost of external finance index along with the actual cross sectional moments from Compustat data to construct an estimate of the empirical cost of external finance in US data. We argue that our index measure of the aggregate cost of external finance, which exploits the revealed preferences implied by firms' financing and savings decisions, is a useful complement to existing measures. For example, the widely used default spread only measures the cost of debt finance, and much of the default spread may be due to a fair return adjustment for risk. Indeed, we show that our index implied cost contains new information relative to the default spread. For example, the index implied cost predicts that external finance was less costly in 1986 and more costly in 2001 than the default spread seems to imply.

Finally, we use a version of SMM to uncover the hidden aggregate state variable which describes the aggregate cost of external finance, and show that the results are similar to the regression method. For this estimation, we find for each date the value of the aggregate state describing the level of the cost of external finance which sets the model moments describing issuance and savings behavior in the cross section closest to their empirical counterparts. Thus, we propose a method for using cross sectional moments, along with a calibrated model, to make inferences about a hidden aggregate state.

## II. Related Literature

The empirical literature documenting the cyclical behavior of macroeconomic quantities has only recently begun to include quantities describing the financing of corporations. Jermann and Quadrini (forthcoming), and Covas and Den Haan (2011a) both document that debt issuances are highly procyclical, and Covas and Den Haan also report procyclical equity issuances. We are the first to incorporate data on firms' liquidity accumulation, as well as their investment, in order to consider the role of pure financing shocks vs. shocks to productivity in explaining firm level and aggregate investment and financing activities.<sup>2</sup> We argue that looking at the joint dynamics of liquidity accumulation and external finance is useful for examining the role of shocks to the cost of external finance, since how firms use funds may help to disentangle financing shocks from shocks that drive investment opportunities. Therefore while previous studies have focused on how external funds are raised, whether by debt or equity financing, our paper shows that how external are used is also useful in understanding the cost of external finance.

Several recent papers develop models which use a shock which originates in the financial sector to better match business cycle facts.<sup>3</sup> Jermann and Quadrini (forthcoming) show how a model with an endogenous credit limit and a shock to capital liquidity can generate realistic business cycles as well as matching the procyclical debt issuance and countercyclical equity issuance which they document using US Flow of Funds data. Covas and Den Haan (2011a) show that in Compustat data both debt and equity issuance are procyclical. In Covas and Den Haan (2011b), they develop a model in which countercyclical equity issuance costs are useful for generating both procyclical equity issuance and a countercyclical default rate.<sup>4</sup> Khan and Thomas (2011) build a quantitative business cycle model in which credit shocks drive aggregate productivity down by inhibiting productive investment reallocation across firms. This effect shows up in our model as well, and we show that estimated TFP is below actual TFP when external finance is costly. Hugonnier *et al.* (2011) build a search theory of external finance and show how idiosyncratic external finance risk affects corporate savings, investment, and payout policy. Bolton *et al.* (2011) develop a dynamic theory of firm finance and risk management with stochastic financing costs, and show analytically that such costs can increase savings and can delink external finance from

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<sup>2</sup>Eisfeldt and Rampini (2009) builds an aggregate model of internal and external finance to study the implications of corporate liquidity demand for the observed low return on liquid assets, but does not consider shocks to the cost of external finance. Covas and Den Haan (2011a) focus on debt and equity issuances, but they do note that, empirically, firms tend to both accumulate financial assets and invest when they issue external finance.

<sup>3</sup>These papers build on the seminal contributions of Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Carlstrom and Fuerst (1997) on the role of financial market conditions on firm investment and business cycle dynamics.

<sup>4</sup>Choe *et al.* (1993), and Korajczyk and Levy (2003) also study issuances over the business cycle. Both find that equity issuance is procyclical. Korajczyk and Levy (2003) report countercyclical debt issuance.

investment at the firm level in a model with constant investment opportunities. Our model confirms these effects in a calibrated, quantitative model with stochastic investment opportunities, and we document their empirical relevance.

We also use our model to estimate the cost of external finance in the US time series. Thus, our paper is most closely related to Jermann and Quadrini (forthcoming), with two key differences. First, Jermann and Quadrini (forthcoming) focus on the distinction between debt vs. equity in their estimation, and estimate a debt financing cost shock, whereas we do not distinguish between sources of external finance and instead incorporate information regarding how all external funds are used into our estimation strategy. Second, Jermann and Quadrini (forthcoming) use an assumed binding constraint to identify their shock. While we cannot solve our model for the cost of external finance shock in closed form, we think that the use of cross-sectional moments to identify a hidden aggregate state is a methodology with other potential uses.

Despite this renewed interest, the fact that financial constraints, or shocks originating in the financial sector, are important for either firm level investment, or business cycle dynamics, is not a foregone conclusion amongst economists. While Ivashina and Scharfstein (2010), Duchin *et al.* (2010), Campello *et al.* (2010), Matvos and Seru (2011), and Almeida *et al.* (2009) provide evidence that the financial crisis hindered external finance and investment activity at the firm level, Paravisini *et al.* (2011) find only small effects of credit supply shocks on trade. Moreover, Chari *et al.* (2008) argue that aggregate data do not support the occurrence of a credit crunch and question the appropriateness of government interventions aimed at improving access to external finance.<sup>5</sup>

Another striking empirical fact is that aggregate corporate investment closely tracks aggregate corporate internal funds.<sup>6</sup> Moreover, aggregate investment rarely exceeds internal funds. Interestingly, this observation has been used both to motivate theories of costly external finance, such as the pecking order (Myers (1984) and Donaldson (1961)), and conversely to argue that perhaps frictions between the household and corporate sector are unimportant for corporate investment (Chari *et al.* (2007)). Chari, *et. al.* do, however, acknowledge that reallocation of funds within the corporate sector, and frictions therein, may play a role.<sup>7</sup> We show using our calibrated, dynamic model, that costly external finance is consistent with aggregate shortfalls being rare. In our model, the corporate sector as a whole is rarely raising external finance,

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<sup>5</sup>Likewise, Chari *et al.* (2007) use business cycle accounting to argue that shocks to the cost of installing capital, or to the return on capital, are only of tertiary importance for explaining the US fluctuations output, investment, and employment. However, papers such as Justiniano *et al.* (2010), and Christiano *et al.* (2010), assert that such shocks explain a large fraction of business cycle fluctuations.

<sup>6</sup>See Eisfeldt and Rampini (2009).

<sup>7</sup>See also Shourideh and Zetlin-Jones (2012).

although the likelihood for individual firms raising external finance is more than an order of magnitude higher than the aggregate likelihood.

It is important to note that even if costly external finance is an important driver of investment over the business cycle, it does not necessarily follow that government policies aimed at lowering such costs in recessions are useful. Gomes *et al.* (2006) point out that the shadow cost of external finance is procyclical in a standard business cycle model with agency costs of external finance. Gomes *et al.* (2006) estimate an aggregate production based asset pricing model in which the stochastic discount factor varies with the default premium, and find that the estimated shadow cost of funds is procyclical.<sup>8</sup> This makes sense if the shocks which drive firms' demand for external funds are procyclical. In our model with investment in both liquid assets and physical capital, lowering the cost of external finance without affecting the relative returns to liquid and physical capital does not spur investment in physical capital since firms can instead save funds for when investment opportunities improve. That this may be empirically relevant was evident in the financial crisis when government subsidized funding was provided to banks, and banks responded by hoarding the funds instead of by making more new loans.

Our paper is also related to papers which develop dynamic models of corporate saving. The main difference is in focus; these papers are focused on understanding firm level dynamics or making inferences about firm level of financial constraints. In contrast, our paper, which is focused on understanding the dynamics and the effects of the aggregate component of the external finance fits between this literature and the macro finance literature which studies business cycles with financial frictions. Kim *et al.* (1998) develop a three date model and show that cash accumulation is increasing in the cost of external finance, the variance of future cash flows, and the return on future investment opportunities, but decreasing in the return differential between physical capital and cash.<sup>9</sup> Almeida *et al.* (2004) study the cash flow sensitivity of cash and empirically document a link between the propensity to save out of cash flow and financial constraints.<sup>10</sup> Riddick and Whited (2009) construct a fully dynamic model of corporate savings and emphasize the importance of uncertainty for determining corporate savings, and argue that in such a model, the propensity to save is not an accurate measure of financial constraints. Thus, the link between financial constraints and investment in financial assets is also unresolved. Our

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<sup>8</sup>A related finding in Chari *et al.* (2007) is that using business cycle accounting it actually appears that financial frictions improved during the great depression.

<sup>9</sup>For a model which instead focuses on the value of the flexibility of cash for adjusting net leverage, see Gamba and Triantis (2008).

<sup>10</sup>See also Faulkender and Wang (2006) for evidence that cash is more valuable when held by financially constrained firms. Harford *et al.* (2011) argue that firms save to insure against refinancing risk and document an inverse relationship between debt maturity and cash holdings which is stronger when credit market conditions are tighter.

two date model can be used to shed light on the controversy over the cash flow sensitivity of cash. A contemporaneous paper with a related focus to ours, but again directed at understanding firm level behavior, is Warusawitharana and Whited (2011), which uses simulated method of moments to show that equity misvaluation shocks can help explain firm level corporate issuance and savings policies. Note that because both Riddick and Whited (2009), and Warusawitharana and Whited (2011) are focused on firm level moments, the parameter estimates they form ignore any information in aggregate moments. However, our calibration focusing on aggregate moments is not too dissimilar, and supports the generality of the basic Riddick and Whited (2009) framework. Our paper focuses on understanding the role of costly external finance in aggregate issuance and savings waves using both aggregate and cross-sectional moments for external finance, liquidity accumulation, and investment, and hence is closest in spirit to quantitative macro-finance models of financial frictions and business cycles.

Finally, our paper is related to dynamic models of capital structure. The fact that firms tend to simultaneously raise external finance and accumulate liquidity is at odds with standard static pecking order intuition. Static pecking order theories based on Myers (1984) predict that firms will first draw down cash balances and only once these are exhausted will they seek external finance. Thus, such theories predict a counterfactually negative correlation between external finance and liquidity accumulation. Our dynamic model features a pecking order in the sense that internal funds are less costly than external funds, and generates the observed positive correlation between external finance and liquidity accumulation. This result is similar to the implications of the models in Hennessy and Whited (2005) and Strebulaev (2007) for the trade off theory of capital structure. Those papers show that data which appear to be inconsistent with static trade-off theories of capital structure can be generated by dynamic models in which firms' objectives are based precisely on the trade-off between the tax benefits and distress costs of debt.

### III. Stylized Facts

#### A. Data Description

Our main data set consists of annual firm level data from Compustat from 1980-2010. We focus on Compustat data since we are able to analyze firm level, as well as aggregate, facts. Thus, our sample selection criterion closely follows that in Covas and Den Haan (2011a). When matching the aggregate facts, we show the results obtained using Flow of Funds data are qualitatively similar. The Data Appendix gives a detailed description of the construction of our data.

We use firm level cash flow statements to track corporate flows. We define liquidity accumulation as changes in cash and cash equivalents.<sup>11</sup> We define net external finance raised as the negative of the sum of net flows to debt and net flows to equity. We define flows to debt as debt reduction plus changes in current debt plus interest paid, less debt issuances, and flows to equity as purchase of common stock plus dividends less sale of common stock. Following Covas and Den Haan (2011a), and Fama and French (2005), we also consider using the negative of the change in total liabilities as flows to debt and negative changes in book equity as flows to equity. We find similar results using these stock measures. We focus on the flow measures in the interest of brevity, and since our model does not feature issuances which are not truly “external” like those related to mergers or employee compensation which are emphasized in Fama and French (2005). Finally, we have also verified that the results are similar if we just focus on issuances of debt and equity, rather than the total net flows from these claim holders. We define investment (in physical capital) as capital expenditures. We do not include acquisitions in our investment measure. Firm level acquisitions are very lumpy, which can bias the correlations we compute. Including acquisitions does not change our aggregate results, since the aggregate series smooths out individual firm lumpiness.

When computing most aggregate and firm level moments, we normalize firm level variables by current total book assets. When computing aggregate correlations, we instead normalize by the lag of book assets, to avoid inducing spurious correlations. Book assets are slow moving and fairly acyclical and thus shouldn’t induce any trends in our data. Our results are robust to alternative normalizations, such as aggregate output or aggregate gross-value added from the corporate sector. We use the Hodrick and Prescott (1997) filter to remove any remaining series trends when computing aggregate correlations, since, for example cash holdings have trended upwards as a share of assets over our sample (Bates *et al.* (2009)). The filter ensures that the empirical series are stationary, which is consistent with the stationary model we study. Thus, our focus is on the business cycle dynamics of the cost of external finance.

As in Covas and Den Haan (2011a), our main analysis drops the top 10% of firms by asset size. There are several reasons to do this. First, the very largest firms present unique measurement problems. More of the investment for these firms falls under the accounting category “other investments”. These other investments are typically long term receivables to unconsolidated subsidiaries. Thus, a large firm may raise funds on behalf of a smaller subsidiary, which in turn

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<sup>11</sup>We do not use the balance sheet measure of cash since the stock measure is affected by acquisitions. Covas and Den Haan (2011a) instead remove firms involved in mergers which increase sales by more than 50%. We have checked that our findings are similar using stock measures and the non-merger sample. All non-reported robustness checks are available from the authors upon request.

may use the funds to build a new factory, or may store the funds as liquid assets. Since we are not able to measure these funds' ultimate use, we are not able to identify accumulated liquidity vs. physical investment, the main goal of this paper. Second, the largest firms tend to have a much larger share of foreign earnings. Cash accumulation for firms with large foreign earnings may be influenced by tax motives and repatriation timing. Third, as Covas and Den Haan (2011a) point out, external finance for the largest firms is not representative of the rest of the sample. They show in particular that one incidence of AT&T raising equity during a recession in 1983 has implications for the cyclicity of *aggregate* equity issuance. They advocate dropping the top firms because they have an unusually large influence on the aggregate series. Fourth, it is possible that the very largest firms face little or no financial constraints. Finally, we note that in the type of stationary model we study, the distribution of firm sizes will be much less skewed than that in the data. Although the model will generate the decreasing correlation between external finance and liquidity as firm size increases, aggregate model data will not be as heavily driven by the activities of a few large firms.

