Heterogeneous Global Cycles *

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Abstract

We study the heterogeneous effect of capital supply fluctuations on real outcomes across countries. We show that frictions in global intermediation lead to an endogenous partitioning of economies into groups with low and high exposure to global credit cycle, because low skilled investors dramatically re-balance their portfolios as the aggregate state changes. The differential response of investors invites differential strategies of firms, shaping heterogeneous global cycles. We connect the implications of our model to stylized facts on credit spreads, investment, safe asset supply, concentration of debt ownership, and the return on debt during various boom-bust episodes, both in the time series and in the cross-section. We demonstrate that a global savings glut implies that some countries are pushed from the low exposure to the high exposure group and exacerbates both booms and busts in the high exposure group.

1 Introduction

Before 2008, boom-bust cycles had been associated almost exclusively with emerging markets. The pattern – a boom phase started by poorly regulated financial liberalization leading to a surge in foreign capital, large credit flows to the non-financial sector, build-up of debt at low interest rates, and rapidly increasing investment abruptly turning to a bust phase where interest rate spike and credit flies to safety triggering a collapse in output, — has been connected to a large catalog of structural weaknesses in Latin American, East Asian and Eastern European economies.

However, the Global Financial Crisis in 2008, and especially the Eurozone Crisis in 2010 have dramatically exposed similar vulnerabilities in a group of advanced economies. This led to a shift

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in focus to the role of increasingly globalized financial intermediation and the implied changes in global capital supply.[1]

In this paper, we conceptualize how changes in global capital supply determine heterogeneous country outcomes. We argue that frictions in global intermediation lead to a partition of economies to groups with low and high exposure to global investment cycles. We connect the implications of our model to stylized facts on credit spreads, investment, safe asset supply, concentration of debt ownership, and the return on debt during various boom-bust episodes, both in the time series and in the cross-section. We demonstrate that a global savings glut implies that some countries are pushed from the low exposure to the high exposure group and exacerbates both booms and busts in the high exposure group.

In our model, firms operate a Holmström and Tirole (1998) technology. They allocate their endowment between investment and savings, to manage the risk of future liquidity shocks. A firm that is hit by a liquidity shock has to either pay a maintenance cost, or abandon production. To pay the additional cost, the firm can access international capital markets for credit. A pledgeability constraint implies that the firm has to cover part of its financing needs from its own savings as downpayment.

A key friction of our model is that international investors are heterogeneous in their skill to identify whether a firms collateral is good or bad. Obtaining reliable information about firms requires more skill in some countries, which we refer to as opaque countries. While firms are heterogeneous in collateral quality, all countries are identical in the composition of their firms.

Moreover, we assume that investors *prudence* varies with the aggregate state. In the high state investors are *bold* and are not willing to miss out on any good opportunities, even at the expense of extending loans to some bad firms by mistake. On the contrary, in the low state they are *cautious* and strictly avoid financing any firm that might not repay, even if doing so leads them to forgo some profitable investment opportunities. An important implication is that investors lend to different firms as the aggregate state changes. The differential response of investors across countries invites differential strategies of firms, shaping heterogeneous global cycles.

In particular, as a rational response to their imperfect information, low-skilled investors disproportionally lend to firms in opaque countries in the high state, and to firms in transparent countries in the low state. As a result, our model features boom-bust cycles with heterogeneous exposures across countries.[2] The most opaque countries form a high exposure group. In booms, these countries enjoy large credit inflows at low interest rates and high growth. However, in busts firms in these countries can obtain new credit only at high rates, if at all, and their output and credit flows

[1] For instance, see Caballero et al. (2017) and citations therein on the role of global scarcity of safe assets, Caballero and Simsek (2016) on fickle capital flows, and Avdjiev et al. (2016) emphasizing that globalization pushed decisions on credit supply outside of the boundaries of affected countries, a new phenomenon for advanced economies.

[2] Ivashina et al. (2015) and Gallagher et al. (2018) find that a group of money market funds stopped lending only to European banks, but not to other banks of similar risk in 2011. Ivashina et al. (2015) find evidence that this lead to significant disruption in the syndicated loan market. These facts are broadly consistent with our proposed mechanism.

collapse. Instead, international capital floods a group of more transparent countries at low interest rates, their transparency effectively shielding them from negative exposure to the global cycle.

Our model implies a qualitative difference in the functioning of credit markets between booms and busts. In booms, firms borrow at the same rate in high and low exposure countries. In this state, investors have heterogeneous portfolios, and highly skilled investors derive excess returns by extending credit to higher quality borrowers across all countries. In contrast, during busts there is a significant spread for borrowing in the high and low exposure countries. In this state lenders are cautious, which implies the same credit quality across their portfolios. As such, highly skilled investors derive excess returns by lending at higher rates to good, but opaque firms in high-exposure countries. This picture rationalizes the – sometimes puzzlingly – low premium on emerging market assets before the East-Asian and Russian Crises and those on periphery country assets before the Eurozone Crisis (e.g. Kamin and von Kleist, 1999; Duffie et al., 2003; Gilchrist and Mojon, 2018).

The real investment and output in each country is determined by how firms trade off investment and liquidity risk management, which is in turn driven by credit market conditions they face. This trade-off leads to risky investment decisions by firms in high exposure countries, and these firms make similar investment decision to firms in low exposure countries. It follows that when exports are bold, both low and high exposures countries enjoy high output. However, when investors turn cautious, international credit markets are plagued by funding mismatch, and the high exposure countries undergo a drastic output collapse. Thus the two groups of countries experience very different real outcomes in downturns. Put differently, firms in high exposure countries *gamble*: they produce at high scale in booms, at the expense of a abandoning production in busts. This further implies that aggregate economic activity is non-monotonic across countries.

Our model further implies that most of non-performing debt is issued in booms, in high exposure countries, and financed by low-skilled investors. We connect this pattern to the heterogeneous borrowing and investment patterns in Spain preceding the Eurozone crisis, where the real estate boom was fueled by the borrowing and lending activity between a politically connected group of firms and banks, and to crony lending in Thailand and Phillipines preceding the East-Asian Crisis (see Corsetti et al., 1999; Cuñat and Garicano, 2009).

The model gives further, yet-to-be-tested predictions that ownership of debt is most concentrated in busts, especially in high exposure countries. Also the realized return on the sovereign bonds issued in booms (busts) in a given country is higher (lower) in low than high exposure countries.

Our framework emphasizes the roles of sophisticated and unsophisticated capital in international capital markets. We argue that this also sheds new light on the effects of global saving glut on investment cycles, and on safe asset supply. To highlight this, we analyze the effect of increased capital supply by low-skilled investors. Consistent with the literature on rising global imbalances (see for a review Caballero et al., 2017), this decreases the yield on bonds in a boom. It also increases the supply of safe assets as defined by (He et al., 2016), but not nearly by enough to satisfy the

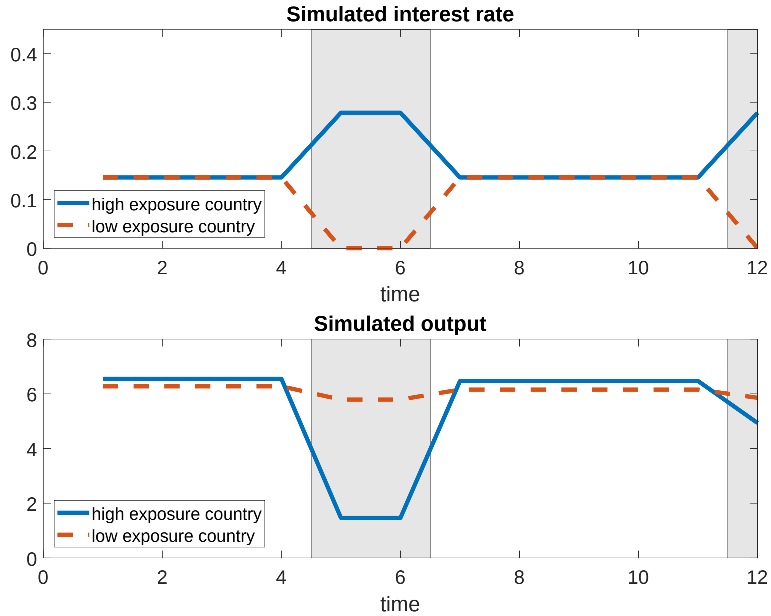


Figure 1: Simulated interest rate and output paths for a low(solid) and a high (dashed) transparency country. Shaded areas correspond to low aggregate states.

increasing demand for safe assets in busts. Additionally, during a bust some countries are pushed from the low to the high exposure group. High exposure countries experience exacerbated boom-bust cycles as well.

Finally, to best illustrate how our model generates the heterogeneous global cycles, we build a simple dynamic version of the model with consecutive generations of firms and investors. Figure 1 illustrates the simulated path for yields and output, for a representative high and a low exposure country. The output of the high exposure country is larger when investors are bold, but collapses sharply when investors turn cautious (shaded areas), and the yield at which its credit is traded spikes. The low exposure country experiences only moderate drop in its output in bust, while the yield on its bonds can even drop in the low state.

Related Literature. Our paper is the first to show that frictions in global supply of capital partitions countries into a low and a high exposure groups, creating heterogeneous global cycles. Still our work builds on a large and diverse literature.

Our work builds on the extensive literature started by Kiyotaki and Moore (1997) which originates boom and bust patterns from financial frictions. The main mechanism in these papers is that the collateral constraint tightens in recessions due to the collapsing value of the collateral (e.g. Kiyotaki and Moore, 1997; Lorenzoni, 2008; Mendoza, 2010). The most related paper within this group is (Gorton and Ordonez, 2014) where more learning in recessions leads to a drop in the perceived quality of the collateral. While we also emphasize that investors' information differs across aggregate states, our mechanism is not via tighter collateral constraints in recessions. Instead, due to prudence shock low-skilled investors rebalance towards firms in more transparent countries. As a consequence, on top of the time-series pattern, we also derive predictions on the cross-sectional differences of real outcomes.

There is also a group of papers[3] which connects flight-to-quality episodes to international risk-sharing. For instance, Gourinchas et al. (2017) and Maggiori (2017) argues that the US financial sector is less risk averse, or less constrained then others, therefore it takes a leveraged position in the global risky asset in booms and deleverages in busts. Given the two country representative agent approach of these papers, they are better fitted to capture the characteristics of capital flows between US and the rest of the world. At the same time, they are silent on a number of dimensions by construction – for instance, on the cross-sectional differences in returns, stock of non-performing debt or on concentration in debt ownership – of which our novel structure can give a detailed picture. Therefore, we think of our modeling approach as complementary to this literature.

Another stream of the literature studies why sudden stops are more frequent in emerging market countries. Aguiar and Gopinath (2007) and Rey and Martin (2006) point to technological differences, while Eichengreen and Hausmann (1999), Caballero and Krishnamurthy (2003) and Broner and Ventura (2016) point to differential incentives of saving in foreign or domestic currency as a consequence of differences in country fundamentals. In contrast, we propose a mechanism under which heterogeneous patterns can arise within advance economies also, where the available technology, the level of human and physical capital and the legal-economic-political system are similar.

Turning to the Eurozone crisis, a series of papers emphasize a wide range of issues from misallocation of credit (Reis, 2013), the political connections of some banks and firms (Cuñat and Garicano, 2009), compromised structural reforms (Fernandez-Villaverde et al., 2013), the role of downward wage rigidity (Schmitt-Grohé and Uribe, 2016), the role of perceived risk of a euro-zone breakup (Battistini et al., 2014), the interaction between risk-shifting incentives of banks and sovereigns (Farhi and Tirole, 2016), coordination problems between monetary and fiscal policy (Aguiar et al., 2015), and the role of private debt expansion (Martin and Philippon, 2017). Our mechanism is complementary to these papers. Moreover the loose financing conditions and the resulting ex-ante expansion of debt in periphery countries is exogenous in this stream of papers, while our focus is to generate this pattern endogenously.

By topic, we are related to the literature which connects the international capital flows and safe asset scarcity (He et al., 2016; Caballero et al., 2017; Farhi and Maggiori, 2017). Our model derives the equilibrium supply and demand for safe assets from informational frictions, a new element in this literature. As we demonstrate in Sections 4.3 and 5.3, this approach translates to novel predictions.

There is also a group of finance papers rationalizing flight to quality on financial markets due to Knightian-uncertainty shock (Caballero and Krishnamurthy, 2008), to fund managers incentives,

[3]See Gourinchas and Rey (2014) for a detailed review

(Vayanos, 2004), or by adverse-selection (Fishman and Parker, 2015). In contrast, we emphasize the interaction of flight-to-quality with real investment decisions and differential credit booms across countries.

Finally, in our formalization and construction of the credit market equilibrium, we build heavily on Kurlat (2016).

2 Model

Consider a three-period model, $t = 0, 1, 2$, with a single perishable good. There are two main groups of agents in the model. The first group are firms, who invest and produce, and are located across a continuum of countries. The second group are international investors, who provide financing for firms. There is a third type of agents in the model, bankers, who are competitive and only provide a frictionless contingent saving technology to the local firms. All agents are risk neutral, and there is no discounting. Agents maximizes expected sum of consumption across all periods.

We start this section with a description of the components of the model. We then proceed to agents' optimization problem and equilibrium definition, and finally provide a discussion of model ingredients.

2.1 Set up

Shocks. The model has one aggregate shock (θ) and one idiosyncratic shock, both publicly observable. Shocks are sequentially realized at $t = 1$, aggregate shock being realized first.

The aggregate state is either $\theta = H$ (high) or $\theta = L$ (low). Let π_θ denote the probability of aggregate state θ. The idiosyncratic (liquidity) shock is at the firm level, and a firm hit by the idiosyncratic shock needs a liquidity injection to continue investment. Let ϕ denote the iid probability of receiving a liquidity shock. We will explain both shocks in further detail below.