For the Flow of Funds data, we normalize each series by the HP filter implied trend in gross-value added of the corporate sector.<sup>12</sup> If we very narrowly define the accumulation of liquid assets as the net acquisition of financial assets minus trade receivables minus miscellaneous assets, the flow of funds data display a counterfactual decrease over time in this series for liquid assets held within the corporate sector.<sup>13</sup> Thus, the Flow of Funds data do not do a good job of identifying and classifying all corporate investment in marketable securities. There is a large, and growing, category “miscellaneous assets,” which contains both marketable and non-marketable assets. To account for this, we also include 1/3 of miscellaneous other assets as liquid.<sup>14</sup>

## B. Main Facts

We document two new stylized facts describing aggregate issuance and savings waves. First, the aggregate time series correlation between external finance raised and liquidity accumulation is strongly positive. For all but the top 10% of Compustat firms, the aggregate correlation is 0.60 and is statistically significant at the 5% level. Figure 2 plots cash flows to liquid assets vs. cash flows to external finance at this aggregated level and clearly illustrates our first stylized fact. This aggregate correlation is higher (0.74) if one conditions on firms that are currently raising external

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<sup>12</sup>Results using total GDP are similar.

<sup>13</sup>See Bates *et al.* (2009).

<sup>14</sup>The decision to use 1/3 of other miscellaneous assets was based on personal communication with staff at the Board of Governors. Their rough estimate using recent IRS data is that about 1/3 of miscellaneous other assets were marketable securities.

finance, so the positive aggregate correlation does not seem to be driven by some firms saving, and other firms issuing external finance. The aggregate correlation is also higher when one excludes more of the largest firms. For the top half of firms, the correlation between aggregated external finance raised and liquidity accumulated is 0.84. This is in contrast to conditioning on other measures of financial constraints, such as whether a firm pays no dividends, or has no credit rating, in which case we find correlations close to that for the larger sample (0.68 and 0.56 respectively). This could be due to the importance of fixed costs in accessing external financial markets, or it could be that size is simply a better proxy for financial constraints. Finally, we also find a positive correlation using flow of funds data. If we very narrowly define liquid assets as the net acquisition of financial assets minus trade receivables minus miscellaneous assets, we find a correlation between external finance and liquidity accumulation of 0.33. Including 1/3 of miscellaneous other assets as liquid helps align the flow of funds data with the fact that the net accumulation of liquid assets within the financial sector has been positive over recent history. and find a correlation of 0.38 which is statistically significant. Table I displays our main aggregate issuance and savings stylized facts. The Subsection A. and the Data Appendix contain details of our data and variable construction.

Table V displays the correlations between liquidity and investment with debt vs. equity separately. While we see that the correlation with liquidity accumulation is stronger for equity (0.69) than debt (0.16), both are positive. Conditional on firms raising external finance, we see both correlations increase to 0.77 and 0.33, respectively, and both are statistically significant. We also note that investment is more correlated with debt (0.60) than equity (-0.15). This fact has been pointed out by DeAngelo *et al.* (2010) who argue that debt might be used more frequently for investment. Also, we note that debt drives most of the variation in external finance, with a correlation with external finance of 0.77 vs 0.43 for equity. For parsimony, and to match our model, we focus on the overall correlation with external finance and abstract from debt vs equity. Studying total external finance allows us to focus on what is new in our work, namely the relationship between external finance and liquidity accumulation at the aggregate level.

The second main new fact that we document is that in the cross-section, firms are more likely to raise external finance and save the proceeds when the default spread is low, and when lending standards are less tight. This is consistent with the intuition we will illustrate theoretically that when financing costs are high, firms are unlikely to raise costly external finance only to save the proceeds in low-return, liquid, assets. At each date, we compute the cross-sectional correlation between aggregate net external finance raised and liquidity accumulation (each normalized by

lagged book assets), and construct a time series of this cross-sectional correlation, which we call  $xsrho_{i,e}$ . We then show that the correlation between  $xsrho_{i,e}$  and the negative of the Baa-Aaa default spread is 0.64. Similarly, the correlation between  $xsrho_{i,e}$  and the negative of the fraction of banks reporting tighter lending standards is 0.58. Both correlations are statistically significant at the 5% level. Figure 4 illustrates the strong relationship between  $xsrho_{i,e}$ , the default spread, and lending standards by plotting the time series for  $xsrho_{i,e}$  along with the negative of the default spread and lending standards. Although all the series are highly correlated, there is independent information in  $xsrho_{i,e}$ . For example, the high  $xsrho_{i,e}$  indicates a low cost of external finance in the boom of 1986, however the default spread was not particularly low then. The tech bust of 2001 is also more apparent in the drop in  $xsrho_{i,e}$  than it is in the relatively small increase in the default spread, potentially suggesting that this was largely an increase in the cost of equity issuance not captured by the default spread. Finally, we show that, by contrast,  $xsrho_{i,e}$  is less correlated with TFP (0.48). These facts together motivate our estimation exercise in section VI. Building on the idea of combining the information in both firms' *sources* and their *uses* of funds to learn about the cost of external finance, we use firms' financing and liquidity accumulation decisions in the cross-section to make inferences about the aggregate cost.

In sum, we present two new stylized facts, namely the strong positive correlation between aggregate issuance and savings, and the strong positive relation between issuance and savings in the cross-section and traditional measures of the cost of external finance. In the following sections, we explore the role of shocks to the cost of external finance in generating these and related stylized facts about the joint dynamics of internal and external finance, and provide our estimate of the time series of the cost of external finance in the US time series 1980-2010.

## IV. Model

### A. Two Date Model

We present a two date model of investment, external finance, and savings and analytically characterize the relationship between the cost of external finance, the amount of external finance raised, and investment in capital and liquid assets. In particular, we show how the optimality conditions for financing, investment, and liquidity accumulation in the two date model motivate the use of the cross sectional correlation between liquidity accumulation and external finance in identifying the level of the cost of external finance.

We study a firm which maximizes the present value of cash flows over two dates, zero and

one. For simplicity, we set the interest rate to zero. At date zero, the firm receives an endowment of liquid assets,  $l$ , and internal funds from operating cash flows,  $y$ , and chooses how much to invest in both physical capital ( $i_k$ ) and liquid assets ( $i_l$ ). At date one, the firm receives cash flows from its productive physical capital and from its liquid assets. Liquid assets produce  $r_l > 1$  at date one. We motivate  $r_l$  larger than one despite a unit discount rate by considering that liquid assets may provide a hedge for investment opportunities at date one, as in the dynamic model. Physical capital produces output according to  $zi_k^\theta$  and does not depreciate. We define  $e = y - i_l - i_k$  as internal funds minus investment in physical capital and liquid assets. If  $e < 0$ , the firm is raising external finance, and pays a cost  $\xi \frac{1}{2}(e)^2$ , where  $\xi$  is interpreted as the current “level” of the cost of external finance.

The firm’s objective over date zero and date one cash flows, respectively, is then:

$$\begin{aligned} & \max_{i_k, i_l} \left\{ \left[ e - \mathbb{I}_{(e < 0)} \xi \left( \frac{1}{2} e^2 \right) \right] + \left[ (i_k + zi_k^\theta) + (l + i_l)r_l \right] \right\} \\ \text{s.t. } & e = y - i_l - i_k \\ & i_l \geq -l. \end{aligned}$$

We focus on the case in which the constraint  $i_l \geq -l$  is not binding. In this case, the first order condition with respect to investment in liquid assets,  $i_l$ , is:

$$r_l - 1 = -\xi (y - i_l - i_k).$$

The first order condition with respect to capital investment,  $i_k$ , is:

$$\theta z (i_k)^{\theta-1} = -\xi (y - i_l - i_k).$$

Intuitively, the first order conditions equate the marginal product of capital, the return on liquid assets, and the marginal cost of raising external finance. These first order conditions imply the

following optimal financing and investment policies:

$$\begin{aligned}
i_l &= y + \frac{r_l - 1}{\xi} - \left( \frac{r_l - 1}{\theta z} \right)^{\frac{1}{\theta - 1}} \\
i_k &= \left( \frac{r_l - 1}{\theta z} \right)^{\frac{1}{\theta - 1}} \\
-e &= \frac{r_l - 1}{\xi}.
\end{aligned}$$

Thus, the amount of external finance raised, and the amount of liquidity accumulated are both decreasing in  $\xi$ . By contrast, capital investment is independent of  $\xi$  and is instead pinned down by productivity and the other return and production function parameters. Formally, we have the following comparative statics:

$$\begin{aligned}
\frac{\partial i_k}{\partial \xi} &= 0 \\
-\frac{\partial e}{\partial \xi} &= \frac{1 - r_l}{\xi^2} \\
\frac{\partial i_l}{\partial \xi} &= \frac{1 - r_l}{\xi^2} \\
-\frac{\partial i_l}{\partial e} &= -\frac{\partial i_l}{\partial \xi} \frac{\partial \xi}{\partial e} = 1.
\end{aligned}$$

At the optimum, capital investment is independent of  $\xi$ . On the other hand, both external finance and liquidity accumulation have the same, negative, partial derivative with respect to the level of the cost of external finance,  $\xi$ . Moreover, at the optimum, the partial derivative of liquidity accumulation with respect to external finance raised is equal to one. Thus, as long as  $i_l > -l$ , any additional dollar will increase liquidity accumulation, and, at the margin, liquidity accumulation and external finance will increase one for one if  $\xi$  decreases since all additional funds raised will be used to augment cash balances.

Figure 1 illustrates the firm's investment and financing decisions graphically by plotting the net marginal benefit of capital investment and investment in liquid assets, along with the marginal cost of external finance. The graph depicts the case in which both types of investment are strictly positive. First, the firm uses its internal funds,  $y$ , to invest in physical capital. As the firm invests more, the marginal product declines. When the firm runs out of internal funds for investment, it raises external funds, and the marginal cost of external funds increases linearly in the amount of funds raised. Once the firm invests enough such that the

marginal product of capital declines to the level of the marginal product of liquid assets, which is constant, the firm begins to invest in liquid assets. The firm then raises external finance and invests in liquid assets until the marginal cost of external funds rises linearly to equal the marginal return on liquid assets. Clearly, there will always be some investment in physical capital. Then, if  $z$  and or  $\xi$  are low enough, there will also be positive liquidity accumulation.

The figure clearly illustrates that investment in physical capital is independent of the cost of external finance in the region with positive liquidity accumulation; as one changes the slope of  $\xi(-e)$  in this region, only  $i_l$  is affected. Thus, as long as  $z$  or  $\xi$  are low enough to induce positive liquidity accumulation then any change in  $\xi$ , implemented through government policies, for example, will not change investment in physical capital  $i_k$ , but will only affect investment in liquid assets  $i_l$ . Similarly, the figure also shows that the cutoff  $\xi$  below which there is positive liquidity accumulation is decreasing in  $z$ . Specifically, for positive liquidity accumulation, we need the intersection of the two curves  $\theta z (i_k)^{\theta-1}$  (the marginal product of capital) and  $\xi(-e)$  (the marginal cost of external finance) to occur below the line  $r_l$ . Lowering  $\xi$  has the effect of lowering the point at which these two curves intersect, and hence increases the amount of liquidity accumulation but does not change the level of investment. Lowering  $\xi$  only has an impact on investment in cases where there is zero liquidity accumulation and the marginal product of capital is high relative to the return on liquid assets:  $\theta z (i_k)^{\theta-1} = \xi(-e) > r_l - 1$ .

Note also that in this simple model, the amount of external finance is independent of internal funds. This can be seen from the optimality conditions above, or in the figure for the case that  $i_l > 0$ . A higher  $y$  shifts the  $\xi(-e)$  line to the right, but the amount of external finance will still be pinned down by setting  $\xi(-e)$  equal to  $r_l - 1$ .

The comparative statics show that external finance and liquidity accumulation move one for one together at the margin if the firm is raising external finance and has positive liquidity accumulation. We further motivate the use of the correlation between external finance and liquidity accumulation in the cross-section to uncover the aggregate level of the cost of external finance by examining this correlation directly in the two date economy. For  $i_l > -l$ , the cross-section correlation between liquidity accumulation and external finance is given by:

$$xsrho_{i_l,e} = corr \left( \frac{r_{l,i} - 1}{\xi}, y_i + \frac{r_{l,i} - 1}{\xi} - \left( \frac{r_{l,i}}{\theta z_i} \right)^{\frac{1}{\theta-1}} \right),$$

where, consistent with our dynamic model, internal funds from operating cash flows  $y_i$ , the physical plus the shadow return to liquid balances  $r_l$ , and the productivity of physical capital  $z$ , vary across firms. All else equal, when the cost of external finance,  $\xi$ , is low, external finance and liquidity accumulation will both be dominated by the  $\frac{r_l-1}{\xi}$  term, and as a result these two

flows will be more correlated. This intuition is corroborated empirically by the high correlation between  $xsrho_{i_l,e}$  and the Baa-Aaa default spread (0.64) and the fraction of banks reporting tighter lending standards (0.58) in the data. It is also supported by the high correlation between  $xsrho_{i_l,e}$  and  $\xi$  in our calibrated solution to the infinite horizon model in section V.A.

It is clear from the comparative statics above that conditional on being away from a corner solution, policies which lower  $\xi$  in this model result in no change in investment, and only a potential change in liquidity accumulation. In this context, it is also interesting to note the effect of a change in productivity  $z$  on liquidity accumulation. All else equal,  $i_l$  is decreasing in  $z$ . As capital becomes more productive, holding  $\xi$  constant, the firm will exhaust its desired demand for external finance at a lower level of liquidity accumulation.<sup>15</sup> Moreover, the cutoff  $\xi$  below which liquidity accumulation is positive is decreasing in  $z$ . This means that the lower is  $z$ , the more likely it is that a change in  $\xi$  will mainly affect liquidity accumulation. These comparative statics are consistent with recent events in which the US government's efforts to reduce financing costs have been met mainly with increased savings by firms.

Finally, we construct the investment returns for physical capital and liquid assets in this simple model, in order to build intuition for the dynamic model, for which we provide analogous returns. Each return is the physical return times an external finance discount factor. The external finance discount factor is the ratio of a firm's marginal value of funds tomorrow relative to today. In this model, all else equal, the external finance discount factor is high when the firm is raising a lot of external finance ( $e \ll 0$ ), and this high cost reduces the return to investment and liquidity accumulation. Specifically:

$$R_k = \frac{\theta z i_k^{\theta-1} + 1}{1} \frac{1}{1 - \xi e},$$

where again we can interpret the first term as the physical return and the second term as the external finance discount factor. For the return on liquidity accumulation, we have:

$$R_l = \frac{r_l}{1} \frac{1}{1 - \xi e}.$$

As discussed, if liquidity accumulation is positive, then  $\frac{r_l}{1 - \xi e}$  is also the return to an additional dollar of external finance, since that dollar will be invested in liquid assets. At the optimum, all returns are equated, and equal to one since there is no discounting. Starting at this optimum, consider perturbing  $\xi$  to  $\xi - \epsilon$ . Quantities must adjust so that returns equate to one under the new cost of external finance. The lower  $\xi$  lowers the marginal cost of an additional dollar and so

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<sup>15</sup>Thus, *ceteris paribus*, in this simple static model, higher aggregate productivity implies lower liquidity accumulation, but, as we will show in section V.A. this is not necessarily the case in the dynamic model.

the firm will raise more external finance. The firm will also invest in capital and liquidity until those returns equal the new marginal cost. Due to the concavity of the production function, the marginal product of capital declines with greater investment, but the marginal benefit of liquidity accumulation does not since its return is linear. Thus, liquidity accumulation will respond more strongly to the decrease in  $\xi$ . In fact, we showed that at the margin around the optimum, liquidity accumulation will increase one for one with external finance.

## B. Dynamic Model

The economy consists of a continuum of heterogeneous, risk-neutral firms, which differ in terms of their current idiosyncratic productivity shock, and their current stocks of physical capital and liquid assets. The model is partial equilibrium. Firms face a common aggregate productivity shock, and take the exogenous risk-free rate as given. Each firm chooses its investment in physical and liquid assets in order to maximize the discounted value of its net payouts, subject to financing and investment adjustment costs.

Firms produce output or cash flows using physical capital  $k$  according to:

$$y = zk^\theta$$

where  $z$  is the level of the firm's productivity and  $\theta \in (0, 1)$ . Capital evolves according to the standard law of motion:

$$k' = (1 - \delta)k + i_k,$$

where  $i_k$  is investment and  $\delta \in (0, 1)$  is the depreciation rate. Investment in physical capital is subject to adjustment costs  $\phi_i(i_k, k)$  given by:

$$\phi_i(i_k, k) = ck\Phi_i + \frac{a}{2} \left( \frac{i_k}{k} \right)^2 k.$$

Thus, the investment adjustment cost has both a fixed and convex component, governed by the parameters  $c > 0$  and  $a > 0$ , respectively. We specify that,  $\Phi_i = 0$  whenever  $i_k = \delta$  and  $\Phi_i = 1$  otherwise. Liquid assets  $l$  evolve according to:

$$l' = (l + i_l)((1 + r(1 - \tau))$$

where  $i_l$  is investment in liquid assets and  $r$  is the risk free rate. Following the recent corporate finance literature on firm dynamics, corporate payouts are motivated by a tax wedge,  $\tau > 0$ ,

as in Riddick and Whited (2009). We note, however, that in practice payout policy is likely to also be driven by agency and asymmetric information considerations, as in Eisfeldt and Rampini (2009).

Pre-financing cost, after tax, net payouts are then internal cash flows minus investment in physical capital and liquidity accumulation, less investment adjustment costs. We have:

$$e \equiv zk^\theta(1 - \tau) - i_l - i_k - \phi_i(i_k, k), \quad (1)$$

If  $e > 0$  the firm is paying out funds and if  $e < 0$  the firm is raising external finance. Intuitively, the firm raises external finance if after tax operating profits do not cover the firms' total investment in physical and liquid assets, net of physical adjustment costs.

Firms maximize this net payout, less their financing costs. Following Gomes (2001), we parameterize the cost of external finance exogenously with a fixed and linear cost. Following Hennessy and Whited (2005), Hennessy and Whited (2007), Riddick and Whited (2009), and Eisfeldt and Rampini (2009), we also include a quadratic component. We assume the following functional form for the cost of external finance

$$\phi_e(e, \xi) = \Phi_e(-\lambda_0 + \xi(\lambda_1 e - \frac{1}{2}\lambda_2 e^2)),$$

where  $\lambda_0, \lambda_1, \lambda_2 > 0$ ,  $\Phi_e$  is an indicator that takes the value 1 when  $e < 0$  and 0 otherwise, and  $\xi > 0$  denotes aggregate state for external financing costs. Our model is thus standard in the dynamic corporate finance literature, except for our focus on aggregate outcomes, and the addition of the stochastic “level” of the cost of external finance,  $\xi$ . To bring the model to the data, we specify the dynamics of the cost exogenously. Microfoundations of a time varying cost of external finance have been considered in the context of an agency friction that varies over time, along the lines of Bernanke and Gertler (1989) and Carlstrom and Fuerst (1997), as collateral constraints as in Kiyotaki and Moore (1997), or as a time-varying adverse selection problem as in and Eisfeldt (2004), or Kurlat (2011). Note that the fixed cost of external finance is constant, a feature that we will show helps bring the model in line with the data.

In recursive form, the firm's problem is:

$$V(k, l, z, \xi) = \max_{k', l'} \left( e + \phi_{e, \xi}(e(k, k', l, l', z), \xi) + \frac{1}{1+r} E_t [V(k', l', z', \xi')] \right) \quad (2)$$

where we denote the firm's value function by  $V$ . Each firm's state is given by their individual stocks of physical capital  $k$ , and liquid assets  $l$ , along with their productivity  $z$  and the current state of external financing costs  $\xi$ .

Each firm’s productivity  $z$  is the product of an idiosyncratic shock  $z_i$ , and an aggregate shock  $z_{agg}$ . The aggregate productivity level, and each idiosyncratic productivity level, follow AR(1) processes with identical persistence parameters. However, we allow for the idiosyncratic and aggregate processes to have different volatilities. We discuss these choices further when we detail our calibration. These assumptions allow us to construct each firm’s productivity level as follows:

$$z = z_i z_{agg} \tag{3}$$

$$\ln(z'_i) = \rho \ln(z_i) + \epsilon'_i \tag{4}$$

$$\ln(z'_{agg}) = \rho \ln(z_{agg}) + \epsilon'_{agg} \tag{5}$$

$$\ln(z) = \rho \ln(z) + \epsilon'_i + \epsilon'_{agg}. \tag{6}$$

Finally, we specify that the aggregate state of external financing costs,  $\xi$ , follows an AR(1) in logs.

$$\ln(\xi') = c + \gamma \ln(\xi) + \eta' \tag{7}$$

We choose the value  $c$  so that the cost,  $\xi$ , is on average 1.

## C. Investment Returns

We solve our model numerically, however we describe the basic intuition for firm investment and liquidity accumulation policies by describing the investment returns to each. We show that each return is equal to its “physical return” times an external finance discount factor. That is, physical capital and liquid assets each have a physical return, but the value of this physical return varies with how financially constrained the firm is. If the firm would otherwise be accessing funds through costly issuances, it places a particularly high value on funds generated internally through production or savings. In the model with only productivity shocks, these shocks alone drive the returns to capital and liquidity accumulation. With shocks to  $\xi$ , we show that, away from the optimum, the returns to liquidity accumulation in particular vary with the cost of external finance, which supports our use of information in the cross-sectional correlation between external finance and liquidity accumulation to uncover the aggregate cost of external finance.

To construct investment returns using firms’ marginal rates of transformation, we combine firms’ first order conditions with their envelope conditions as in Cochrane (1991) and Cochrane (1996). Thus, in what follows, we analyze the solution for firms with interior investment and financing policies at each date. Due to the fixed costs of investment and external finance, in

general the solution will exhibit regions of action and inaction.

The first order condition with respect to  $k'$  is

$$\left(1 + a \frac{i_k}{k}\right) (1 + \Phi_e \xi (\lambda_1 - \lambda_2 e)) = \frac{1}{1+r} E_t \left( \frac{\partial V'}{\partial k'} \right)$$

Using the envelope condition  $\frac{\partial V}{\partial k} = \frac{\partial e}{\partial k} (1 + \phi'_e(e))$ , we have

$$\begin{aligned} & \left(1 + a \left(\frac{i_k}{k}\right)\right) (1 + \Phi_e \xi (\lambda_1 - \lambda_2 e)) \\ &= \frac{1}{1+r} E_t \left( \left( (1 - \tau) \theta z' k'^{\theta-1} + (1 - \delta) - c \Phi_i + a \left(\frac{i'_k}{k'}\right) \left(\frac{1}{2} \frac{i'_k}{k'} + 1 - \delta\right) \right) (1 + \Phi'_e \xi' (\lambda_1 - \lambda_2 e')) \right) \end{aligned} \quad (8)$$

Rearranging, we have the familiar pricing equation for a risk-neutral investor

$$1 = \frac{1}{1+r} E_t (R_k) \quad (9)$$

where  $R_k$ , the return on capital, is given by

$$R_k = \frac{\left( (1 - \tau) \theta z' k'^{\theta-1} + (1 - \delta) \left(1 + a \left(\frac{i'_k}{k'}\right)\right) - c \Phi'_i + a \left(\frac{i'_k}{k'}\right) \left(\frac{1}{2} \frac{i'_k}{k'}\right) \right) (1 + \Phi'_e \xi' (\lambda_1 - \lambda_2 e'))}{\left(1 + a \frac{i_k}{k}\right) (1 + \Phi_e \xi (\lambda_1 - \lambda_2 e))} \quad (10)$$

We can understand the return on capital by thinking of the marginal benefit of increasing capital one unit today relative to the marginal cost. Define  $R_k = (\text{marginal benefit} / \text{marginal cost})$  as the return from this strategy. The marginal benefit to increasing capital by one unit is: a marginal increase in output, the value of the additional depreciated capital, and a lower convex cost less the higher fixed costs of investment in the following period. This is the physical return to capital. The total return is the physical return multiplied by how much a dollar will be worth inside the firm tomorrow relative to today, namely, the marginal cost of funds in the following period relative to the current period. Thus, additional capital is more valuable if internal funds are expected to be scarce, i.e. if the firm will be raising funds externally. The physical marginal cost of increasing capital by one unit is a dollar, plus adjustment costs. Again, the total marginal cost is the physical cost multiplied by the shadow value of internal funds today. Since the conditional expected return on capital in equation (9) is a constant, quantities must adjust for this asset pricing Euler equation to hold. The return to capital is decreasing in investment and increasing in productivity. Thus, the firm will increase investment in high productivity states until this optimality condition holds.

The first order condition with respect to  $l$  is

$$\frac{1}{1+r(1-\tau)}(1+\Phi_e\xi(\lambda_1-\lambda_2e)) = \frac{1}{1+r}E_t\left(\frac{\partial V'}{\partial l'}\right) \quad (11)$$

Using the envelope condition for  $l$ ,  $\frac{\partial V}{\partial l} = 1 + \xi\Phi_e(\lambda_1 - \lambda_2e)$ , and rearranging yields

$$1 = \frac{1}{1+r}E_t(R_l) \quad (12)$$

where  $R_l$  the return on liquid assets, is given by

$$R_l = (1+r(1-\tau))\frac{1+\Phi'_e\xi'(\lambda_1-\lambda_2e')}{1+\Phi_e\xi(\lambda_1-\lambda_2e)} \quad (13)$$

The return on liquid assets is made up of two components. The first is simply the risk-free rate earned by liquid assets (the risk-free rate less any taxes paid). The second piece gives the marginal value of a dollar of internal funds tomorrow versus today. The return on savings will be high when a dollar of internal funds is more valuable tomorrow than it is today.

It is convenient to define the *external finance discount factor* that governs firms' state-pricing as follows

$$\mathcal{F} = \frac{1+\Phi'_e\xi'(\lambda_1-\lambda_2e')}{1+\Phi_e\xi(\lambda_1-\lambda_2e)} \quad (14)$$

Intuitively, the discount factor is the ratio of a firm's marginal value of funds tomorrow versus today. Assets that pay off when the firm is raising costly external finance are more valuable since they provide internal funds. Notice that  $\Phi_e(\lambda_1 - \lambda_2e) > 0$  if and only if  $e < 0$ . When the firm is not raising external finance at either date, the marginal value of a dollar inside the firm is the same as it is outside the firm. In contrast, when a firm is currently raising more external finance, the marginal value of a dollar inside the firm is greater than one and is increasing in the amount of external finance raised. A similar effect would appear in a model with a constraint on funds raised; the marginal value of a dollar would increase with the tightness of that constraint. The external finance discount factor implies that although the firm is risk neutral, it can behave as if it is risk averse. Define  $\widehat{R}_l, \widehat{R}_k$ , as the returns to capital and liquidity without external financing costs (the neoclassical case). Then, we can write the firm's investment return moments as

$$1 = \frac{1}{1+r}E_t\left(\mathcal{F}\widehat{R}_l\right) \quad (15)$$

$$1 = \frac{1}{1+r}E_t\left(\mathcal{F}\widehat{R}_k\right) \quad (16)$$

showing that indeed  $\mathcal{F}$  acts as a type of external finance induced stochastic discount factor.

Looking at 10 and 13, one can see that while the return to physical capital depends directly on TFP shocks, the return to liquidity accumulation varies directly with  $\xi$  only (but depends on TFP through  $e$  and the shadow value of funds at different dates.) Intuitively, both external finance raised and investment in liquid assets are more sensitive to the cost of external finance than investment in capital is. One can see this by considering the following perturbation argument: At the optimum, all returns are equated, and equal to  $(1 + r)$ . Starting at the optimal policies at date  $t$  for a firm with positive investment and liquidity accumulation, consider perturbing  $\xi$  to  $\xi - \epsilon$ . This lowers the marginal cost of an additional dollar and the firm will raise more external finance. The firm will also invest in capital and liquidity until those returns equal the new marginal cost. Due to the concavity of the production function, the marginal product of capital declines with greater investment. The true return on liquidity accumulation is likely to be concave, due to the effect of the hedging benefits which are tied to the value of funds used in production on the external finance discount factor. Clearly, however, the return to liquidity accumulation is less concave than that for capital since the return to liquid assets is the product of a linear function and the external financing discount factor, whereas the return to capital is the product of a concave function and the external finance discount factor. Thus, the investment returns indicate that liquidity accumulation should respond more strongly to the decrease in  $\xi$ , just as in the two date model.

## V. Calibrated Solution

### A. Numerical Methods and Calibration

We calibrate our model and solve it using standard discrete state space dynamic programming techniques. Specifically, we use the value function iteration method in Ljungqvist and Sargent (2004). Given the policy functions implied by our model solution, along with values for the model state variables and the stochastic processes for the exogenous states, we simulate a panel of firms, and analyze the data from that panel as in Gomes (2001).

To discretize the state space, we approximate the realization of the productivity and stochastic cost of external finance shocks using standard Gauss-Hermite quadrature techniques (see Tauchen and Hussey (1991)). There are six productivity states, which govern both idiosyncratic and aggregate productivity, and two aggregate states for  $\xi$ . For capital and liquid assets, we choose a large enough grid such that the stationary probabilities of being at the upper bound of the grid are negligible, something we verify ex-post.