Firms and Production Technology. There is a double continuum of firms, indexed by $j = (\omega_f, \tau)$. Firms invest and produce, and are subject to liquidity shocks.

$\omega_f \in [0, 1]$ denotes the *transparency* of the firm, where $\omega_f = 0$ is the most opaque and $\omega_f = 1$ is the most transparent firm. Firm transparency relative to the expertise of investors is the source of information friction in our model. We will explain this friction in detail below.

$\tau \in \{g, b\}$ denotes the *(pledgeability) type* of the firm, where g (b) is a good (bad) firm. λ $(1-\lambda)$ fraction of all firms are good (bad), and they are distributed iid across transparency classes. The type of the firm determines the fraction of its output lenders can seize.

Each firm is endowed with a technology akin to Holmström and Tirole (1998) and Lorenzoni (2008), and one unit of good. At $t = 0$, firm $j = (\omega_f, \tau)$ chooses the initial investment $I(\omega_f, \tau) \leq 1$, and saves the reminder of his endowment using the bankers.

At $t = 1$, after the realization of the aggregate state θ, a fraction ϕ of firms are hit by a liquidity shock. The liquidity shock is observable and verifiable by all agents. Any such firm has to inject extra ξ units, per unit of initial investment that the firm wants to maintain. Any unit that does not receive the liquidity injection fully depreciates. Thus such firm chooses to drive $i(\omega_f, \tau; \theta)$ unit of investment to completion, where

$$i(\omega_f, \tau; \theta) \leq I(\omega_f, \tau),$$

and abandon the rest of his initial investment. The firm finances the liquidity injection (maintenance cost) from his savings and/or by issuing bonds to international investors.

At $t = 2$ each unit of completed investment produces ρ_τ units of good, where $\rho_g \geq \rho_s > \xi$. This implies that for a firm hit by the liquidity shock, the total cost of producing $\rho_\tau i(\omega_f, \tau; \theta)$ units is $I(\omega_f, \tau) + \xi i(\omega_f, \tau; \theta)$, while for a firm not hit by the liquidity shock the total cost of producing $\rho_\tau I(\omega_f, \tau)$ units is $I(\omega_f, \tau)$.

In line with Holmström and Tirole (1998), we make the following assumption on the production technology.

Assumption 1 *Continuing with full scale, and abandoning production after a liquidity shock are socially positive NPV for both good and bad firms,*

$$\rho_\tau > \max(1 + \phi\xi, \frac{1}{1-\phi}), \qquad \tau = g, b.$$

Countries. Firms are distributed among a unit mass of countries. We assume that the distribution is iid with respect to the firm pledgeability type τ, but not with respect to the transparency ω_f. Let $\omega = \mathbb{E}[\omega_f]$ denote the implied transparency of the country, which is defined as the average transparency of the firms in that country. We assume that the investors do not know the distribution of firms across countries, and have an uninformative prior about ω. Thus from investor perspective, all countries are identical.[4]

To isolate our main mechanism, we assume that each country is populated with firms of a single transparency class and no other firms. Thus, we can index the countries by ω up to a random permutation. Since all the firms in country ω have the same transparency level, to save on notation, in the reminder of the paper we will use ω to index firm transparency as well.

Banks and Saving Technology. A state-contingent saving technology is available at actuary fair terms to all firms through local banks. Bankers are competitive, deep-pocketed agents who do not have the expertise to seize any future income of firms. Thus they cannot lend to firms, but firms can save towards future aggregate or idiosyncratic states with bankers.

[4]See section 2.3 for a discussion.

International Experts. There is a continuum of investors, indexed by their *skill* level, $s \in [0, 1]$. Each investor is endowed with one unit of good in period $t = 1$, and can provide financing to (a selected subset of) firms who demand liquidity. Experts can seize exactly ξ per unit of investment only from good firms, that is, with perfect information, investors can provide full financing for the liquidity shock.[5]

However, investors are subject to an information friction. They have imperfect and heterogeneous information about firm type. Higher s investors have higher quality information, as we specify below. They use their expertise to lend to (potentially a subgroup of) firms hit by a liquidity shock in $t = 1$.

Let $w(s)$ denote the type-density of investors. We assume $w(s)$ is continuous, and strictly monotonically decreasing, $w'(s) < 0$. This assumption means that smart capital is in short supply.

Aggregate Shock and Information Friction. Each investor has a prior $1 - \lambda$ (λ) that a given firm is good (bad). She searches for evidence about the true type of each firm who demands liquidity. Then, the investor chooses an acceptance rule $\chi(\omega, \tau; \theta) \in \{0, 1\}$, which specifies which bonds she is willing to buy. Beyond that, he rule has to be measurable with respect to the information the investor obtains by collecting evidence. Let X_s denote the set of such acceptance rules for investor s.

The information of investor s about firm $j = (\omega, \tau)$ depends on her expertise level, s, transparency of firm country of origin, ω, and the aggregate state, θ. Let $x(\tau; \omega, s, \theta)$ denote this information.

The type of evidence that investors are searching for is determined by the aggregate state. We call the aggregate shock *prudence shock* as it determines how prudent the investors are. If the aggregate state is high, $\theta = H$, investors are *bold*. They are willing to buy the bond issued by a firm, unless they find conclusive evidence that the issuer is a bad firm. Thus they search only for evidence whether a firm is bad. If the investor is sufficiently skilled relative to how opaque a bad firm is, the evidence is conclusive. Otherwise, the search is uninformative. Thus we have

$$x(\tau; \omega, s, H) = \begin{cases} b & \text{if } s > 1 - \omega \text{ and } \tau = b \\ \varnothing & \text{otherwise} \end{cases} \tag{1}$$

where $x = b$ is evidence that the firm is bad, while $x = \varnothing$ implies that the investor have not found evidence. Thus a bold investor of skill s identifies the bad firms that are sufficiently transparent, $\omega \geq 1 - s$. However, she does not find such evidence for bad firms of lower transparency, neither for any good firms. As such, bold investors only make false positive mistakes.

In contrast, if the prudence shock is low, $\theta = L$, investors are *cautious*. They are willing to buy a bond only if they find conclusive evidence that the bond is issued by a good firm. Thus they search only for evidence whether a firm is good. Again, the evidence is conclusive only if the

[5]The model can easily be extended to the case where investors can seize $\xi_\Delta > \xi$.

investor is sufficiently skilled and the firm is good. That is,

$$x(\tau;\omega,s,L)=\begin{cases} g & \text{if } s>1-\omega \text{ and } \tau=g \\ \varnothing & \text{otherwise} \end{cases} \tag{2}$$

Thus a cautious investor of skill s identifies the good firms that are sufficiently transparent, $\omega \geq 1-s$. However, she does not find such evidence for good firms of lower transparency, neither for any bad firms. As such, cautious investors only make false negative mistakes.[6]

Liquidity Shock and Bond Issuance. At $t=1$, a firm $j=(\omega,\tau)$ can issue bonds on the international market to raise financing from a subset of investors who are willing to lend to him. The firm receives one unit of financing per bond issued, and promises to pay back $1+r(\omega,\tau;\theta)$ at date $t=2$. The repayment is subject to the pledgeability constraint. The interest rate $r(\omega,\tau;\theta)$ is determined in equilibrium.

Market Structure. We model the international credit market in the spirit of Kurlat (2016). At $t=1$ many markets open. Each market m is defined by an interest rate $\tilde{r}(m)$, and it can be active or inactive in equilibrium. The set of all markets is denoted by M. A market is active if both firms and investors are present at that market.

Firms can go to as many markets as they desire, and demand credit for the corresponding interest rate $\tilde{r}(m)$. The firm chooses how many bonds (σ) it demands to issue at each market, such that

$$\sigma(m,\omega,\tau;\theta) \leq \bar{L}, \tag{3}$$

where $\bar{L}$ is the exogenous capacity limit for all markets. We assume that the capacity limit $\bar{L}$ corresponds to the maximum demand any good firm would submit.[7]

Each investor s chooses (at most) one market m to buy bonds from, number of bonds he wants to finance δ, and an acceptance rule $\chi(\omega,\tau;\theta)\in\{0,1\}$ he will use. The acceptance rule specifies which bonds the investor is willing to buy, and has to be measurable with respect to the investor's collected evidence $x(\tau;\omega,s,\theta)$. Note that investors cannot observe, thus cannot condition their decisions on the total number of bonds a given firm is issuing.

If there are investors of multiple skills offering credit at a given m, the transactions of the least selective investors, i.e. those with least informative evidence clears first. The formal definition is provided in the Appendix.

Markets do not have to clear. In particular, firms understand that in each state θ, for each firm

[6] See section 2.3 for a discussion.

[7] This assumption is intuitive since markets should allow for orders consistent with the needs of all good firms, but should not increase maximum order size to a range where only bad firms would submit. We also believe the assumption could be derived, but at this point we keep it exogenous.

$j = (\omega, \tau)$ and each market m, there is an equilibrium measure $\eta(m, \omega, \tau; \theta)$ such that firm j issues $\eta(m, \omega, \tau; \theta)$ units of bond per unit of demand submitted to market m. We call $\eta(m, \omega, \tau; \theta)$ the rationing function. As such, we define the total number of bonds issued by firm j, $\ell(\omega, \tau; \theta)$, and j's effective interest $r(\omega, \tau; \theta)$ as

$$\ell(\omega, \tau; \theta) \equiv \int_M \sigma(m, \omega, \tau; \theta) d\eta(m, \omega, \tau; \theta) \tag{4}$$

$$r(\omega, \tau; \theta) \equiv \frac{\int_M \tilde{r}(m)\sigma(m, \omega, \tau; \theta) d\eta(m, \omega, \tau; \theta)}{\ell(\omega, \tau; \theta)}. \tag{5}$$

Thus an investor s who chooses market m with interest rate $\tilde{r}(m)$, buys a representative portfolio of bonds issued by the pool of firms who 1) demand to issue bonds at m, 2) satisfy investor s acceptance rule based on her evidence on the firm, and 3) their bonds are not exhausted by investors less selective than s.

Finally, equilibrium supply and demand determines the allocation function $A(\omega, \tau; \chi, m, \theta)$, the measure representing the fraction of bonds of firm $j = (\omega, \tau)$ financed by investor s with acceptance rule χ in market m. Expert s choice of how many bonds he wants to finance, δ, has to satisfy his budget constraint. That is, the total number of bonds allocated to an investor has to at most equal to his unit endowment.

This market structure allows for many-to-many matching. A given firm might obtain credit from a group of heterogeneous investors (as described by the rationing function, η), and a given investor might buy the bonds issued by a pool of heterogeneous firms (as described by the allocation function, A).

2.2 Equilibrium Definition

Firm and Expert Problems, and Timing. We next summarize the time line of the model, along with firm and investor optimization problems.

At $t = 0$, each firm chooses how much to invest and how much to save to insure the risk of liquidity shock. As long as the interest rate is not prohibitively high, the optimal decision for firm $j = (\omega, \tau)$ who is hit by a liquidity shock is to issue the maximum number of bonds, at interest rate $r(\omega, \tau; \theta)$, without violating the pledgeability constraint.[8] Thus for a given continuation decision $i(\omega, \tau; \theta)$ in state θ,

$$\ell(\omega, \tau; \theta) = \frac{1}{1 + r(\omega, \tau; \theta)} \xi i(\omega, \tau; \theta), \tag{6}$$

[8]As firms cannot increase their investment above $I(\tau_i, k)$ and investors lend against units of maintained investment, firms cannot borrow more from investors. See appendix for the detail of the other direction of inequality.

where

$$i(\omega, \tau; \theta) \leq I(\omega, \tau). \tag{7}$$

The firm finances the rest of the liquidity needs by state-contingent saving from own initial endowment. Thus the firm ex-ante budget constraint can be written as

$$I(\omega, \tau) + \phi\xi \sum_{\theta} \pi_\theta \frac{r(\omega, \tau; \theta)}{1 + r(\omega, \tau; \theta)} i(\omega, \tau; \theta) = 1 \tag{8}$$

At $t = 1$, after the realization of the aggregate state and the liquidity shock, each firm submit his demand for bond issuance to a subset of markets, taking each market's interest rate and rationing function as given.

The problem of a firm $j = (\omega, \tau)$ can be written as

$$\max_{I(\omega,\tau),\{i(\omega,\tau;\theta)\}_\theta,\{\sigma(m,\omega,\tau;\theta)\}_{\theta,m}} \sum_{\theta} \pi_\theta \Big[\rho_\tau \big(\phi i(\omega, \tau; \theta) + (1 - \phi) I(\omega, \tau)\big) - \mathbb{1}_{\tau=g} \phi\xi i(\omega, \tau; \theta)\Big] - 1 \tag{9}$$

subject to (3)-(8).

It is important to note that firms face a different problem only to the extent that they face a different interest rate $r(\omega, \tau; \theta)$. Therefore, heterogeneous decisions about initial and continued investment is driven by heterogeneous expected financing conditions in credit markets.

Experts arrive in $t = 1$, once both the aggregate and idiosyncratic shocks are realized. They have unit wealth and consume in $t = 1, 2$. Each investor picks a market, m, and submit her acceptance rule, χ, and supply of credit, δ. Thus the problem of investor s in aggregate state θ can be written as

$$\max_{m,\chi,\delta} \delta \left[(1 + \tilde{r}(m; \theta)) \int_{\omega} dA(\omega, g; \chi, m, \theta) - \int_{(\omega,\tau)} dA(\omega, \tau; \chi, m, \theta)\right] + 1 \tag{10}$$

s.t.

$$\chi \in X_s$$

$$\delta \int_{(\omega,\tau)} dA(\omega, \tau; \chi, m, \theta) \leq 1$$

Since investors' objective function is linear in δ, they choose $\delta = 0$ if the net return on bond is negative; and δ is determined by their resource constraint otherwise.

We end this section with the formal definition of equilibrium.

Definition 1 *A global equilibrium is a set of firm investment plans* $I(\omega, \tau), \{i(\omega, \tau; \theta)\}_{\theta=H,L}$ *and demand function for credit* $\{\sigma(m, \omega, \tau; \theta)\}_{\theta=H,L}$*, investors choice of interest rate,* $\tilde{r}(m; \theta)$*, and acceptance rule* $\chi(s, \omega)$*, along with a rationing function* $\{\eta(m, \omega, \tau; \theta)\}_{\theta=H,L}$*, allocation function*

$A(\omega, \tau; \chi, m, \theta)$, and interest rate schedule $\{\tilde{r}(m;\theta)\}_{\theta=H,L}$ and the corresponding $\{r(\omega,\tau;\theta)\}_{\theta=H,L}$, such that

(i) firm investment plan and demand function solves firm optimization problem (9)*, given the rationing function and the interest rate schedule;*

(ii) investor choice of interest rate and the corresponding acceptance rule maximizes investor optimization problem (10)*, given the allocation function and the interest rate schedule;*

(iii) rationing functions, allocation functions and interest rate schedules are consistent with investor and firm optimization.

2.3 Comments

In this section we discuss two simplifying assumptions in detail which the reader might find surprising, and point the reader to relevant extensions in later sections.

We start with the assumption that, from the point of view of investors, countries are ex-ante identical. This assumption has two parts.

First, we assume that the production fundamentals in each country are the same. There is the same fraction of good and bad firms and those firms are identical across countries. We make this assumption solely for expositional reasons. While we do not doubt that there are fundamental differences between emerging and developed markets, or core and periphery countries in the European context, we are interested in the heterogeneous implications of frictions in the availability of international capital (the supply side) across countries. Therefore, we suppress difference in production fundamentals (the demand side).

Second, we assume that investors have an uninformative prior about ω, the average transparency of firms in a given country. That is, if they do not find conclusive evidence on a firm with a given country of origin, the name of the country does not help them to understand whether it is because there is no evidence, or it is because they are not skilled enough to find that evidence.

While it is important for our results that investors' prior information is coarse on ω, we show in section 7.2 we do not need the extreme assumption that their prior is uninformative. For example, in the European context, they might know that it tends to be harder to learn about Italian and Spanish firms than German firms, as long as they are uninformed on how Spanish and Italian firms compare to each other. In section 7.2, we show that all our results are robust to such a generalization.

Intuitively, we think of the coarseness of prior information on ω of a country, as an assumption capturing the fact that boom-bust patterns are often preceded by major changes in the affected countries contributing to investors' uncertainty on ω. For example, the introduction of the European Monetary Union, or the major economic reforms preceding the fast growth of East Asian countries perhaps led investors to rely less on their existing knowledge of these markets.

Another simplifying assumption is that we treat the realization of the aggregate state as an exogenous "prudence shock " throughout the main body of the paper. This exogenous treatment allows us to focus on the credit market and real outcomes in each state, and explore the interactions and spillovers across the two states. Later in section 7.1 we provide two alternative micro-foundations for the different prudence shocks to arise endogenously. In the first micro-foundation, an aggregate productivity shock triggers different prudence shocks , while in the second micro-foundation different prudence shocks arise due to changing sentiment. See also Philippon (2006) and Bouvard and Lee (2016) for theory and evidence suggesting that firms are indeed less prudent in economic booms.

3 Global Equilibrium

In this section, we characterize the global equilibrium. We start with the analysis of two simple benchmarks, and then move to the full model. All the proofs are in Appendices B.1.3 and B.2.

3.1 Benchmark Economies

Here we study two benchmarks: when international capital markets are completely shut down, and when there is abundant skilled capital. The key feature shared by the two benchmarks is that heterogeneity in investor skill level does not affect the ability of firms acquiring financing. We show that in both cases, ex-ante identical countries are ex-post identical as well. Moreover, the aggregate shock does not affect any of the outcomes.

Credit Market Shutdown. In this benchmark, firms are unable to raise any financing, thus each firm is in autarky. Formally, assume $w(s) = 0, \forall\, b$.

Abundant Smart Money. In this case, the only constraint that good firms face in raising funding is the pledgeability constraint. Formally, we assume each investor has K units of wealth, $K \to \infty$. In particular, $w(1) \to \infty$, thus the most skilled investors are sufficiently wealthy to absorb the liquidity demand by all good firms.

The first proposition describes the equilibrium quantities and prices in the two benchmarks.

Lemma 1 [Benchmark Economies]

(i) Credit market shutdown: $\forall\, \theta,\ \forall\, j = (\omega, \tau)$

(a) If $\xi > \frac{1}{1-\phi}$, *then* $I^A(\omega, \tau) = 1$ *and* $i^A(\omega, \tau; \theta) = 0$,

(b) Otherwise, $i^A(\omega, \tau; \theta) = I^A(\omega, \tau) = \frac{1}{1+\phi\xi}$.

Moreover, the total output is identical across countries and across states,

$$Y^A(\omega,\theta) = \rho_\tau \max\left(1-\phi, \frac{1}{1+\phi\xi}\right).$$

(ii) Abundant smart money: $\forall\,\theta, \forall\,\omega$

(a) $i^{FL}(\omega, g; \theta) = I^{FL}(\omega, g) \to 1$. *Moreover, good firms face zero interest rate,* $r(\omega, g; \theta) \to 0$.

(b) $I^{FL}(\omega, b) \to 1$ *and* $i^{FL}(\omega, b; \theta) = 0$.

The total output is identical across countries and across states,

$$Y^{FL}(\omega,\theta) = \rho_\tau\left(1-\lambda+\lambda(1-\phi)\right)$$

While unsurprisingly, total output in each country is smaller when the credit market is shut down, it is identical across countries and aggregate states in both cases. Thus these benchmarks emphasize that in our model, all the fluctuations across countries, and for different aggregate prudence shock , comes from the fact that credit is provided by investors with scarce capital and imperfect information.

Moreover, Lemma 1 clearly exhibits one of the key trade offs of the model, the trade off between investment scale and liquidity risk management. Even when firms do not have access to capital markets, they can insure against the future liquidity shock by saving some of their own endowment using the bankers. The first part of the lemma shows that if the liquidity shock is large relative to the probability that it hits the firm, then it is too costly for the firm to insure against it. Thus the firm foregoes risk management, enjoys high output when it is not hit by the shock but has to liquidate when it is.

3.2 Simple Global Equilibrium

We now turn to equilibrium characterization for the full model, when there is heterogeneity in liquidity supply. In the Appendix, we discuss different variants of the equilibrium that arise depending on the choice of parameters. However, in order to highlight the main mechanism of the model we restrict focus on the simplest variant, and call it the "simple global equilibrium". To construct the equilibrium we proceed by backward induction. We start by analyzing the credit market outcome given initial investment choices as given, and then characterize the equilibrium real quantities.

3.2.1 Equilibrium Interest Rates and Credit Allocation

The first step to solve for the equilibrium is to characterize the interest rates that different firms face when they raise liquidity, and the corresponding number of bonds they are able to issue, for the two different aggregate states.

The next two propositions describe the interest rates and corresponding allocation of financing in the international credit market, given the ex-ante investment functions $\{I(\omega,\tau)\}_{\omega,\tau}$. Figure 2 illustrates the equilibrium for a given set of parameters. A more detailed statement, as well as the relevant parameter restrictions, are provided in the Appendix.

Proposition 1 [International Rates]

(i) If $\theta = H$, there is a single common interest rate, r_H. There is a $s_H \in [0,1]$ such that only more skilled investors, $s \geq s_H$ participate in the bond market.

(ii) If $\theta = L$, there is an interest rate schedule, $r_L(\omega)$, at which good firms from country ω raise credit. The interest rate schedule is characterized by endogenous thresholds $0 \leq \omega_2 \leq \omega_3 \leq 1$, such that

$$r_L(\omega) = \begin{cases} 0 & \text{if } \omega \in (\omega_3, 1] \\ \hat{r}(\omega) & \text{if } \omega \in (\omega_2, \omega_3] \\ \bar{r} & \text{if } \omega \in (0, \omega_2] \end{cases} \tag{11}$$

where $\hat{r}(\omega)$ is a continuous, decreasing function, with $\hat{r}(\omega_2) = \bar{r}, \hat{r}(\omega_3) = 0$.

Proposition 2 [International Credit Allocation]

(i) If $\theta = H$, each good firm issues $\xi I(\omega,g)/(1+r_H)$ bond and maintains investment at full scale, $i(\omega,g;H) = I(\omega,g)$.

Bad firms partially liquidate investment. Specifically, there is a weakly decreasing function, $\eta_H(\omega)$, such that $i(\omega,b;H) = \eta_H(\omega)\bar{L}/\xi \leq I(\omega,b)$, with $\eta_H(0) = 1$ and $\eta_H(1) = 0$.

(ii) If $\theta = L$, bad firms do not obtain any credit, and terminate investment.

There is $\omega_1 \in (0,\omega_2)$ such that good firms from countries $\omega \in (\omega_1, 1]$ maintain investment at full scale, while good firms from countries $\omega \in [0,\omega_1]$ partially liquidate. Specifically, there is a weakly increasing function, $\eta_L(\omega)$, such that $i(\omega,g,L) = \eta_L(\omega)I(\omega_1,g,N)/\xi$ for $\omega \in [0,\omega_1]$, with $\eta_L(0) = 0$ and $\eta_L(\omega) = 1$.

As the proposition states, when investors are bold only those are active who has a sufficiently high expertise, $s \geq s_H$ and all of them pick the same market. That is, all credit are obtained at the same interest rate r_H. Recall that in this state all good firms are known, but low s investors cannot distinguish bad firms in low ω countries from good ones. This is why all bad firms are rationed: they obtain credit from only those investors who mistake them for good firms. However, good firms are fully financed. The interest rate r_H and the marginal type s_H are pinned down by two conditions. First, the interest rate has to compensate for the default rate in the marginal type's obtained pool. Second, the wealth of all participating investors has to be sufficient to cover the required credit of good firms.

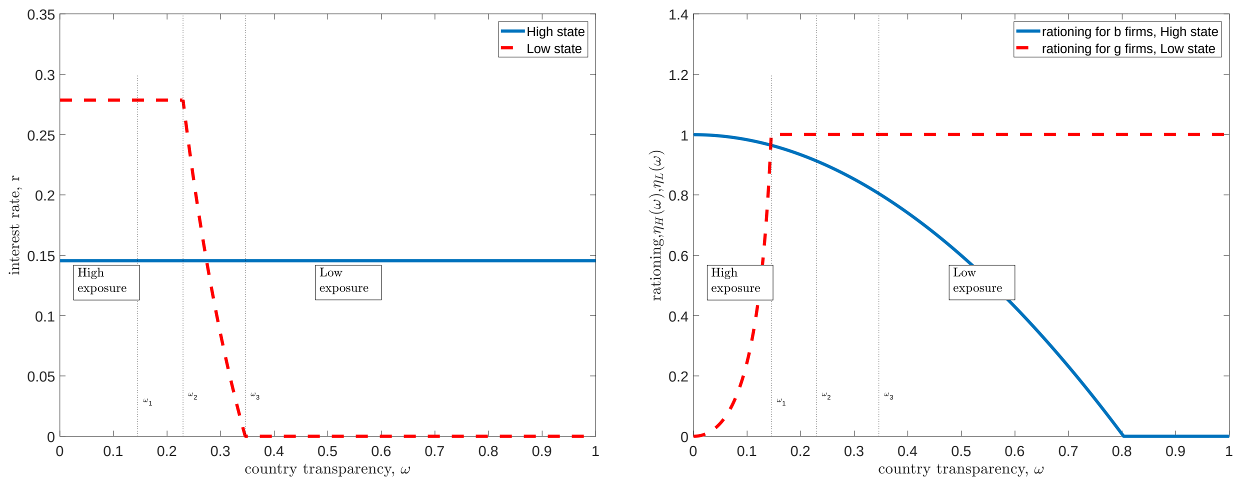


Figure 2: Premia for credit and fractions of bad and good countries financed by country in high state (solid) and low state (dashed) regimes.

Apart from this particular market, m_H, good firms demanding maximum quantity, $\bar{L}$ credit at every market with lower interest rate, while bad firms are demanding the same in all markets with lower and higher interest rate. Experts would like to get higher interest rate, but at those markets good firms are not present. Separation is not possible because (i) all good firms look the same to all investors, so they cannot be served at different markets (ii) for any interest rate the demand of bad firms is weekly higher than that of bad firms as their effective cost of credit is lower due to not paying back. Hence, bad firms would follow good ones to any market.