Table II displays our calibration and compares our parameters choices to those in the literature. In order to further motivate our model with a stochastic cost of external finance, we provide a comparison “baseline” model with constant costs of external finance. The main purpose of this model is to demonstrate the dynamic, intertemporal effects of time invariant investment and financial costs in an aggregate model with productivity shocks alone. Calibration for the full model then appears in the column labeled “SC Model”. The second to last column offers a comparison to the parameters in Riddick and Whited (2009), (RW). Since our focus is on aggregate moments, where available we also report the parameters used in the real business cycle (RBC) literature as reported in Cooley and Prescott (1995).

We begin by describing the calibration of parameters common to both models. For the tax rate, we choose 10%. In our model, liquid assets are only accumulated in order to ultimately hedge investment opportunities in physical capital. Moreover, firms can simply over-accumulate physical capital and hedge via the additional cash flows that capital produces. Thus, if the tax rate is too high, firms do not accumulate any liquidity. In practice, most firms do not pay the 34% statutory rate. Moreover, we do not include a major benefit of cash in practice, which is that cash insures against costs of financial distress. Similarly, our model firms do not experience shortfalls from operating or financial leverage as in Eisfeldt and Rampini (2009). We use a standard RBC value for depreciation, 8%. For the production function curvature parameter, we specify 0.65, which is consistent with evidence in Cooper and Haltiwanger (2006). Higher curvature parameters, i.e. production functions closer to linear, imply too large investment volatilities and disinvestment frequencies. We calibrate the persistence of both idiosyncratic and aggregate productivity shocks to be 0.66, which allows us to conserve on one state variable since firms only care about total productivity in our partial equilibrium model. This value is equal to that used by RW for the firm level. Khan and Thomas (2008), page 407, contains a detailed discussion of the disagreement in the literature about this parameter, however based on our reading 0.66 is a good modal value. Moreover, we also found the average industry level persistence in the data from Basu *et al.* (2006) to be close to this value (0.65). Finally, using two trend breaks, as advocated in Fernald (2007), we find an aggregate persistence of about 0.62 using the data from Fernald (2009). We set the total volatility of firm level productivity equal to the value in RW, which is also near the value used by Khan and Thomas (2011). We then specify that aggregate volatility is about one fourth of total volatility as in Khan and Thomas (2008) and Cooper and Haltiwanger (2006). We set the risk free rate to 0.04 as in the RBC literature, and in RW.

Finally, we choose the parameters governing the cost of external finance ( $\lambda_0, \lambda_1$ , and  $\lambda_2$ ),

and the investment adjustment costs ( $a$  and  $c$ ). The main moment we target is the aggregate correlation between liquidity accumulation and external finance of 0.60. Specific parameter values vary between the baseline and the SC model.

For the baseline model, we focus on the choice of two key parameters, the fixed cost of external finance,  $\lambda_0$ , and the convex cost of investment adjustment,  $a$ . The baseline model generates a positive correlation between external finance and liquidity accumulation through the following intertemporal effects of persistent productivity with physical and financial adjustment costs: When firms receive a positive productivity shock, they would like to invest in the current and future periods. A relatively high fixed cost leads external finance to be “lumpy”, and firms raise more external finance than they currently need. They save some of the issuance proceeds as liquid assets in order to reduce convex adjustment costs by smoothing investment over time. Thus, the baseline model effectively employs higher values for the fixed cost of external finance and the quadratic costs of investment adjustment than used by RW.<sup>16</sup> We choose the quadratic adjustment cost,  $a$ , to match the first order autocorrelation of investment in the aggregated Compustat data of 0.38. Note that this persistence is fairly high, consistent with the evidence in Eberly *et al.* (forthcoming), which justifies a relatively important convex component of investment adjustment costs. We set the fixed investment cost  $c$  to a small value such that investment adjustment costs are not too high. The average cost of investment is less than 1% of investment dollars spent. Next, we choose the fixed cost of external finance,  $\lambda_0$ , to generate  $\rho(i_t, -e) = 0.60$ , and set the linear and quadratic cost parameters  $\lambda_1$  and  $\lambda_2$  to small values such that external finance costs are not too high. Our baseline calibration implies an average percentage issuance cost of 1.7%, which is well within empirical estimates.<sup>17</sup> Relative to RW, our calibration features a higher fixed cost of raising external funds and lower linear and quadratic components. The higher fixed cost is consistent with the importance of the variation in the extensive margin of external finance over the business cycle, as well as with the evidence in Bazdresch (2005) and Cummins and Nyman (2004) which emphasize the importance of lumpy external finance. Figure 6 plots the time series variation in the percent of firms raising external finance, along with GDP growth, and shows the importance of the extensive margin of external finance in explaining aggregate variation over the business cycle.

The SC model requires us to calibrate the stochastic process for  $\xi$ . We choose the persistence

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<sup>16</sup>Note that the higher curvature of firms’ production function in our model implies that firms will be smaller than in RW, and thus smaller values for cost parameters can imply higher costs relative to firm size. RW uses the estimates from Hennessy and Whited (2007) for the costs of external finance, however they do not incorporate aggregate moments as we do. In general, different moments lead to different parameter values.

<sup>17</sup>For example, Asquith and Mullins (1986) find that abnormal stock returns around secondary equity offerings are about 3%.

of the stochastic cost to match the persistence of the annual Baa-Aaa default spread over our sample period. We assume the stochastic cost is uncorrelated with the aggregate productivity shock, an assumption consistent with our empirical estimates. The empirical correlation between innovations in the default spread and TFP shocks is -0.2 and is not statistically significantly different from zero. Similarly, the empirical correlation between innovations to the expected default adjusted excess bond spread from Gilchrist and Zakrajsek (2012) and TFP shocks is -0.1 and is again not statistically significantly different from zero. The autocorrelation of innovations in this series based on annualized quarterly data is 0.35, so this is also consistent with our persistence parameter for  $\xi$  of 0.4. We choose the volatility of the  $\xi$  shock to approximately match the standard deviation of external finance raised, 0.03. We also check that we do well on this at the firm level, and match the firm level standard deviation of 0.17. Finally, given this volatility, we choose the mean of the  $\xi$  shock to generate an average credit shock ( $\exp\left(\mu - \frac{\sigma^2}{2}\right)$ ) of one. This is without loss of generality, but makes the cost parameters  $\lambda_1, \lambda_2$  more easily comparable to the other models.

For the parameters governing the cost of external finance, and the investment adjustment costs in the SC model, we again begin by setting the quadratic adjustment cost,  $a$ , to approximately match the first order autocorrelation of investment in the aggregated Compustat data of 0.38. We then choose the fixed cost of external finance,  $\lambda_0$  in order to match  $\rho(i_t, -e) = 0.60$ . Finally, we again set  $\lambda_1, \lambda_2$ , and  $c$  to small values such that average issuance and investment adjustment costs are small, and so that external finance has the empirically observed volatility given our calibration of  $\xi$ . For the SC model, the average issuance cost is 3% of proceeds. This is higher than our baseline model, but is smaller than what is suggested by evidence in Hennessy and Whited (2007) who estimate that firms face an issuance cost of 8.3% on the first million dollars raised. Average investment adjustment costs are smaller than in the baseline model.

## B. Aggregate Analysis

We begin with the aggregate implications of our model, since we are mainly interested in aggregate issuance and savings waves. With our numerical solutions in hand, we can construct a panel of firm data from each model. Specifically, we simulate 1,000 idiosyncratic productivity processes, 1 aggregate productivity process, and 1 aggregate stochastic cost process following the persistence and volatility given in Table II. We then create 1,000 total firm productivity shocks by summing each firm specific and aggregate productivity series and taking the exponential. We simulate 600 years of data, throwing away the first 100 years to avoid any initial dependencies. We then aggregate across firms to form aggregate corporate flows, analogous to our procedure

in Compustat.

Table III presents a comparison of aggregate issuance, savings, and investment moments in the data and in the model. We begin with the results from the baseline model, in order to illustrate that even if the specified cost of external finance is constant, in a dynamic model with rich financial and physical capital adjustment costs it is natural to observe waves of aggregate issuance and savings. This is in contrast to the intuition from a static pecking order model. As discussed in section V.A., this model features persistent investment opportunities, fixed costs of external finance, and convex costs of investment adjustment. With these three ingredients, when a positive productivity shock arrives, firms issue more external finance than they need in order to avoid paying the fixed cost more than once. Firms also save a fraction of issuance proceeds in order to smooth investment over time, and to avoid paying high convex costs of investment adjustment.

This baseline model with constant costs of external finance in fact generates aggregate issuance and savings waves that are fairly empirically realistic. The correlation between external finance and liquidity accumulation matches the empirical correlation of 0.60. To compare the output from our model to the actual US time series 1980-2010 graphically, we perform an additional simulation using the actual TFP shock realizations from Fernald (2009) for our aggregate shock. This way, we can examine the performance of the baseline model over specific US episodes, such as the recent financial crisis, the recession of the early 1980s, and the tech boom and bust. For the simulation using actual TFP shock realizations, we use a TFP series which consists of 500 years of calibrated shocks, plus the actual shocks realized for the 60 years 1950-2010. Then, we discard all simulated data except that corresponding to the last 30 years, 1980-2010. We plot the aggregate flows to liquid assets against flows to external finance in the data and the model using the empirical TFP shocks from 1980-2010 in figures 2 and 3, respectively. Comparing the two figures, one can see that the early 1980s recession is associated with relatively low external finance and liquidity accumulation both in actual and simulated data. The mid 1980s was a period of relatively high productivity and the data and model show relatively high external finance and liquidity accumulation. The early 1990s are recessionary, and again financing and liquidity accumulation are low. However, in the mid to late 1990s, the model predicts less external finance and liquidity accumulation than were observed empirically. Similarly, the simulated data do not capture the strong decline in external finance and liquidity accumulation after the tech bust in 2001. However, the simulated data do seem to capture the financing and savings wave observed in the mid 2000s, as well as the observed decline during the financial crisis. Thus, compared to the actual data, the baseline model using actual TFP matches the time series for liquidity

accumulation and external finance fairly well. However, the baseline model with productivity shocks alone seems to miss or underestimate the depth of the early 1990s recession, and the magnitude of the tech boom and bust.

Table III also shows that, in addition to matching the empirical correlation between external finance and liquidity accumulation, the baseline model’s moments are close to their empirical counterparts for the standard deviation of liquidity accumulation, expected investment and the standard deviation of investment, the serial correlation of investment, and the standard deviation of external finance. The baseline model also matches the fact that the aggregate shortfalls, i.e. years when the corporate sector is a net receiver of funds, are rare. The probability of an aggregate shortfall is 5% in the data, and 0.4% in the baseline model. Finally, the baseline model also replicates the relationship between firm size and the correlation between external finance and liquidity accumulation, as shown in table IV. This is because the fixed cost of external finance is relatively more important for small firms.

The baseline model performs surprisingly well for its simplicity. However, this model fails to match what we argue are key moments regarding aggregate issuance and savings waves. First, this model fails on business cycle correlations; it implies too high of a correlation between liquidity accumulation, external finance, and capital expenditures and the single aggregate shock, TFP. This is apparent in the third panel of table III, which displays the correlation between these flows and GDP. Thus, the model displays “stochastic singularity”. Clearly, adding another shock should reduce these correlations, and improve the fit of the model along this dimension. Thus, we use the correlation between moments that describe firms’ financing and savings activities and empirical proxies for TFP and  $\xi$  to motivate including a shock to the cost of external finance in particular.

The second key failure of the baseline model can be seen in the bottom panel of table III. The first column of data in the bottom panel of table III describes the empirical relationship between the time series of the cross-sectional correlation between liquidity and external finance,  $xsrho_{i,e}$  and the Fernald (2009) series for TFP, as well as a traditional empirical proxy for the cost of external finance, the default spread. The correlation between TFP and  $xsrho_{i,e}$  is 0.48. The correlation between the proxy for  $\xi$  and  $xsrho_{i,e}$  is 0.64. Thus,  $xsrho_{i,e}$  is more correlated with the default spread than with TFP, consistent with the intuition from both our two date model, and our dynamic model with stochastic costs of external finance. Similarly, the fraction of firms raising external finance is also more correlated with the default spread than with TFP (0.59 vs. 0.25). These moments indicate that not only is another shock needed to match the empirical moments about issuance and savings, but a shock which is close in spirit to the default spread

appears particularly promising in terms of matching model to the data. Moreover, carefully identified studies of differences in changes in investment across firms that are more and less dependent on external finance show that credit supply shocks do affect firm level investment, suggesting an important role for stochastic variation in the cost of external finance.<sup>18</sup> Finally, our quantitative model is consistent with the analytical results regarding firms' market timing with stochastic costs in Bolton *et al.* (2011).

Looking at the second column of data in table III, the SC model retains the successes of the baseline model, but improves key moments. The SC model again matches the correlation between liquidity accumulation and external finance of 0.60. This correlation is higher, as it is in the data, when one conditions on firms raising external finance. The SC model improves the correlations between liquidity accumulation, external finance, and investment with GDP by lowering them. It also matches the relationship between size and the correlation between liquidity accumulation and external finance, as shown in table IV. This table also illustrates the difference in firm size skewness in the data and in the model. Neither model replicates the extreme importance of the largest 10% of firms in driving the aggregate moments in Compustat data. In both models and in the data the top 10% of firms display a much lower correlation between external finance and liquidity accumulation. However, only in the Compustat data does including those firms change the total aggregate correlation by more than an order of magnitude. This is because the model is not designed to match the extreme empirical skewness in firm size. This is why we compare the model data to the firms in the bottom 90% of the Compustat sample as in Covas and Den Haan (2011a).

Most importantly, the SC model generates the observed correlations between  $\xi$  and  $xsrho_{i,e}$  and the percentage of firms raising external finance. This is for the same reasons that firms issue and save more in the two date model when  $\xi$  is low. Only when the cost of external finance is low do firms find it optimal to issue costly external finance and save the proceeds. The change in the internal margins of external finance and savings when  $\xi$  is low manifests in an increase in  $xsrho_{i,e}$ . The change in the external margin results in an increase in the percentage of firms raising external finance. The high correlations between the  $\xi$  shock in the model, and these cross-sectional moments, motivate our use of these moments to estimate the time series for the cost of external finance in section VI.. In fact, we show there that a time series regression of  $\xi$  on these two moments yields an  $R^2$  of 77%.

We also note that the correlation between TFP and these cross sectional moments describing

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<sup>18</sup>See Ivashina and Scharfstein (2010), Duchin *et al.* (2010), Campello *et al.* (2010), Almeida *et al.* (2009). See also Matvos and Seru (2011) for evidence of financing shocks from their estimation of a structural model comparing resource allocation by diversified and undiversified firms during the financial crisis.

financing and savings by firms are lower than the correlations with  $\xi$ , as is true empirically. In the SC model, the correlation between  $xsrho_{i,e}$  and TFP is either zero (using the actual  $z$  series), or only slightly positive (using TFP estimated with a log linear production function, treating financial and investment costs as deadweight costs, which matches the empirical estimation of TFP used in the data). In the two date model, we showed that liquidity accumulation is decreasing in TFP if there are no fixed costs to external finance or investment. Thus, without fixed costs, the SC model can actually generate a negative correlation between TFP and  $xsrho_{i,e}$ . However, the intuition from the baseline model shows how the fixed cost of external finance lends a force toward a positive correlation (as seen in the data). For this reason, we specified the SC model to have a constant fixed cost, with a stochastic linear and quadratic component.

In sum, in our SC model, the cross-sectional correlation between external finance and liquidity ( $xsrho_{i,e}$ ) tends to be high when the cost of external finance is low, more so than when TFP is high. Figures 4 and 5 illustrate this fact graphically in the data and in the model, respectively, by plotting the empirical time series for  $xsrho_{i,e}$  against the empirical proxy for  $\xi$ , and the simulated time series for  $xsrho_{i,e}$  against the  $\xi$  shock in the SC model. As discussed in section B.,  $xsrho_{i,e}$  is correlated with the default spread, but contains independent information. This makes sense since it reflects total issuance, and not only debt financing.

Finally, we note that both models have one main shortcoming. The only ultimate use of funds in our model is for payouts or for investment in physical capital. This shortcoming results in too little liquidity accumulation relative to the data, and too high of a correlation between external finance and investment. In practice, firms have other uses of funds, including, importantly, labor expenses. As long as labor and capital are close complements at the business cycle frequency, however, the fact that the firms in our model use only capital for production should not significantly alter the external finance and savings moments we focus on. In a model such as ours, in which liquidity accumulation is driven only by a desire to hedge investment opportunities in physical capital, it is difficult to generate a realistic amount of liquidity accumulation. This is because firms can hedge by investing in physical capital itself, and the returns to capital tend to be higher than the returns to liquid assets except for firms with low  $z$  or large capital stocks.<sup>19</sup> We also note that much of the high level of liquid assets is due to the 10 or so years prior to the financial crisis, and our stationary model is not aimed at matching this lower frequency phenomenon.

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<sup>19</sup>Warusawitharana and Whited (2011) generate higher liquidity accumulation by allowing issuances to be overvalued. If firms can profit from issuances, it is necessary to parameterize how quickly price impact or other constraints limit issuance profits. Since we found the question of whether issuances are costly or profitable to be mostly a level effect, we chose to save on parameters and maintain costly issuance.

## C. Firm Level Analysis

We use the optimal policy functions for a given firm to compare the model generated firm level moments to those in the data. We report the relevant moments in Table VI. Although our focus is on aggregate moments, the firm level moments provide an additional check on the model. Most correlations increase with aggregation, both in the data and in the model. This is not surprising, given that all firms are subject to independent idiosyncratic shocks, and common aggregate shocks. We provide moments from both the baseline and SC model in the table, but for brevity discuss only on the SC model in the text. For firm level issuance and savings activity, we find a correlation between external finance and liquidity accumulation of 0.39 in the SC model, and 0.18 in the data. Thus, the model overshoots on this correlation at the firm level, but get the aggregate correlation just right, since this aggregate moment was one of the main moments targeted by our calibration. We note that the lower correlation at the firm level in Compustat data may be due to noise, or accounting statement timing issues.

We find a slightly positive correlation between liquidity accumulation and investment in the SC model (0.06), while the correlation is slightly negative in the data (-0.06). At the firm level, as in the aggregate, the correlation between external finance and investment is too high in the model (0.94) relative to the data (0.20). Similarly, liquid asset balances are too small (0.03 vs 0.15). The (unconditional) probability of raising funds is about 15% in the baseline model and 10% in the SC model. These probabilities are roughly a fourth to a third of the number in the data (43%). However, we note that the high empirical probability of issuance may be due to small issuances with low costs, such as drawdowns on lines of credit. Consistent with this, Bazdresch (2005) provides evidence that a small fraction of observations account for most of firms' external financing activity. Likewise, the average amount of external finance raised is smaller in both models than in the data. The model does well in matching the average level of investment and its volatility, and on the standard deviation of external finance. The model produces too low autocorrelation of investment at the firm level, but again this correlation increases with aggregation. Overall, given that our calibration targets aggregate moments, and that our study is focused on understanding aggregate moments, the performance of the model at the firm level seems satisfactory.

## VI. Estimating The Cost of External Finance

Intuitively, if at a given date firms are simultaneously raising funds and saving them, it is likely that costs of raising external finance are low. Empirically we find correlations between

the cross-sectional correlation between liquidity accumulation and external finance,  $xsrho_{i,e}$ , and the default spread and lending standards of -0.64 and -0.58, respectively. We also show empirically that the percentage of firms raising external finance at any given date,  $\%raise_t$ , is highly correlated with these measures as well. Analytically, we show in a two-date model that  $xsrho_{i,e}$  will be higher when  $\xi$  is lower, and we show that this intuition holds in the calibrated infinite horizon model as well. In the model, the correlation between  $xsrho_{i,e}$  and  $\xi$  is 0.86. Thus, we argue that focusing on times when firms simultaneously raise funds and save them (as measured by the cross-sectional correlation) can be informative about aggregate credit conditions.

Expanding on this idea, we use the relationship in the model between financing and savings activity in the cross section, and the aggregate cost of external finance, to estimate the average cost of external finance per dollar raised in the US time series 1980-2010. Specifically, we construct an aggregate external finance index in the model, in which the cost is predicted to be a weighted average of cross sectional moments describing firms' issuance and savings. We then use the coefficients from this index, along with the empirical cross sectional moments at each date, to construct our estimates for the US time series.

In the model, we can express the average cost of a unit of external finance per dollar raised as  $\frac{E[\phi_e(e,\xi)]}{E[e|e<0]}$ . This is the average cost of external finance paid across firms at any given date, divided by the average amount raised. We compute this expression at each point in time in the model and then use this series as the dependent variable in the following time series regression estimated on model data:

$$\frac{E[\phi_e(e,\xi)]}{E[e|e < 0]}_t = \alpha + \beta_1 \%raise_t + \beta_2 xsrho_{i,e,t} + \varepsilon_t$$

We normalize the independent variables,  $\%raise_t$  and  $xsrho_{i,e,t}$ , to have mean zero and unit standard deviation. We report regression results in Table VII. We find that both the coefficients,  $\beta_1, \beta_2$  are negative, as expected, and around -0.01 in magnitude. This implies that a standard deviation increase in either variable is associated with a 1% decrease in the average cost paid per dollar raised. The constant term,  $\alpha$ , is 3% and represents the average cost of external finance paid per dollar raised in the model. Thus, each of these two cross sectional moments is economically important for explaining variation in external financing costs in the model data. Moreover, the  $R^2$  in this regression is 71%, and thus these two variables alone do a very good job explaining variation in the cost paid by firms in the model economy. We also show the explanatory power of each variable individually, and finally, we show that these variables also measure  $\xi_t$ , the stochastic cost of external finance 'level' series, in the model. In both cases, the

explanatory power of these variables jointly exceeds using each one individually, showing that each contains additional information to uncover the cost of external finance. This makes sense given the fact including both moments helps sort out the effects of TFP shocks vs.  $\xi$  shocks, as discussed above.

We then apply the regression coefficients,  $\alpha, \beta_1, \beta_2$ , to the analogous objects in the data to form a time series of estimates of the cost of external finance paid per dollar raised at each point in time. We plot this series in Figure 10. This series has very intuitively appealing properties. The estimated cost is high in the early 1980's, late 1980's, around the 2001 dot com crash, and in the recent financial crisis. Moreover, each spike in this cost is associated with a recession. Economically, the cost per dollar raised varies from essentially zero in the mid to late 1990's, up to 4.5% during the recent financial crisis. This value seems economically large if we consider a firm debating whether to take on an investment project for which it must raise external finance.

We next turn to a more formal estimation of the cost of external finance using the SC model. As in the regression method, we use our calibrated model, along with cross sectional moments to make inferences about the aggregate cost of external finance. In particular, at each date in time, we use a version of Simulated Method of Moments (SMM) to infer the value of the stochastic cost that generates simulated cross-sectional moments as close as possible to the data. The difference between our estimation exercise and a typical SMM estimation is that we are looking to uncover a hidden state instead of estimating a parameter. This distinction matters since state variables, unlike parameters, influence the model's transition dynamics over time, not just through policy functions, but also directly.<sup>20</sup>

As moments, we choose the correlation between liquidity accumulation and external finance as well as the percentage of firms raising external finance at each date, as these are each informative about this cost in the model. More specifically, we define the vector  $M_t$  as follows:

$$M_t = \begin{bmatrix} \rho_{N,\text{mod}} \left( \frac{i_t}{TA} (\xi_t), \frac{-c}{TA} (\xi_t) \right) - \rho_{N,\text{data}} \left( \frac{i_t}{TA}, \frac{-c}{TA} \right) \\ E_{N,\text{mod}} [\mathbf{1}_{e<0} (\xi_t)] - E_{N,\text{data}} [\mathbf{1}_{e<0}] \end{bmatrix}$$

where  $\rho_{N,\text{mod}}(x, y)$  represents the cross-sectional correlation between  $x$  and  $y$  in the model, which is a function of  $\xi_t$ , and  $\rho_{N,\text{data}}(x, y)$  represents the empirical counterpart. We use  $N$  to emphasize that these are cross-sectional, rather than time-series moments.<sup>21</sup>

<sup>20</sup>See Lee and Ingram (1991) for the use of SMM to estimate parameters in dynamic settings. An early example of using a model along with empirical moments to make inferences about a hidden aggregate state is Eisfeldt and Rampini (2006), which uses a calibrated model along with moments describing capital reallocation to infer the business cycle dynamics of capital liquidity.

<sup>21</sup>We have also estimated the series adding two additional moments:  $(\xi_t - \mu_\xi) - \rho(\xi_{t-1} - \mu_\xi)$  and  $((\xi_t - \mu_\xi) - \rho(\xi_{t-1} - \mu_\xi))^2 - \sigma_\xi^2$  which help ensure that the series  $\xi_t$  follows an AR(1) with the parameters we cali-

At every date  $t$  we choose the value  $\xi_t$  that minimizes deviations of the cross-sectional model implied moments and empirical moments. Specifically, we choose  $\xi_t$  to minimize the following objective function

$$\min_{\{\xi_t\}} M_t' W M_t$$

where we set  $W = I_{2 \times 2}$  as the identity matrix which weights all moments equally.

We initialize the series by first starting capital and liquidity at their steady state values, and then feeding in the observed aggregate TFP and the default spread series beginning in 1951. We do this to ensure that the model distribution over capital and liquidity stocks reflects the history of TFP shocks in the US given calibrated  $\xi_t$  shocks. We use the empirical realizations of these series to proxy for  $z_{t,agg}$  and  $\xi_t$  until 1980 when our Compustat sample begins. Thereafter, we estimate  $\xi_t$  date-by-date by setting the value of aggregate productivity to equal to our observed TFP series and choosing  $\xi_t$  to minimize our objective function. This allows us to estimate a rolling time series for the stochastic cost of external finance that firms most likely face in a given year using the empirical cross-sectional moment.

The resulting series identifies the following years as having high cost of external finance: 1980-1986, 1988-1990, 2001-2002, 2008-2010. Our estimation procedure appears to pick up events that our priors suggest might be associated with costly external finance, such as the recession of the early 80's, the crash of the tech boom in 2001, and the recent financial crisis. Therefore we take this as additional evidence that the model can be used to identify times when the cost of external finance is high or low. The main limitation of the SMM procedure, however, is that it only chooses between the two states used in our model. Comparing these dates to the regression implied estimate, one sees that the estimates in both methods are very similar; the SMM method picks a high  $\xi$  when the regression method implies a high average cost.

## VII. Conclusion

We document the empirical regularity of aggregate issuance and savings waves. We also show that cross sectional moments describing firms's issuance and savings behavior are informative about the aggregate cost of external finance. We document that, empirically, the time series of the cross-sectional correlation between external finance and liquidity accumulation, and the time series of the percentage of firms raising external finance, are highly correlated with standard measures of the aggregate state of the cost of external finance, such as the default spread and the fraction of banks tightening lending standards.

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brated. The results are qualitatively similar.

We argue that both the observed realization of the correlation between external finance and liquidity accumulation in the cross-section,  $xsrho_{i,e}$ , and our model implied estimate of the level of the cost of external finance in the US time series 1980-2010, are useful measures of the state of the aggregate level of the cost of external finance. Using firms' actual decisions about how much external finance to raise and how they use the proceeds from external finance is a revealed preference method of making inferences about the true cost of external finance. Such a measure might provide useful policy guidance as to the likely impact of interventions aimed at lowering the cost of external finance since the macroeconomic benefit of lowering the cost of external funds depends (amongst other things) on whether those funds will be used for investment, or accumulated as cash.

Understanding the role of a potentially time varying cost of external finance is important for several reasons. First, studying whether shocks to the cost of external finance are important for firm financing, liquidity accumulation, and investment dynamics may help to uncover the role of the financial sector in business cycles. From a theoretical standpoint, many models featuring endogenous variation in the cost of external finance in business cycles feature a tight link between variation in fundamentals such as productivity and variation in the informational frictions which make external finance costly. In contrast, shocks to the cost of external finance in recent DSGE models are often only partially correlated with other fundamental shocks. By using our dynamic model as a filter on the US time series for firm financing, liquidity accumulation, and investment, we provide implied estimates of the cost of external finance at each date.

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## VIII. Data Appendix

### Data Appendix

Our data construction closely follows Covas and Den Haan (2011a). Our primary source of data is the Compustat fundamentals annual file. Our main results use data from 1980-2010. We exclude financials, utilities and firms with SIC codes starting with 9. We also exclude firms with missing assets, equity, debt, and those with missing or negative PPE and cash balances. As in Covas and Den Haan (2011a), we also remove GM, GE, Chrysler, and Ford, since these firms were the most affected by the accounting change in 1988 requiring firms to consolidate the balance sheets of their wholly owned subsidiaries.