Recall that when investors are cautious, all bad firms are identified, so none of them can obtain any credit. However, low s investors cannot distinguish good firms in low ω countries from the bad ones. As a result, good firms from high ω countries can exploit these investors by pooling at the 0 interest rate market. These lowest s investors serve this group even for 0 interest rate as these firms are the only ones they can identify as good ones. As $w(s)$ is decreasing, there is a critical country, ω_3 that total amount of the endowment in $t = 2$ of investors who can just identify good firms from this country is just sufficient to satisfy their demand at 0 interest rate. Therefore, in a range $[\omega_2, \omega_3]$ the market clears at interest rate $\hat{r}(\omega) > 0$ which equates the credit supply and credit demand. Essentially, in this region we have cash-in-the-market pricing in the spirit of Allen and Gale (2005). However, as we will argue, there is an upper bound $\bar{r}$ of how much any good firm is willing to pay for a unit of credit. Threshold ω_2 is determined by $\hat{r}(\omega_2) = \bar{r}$. Intuitively, the mass of investors who are sufficiently skilled to be able to identify good firms in countries $\omega < \omega_2$ are sufficiently scarce that these investors can ask a maximum interest rate, $\bar{r}$ as a rent. What is more, the total endowment of this group of skilled investors might not be sufficient to satisfy all the demand of good firms in countries $\omega \in [0, \omega_2]$. In this case, there is a positive ω_1 that firms from the lowest ω of countries $\omega \in [0, \omega_1]$ are rationed. The smaller the level of capital of the investor

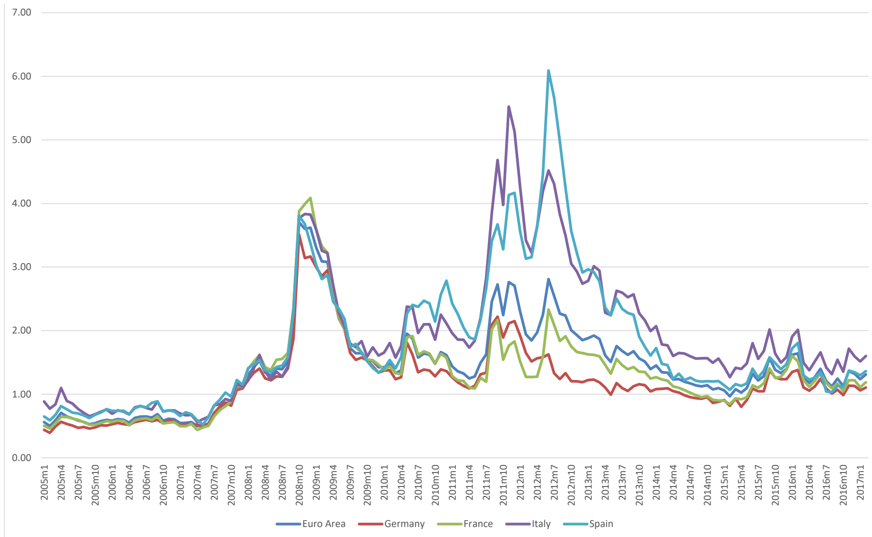


Figure 3: Euro area non-financial corporate credit spreads compared to German treasuries of equal maturity as estimated by Gilchrist and Mojon (2018)

who can identify a good firm from a given country in this group, the less capital this firm obtaines.

Clearly, when investors are cautious firms cannot be served at the same interest rate. If this interest rate were positive, a good firm from the highest ω country would be motivated to demand credit at 0 interest rate only knowing that she is the only good firm the worst type of investor can identify. Pooling at 0 is also impossible as investors do not have enough funds to satisfy all firms at 0.

As Figure 2 illustrates, we refer to the group of countries served at 0 interest rate, $\omega \in [\omega_3, 1]$ as *low exposure*, while the group where even good firms are rationed, $\omega \in [0, \omega_1]$ as *high exposure*.

We connect the model implied interest rate schedule on Figure 2 to the dynamics of Euro area corporate credit spreads on Figure 3. The shift to the low aggregate state in our model corresponds fragmentation of the corporate bond market around 2010. The figure illustrates that by 2010 non-financial firms active in the corporate bond market, treated close to equal before the Greek crisis (just as in the high aggregate state in our model), suddenly faced very different market conditions depending on their country of origin. Whether an investment grade firm was French or Italian did not seem to matter before or even during the crisis in 2008-2009. By 2011, Italian firms paid a much higher interest rate for credit then French firms.

3.2.2 Equilibrium Quantities

In this section, we show how each firm (ω, τ), foreseeing the equilibrium in the credit market as we described in Proposition 1, choose their investment plan, $I(\omega, \tau_j), \{i(\omega, \tau; \theta)\}_\theta$.

The next Proposition describes the good firms' optimal choice.

Proposition 3 [Good Firm Investment] *In a simple global equilibrium a good firm* (ω, τ) *chooses*

$$I(\omega, g) = \frac{1 + (1 - \eta_L(\omega)) \frac{\phi \xi \pi_L \frac{\bar{r}}{1+\bar{r}}}{1 + \phi \xi \pi_H \frac{r_H}{1+r_H}}}{1 + \phi \xi \left(\pi_H \frac{r_H}{1+r_H} + \pi_L \frac{r_L(\omega)}{1+\bar{r}_L(\omega)} \right)} \tag{12}$$

$$i(\omega, g; H) = I(\omega, g) \tag{13}$$

$$i(\omega, g; L) = \eta_L(\omega) I(\omega, g) \tag{14}$$

The Proposition states that all firms continue with full scale when investors are bold. Most good firms also continue with full scale when investors are cautious, except those from high exposure countries. As we described above, the market for firms from high exposure countries (partially) dries up when investors are cautious because of the scarcity of investor capital. Importantly, firms foresee this and choose their initial investment, $I(\omega, g)$, accordingly. Given the firm's investment policy and market conditions the firm expects to face, initial investment is determined by constraint (8). This constraint demonstrates a simple trade-off. The cost of credit for maintanance after a liquidity shock limits the initial investment the firm can afford. An immediate consequence of (8) is the scale of economic activity of good firms is non-monotonic across the exposure spectrum. Good firms in low exposure and high exposure countries invest more than firms in countries in between, because they spend little on credit when investors are cautious: firms in the low exposure region obtain them at zero cost, while firms in the high exposure region can obtain very little and abandon most of their production in state $\theta = L$ when they receive a liquidity shock.

Note also that the maximum interest rate $\bar{r}$ in the low state is the level for which a firm is indifferent between a large initial investment at the expense of abandoning a part or all of its production when $\omega_j = H$, and obtaining sufficient credit for this high interest rate to continue with full scale in all states on the expense of a smaller initial investment. In this sense, those firms who opt for the earlier are gambling.

Figure 4a shows total investment in each country, $I(\omega, g) + \phi \xi i(\omega, g, \theta)$ across aggregate prudence shocks . As when investors are bold good firms produce with full capacity, total investment inherits the non-monotonic pattern of the initial investment, $I(\omega, g)$, across countries. When investors are cautious, there is a collapse in investment in the high exposure countries as firms hit by the liquidity shock abandon production.

Bad firms' investment plan choice differs from that of good ones because they face different conditions in the market for credit. They understand that they are not be able to obtain any credit when investors are cautious (as all investor recognize them as bad firms) and they are rationed when investors are bold. The next proposition describes their optimal choice.

Proposition 4 [Bad Firm Investment] *In a simple core periphery equilibrium bad firms choose* *investment plan*

$$
\begin{aligned}
I(\omega, b) &= 1 - \frac{r_H}{1+r_H}\phi\pi_H\eta_H(\omega)\bar{L} \\
i(\omega, b; H) &= \frac{\eta_H(\omega)\bar{L}}{\xi} \\
i(\omega, b; L) &= 0.
\end{aligned}
$$

Bad firms' choice of the size of their land is also determined by the trade-off embedded in the financing constraint (8). Bad firms in low ω countries can obtain more credit, which they do not plan to pay back. Their initial investment is somewhat lower than for bad firms in high ω countries as part of their capital is used up by the interest rate they pay for the obtained credit. However, as Figure 4b, the total investment of bad firms in low ω countries are higher, because their smaller initial investment is overcompensated by the fact that these credit help them to maintain investment after hit by a liquidity shocks. In contrast, when investors are cautious the latter effect is mute. Hence, total investment of bad firms is higher in high ω countries in the low aggregate state. Figure 4b also plots the total face-value of credit issued to bad firms in the high state. This is the total facevalue of non-performing credit in a given country.

3.2.3 Existence

We start by a set of sufficient conditions to ensure the existence of a simple global equilibrium.

Assumption 2 *Assume the parameters are such that*

(i) $\xi > \frac{1}{1-\phi}$

(ii) $\frac{\lambda}{1-\lambda} \leq \frac{(\rho_g-\xi)}{\rho_g\xi(1-\phi)+(\rho_g-\xi)(\phi\pi_L\xi-1)}$

(iii) $w(s)$ *is continuous, with* $w'(s) < 0$, $w(0) \geq \phi(1-\lambda)\xi$ *and* $\lim_{s\to 0} w(s) = 0$.

(iv) $\min\left\{\frac{(\rho_g-\xi)(1+\lambda\phi\xi\pi_H)}{(\rho_g(1-\phi)+\phi(\rho_g-\xi)\pi_H)\xi}, \frac{\xi\phi(1-\lambda)-w(1-\omega)}{\xi\phi((1-\lambda)+w(1-\omega)\pi_L)}\right\} \leq \frac{\lambda}{\lambda+(1-\lambda)\omega} \qquad \forall\omega$

Condition (i) ensures that without access to credit markets, firms choose to invest all of their initial endowment and do not use part of it to manage liquidity risk. It also implies that without access to credit markets firms do want to invest (rather than consume right away), which requires $\rho_\tau > \frac{1}{1-\phi}$, and follows since $\rho_\tau > \xi, \forall\, \tau$. Condition (ii) ensures two properties of the wealth function. First, low-expertise investors have sufficient wealth that some bonds are issued at zero interest rate. Second, expert capital in short supply. Condition (iii) ensures that the common interest rate is not prohibitively high when $\theta = H$, so that firms use the international markets and part of their own endowment to manage liquidity risk, as opposed to investing all of their initial endowment. Condition (iv) ensures that, when investors are cautious, there is no equilibrium interest rate for which some investors are willing to buy up all the offered securities independently of their signal.

The next proposition spells out how the equilibrium objects $r_H, s_H, \omega_1, \omega_2, \omega_3, \eta_L(\omega)$ and $\eta_H(\omega)$ are pinned down, and proves that a simple global equilibrium exists.

Proposition 5 [Existence] *For parameters satisfying assumptions 1 and 2, there exists a simple global equilibrium in which* $x^* \equiv \frac{r_H}{1+r_H} \in [0,1]$ *is the fixed point of the following equation,*

$$F\left(x\right) = \frac{\lambda \left(1 - s_H(x)\right) D\left(0; x\right)}{\lambda \left(1 - s_H(x)\right) D\left(0; x\right) + \left(1 - \lambda\right) \bar{D}(x)} \tag{15}$$

where $s_H(x)$ *solves*

$$\int_{s_H(x)}^{1} \frac{1}{\lambda(1-s)D(0;x) + (1-\lambda)\bar{D}(x)} w(s) ds = (1-x)\phi, \tag{16}$$

if equation (16) *has a positive solution, and* $s_H(x) = 0$ *otherwise.*

Moreover

$$\bar{y}(x) = \frac{\left(\rho_g - \xi\right)\left(1 + \phi \xi \pi_H x\right)}{\left(\rho_g \left(1 - \phi\right) + \phi \left(\rho_g - \xi\right) \pi_H\right) \xi} \tag{17}$$

$$D(y; x) = \frac{\xi}{1 + \phi \xi (\pi_H x + \pi_L y)} \tag{18}$$

$$\begin{aligned} \bar{D}(x) =& (1 - \omega_3(x)) D(0; x) + \int_{\omega_2(x)}^{\omega_3(x)} D(y^C(\omega); x) d\omega \\ &+ \left(\omega_2(x) + \frac{\phi \xi \pi_L \bar{y}(x)}{1 + \phi \xi \pi_H x} \int_0^{\omega_1(x)} (1 - \eta_L(\omega)) d\omega \right) D\left(\bar{y}(x); x\right). \end{aligned} \tag{19}$$

where

$$y^C(\omega) \equiv \frac{\xi \phi (1 - \lambda) - w(1 - \omega)(1 + \phi \xi \pi_H x)}{\xi \phi ((1 - \lambda) + w(1 - \omega) \pi_L)} \qquad \omega \in \left[\omega_2(x), \omega_3(x)\right]. \tag{20}$$

The rationing functions are given by

$$\eta_L(\omega) = \min \left(1, \int_{1-\omega}^{1} \frac{1}{\phi(1-\lambda)(1-\bar{y}(x))D(\bar{y}(x);x)(\omega_2(x) - (1-s)) - \int_{1-\omega_2(x)}^{1-\omega_1(x)} w(s) ds} w(s) ds \right)$$

$$\eta_H(\omega) = \min \left(1, \int_{s_H(x)}^{1-\omega} \frac{1}{\lambda(1-s)D(0;x) + (1-\lambda)\bar{D}(x)} \frac{w(s)}{\phi(1-x)} ds \right)$$

and $\omega_1(x), \omega_2(x), \omega_3(x)$ *are defined as follows.*

Let $\hat{\omega}_2(x)$ and $\hat{\omega}_3(x)$ be the solution to the following two equations, respectively:

$$w\,(1-\omega_2) - \phi\,(1-\lambda)\,(1-\bar{y}(x))\,D\,(\bar{y}(x);x) = 0, \tag{21}$$

$$w\,(1-\omega_3) - \phi\,(1-\lambda)\,D\,(0;x) = 0. \tag{22}$$

Then

$$\omega_2(x) = \min\{\max\{\hat{\omega}_2(x), 0\}, 1\}, \tag{23}$$

$$\omega_3(x) = \min\{\max\{\hat{\omega}_3(x), 0\}, 1\}. \tag{24}$$

Moreover, let $\hat{\omega}_1$ be the solution to

$$1 = \int_{1-\omega_1}^{1} \frac{1}{\phi(1-\lambda)(1-\bar{y}(x))D(\bar{y}(x);x)\left(\omega_2(x)-(1-s)\right) - \int_{1-\omega_2(x)}^{1-\omega_1} w(s)ds} w(s)ds \tag{25}$$

$$\omega_1(x) = \min\{\max\{\hat{\omega}_1(x), 0\}, 1\}. \tag{26}$$

The statement shows that the equlibrium is defined by a solution of a one-variable fixed point problem. We show in the Appendix that the Brower Fixed Point Theorem applies, hence, a fixed point exists.

We focus on the simple global equilibrium, because for the set of parameters it does not exist, our problem tends to be uninteresting. We spell out the details of the other cases in the Appendix.

In the next section, we analyze the implications of our model.

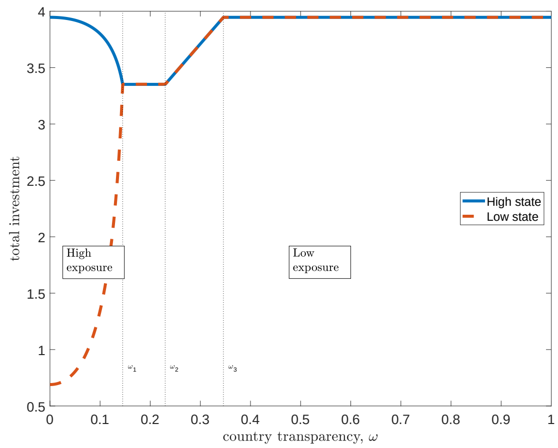
4 Booms and Busts in Core and Periphery Economies

In this section, we analyze the implications of the model. First, we focus on our implications related to the real economy: investment, growth, debt and default. Second, we explore our implications to the credit market. Each part we conclude with a corollary summarizing our predictions.

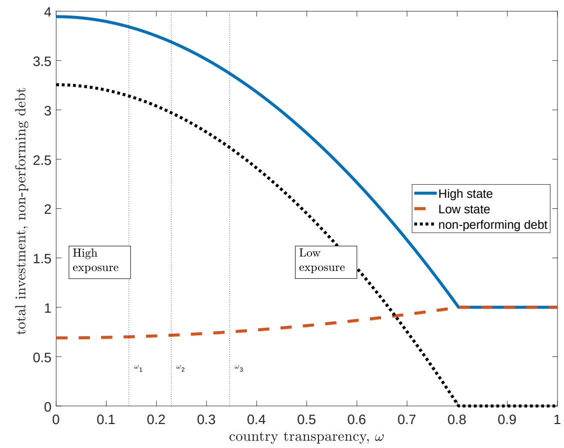
4.1 The Real Economy: Investment, Growth, Debt and Default

We argue that when aggregate state is high (low), the three periods of our model corresponds to the boom (bust) phase of the global cycle.[9] To see this, we calculate the total output, $Y(\omega, \theta)$ in each country in each regime by integrating good and bad firms affected and unaffected by the

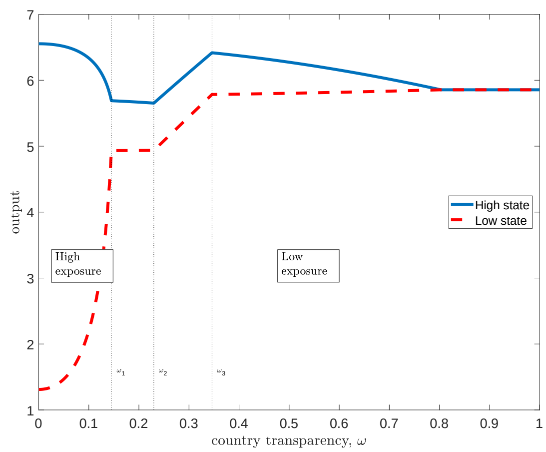
[9] In section 6, we explicitly consider a dynamic version of our set up by repeating our three period model indefinitely.



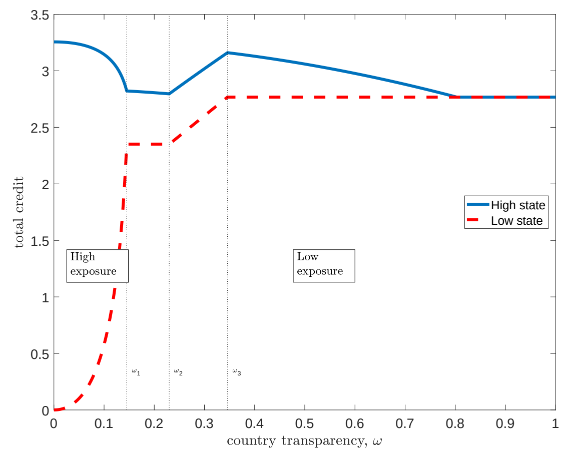
(a) Good firms' total investment, $I(\omega, g) + \phi\xi i(\omega, g, \theta)$ in high and low states



(b) Bad firms' total investment, $I(\omega, b) + \phi\xi i(\omega, b, \theta)$, non-performing debt, $\phi\xi i(\omega, b, H)$.



(c) Output, $Y(\omega, \cdot)$ per country in high (solid) and low (dashed) states



(d) Total credit, $C(\omega, \cdot)$ per country in high (solid) and low (dashed) states

Figure 4: Capacity, investment, output and debt

liquidity shock.

$$Y(\omega, H) \equiv \rho_\tau \left((1-\lambda) I(\omega, g) + \lambda((1-\phi) I(\omega, b) + \phi i(\omega, b, H)) \right) \tag{27}$$

$$Y(\omega, L) \equiv \rho_\tau \left((1-\lambda)(((1-\phi) + \phi\eta_L(\omega)) I(\omega, g) + \lambda((1-\phi) I(\omega, b)) \right). \tag{28}$$

Figure 4c we illustrate these objects. Note that production is higher in each country when investors are bold than when investors are cautious. This is why we can associate the earlier with booms and the latter with busts.

It is also apparent that output crashes in the low state in high exposure countries, while the effect of the prudence shock is much less pronounced for other countries. It is so, because high exposure countries can invest little after a liquidity shock in the low state.

Total face-value of credit obtained from investors by good and bad firms in country ω in state θ is

$$C(\omega, H) \equiv \phi\xi\left((1-\lambda)I(\omega, g) + \lambda i(\omega, b, H)\right) \tag{29}$$

$$C(\omega, L) \equiv \phi\xi\left((1-\lambda)\eta_L(\omega)I(\omega, g)\right). \tag{30}$$

As it is illustrated on Figure 4d our mechanism implies higher credit flows to high exposure countries than to any other countries in booms, and collapsing flows in busts. This is consistent with the stylized facts of sudden stop crises in emerging markets in general (Calvo et al., 2004) and, with the experience of periphery countries during the European sovereign debt crisis (Lane, 2013; Martin and Philippon, 2017). Note that, models which assume larger frictions in emerging markets tend to predict lower credit flows to these countries than to more developed markets in each state.

From Figure 4a and 4b observe also that good and bad firms invest and borrow differently across booms and bust and within countries. All bad firms in all countries gamble in the following sense. They know that that investment is abandoned in bust and liquidity will be rationed in booms. They also do not pay back their debt to investors. Hence, they choose to leverage up as much as they can and invest with as high capacity as possible. With this decision they can produce more in each phase if they are not hit by a liquidity shock at the expense of abandoning production otherwise. In this sense, good firms are gambling partly with their creditors funds. Figure 4b also reveals that this gambling is more extreme in more peripheral countries in booms, because in less transparent countries more investors mistake bad firms for good ones under the high state prudence shock .

As an example, we connect this picture with borrowing and investment patterns in Spain before the European crisis. As Cuñat and Garicano (2009) describes, in Spain a large part of the lending and real estate boom can be connected to a specific group of banks called Cajas. Their lending activity to politically connected firms fueled the bubble which later lead to a devastating crisis. A large part of these credit turned to non-performing. We interpret this group of firms as the bad firms in our model.

Figure 4b reveals the dynamics of non-performing debt across states and countries reflecting that only bad firms default on their debt to international investors and they can obtain credit only in booms. Our model implies that within each country, the stock of non-performing debt is higher among those issued in booms than those issued in busts. Furthermore, among those issued in booms, it is higher in periphery countries than in core countries.

As Figure 4a illustrates, good firms in high exposure countries gamble too in the sense of investing a lot to maximize their production in the boom phase and in the bust phase when not hit by the liquidity shock at the expense of collapsing investment when hit by the liquidity shock in bust. However, unlike bad firms, they only gamble with their own funds in the sense that they pay back fully.

The following Corollary summarizes the results.

Corollary 1 [Testable Predictions: Real Economy]

(i) Total output, total debt and total investment by country (over initial gdp)

(a) are more cyclical in periphery countries than in other countries,

(b) in booms, are higher in periphery countries than in low exposure countries.

(ii) The face value of non-performing debt (over initial gdp)

(a) within each country, is higher among those issued in booms than those issued in busts,

(b) among those issued in booms, is higher in periphery countries than in core countries.

4.2 Credit Market

Turning to credit markets, the main prediction of our model is that while these market might look integrated in booms, they become fragmented in recessions. In relation to the European Sovereign Debt crisis, this fragmentation was observed not only on the market of sovereign bonds, but also on financial and non-financial corporate debt (Battistini et al., 2014; Farhi and Tirole, 2016; Gilchrist and Mojon, 2018), and bank credit (Darracq Paries et al., 2014).

The basic manifestation of fragmentation in our model is that yields to perceived low quality borrowers spike especially relative to perceived high quality borrowers.

On a deeper level, our model also highlights that the interest rate on bonds are determined by different principles across states and countries. In a boom, the common interest rate, r_H is determined by the indifference condition of the marginal investor needed to clear the market. This is the standard way how assets are priced in models with heterogeneous investors. However, in bust the spread is determined differently. For low exposure countries it is determined by the zero lower bound, that is, by investors opportunity cost of holding bond. For firms in countries $\omega \in [\omega_2, \omega_3]$ bonds are priced by the scarcity of investor capital. For firms in countries $\omega \in [\omega_1, \omega_2]$ bonds are priced by indifference of investors.

Relatedly, our model predicts a funding mismatch in bust. On one hand, investor capital is scarce for low ω economies to the extent that financing for firms in the low exposure region dries up. On the other hand, low s investors are "queuing" to lend to low exposure countries even for zero compensation. Too see this, observe that the total capital of investors who can identify good firms only in low exposure countries is always strictly larger than the total amount of credit these firms are willing to purchase (from the definition of ω_3 in (24) and $w'(s) < 0$):

$$\int_0^{1-\omega_3} w(s)ds > (1-\omega_3)\phi(1-\lambda)I(\omega, g)_{\omega\in[\omega_3,1]}, \tag{31}$$

implying that some of these investors do not invest at all.

Looking together at the pattern of interest rate and non-performing credit, Figures 2 and **??**, we obtain predictions on the realized return across countries and states. While in booms yields are the same, more credit contracts issued in booms default in periphery countries. In contrast, the default rate among contracts issued in busts is zero in our model, but the yields for periphery bonds are higher. That gives the prediction that the realized returns are higher on periphery bonds if issued in busts, while it is higher in core bonds if issued in booms.

Note also, that a direct testable implication of our assumed information structure is that the portfolio of securities changes significantly across states, countries and investors. In a boom, credit from each country is held by a wide range of investors with various skills. Hence, the concentration of ownership of bonds is small in each country, as even low-skilled investors are ready to lend to firms in high-exposure countries. However, in busts, these investors stop lending in high exposure countries rebalancing their portfolio towards low exposure countries where they can confidently identify good firms. In contrast, high skilled investors rebalance toward high exposure countries where they can earn high returns. This leads to high concentration of ownership of credit in high exposure countries.

Indeed, in the context of the Eurozone Crisis, Ivashina et al. (2015) and Gallagher et al. (2018) find that a group of money market funds stopped lending only to European banks, but not to other banks of similar risk in 2011. With this group in the role of low-skilled investors, this is consistent with our mechanism. Ivashina et al. (2015) also find evidence that this process lead to significant disruption in the syndicated loan market, a possible channel for the real effects our model predicts. Gallagher et al. (2018) finds that funds with active investor base shed their Eurozone assets the most aggressively. This suggests that fund managers' reputational concerns might be behind the switch from bold to cautious, a channel which is consistent with our simple microfoundation in Section 7.1.

From the point of view of concentration of ownership, a pattern consistent with our predictions was observed in the context of sovereign lending in the Eurozone (Battistini et al., 2014). However, we are not aware of any test of these predictions on corporate credit.

Finally, considering the realized returns of investor types on various holdings, we notice that the source of dispersion in realized returns has a different structure in booms and recessions. In busts, each investor type holds only those assets which she knows will not default. That is, in our model, there is no dispersion in performance within assets of a given country in busts. However, in booms while each participating investor holds a portfolio of bonds issued by firms in every country $\omega > s_H$, lower skilled investors are holding a larger fraction of defaulting assets within those countries. Thus, there will be considerable dispersion these investors performance within assets of a given country in booms.

We summarize these observations as testable predictions in the following Corollary.

Corollary 2 [Testable Predictions: Credit Market]

(i) Credit markets are fragmented in busts, but integrated in booms. That is, nominal yields

   (a) in booms, (close to) equal across countries,

   (b) in busts, higher in high than in low exposure countries.

(ii) Ownership of debt

   (a) in each country, is more concentrated in busts than in booms ,

   (b) in busts, is more concentrated in high than in low exposure countries .

(iii) Portfolio rebalancing is heterogeneous across investors in bust. Namely,

   - *skilled investors rebalance toward high exposure countries with high yields*
   - *unskilled investors shed assets in high exposure countries and lend to low exposure countries only.*

(iv) Realized return on the representative portfolio of bonds

   (a) issued in booms in a given country, is higher in periphery countries than in core countries,

   (b) issued in bust in a given country, is higher in core countries than in periphery countries.

(v) The difference in dispersion of funds' performance between recessions and booms is higher within lower quality asset classes than higher quality asset classes.

To sum up, our model is consistent with the stylized picture than in booms only periphery countries experience a credit boom, building up more debt than core countries. A large part of the debt issued in booms defaults later. Then, in bust periphery countries experience a much larger contraction in credit, investment and output than periphery countries. Also, while periphery countries get credit for similar interest rate as core countries in booms, in bust spreads between core and periphery credit interest rate spike, and the credit market becomes fragmented.

Let us emphasize that we generate these facts in a model where countries are ex-ante identical in their fundamentals, and where the aggregate shock would not have any effect without investors being heterogeneously informed on firms in different countries. While we do not doubt fundamental differences across European countries also contributed to their differential performance, we do not introduce such heterogeneity for two reasons. First, it makes are argument that heterogeneity in investor's information is sufficient to create our predictions very clear. Second, we want to raise the possibility that the initial differences might have been second order. After all, periphery and core countries within Europe are not that different in their deep fundamentals. They are all developed countries, with similar stock of human and physical capital, operating under similar legal, economic, and political institutions.