#### *Computstat Data*

We first define liquidity accumulation, investment, and external finance as:

$$Investment = i_k = CAPEX \text{ (Capital Expenditures)}$$

$$Liquidity \text{ Accumulation} = i_l = CHECH \text{ (Cash and cash equivalents, change)}$$

$$External \text{ Finance} = -e = -(CF_D + CF_E)$$

For flows to debt and equity and operating cash flows we use the statement of cash flows:

*For statements of cash flows:*

$$CF_O = \text{Income before extra items (IBC)} + \text{Depreciation and amortization (DPC)} + \text{EI \& Discontinued Oper (XIDOC)} + \text{Deferred Taxes (TXDC)} + \text{Equity in net loss (ESUBC)} + \text{Funds from operations: other (FOPO)} + \text{Income taxes: accrued inc(dec) (TXACH)} + \text{Assets \& Liab: other (net change) (AOLOCH)} + \text{Accounts receivable dec(inc) (RECCH)} + \text{Inventory dec(inc) (INVCH)} + \text{Accounts payable inc(dec) (APALCH)} + \text{Interest paid (net) (XINT)}$$

$$CF_E = - \text{Sale of common and pref. stock (SSTK)} + \text{Purchase of common and pref. stock (PRSTKC)} + \text{Cash dividends (DV)}$$

$$CF_D = - \text{Long-term debt issuance (DLTIS)} + \text{Long-term debt: reduction (DLTR)} + \text{Changes in current debt (DLCCH)} + \text{Interest paid (net) (XINT)}$$

*For statements by source and use of funds:*

$$CF_O = \text{Income before extra items (IBC)} + \text{Depreciation and amortization (DPC)} + \text{EI \& Discontinued Oper (XIDOC)} + \text{Deferred Taxes (TXDC)} + \text{Equity in net loss (ESUBC)} + \text{Funds}$$

from operations: other (FOPO)+ Interest expense (XINT)

$CF_E = -$  Sale of common and pref. stock (SSTK)+ Purchase of common and pref. stock (PRSTKC) + Cash dividends (DV)

$CF_D = -$  Long-term debt issuance (DLTIS)+ Long-term debt: reduction (DLTR) + Changes in current debt (DLCCH) + Interest paid (net) (XINT)

*For working capital statements:*

$CF_O =$  Income before extra items (IBC) + Depreciation and amortization (DPC) + EI & Discontinued Oper (XIDOC) + Deferred Taxes (TXDC) + Equity in net loss (ESUBC) + Funds from operations: other (FOPO)+ Interest expense (XINT)

$CF_E = -$  Sale of common and pref. stock (SSTK)+ Purchase of common and pref. stock (PRSTKC) + Cash dividends (DV)

$CF_D = -$  Long-term debt issuance (DLTIS)+ Long-term debt: reduction (DLTR) + Changes in current debt (DLCCH) + Interest paid (net) (XINT)

*For cash statements by activity:*

$CF_O =$  Income before extra items (IBC) + Depreciation and amortization (DPC) + EI & Discontinued Oper (XIDOC) + Deferred Taxes (TXDC) + Equity in net loss (ESUBC) + Funds from operations: other (FOPO)+ Interest expense (XINT)

$CF_E = -$  Sale of common and pref. stock (SSTK)+ Purchase of common and pref. stock (PRSTKC) + Cash dividends (DV)

$CF_D = -$  Long-term debt issuance (DLTIS)+ Long-term debt: reduction (DLTR) + Changes in current debt (DLCCH) + Interest paid (net) (XINT)

### ***Flow of Funds Data***

We use annual data from the electronic ASCII flow of funds seasonally adjusted annual rates table F.102 available at

<http://www.federalreserve.gov/Releases/z1/Current/data.htm>.

Refer to the coded tables for definitions and relationships between entries. Codes appear in parentheses after variable names. Interest payments, not reported in table F.102, are from NIPA table 1.14 line 25 “Net interest and miscellaneous payments” for nonfinancial corporate business.

$CF_O$  =(Total internal funds + IVA) (FA1060000105) - Discrepancy (FA107005005) + Net dividends (FA106120005) + Trade payables (FA103170005) + Taxes payable (FA103178000) + Miscellaneous liabilities (FA103190005) - Trade receivables (FA103070005) + NIPA interest

$CF_D$  =Commercial paper (FA103169700) + Mortgages (FA103065003) - Credit market instruments (FA104104005) + NIPA interest

$CF_E$  =Net dividends (FA106120005) - Net new equity issues (FA103164003)

*Liquidity Accumulation* =  $i_l$  = Net acquisition of financial assets - Commercial paper - Mortgages - Trade receivables - Other Assets

*Investment* =  $i_k$  =Capital expenditures

### ***Other Data***

The following series used can be found in the FRED database at the St Louis Fed website.

*Gdp*= Real gross domestic product

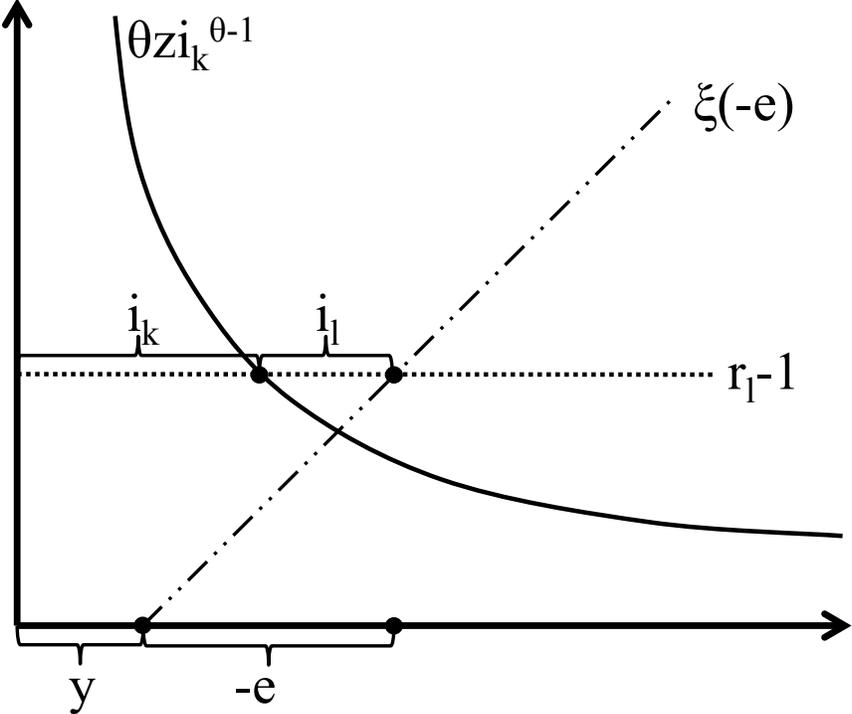
*Default Spread*= Difference between Moody's Seasoned Baa and Aaa yield. We use end of year values.

*Lending Standards*= Net Percentage of Domestic Respondents Tightening Standards for Commercial and Industrial Loans Large and Medium Firms (DRTSCILM). We use end of year values.

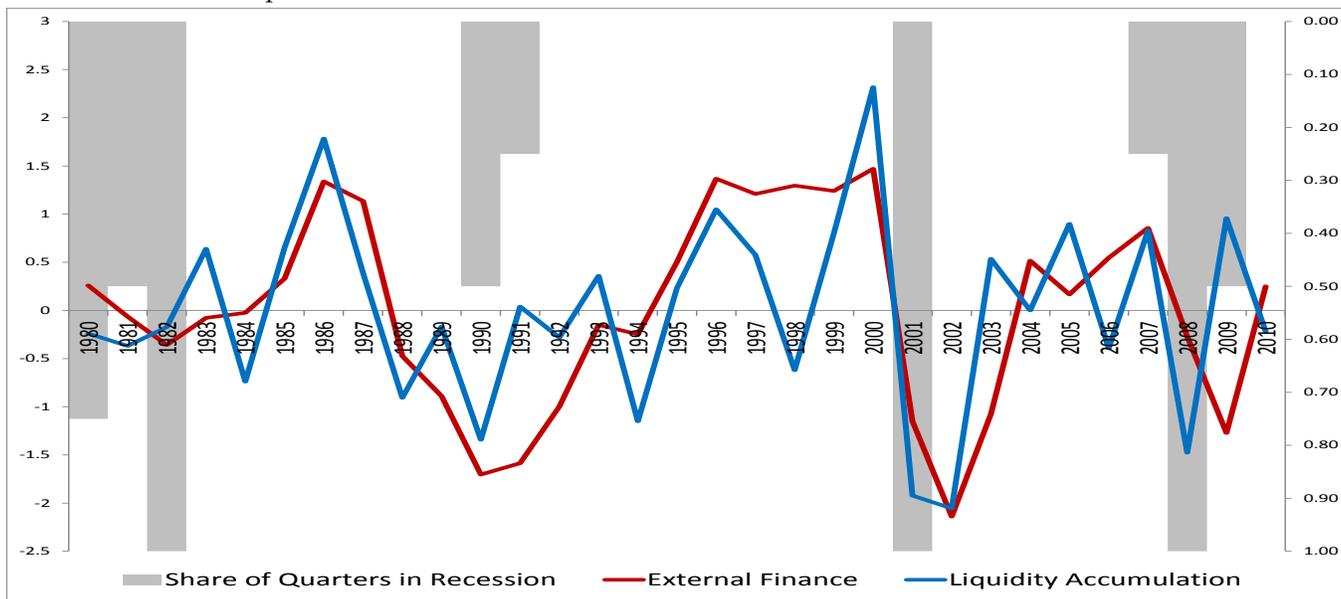
Finally, we obtain TFP data from John Fernald's website at: <http://www.frbsf.org/economics/economists/staff.php?jferald>. We construct the log level series from the series of annual changes provided, and detrend the series with two breaks as in , which advocates breaks after 1974 and 1995. Shocks are then residuals from an AR(1) regression on the log level series.

# IX. Figures, Tables. View in color.

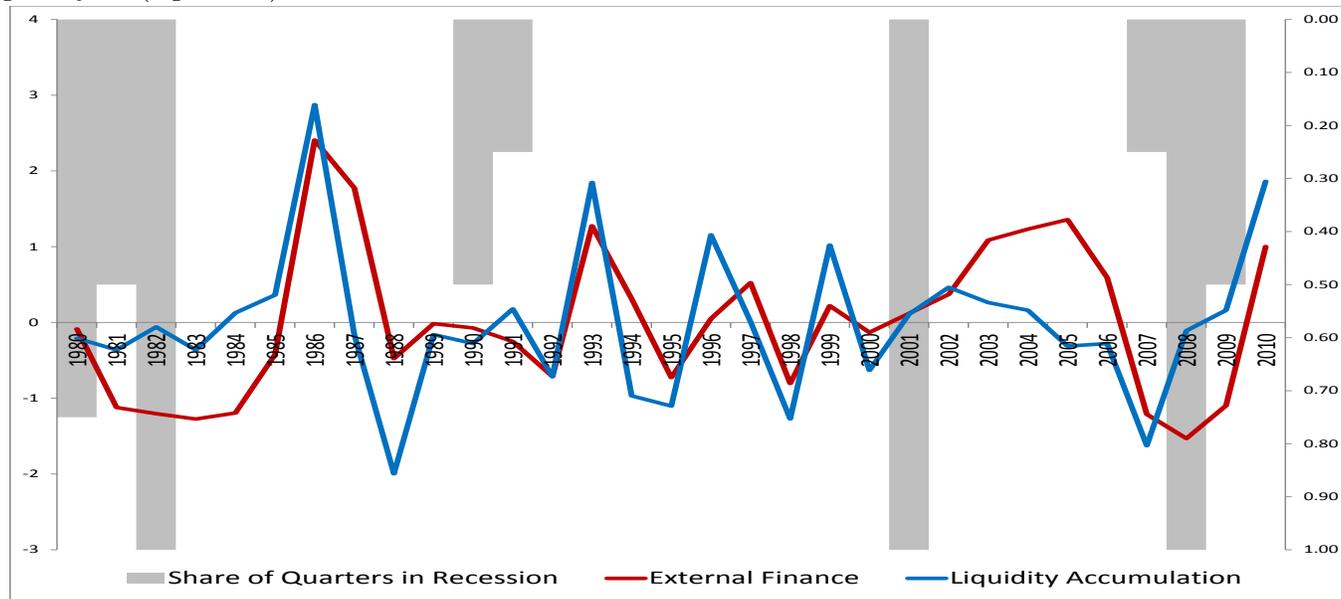
**Figure 1:** Two Period Model. This figure provides the intuition for our two period model.  $\theta z i_k^{\theta-1}$  represents the marginal product of capital,  $\xi(-e)$  the marginal cost of external finance, and  $r_l - 1$  the net return on liquid assets.  $i_l$  and  $i_k$  represent investment in liquid assets and physical capital,  $-e$  represents external finance raised, and  $y$  represents internal funds from operations.



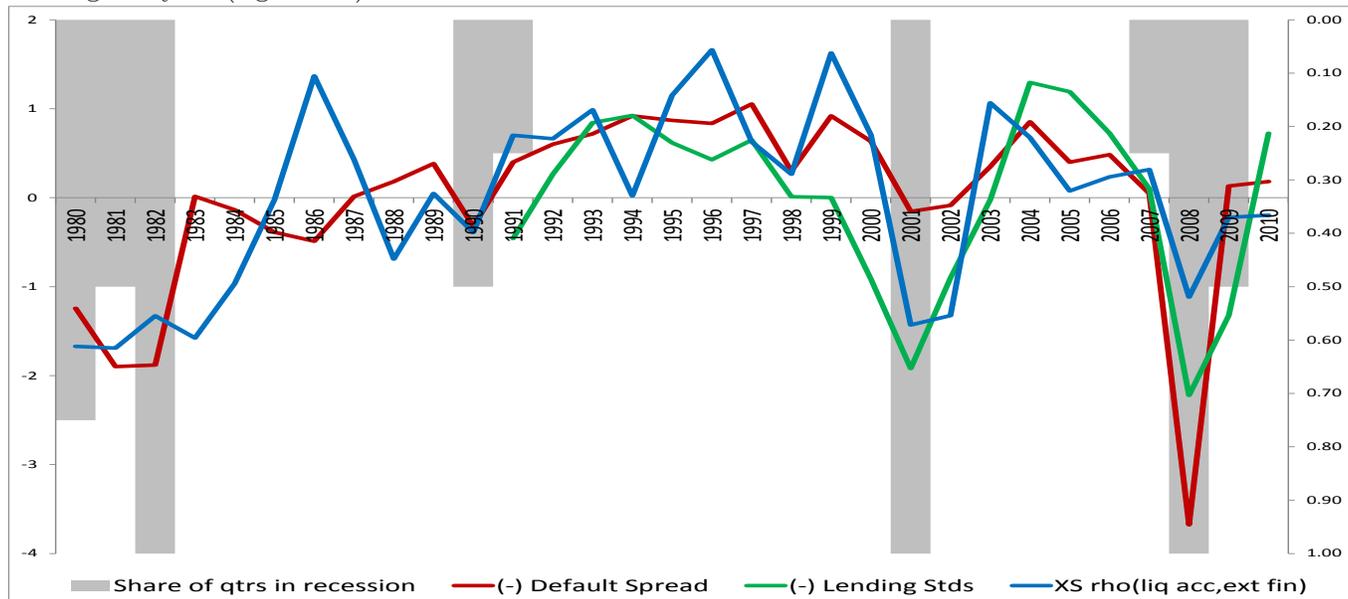
**Figure 2:** We plot aggregate accumulation of liquid assets against aggregate external finance. Sample excludes largest 10% of firms. Data are normalized by lagged assets, HP-filtered, and then scaled to have unit variance. Gray bars indicate fraction of quarters economy is in a recession in the given year (right axis). Correlation between plotted series is 0.6.