4.3 Corporate credit, sovereign bonds and safe asset determination

In our economy, we do not model governments as decision makers. Therefore, we cannot introduce a separate role for sovereign bonds and corporate credit. Still, it is reasonable to think of the spread of the bond portfolio of a given country in our model as a prediction for the sovereign spread of that country. This is so, because sovereign bond spreads and average corporate spreads move in tandem in the data. For example, the average correlation between the spreads depicted on Figure 2 and the sovereign bond spreads of the respective countries was 0.92 between 1999 and 2017 (ranging from 0.88 in Italy to 0.95 in Germany).

With this caveat in mind, our model gives predictions on the set of countries where safe assets, public or private, can be issued. We follow the definition of He et al. (2016) and Maggiori (2017) who define safe assets as those which are traded at lower yield in bad times, often due to flight-to-quality episodes. We state our observations in the following Corollary.

Corollary 3 [Safe asset determination] *Only sufficiently transparent countries, with $\omega > \omega_3$, are able to issue safe assets. These countries have a low-exposure to credit cycles.*

Note that our mechanism provides a novel mechanism for safe asset determination. He et al. (2016) emphasizes the role of coordination of investors. Farhi and Maggiori (2017) focuses on the issuer's limited commitment to not to default on the asset (or devalue the underlying currency). There is also a tradition (e.g. Caballero et al., 2008; Caballero and Farhi, 2013) where a given country is capable of issuing safe assets and/or certain investors demand safe assets by assumption and only the quantity is determined in equilibrium. In contrast, in our paper the set of issuers and the set of buyers are endogenously determined by the level of transparency, ω_3, at which the supply of sufficiently transparent assets just equals the demand of not sufficiently sophisticated investors.[10] In Section 5.3, we highlight how our mechanism leads to new predictions on the implications of excess global savings and the scarcity of safe assets.

5 Core, Periphery, and the Increase in Global Savings

The main insight from the characterization of the equilibrium is that firms' competition for the scarce capital of heterogeneously informed investors leads to heterogeneous boom and bust patterns. Even if all countries share the same production technology, they understand that they will face different financing conditions in bust. Therefore, they choose different investment plans leading to different economic outcomes.

[10] It is useful to recognize that in the existing literature the distinction between the concepts of "a reserve currency" and "a safe asset" is not always clear. A reserve currency is traditionally defined as assets which serve three roles simultaneously: it is an international store of value, a unit of account, and a medium of exchange. Because of the second and third role, there can be only very few reserve currencies in the world almost by definition. While reserve currencies usually qualify, our definition of safe assets can also include a large set of other securities, potentially ranging from sovereign bonds and currencies of some small developed countries (e.g. Swedish sovereign bond, Swiss franks) to highly rated commercial papers or certain asset backed securities.

In the previous sections, we categorized countries in exposure groups based on the cyclicality of their credit spreads. We showed that high exposure countries with counter-cyclical spreads also have more volatile output, investment and more non-performing credit after a boom, and higher concentration of debt in a recession.

In this section, we turn to the determinants and characteristics of the heterogeneity in outcomes through a series of comparative statics excercises. What determines that a country with a given ω belongs to one or another exposure group? What determines the charateristics of the boom bust cycle within each group?

In the first part, we argue that the determinants of relative demand for high and low expertise capital and the available supply is key to answer these questions.

In the second part, we illustrate this insight with the experiment where we increase the mass of investors with low s, but keep the rest of the $w(s)$ function unchanged. We interpret this as a possible representation of the increase in global savings experienced in the last couple of decades. We show that this leads to less countries becoming low exposure, more countries becoming high exposure and more volatile outcomes in high exposure.

5.1 The Demand and Supply for International Capital

As a starting point, we consider the determinants of the relative size of the low exposure and high exposure groups as given by ω_1 and ω_3. The left panel on Figure 5 illustrates our analysis. As we explain in this part, we interpret the increasing solid and dashed curves as capital supply curves and the horizontal dot-dashed and dashed lines as capital demand curves.

5.1.1 The Low Exposure Group

Using (22), the size of the low exposure group is determined by

$$w(1-\omega_3) = \phi\,(1-\lambda)\,\xi I\,(\omega, g)\,|_{\omega\in[\omega_3,1]}. \tag{32}$$

The left hand side is the supply of capital of investors who can just identify good firms in country ω_3. We think of this as the supply of low expertise capital. In our plot, it is determined by the solid line, $w(1-\omega)$. The right hand side is the amount good firms borrow in a representative low exposure country. We think of this as demand for low expertise capital. In our plot, this is the dash-dotted flat demand curve.

Any change in parameters leading to larger demand of the representative low exposure firm implying an upward shift of the demand curve, would lead to a higher ω_3, that is, a shrinking low exposure group.

Keeping demand constant, and decreasing the supply of low expertise capital would lead to a similar outcome. This corresponds to an upward shift of solid supply curve in our plot.

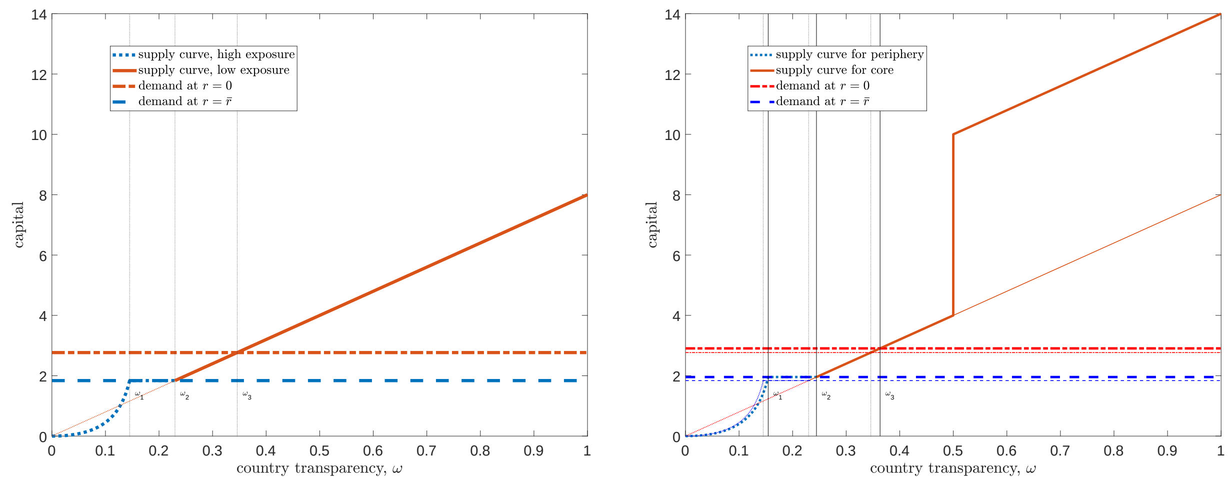


Figure 5: Supply and demand for investors' funds, and the determination of thresholds $\omega_1, \omega_2, \omega_3$.

The intuition is that when good firms in low exposure countries demand more capital relative to supply, the marginal investor who has sufficient funds to cover this demand has lower skill, s. As that investor can identify good firms only in higher ω countries, the low exposure group shrinks.

Note also that the size of the low exposure group is directly related to the size of idle investors' capital when investors are cautious. In our plot, this is the triangle between the dash-dotted demand curve and the solid curve. An upward shift of demand represented by the dash-dotted line or a downward shift in supply both decrease this area.[11]

Typically, changes in parameters affect demand and/or supply via multiple channels. Consider an increase in the fraction of good firms, $1-\lambda$, or in the probability or size of the liquidity shock ξ, ϕ. Using (12), (32) and the fact that $\eta_L(\omega) = 0$ for $\omega > \omega_3$, the direct effect is an increase in demand for low expertise capital. This is an upward shift in the dot-dashed line. We summarize the sign of the direct effect in the following Lemma.

Lemma 2 *In a simple global equilibrium, the direct effect is* $\frac{\partial \omega_3}{\partial \rho_H}, \frac{\partial \omega_3}{\partial \phi}, \frac{\partial \omega_3}{\partial (1-\lambda)}, \frac{\partial \omega_3}{\partial \pi_L}|_{r_H \text{ fixed}} > 0$.

However, note that any of these parameter changes affect the equilibrium interest rate in the high state too. For example, an increase in $\xi\phi$ increases all firms demand for liquidity in booms. Typically, this leads to an increases of the interest rate r_H, introducing an indirect, negative, effect on demand. We illustrate in the next section that this indirect effect might be important in experiments when the direct effect is not present. When both the direct and indirect effect is present, in our numerical simulations the direct effect always dominates.

[11] For the formal argument substitute the definition of ω_3 into (31) implying that

$$\frac{\partial \int_0^{1-\omega_3(Z_3)} w(s)\, ds - w(Z_3)\, Z_3}{\partial Z_3} = -w^{-1}(Z_3) = \omega_3 - 1 < 0.$$

where $Z_3 \equiv \phi(1-\lambda)\, \xi I(\omega, g)\, |_{\omega \in [\omega_3, 1]}$.

5.1.2 The High Exposure Group

The size of the group of high exposure countries is defined implicitly in (25).

Let $Z_1 = \phi(1-\lambda)\frac{\xi}{1+\bar{r}}I(\tau_j = \omega, g)\,|_{\omega\in[\omega_1,\omega_2]}$, the amount an unrationed representative good firm borrow facing the maximum interest rate $\bar{r}$. In the left panel of Figure 5, we plot the supply of capital of a $k \leq \omega_1$ firm, $\eta_L(\omega)Z_1$ as the dashed curve, which, using the definition of ω_2 in (21), we can rewrite as

$$Z_1 \int_{1-\omega}^{1} \frac{1}{Z_1\left(1 - w^{-1}(Z_1) - (1-s)\right) - \int_{w^{-1}(Z_1)}^{1-\omega_1} w(s)ds} w(s)ds. \tag{33}$$

By definition, ω_1 is determined by the point where this curve is equal to the demand Z_1, the dashed line, as this is the least transparent country where firms demand for credit is fully met.

While a change in Z_1 moves both curves, using the implicit function theorem, we can verify that $\frac{\partial \omega_1}{\partial (Z_1)} > 0$. That is, any given parameter change increases the group of high exposure countries if good firms facing the maximum interest rate borrow more. When they do, the scarce capital of sufficiently smart investors who can identify good firms in these low ω countries runs out faster. Hence, the high exposure group increases. The following Lemma summarizes the direct effect of the underlying parameters.

Lemma 3 *In a simple global equilibrium, the direct effect is* $\frac{\partial \omega_1}{\partial \rho_H}, \frac{\partial \omega_1}{\partial (1-\lambda)}|_{r_H\ \mathit{fixed}} > 0$*,while* $\frac{\partial \omega_1}{\partial \rho_\tau}|_{r_H\ \mathit{fixed}} < 0$.

While we cannot sign the total effects analytically, in our numerical simulations the direct effect always dominates.

We keep parameters across countries the same for expositional reasons, our model gives a clear intuition on how a relative shift in parameters across countries would affect the size of the low exposure group. Any fundamental changes affects the peripheryness, only to the extent it affects, directly or indirectly, the borrowing decisions of firms in the highest ω countries as specified in (32). If firms in the highest ω countries take up more credit, the group shrinks and countries close to ω_3 fall out from this group.

Let us emphasize that any intuition implying that better fundamentals in general would correspond to a larger low exposure group and a smaller high exposure group is false in our model. Neither any intuition implying the opposite is true. For example, a larger fraction of good firms shrinks low exposure and increases high exposure, while a smaller idiosyncratic shock has the opposite effect. In contrast, increasing the productivity of firms in terms of higher ρ_τ does not affect ω_1 at all, but increases the high exposure group.

5.2 Excess Savings: the Indirect Effect

In this part, we analyze the experiment of increasing the supply of capital in a particular way. We will interpret the exercise as a possible representation of excess global savings described by

Caballero et al. (2017). While we are not aware of evidence on the the effect of these excess funds on the relative distribution of expertise, in this experiment we entertain the possibility that this trend represents a shift towards low expertise capital.

In particular, we consider the effect of substituting $w(s)$ with a $\tilde{w}(s)$ where $\tilde{w}(s) > w(s)$ for all $s < \bar{s}$, but $\tilde{w}(s) = w(s)$ for all $s > \bar{s}$. We illustrate the exercise on the right panel of Figure 5. To make the analysis simple, we pick a $\bar{s}$ for which $\bar{s} < 1 - \omega_3$. This implies that this shock does not have a direct effect on demand and supply around points ω_1 and ω_3. All the results are driven by indirect effects.

First note that the increase of low expertise capital pushes some countries towards the high exposure groups. The reason is that the larger capital supply decreases the interest rate, r_H, for all firms in booms. This increases initial investment for all firms, which increases the demand for low and high expertise capital alike. As a result, thresholds ω_1 and ω_3 shifts the left.

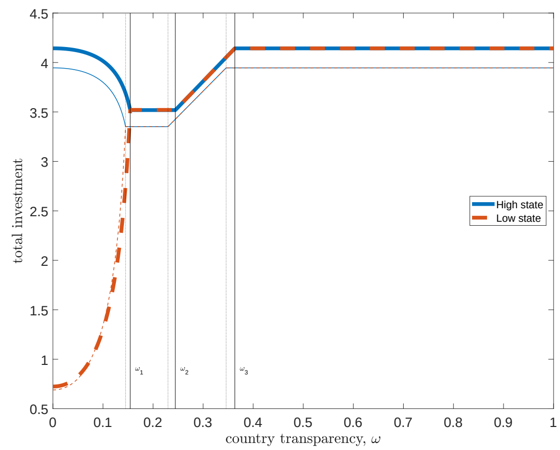
More importantly, the excess capital affects the economic outcome in each country in each state. We illustrate this on Figure 6, where each thin curve is copied from Figure 4, while the thick curves represent the effect of our experiment.