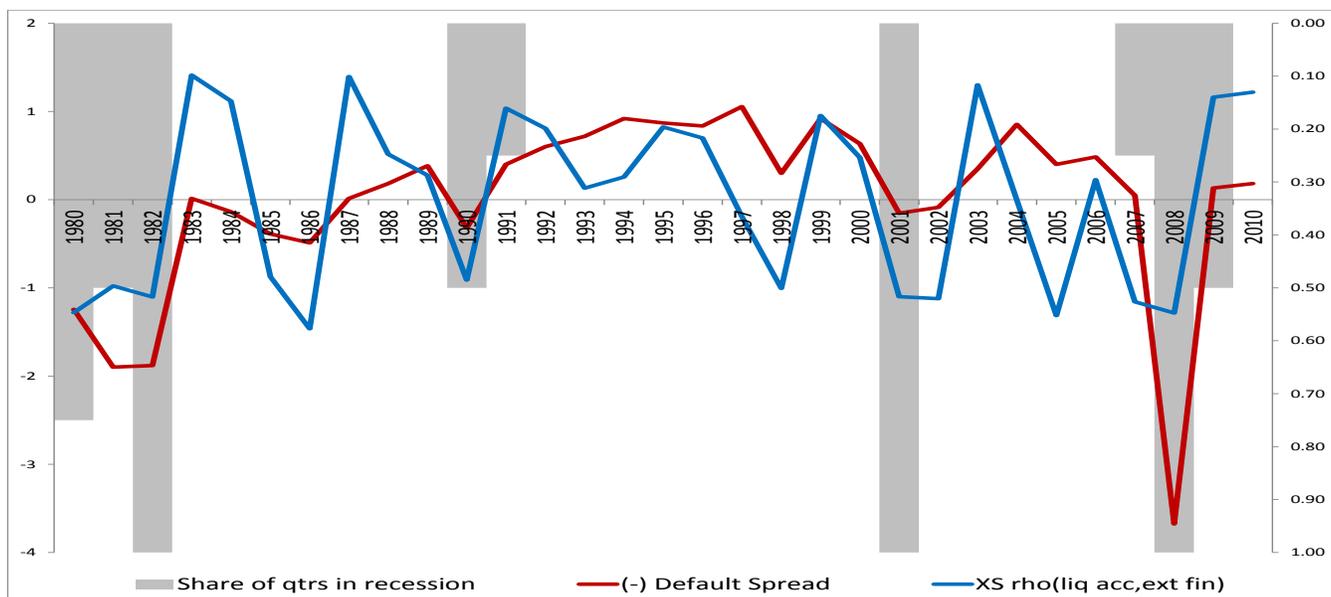
**Figure 3:** We plot aggregate accumulation of liquid assets against aggregate external finance from our baseline model simulation using empirical realized TFP shocks. Both series are normalized by total assets and scaled to have unit variance. Gray bars are the fraction of quarters the economy is in a recession in the given year (right axis).



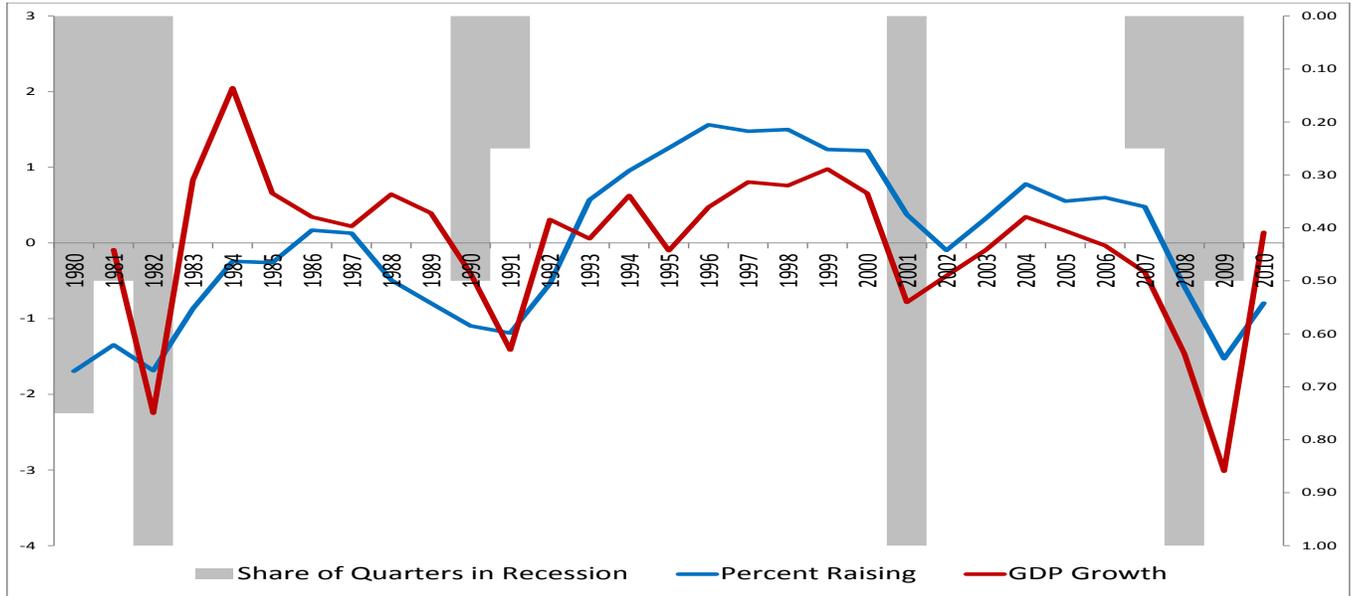
**Figure 4:** We plot the the cross-sectional correlation between liquidity accumulation and external finance and the negatives of the Moody’s Baa-Aaa rate and the net % of banks tightening lending standards for large and medium firms. The correlation between XS rho and the negative of the Default spread and Lending Standards are 0.64 and 0.58, respectively. Gray bars indicate fraction of quarters economy is in a recession in the given year (right axis). Each series is standardized to have mean zero and unit variance.



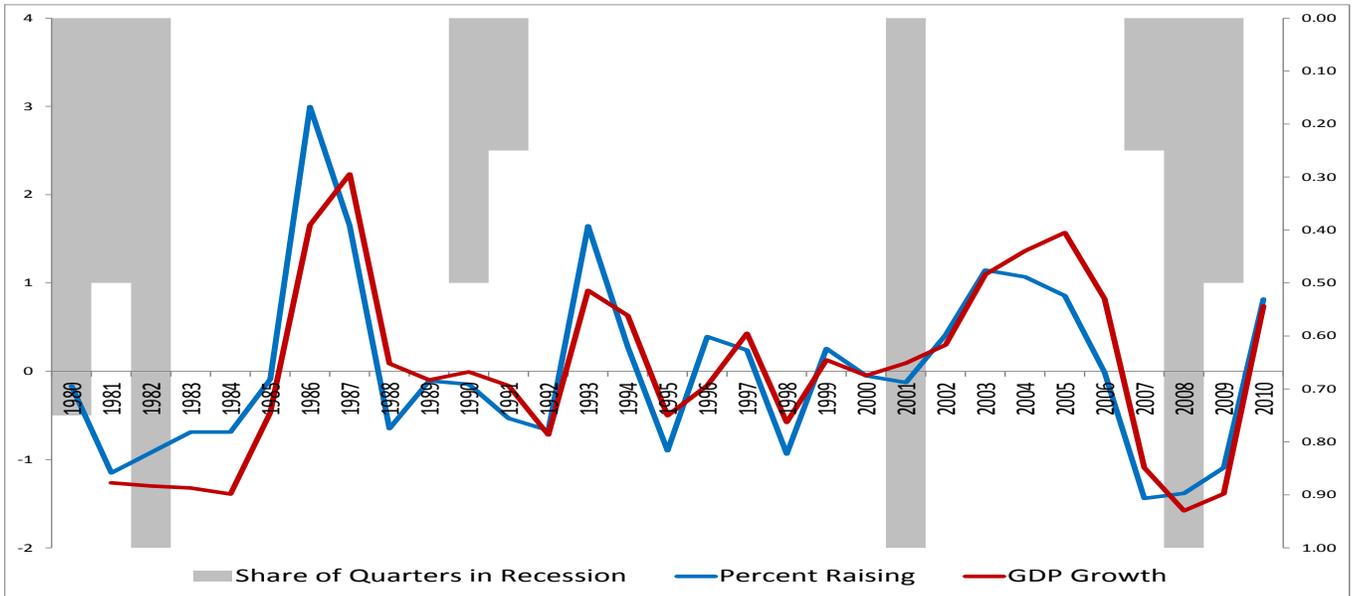
**Figure 5:** We plot the the cross-sectional correlation between liquidity and external finance (XS rho). In our model, this proxy reveals times when external finance is expensive, as measured by the negative of the stochastic cost of external finance ( $-\ln(\xi)$ ). The correlation between XS rho and the negative of the log cost ( $-\ln(\xi)$ ) is 0.80 in the model. Gray bars indicate when gdp growth falls below trend (right axis). Each series is standardized to have mean zero and unit variance.



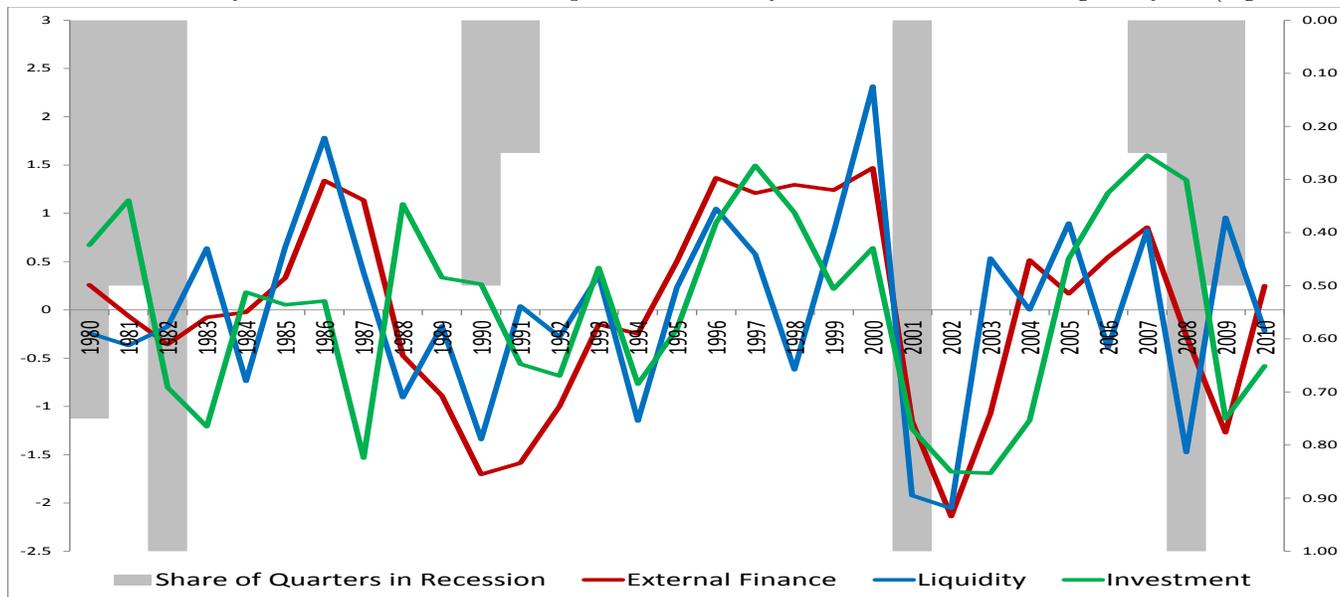
**Figure 6:** This figure plots the time-series of the percentage of firms raising external finance over the business cycle, measured as the growth rate of gdp. The firm level data are from Compustat. A firm is raising external finance if net flows to external finance are negative. Gray bars are the fraction of quarters the economy is in a recession in the given year (right axis). Each series is standardized to have mean zero and unit variance



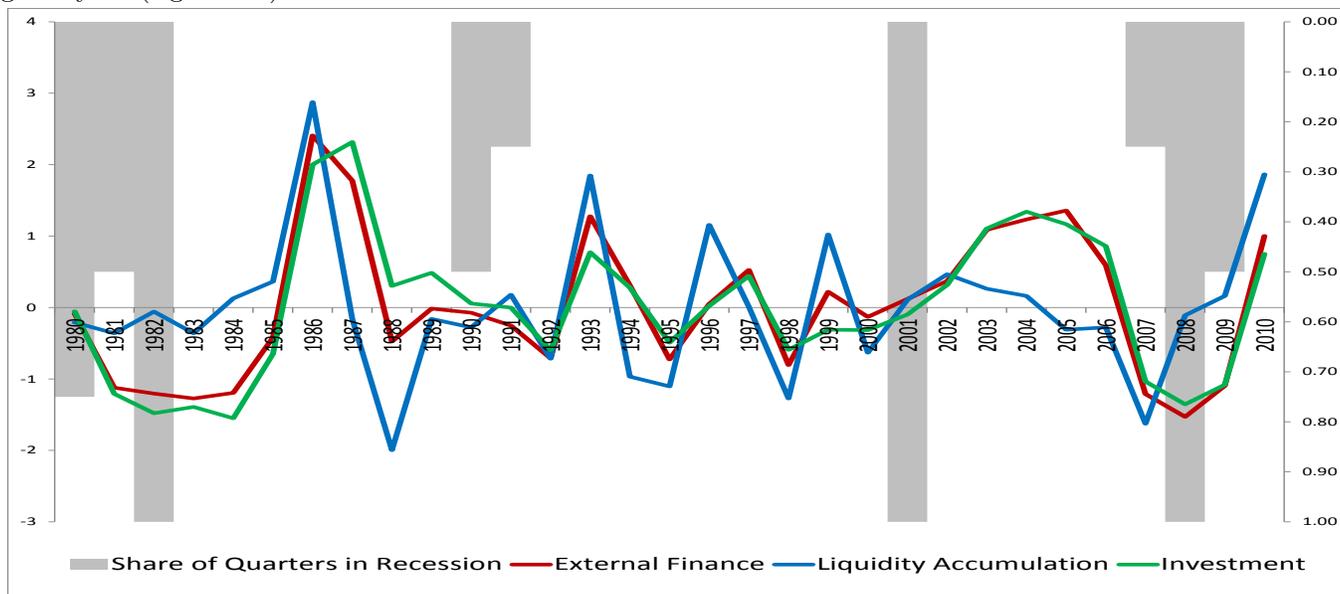
**Figure 7:** This figure plots the time-series of the percentage of firms raising external finance and the growth rate of gdp from our baseline model simulation using empirical realized TFP shocks. Gray bars are the fraction of quarters the economy is in a recession in the given year (right axis). Each series is standardized to have mean zero and unit variance.