As it is apparent, the main effect is that the excess capital increases investment of both good and bad firms in booms. Good firms increase investment, because excess saving decreases interest rate, r_H. Bad firms in non-transparent countries increase investment even more strongly, because the additional capital is channeled to low s investors who mistaken those firms for goods. The more pronounced boom implies a larger collapse and a larger volume of non-performing roles in the high exposure countries.

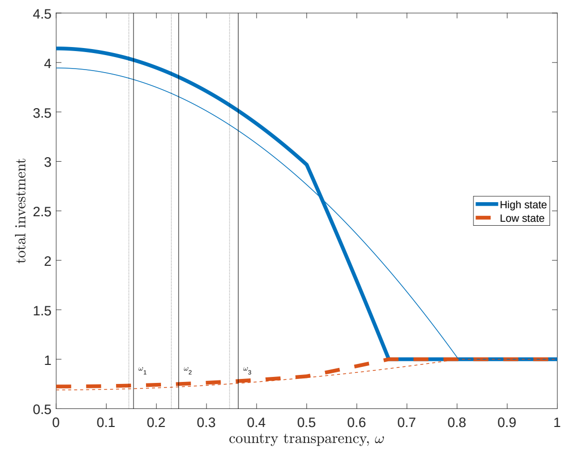
5.3 Excess savings and safe assets

This experiment sheds new light on the problem of scarce safe assets described in Caballero et al. (2017). It is argued that during the last decades the supply of safe assets could not keep up with the increasing demand by firms and the emerging market countries. This led to excessively low interest rate on these assets.

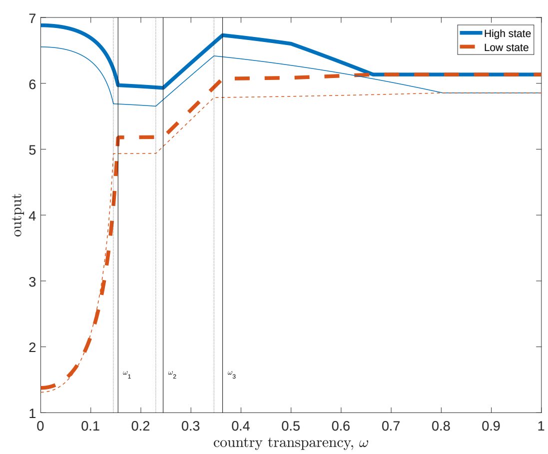
Under our interpretation of safe assets introduced in Section 4.3, an increase in capital of low s investors represents increasing demand for safe assets. Just as it is observed, this increasing demand results in low interest rate, r_H in booms. Similarly to other papers, our model predicts that this increases the supply of safe assets, but not to the extent of the increase in their demand. In our model, this is in the form of more credit to each high exposure firm and a simultaneous increase of idle capital (the area above the horizontal dotted-dashed line, but below the solid curve on the left panel of 5). Our additional prediction is that this also leads to a shrinking low exposure group, increasing periphery group, and more volatile boom and bust patterns in high exposure countries.



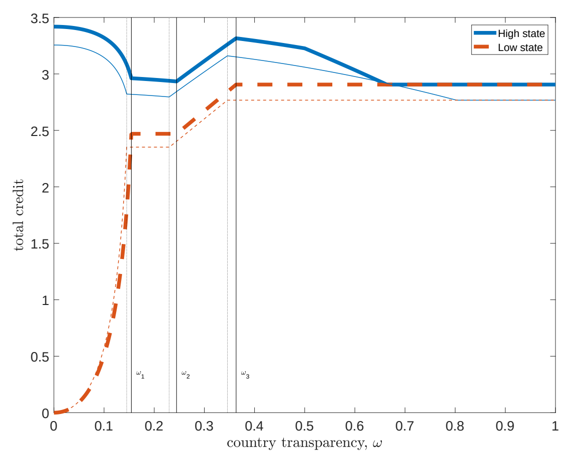
(a) The effect of increase in low s wealth on good firms' investment, $I(\omega, g) + \phi\xi i(\omega, g, \theta)$ in booms (solid) and busts (dashed)



(b) The effect of increase in low s wealth on bad firms' investment, $I(\omega, b) + \phi\xi i(\omega, b, \theta)$, in booms (solid) and busts (dashed)



(c) The effect of increase in low s wealth on output, $Y(\omega, \cdot)$ per country in high state (solid) and low state (dashed) states



(d) The effect of increase in low s wealth on total credit, $C(\omega, \cdot)$ per country in high state (solid) and low state (dashed) states

Figure 6: The effect of increase in low s wealth on investment, output and debt

6 Simple Dynamics: Heterogenous Global Cycles

To illustrate the heterogeneous global cycles implied by our model, in this section we introduce a simple dynamic version of our setting.

The dynamic model consists of consecutive generations of firms and investors, indexed by h. Each generation lives for a random number of periods, explained below, and once dead, it is replaced by a new generation. With some abuse of notation, let t_h denote date t of generation h lifetime, where we start at $t_h = 0$, $\forall\, h$.

In order to accommodate the generations' random life time, we introduce a new shock which we call *"generation shock"*. Consider generation h. The first two period of its lifetime, $t_h = 0, 1$, is identical to $t = 0, 1$ in the baseline model. However, we adjust the date $t = 2$ in the dynamic model. At the beginning of each period when $t_h \geq 2$, the generation shock is realized. With probability ψ_θ the current generation dies at the end of the given period, and with complementary probability $1 - \psi_\theta$ they survive to next period.

Suppose that t_h is the last period of an existing generation's life. We assume that the next generation of (the managers of the) firms will not be able to operate the previous generations' equipment. That is, firms' maintained units of investment pay a final ρ_τ cash-flow. These firms repay their existing credit if $\tau = g$. All firms and investors in this generation consume their remaining wealth. That is, from the point of generation h, this period is similar to period $t = 2$ of the baseline model.

At the same time, generation h of firms and investors is replaced by generation $h + 1$ and $t_{h+1} = 0$. For the new generation of investors, we redraw their state of prudence, θ. For the new generation of firms, we redraw their type, $j = (\omega, \tau)$.

Suppose now that generation h survives, that is, $t_h \geq 2$ is not the last period of their life. Then each unit of firms' existing project which they maintaned in period $t_h - 1$ produces ρ_τ units, before being subject to an additional liquidity shock, with probability ϕ.

Since we have assumed that $\rho_\tau > \xi$, the firms in a surviving generation do not need to borrow again. Instead, they cover the cost of maintenance from the production of their existing units.[12] As a result, once firms reach their $t_h = 2$ period, they continue with their existing units, without any liquidation, until the generation dies.

Finally, we assume that in each period, firms can propose their investors to delay repayment of credit for one more period. As investors do not discount the future, and do not provide further credit after period $t_h = 1$, they are indifferent to grant delay until the last period of generation's life. We assume the investors do so when indifferent.[13] As a result, firms repay their credit (or default) and reveal there pledgeability type only when old.

Based on these assumptions, with slight abuse of notation, we can rewrite the problem of firm in generation h as

$$\max_{I(\omega,\tau),\{\sigma(m,\omega,\tau;\theta),i(\omega,\tau;\theta)\}_{\theta,m}} \sum_{t_h=0}^{\infty} (1-\psi_\theta)^{t_h} \sum_{\theta} \pi_\theta \Big[(\rho_\tau - \xi)\phi i(\omega,\tau;\theta) + \rho_\tau(1-\phi) I(\omega,\tau)\Big] + \mathbb{1}_{\tau=b}\phi\xi i(\omega,\tau;\theta) - 1 \tag{34}$$

subject to (3)-(8).

It is easy to see that (34) is almost identical to (9), and, consequently the equilibrium in the

[12] One can show that good firms weakly prefer to use their own cash for maintenance as borrowing is costly. Because of this, bad firms would not get outside financing either.

[13] A simple intuition for this assumption is that if investors try to seize repayment, they have to write down the loss on their credit to bad firms. It is a weak and realistic assumption that investors prefer to delay the realization of their losses as long as it does not effect their expected cash flows.

dynamic version is almost identical to our baseline model.[14] Indeed, this version nests are baseline model with the choice of $\psi_\theta = 1$.

Figure 1 plots a simulated path of interest rate and output of the dynamic model, for a high and low exposure country, which illustrates the heterogeneous global cycles. The output of the high exposure country collapses sharply in low aggregate states (shaded areas), and its interest rate spikes. The low exposure country suffers only a moderate drop in output in the low aggregate state. The interest rate this group faces can even drop.

7 Generalization

In this section we generalize two aspects of our framework. In the first part, we argue that the prudence shock might be triggered endogenously by an aggregate productivity shock or a sentiment shock. In the second part, we partially relax the assumption that investors' prior is uninformative on the implication of a firm's country of origin on its opaqueness.

7.1 Bold or Cautious Experts? Enodogenous Information

In the baseline case, we modeled the prudence shock to investors' information as exogenous. In this part, we give a simple microfoundation showing that standard aggregate shocks, usually associated with recessions, can turn investors to cautious when information acquisition is endogenous.

Consider that an investor of skill s receives its signals from her analyst of type s. This analyst can choose to be bold or cautious. Consistently with our baseline set up, in the earlier case, the analyst observes a b signal for s fraction of the bad firms and $\varnothing$ for all other firms. In the latter case, he observes a g signal for s fraction of the good firms and $\varnothing$ for all other firms. The analyst is compensated solely based on the basis of her mistaken and correct assessments as specified in Table 1.

Bold	$\varnothing$	b
g	a_g	$-c_g$
b	$-c_b$	a_b

Cautious	g	$\varnothing$
g	a_g	$-c_g$
b	$-c_b$	a_b

Table 1: Pay-off of an analyst as a function of his report and the true type of the given firm.

When the a firm is good, the analyst's bonus if he communicated a favorable opinion (e.g. he states that there is evidence that the firm is good or states that there is no evidence for the contrary) is a_g, while his penalty otherwise is c_g. If the firm's type is $\tau_j = b$, the bonus if he

[14] The steps of the proof of the baseline case in the Appendix go through with trivial modifications. The only caveat is that if $\psi_H \neq \psi_L$, then (17) is replaced with $\bar{y}(x) = \frac{\psi_H}{\psi_L} \frac{\left(\rho_g - \xi\right)(1+\phi\xi\pi_H x)}{\left(\rho_g(1-\phi)\left(\pi_L \frac{\psi_H}{\psi_L} + \pi_H\right) + \phi\left(\rho_g - \xi\right)\pi_H\right)\xi}$.

communicated an unfavorable opinion is a_b while the penalty for contrary is c_b. The pay-offs can be arbitrary as long as $a_b, a_g > 0$ and $c_b, c_g \geq 0$.

To see the optimal decision of the analyst, we compare the expected pay-offs under being bold and cautious. The analyst choses to be bold if and only if

$$\lambda\left(sa_b+(1-s)\left(-c_b\right)\right)+(1-\lambda)\,a_g>(1-\lambda)\left(sa_g+(1-s)\left(-c_g\right)\right)+\lambda a_b$$

which simplifies to

$$\frac{1}{1+\frac{a_b+c_b}{a_g+c_g}}>\lambda. \tag{35}$$

That is, the analyst chooses to be bold when the fraction of bad firms in the economy are relatively small. This is intuitive. For example, if almost all firms are good, then the value of conclusive evidence identifying good firms is lower than conclusive evidence of identifying the very few bad firms. Condition (35) suggests two simple ways to microfund the idea that investors turn to cautious in the low aggregate state.

The first example shows that an aggregate productivity shock triggers the prudence shock.

Example 1 *Suppose that the bonus and penalty depends only on whether the analyst was correct or wrong, $a_g = a_b$, and, $c_g = c_b$. Assume that the low aggregate state, $\theta = L$, is associated with an adverse aggregate productivity shock in the form of $\lambda_H < \lambda_L$. That is, some good firms turn to bad firms in the low state. As long as $\lambda_H < \frac{1}{2} < \lambda_L$, the aggregate productivity shock triggers the prudence shock.*[15]

The second example shows that a particular type of sentiment shock triggers the prudence shock.

Example 2 *Suppose that the bonus for correct recommendations are identical regardless of the recommendation and it is normalized to $a_g = a_b = 1$. However, the (perceived) penalties of the analyst for high state, $c_g(\theta)$, and false negative mistakes, $c_b(\theta)$, are changing with the aggregate state. In particular, suppose that in the high state it is penalized more to miss out on a good firm than in the low state. Clearly, as long as $\frac{1}{1+\frac{1+c_b(H)}{1+c_g(H)}} > \lambda > \frac{1}{1+\frac{1+c_b(L)}{1+c_g(L)}}$, this sentiment shock triggers the prudence shock.*[16]

Finally, we refer the reader to Philippon (2006) and Bouvard and Lee (2016). Both papers argue that firms choose to be less cautious in booms than in recessions when sellecting projects to

[15] We show in Appendix D that our results are robust and our expressions change little with a state dependent λ_θ.

[16] The idea that managers are particularly averse to false negative mistakes in booms is consistent with the infamous quote of Charles Prince, the CEO of Citibank in 2007, at the onset of the financial crisis in the context of the role of Citibank as the major private equity deal provider.