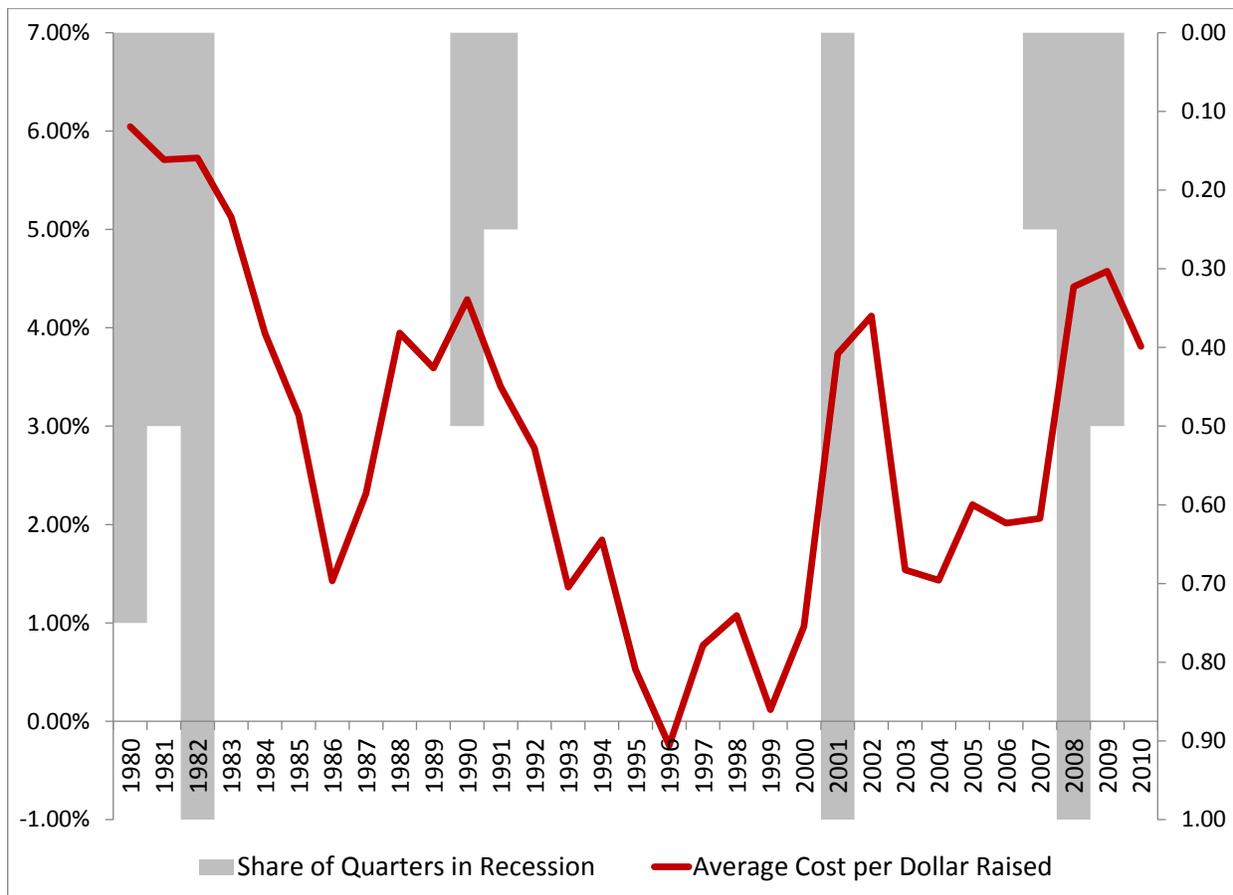
**Figure 8:** We plot aggregate investment, accumulation of liquid assets, and external finance. Sample excludes largest 10% of firms. Data are normalized by lagged assets, HP-filtered, and then scaled to have unit variance. Gray bars indicate fraction of quarters economy is in a recession in the given year (right axis).



**Figure 9:** We plot aggregate accumulation of liquid assets against aggregate external finance from our baseline model simulation using empirical realized TFP shocks. Both series are normalized by total assets and scaled to have unit variance. Gray bars are the fraction of quarters the economy is in a recession in the given year (right axis).



**Figure 10:** The figure plots the average cost of external finance paid per dollar of external finance raised in the US time series estimated using cross-sectional moments and the estimation procedure described in Section VI..



**Table I:** This table displays the main aggregate issuance and savings waves facts. Except where noted, we use annual Compustat data from 1980-2010. We normalize aggregate series by the lag of total assets and hp-filter. Size bins are determined by total asset size. The main results in the paper use the [0,90]% sample. Flow of funds data are normalized by the trend in gross value added for the corporate sector. Narrow liquidity is the net acquisition of financial assets minus trade receivables minus miscellaneous assets. Broader liquidity also includes 1/3 of miscellaneous other assets as liquid assets. \* indicates significance at a 5% level.  $xsrho_{i,e,t}$  is  $\rho_t \left( \frac{ext_i}{TA_i}, \frac{liqacc_i}{TA_i} \right)$

Aggregate Issuance and Savings	
$\rho \left( \frac{\Sigma ext_t}{\Sigma TA_{t-1}}, \frac{\Sigma liqacc_t}{\Sigma TA_{t-1}} \right)$	
[0,50]%	0.84*
<b>[0,90]%</b>	<b>0.60*</b>
[0,100]%	0.12
Conditional on Raising funds: e<0	0.74*
No Dividends	0.68*
No Rating	0.56*
Flow of Funds: Narrow Liquidity	0.33
Flow of Funds: Broader Liquidity	0.38*
$\rho(xsrho_{i,e,t} \text{Aggregate State}_t)$	
Minus Baa-Aaa Spread	0.64*
Minus Lending Standards	0.58*
TFP Shock	0.48

**Table II:** We give our calibrated parameters below along with those in Riddick and Whited (RW) and the standard business cycle literature (RBC). The label e.c.f. denotes external cost of finance and i.a. denotes investment adjustment costs. The lower panel gives the implied average costs of issuance and investment firms pay with the given parameters. For example, the implied average cost of issuance gives the average cost paid for a firm raising external finance as a fraction of the amount of funds raised.

Symbol	Description	Model SC	Model Baseline	RW	RBC
$\tau$	tax rate	0.1	0.1	0.20	—
$\delta$	depreciation	0.08	0.08	0.15	0.08
$\theta$	curvature	0.65	0.65	0.75	0.33
$\rho$	persistence	0.66	0.66	0.66	0.9
$\sigma$	total vol of prod	0.121	0.121	0.121	—
$\sigma_i$	idiosyncratic vol	0.11	0.11	—	—
$\sigma_{agg}$	aggregate vol	0.03	0.03	—	0.022
$\lambda_0$	e.c.f. fixed	0.12	0.2334	0.389	—
$\lambda_1$	e.c.f. linear	0.004	0.004	0.053	—
$\lambda_2$	e.c.f. quad	0.0007	0.00001	0.0002	—
$a$	i.a. quad	0.15	0.147	0.049	—
$c$	i.a. fixed	0.0025	0.01	0.039	—
$r$	risk-free	0.04	0.04	0.04	0.04
$\sigma_\eta$	vol cost of funds	1.75	—	—	—
$\gamma$	persistence	0.4	—	—	—
$\mu_\xi$	mean credit	1	—	—	—

Symbol	Description	Model SC	Model Baseline
$E \left[ \frac{\phi_e(e)}{e} \right]$	implied average issuance cost	0.03	0.017
$E \left[ \frac{\phi_i(i_k, k)}{i_k} \right]$	implied average investment cost	0.005	0.009

**Table III:** This table displays aggregate moments from the model (both baseline and stochastic costs (SC) versions) using a simulated panel of firms. We compare these moments with those from annual Compustat data, 1980-2010. For correlations, we normalize each series by lagged assets and apply the hp-filter. All other series are normalized by current assets.  $xsrho_{i_i,e,t}$  is  $\rho_t\left(\frac{-e_i}{TA_i}, \frac{i_{i,i}}{TA_i}\right)$ . TFP are TFP level shocks. We use the Baa-Aaa default spread as an empirical proxy for  $\xi$ . \* indicates significance at 5% level. We use notation from the model:  $-e$  represents external finance (negative of payouts),  $i_l$  liquidity accumulation,  $i_k$  investment in physical capital, and  $l$  liquid balances. We normalize each series by total assets except investment which is normalized by physical capital  $k$ .

Aggregate Moments			
Moment	Data	Model SC	Model Baseline
$E[l]$	0.11	0.02	0.03
$\sigma(l)$	0.03	0.01	0.05
$\sigma(i_l)$	0.01	0.01	0.01
$E[i_k]$	0.07	0.08	0.08
$\sigma(i_k)$	0.01	0.03	0.03
$\rho(i_{k,t}, i_{k,t-1})$	0.38*	0.34	0.38
$E[-e]$	-0.01	-0.10	-0.10
$\sigma(-e)$	0.03	0.04	0.03
$\rho(i_l, -e)$	0.60*	0.60	0.59
$\rho(i_l, i_k)$	0.12	0.42	0.45
$\rho(-e, i_k)$	0.46*	0.98	0.98
$prob(-e > 0)$	0.05	0.01	0.004
Aggregate Moments: Conditional on $e < 0$			
Moment	Data	Model SC	Model Baseline
$\rho(i_l, -e)$	0.74*	0.92	0.91
$\rho(i_l, i_k)$	0.37*	0.63	0.79
Aggregate Moments: Business Cycle Correlations			
Moment	Data	Model SC	Model Baseline
$\rho(i_l, gdp)$	0.00	0.21	0.41
$\rho(-e, gdp)$	0.28*	0.75	0.99
$\rho(i_k, gdp)$	0.47*	0.82	0.97
TFP vs. $\xi$ Moments			
	Data	Model SC	Model Baseline
$\rho(xsrho_{i_i,e}, \xi)$	0.64*	0.86	—
$\rho(\%raise, \xi)$	0.59*	0.84	—
$\rho(xsrho_{i_i,e}, TFP)$	0.48*	0.00 (0.12)	0.72
$\rho(\%raise, TFP)$	0.25	0.32 (0.44)	0.85

**Table IV:** This table gives the correlation between liquidity accumulation and external finance conditional on firm size and conditional on whether firms are raising external finance ( $e < 0$ ). We use annual Compustat data from 1980-2010. Aggregate series are normalized by the lag of total assets and hp-filtered. Size bins are by total asset size. The main results in the paper focus on the [0,90]% bin. \* indicates significance at 5% level.

Corr $\left(\frac{LiqAcc}{TA}, \frac{ExtFin}{TA}\right)$			
Conditional on Size			
Size	Data	Model SC	Model Baseline
[0,25]%	0.59*	0.81	0.94
[0,50]%	0.84*	0.72	0.86
[0,75]%	0.76*	0.66	0.71
[0,90]%	0.60*	0.61	0.60
[0,100]%	0.12	0.60	0.59
[90,100]%	0.03	-0.05	0.20
Conditional on e			
	Data	Model SC	Model Baseline
e<0	0.74*	0.92	0.91
e>0	0.15	0.28	0.15

**Table V:** This table displays moments for debt and equity separately using our Compustat sample. We normalize each aggregate series by lagged aggregate assets and apply the hp-filter. \* indicates significance at 5% level.

Aggregate Compustat Moments: Debt vs. Equity		
	Unconditional	Conditional on $e < 0$
$\rho(liqacc, debt)$	0.16	0.33*
$\rho(liqacc, equity)$	0.69*	0.77*
$\rho(inv, debt)$	0.60*	
$\rho(inv, equity)$	-0.15	

**Table VI:** Firm Level Facts. The table gives firm level moments. In each case, we compute the relevant moment for the entire panel of firms and then take a median across firms. We use our simulated panel of data (Model column) and Compustat (Data column). We normalize the series by total book assets. \* indicates significance at 5% level. We use notation from the model:  $-e$  represents external finance (negative of payouts),  $i_l$  liquidity accumulation,  $i_k$  investment in physical capital, and  $l$  liquid balances. We normalize each series by total assets except investment which is normalized by physical capital  $k$ .

Firm Level Moments			
Moment	Data	Model SC	Model Baseline
$E[l]$	0.15	0.03	0.01
$\sigma(i_l)$	0.10	0.05	0.07
$E[i_k]$	0.06	0.07	0.11
$\sigma(i_k)$	0.07	0.16	0.16
$\rho(i_{k,t}, i_{k,t-1})$	0.29*	0.04	0.17
$E[-e]$	-0.02	-0.11	-0.07
$\sigma(-e)$	0.17	0.17	0.21
$\rho(i_l, -e)$	0.18*	0.39	0.57
$\rho(i_l, i_k)$	-0.06*	0.06	0.22
$\rho(-e, i_k)$	0.20*	0.94	0.93
$\%raise$	0.43	0.10	0.15

**Table VII:** This table estimates the cost of external finance in the model using regressions of cost measures on  $xsrho_{i,e}$ , the cross-sectional correlation between liquidity accumulation and external finance, and,  $\%raise$ , the percentage of firms raising external finance. We measure the cost of external finance in two ways: first, as the average cost paid per dollar of external finance raised,  $E_N \left[ \frac{\phi_e}{e} \right]$ , and second as the stochastic cost series  $\xi$ .

Inferring the Cost of External Finance via Regressions				
$y$	Constant	$xsrho_{i,e}$	$\%raise$	$R^2$
$E_N \left[ \frac{\phi_e}{e} \right]$	0.030 (52.27)	-0.018 (-31.68)		66.8%
$E_N \left[ \frac{\phi_e}{e} \right]$	0.030 (51.19)		-0.017 (-30.05)	64.3%
$E_N \left[ \frac{\phi_e}{e} \right]$	0.030 (58.00)	-0.011 (-10.78)	-0.009 (-8.66)	71.1%
$\xi$	-0.01 (-1.16)	-0.86 (-36.94)		73.2%
$\xi$	-0.01 (-1.09)		-0.85 (-35.35)	71.5%
$\xi$	-0.01 (-1.34)	-0.49 (-11.27)	-0.41 (-9.40)	77.3%