> When the music stops, in terms of liquidity, things will be complicated. But as long as the music is playing, youve got to get up and dance. We're still dancing. (Financial Times, July 9, 2007)

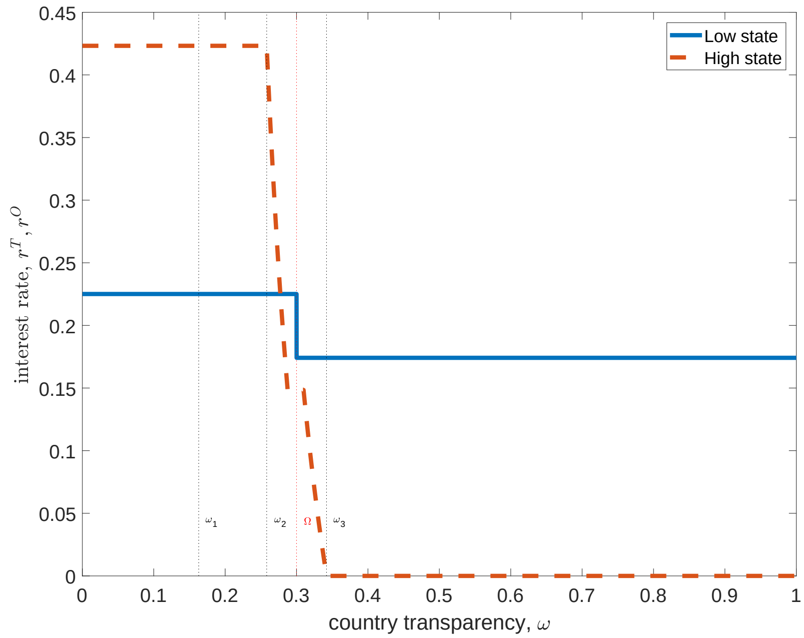


Figure 7: Interest rates for transparent and opaque countries in the high (solid) and low (dashed) aggregate state.

finance. The idea is that the due diligence of these projects takes time and the opportunity cost is higher in booms due to more positive NPV projects present. Bouvard and Lee (2016) shows that in equilibrium competition drives firms to spend inefficiently little time with due diligence, especially in booms. Philippon (2006) presents which supports this point.

7.2 Partitioned Transparency Groups

In the baseline model, we assume that investors have uninformative prior about ω, the average transparency of firms in a given country. That is, if an investor does not find conclusive evidence on a firm, the country of origin does not help her to learn whether it is because there is no such evidence, or because she is not skilled enough to find it.

In this section, we weaken this assumption. In particular, suppose that a public signal partitiones countries into a transparent and into an opaque group. That is, observing the public signal, each investor knows that the transparency, ω of the given country is $\omega > \Omega$ or $\omega < \Omega$ where Ω is an arbitrary cut-off. Intuitively, investors understand that, say, an emerging market firm tends to be more opaque than a developed market firm, but they have no information on how firms in different emerging markets compare to each other.

Figure 7 illustrates the effect of this treatment on the equilibrium interest rate schedules. Comparing the corresponding figure for the baseline case, the left panel of Figure 2, it is apparent that the qualitative difference is small. Here we give the intuition on the effect of this treatment, providing the corresponding analytical forms in the Appendix.

The main effect of this treatment is the partial separation in the high aggregate state. With the public signal, investors have an additional choice. They can choose to accept only firms from the transparent group to lend to. For less skilled investors, this implies a portfolio with less bad firms, as their mistakes are concentrated in opaque countries. Therefore, in equilibrium less skilled investors lend to firms from the transparent group only but for a lower interest rate. More skilled investors instead lend to firms from the opaque group but for higher interest rate. The marginal investor who is just indifferent across this too choices is determined in equilibrium. [17]

While it is an intuitive assumption that investors have some prior knowledge on the average transparency of firms in different countries, we assume this away in the baseline model because of two main reasons. First, we feel that the additional analytical complexity is not validated by additional insights. Second, one of the main focus of our analysis is how investors endogenously classify countries into low and high exposure groups in equilibrium. As this extension illustrates, a public signal on ω classify countries exogenously. Hence, this treatment would obscure our analysis.

8 Conclusion

Our main topic is that in the presence of informational frictions between international investors and firms, financial liberalization impacts countries heterogeneously, even if these countries have similar fundamentals. In particular, in countries where these frictions are more pronounced become highly exposed to aggregate shifts in investors' information. That creates countercyclical credit flows and output. Countries which are less exposed to these informational frictions, experience much smaller drop in output in a bust, low spread and large capital-inflows. We have emphasized that both the existence of the credit cycle and the countries' heterogeneous exposure is implied by the scarcity of investor capital. We have derived several empirical predictions. In future research, we plan to focus on the normative implications of our framework. We are interested in the constrained efficient exposure structure and the optimal policies to achieve that.

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[17] This treatment also introduces a small pooling segment around Ω in the low aggregate state interest rate schedule. As we explain in the Appendix, this comes from the requirement that the interest rate schedule has to be weakly monotonically increasing and obtained by an ironing procedure.

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A Credit Market Formalization

In this section we build on Kurlat (2016) to provide a formal description of the credit markets. The concepts and definitions follow Kurlat (2016) closely, however the proofs have to be generalized in a few key dimensions

(i) In our model firms have heterogeneous liquidity needs in the international markets, while in Kurlat (2016) there is homogeneous scale. We generalize the proofs in Kurlat (2016) to accommodate heterogeneous scale. This generalization requires additional variables that are absent from Kurlat (2016).

(ii) In our model, there is a maximum interest rate that any firm would accept. In Kurlat (2016), there is no minimum acceptable price.

(iii) Since interest rates cannot fully adjust everywhere in our model, credit quantity has to adjust. Thus we have credit rationing when $\theta = L$ as well. There is no rationing in Kurlat (2016) in the corresponding information regime, false negative.

We also adjust Kurlat (2016) properly to work with interest rates payable at $t = 2$ as opposed to prices payable at $t = 0$.

We break the firm problem into two sub-problems. Firm j first chooses its initial and maintained investment levels, $I(\omega, \tau), \{i(\omega, \tau; \theta)\}_\theta$, and then chooses how to raise the required liquidity on the international markets. In this section we formulate the liquidity raising plan on the international credit markets, taking the maximum amount that the firm can raise in total as given. We then solve this problem in section B.1. In section B.2 we relate the the maximum liquidity that the firm can raise to its predgeability constraint, and solve the firm's initial and maintained investment problem given the solution to liquidity raising subproblem.

A.1 International Credit Markets

There are many markets at $t = 1$, m, open simultaneously, where firms can demand bonds. M denotes the set of all markets. Each market in aggregate state θ is defined by two features. The first feature is the market interest rate, $\tilde{r}(m; \theta)$, paid by firms to international investor in exchange for bonds. If in market m only firms from a single country ω are serviced, we use $r_\theta(\omega) = \tilde{r}(m; \theta)$ to denote the interest rate associated with that market m.

The second is a clearing algorithm. A clearing algorithm is a rule that determines which bonds are financed, as a function of demand and supply in a market. Since investors have different information sets, different clearing algorithms result in different allocations and we need to specify what algorithm will be used. We use an adjusted version of the algorithms proposed by Kurlat (2016).

Definition A.2 [LRF Clearing Algorithm] *A clearing algorithm is a total order on X, which determines which acceptance rule is executed first. ζ is a less-restrictive-first (LRF) algorithm if it orders nested acceptance rules according to $\chi_h <_\zeta \chi_{h'}$ if $\chi_{h'}$ is nested in χ_h; i.e. the less restrictive acceptance rule first.*

Thus, acceptance rules of the form $\chi_h(\omega, \tau) = \mathbb{1}(\tau \in T_1 \,||\, (\tau \in T_2 \,\&\, \omega \leq 1-h))$ are ordered according to $\chi_h <_\zeta \chi_{h'}$ if $h < h'$, when ζ is an LRF clearing algorithm. Given the signal structure of investors when $\theta = H$, the less restrictive acceptance rule is also the less accurate.

Definition A.3 [NMR Clearing Algorithm] *ζ is a nonselective-then-more-restrictive-first (NMR) algorithm if it orders nested acceptance rules according to χ_h first if it imposes no restriction, and among acceptance rules with restrictions, the more restrictive acceptance rule first; i.e. $\chi_h <_\zeta \chi_{h'}$ if χ_h is nested in $\chi_{h'}$.*

Let $T_0 = \{g, b\}$. If ζ is an NMR clearing algorithm, acceptance rules of the form $\chi_h(\omega, \tau) = \mathbb{1}(\tau \in T \,\&\, \omega \geq 1-h)$ are ordered according to

(i) $\chi_{1,T_0} <_\zeta \chi_{h,T}$, for all $h < 1$ and T a subset of $\{g, b\}$;

(ii) $\chi_{h,T} <_\omega \chi_{h',T}$ if $h < h'$, for all $h, h' < 1$;

Given the signal structure of investors when $\theta = L$, the more restrictive acceptance rule is the less accurate.

Kurlat (2016) proves that in the presence of markets with different clearing algorithms, there exist an equilibrium where investors self-select into markets using LRF algorithm when the information structure is akin to ours in $\theta = H$, and markets using NMR algorithm when the information struction is that of $\theta = H$. For simplicity, we will directly assume that the clearing algorithm is LRF when $\theta = H$ and NMR when $\theta = L$. These algorithms guarantee that each investor receives a representative sample of the overall supply of bonds he is willing to accept, in the market where he participates.

A.2 Firm Problem

Assumption A.3 [Maximum Market Demand] *There is a maximum number of bonds each firm j can demand to issue in each market m, denoted by $\bar{L}$. We require $\bar{L} \geq \max_\omega \ell(\omega, g; \theta)$.*

Definition A.4 [Rationing Function] *A rationing function η assigns a measure $\eta(., \omega, \tau; \theta)$ on M to each bond demanded by firm $j = (\omega, \tau)$.*

Let $M_0 \subset M$ denote a set of markets. Then $\eta(M_0, \omega, \tau; \theta)$ is the number of bonds firm (ω, τ) issues if he submits one unit of demand to each market $m \in M_0$ in aggregate state θ. The firm receives one unit per bond issued, and $r(\omega, \tau; \theta)$ denote the average interest rate firm $j = (\omega, \tau)$ has to pay back if aggregate state is θ.

Firm Optimization in International Markets. The firm participates in the international markets in each state θ if he is hit by the liquidity shock. We will closely map the problem of the firm in international markets to the seller problem of Kurlat (2016), in order to raise liquidity required to maintain investment. In order to do so, we introduce the following auxiliary variable, $\hat{y}$.

Definition A.5 [Total International Demand] *$\hat{y}(\omega, \tau; \theta)$ is the total number of bonds firm j with (ω, τ) can raise on the international markets, when aggregate state is θ. $\hat{y}(\omega, \tau; \theta)$ is weakly decreasing in r_H.*

Here we define the firm's problem on the international credit market as an independent problem, which takes one state variable, $\hat{y}(\omega, \tau; \theta)$. Later in section B.2 we relate $\hat{y}(\omega, \tau; \theta)$ to the firm's pledgeability constraint, and show that $y(\omega, \tau; \theta)$ in problem (A.1) maps to $\ell(\omega, \tau; \theta)$ as defined in equation (4). Moreover, here we assume $\hat{y}(\omega, \tau; \theta)$ is a weakly decreasing function of r_H. In section B.2 we verify that in equilibrium, this condition is satisfied.

$$V_{\omega,\tau}(\hat{y}(.;\theta)) \equiv \max_{\{\sigma(m,\omega,\tau;\theta)\}_m} (1 + r(\omega, \tau; \theta)) \left(\frac{\rho_\tau}{\xi} - 1\right) y(\omega, \tau; \theta) \tag{A.1}$$

s.t.

$$\begin{aligned}
& y(\omega, \tau; \theta) = \int_M \sigma(m, \omega, \tau; \theta) d\eta(m, \omega, \tau; \theta) \\
& y(\omega, \tau; \theta) \leq \hat{y}(\omega, \tau; \theta) && \text{(A.2)} \\
& 0 \leq \sigma(m, \omega, \tau; \theta) \leq \bar{L} \\
& r(\omega, \tau; \theta) = \frac{\int_M \tilde{r}(m; \theta) \sigma(m, \omega, \tau; \theta) d\eta(m, \omega, \tau; \theta)}{\int_M \sigma(m, \omega, \tau; \theta) d\eta(m, \omega, \tau; \theta)}
\end{aligned}$$

To any unit of bonds that the firm issues to international investors, he adds $r(\omega, \tau; \theta)$ units of what he has saved using the bankers. He then injects this as the required liquidity to maintain investment. Thus by issuing $y(\omega, \tau; \theta)$ bonds, the firm continues at scale $\frac{1+r(\omega,\tau;\theta)}{\xi} y(\omega, \tau; \theta)$, which pays off ρ_τ at date $t = 2$. The firm then has to pay back $1 + r(\omega, \tau; \theta)$ per unit bond issued, which leads to the objective (A.1).

Similar to Kurlat (2016), the choice of $\sigma(m, \omega, \tau; \theta)$ for any single market m such that $\eta(m, \omega, \tau; \theta) = 0$ has no effect on the funding obtained by the firm. Formally, this implies that program (A.1) has multiple solutions. We follow Kurlat (2016) and assume that when this is the case, the solution has to be robust to small positive $\eta(m, \omega, \tau; \theta)$, meaning that the firm must attempt to sell an asset in all the markets where if he could he would want to, and must not attempt to sell an asset in any market where if he could he would not want to.

Definition A.6 [Robust Program] *A solution to program 9 is robust if for each θ and every (m_0, ω_0, τ_0) such that $\eta(m_0, \omega_0, \tau_0; \theta) = 0$ there exists a sequence of strictly positive real numbers $\{z_n\}_{n=1}^{\infty}$ and a sequence of credit demand, and bond issuance decisions $\{\sigma^n(m, \omega, \tau; \theta)\}_m$, and*

$y^n(\omega,\tau)$, *such that defining*

$$\eta^n(M_0,\omega_0,\tau_0;\theta) = \eta(M_0,\omega_0,\tau_0;\theta) + z_n \mathbb{I}(m_0 \in M_0)\mathbb{I}(j = j_0)$$

(i) In aggregate state θ, $\{\sigma^n(m,\omega,\tau;\theta)\}_m$ *solve program*

$$\begin{aligned} \max_{\{\sigma(m,\omega,\tau;\theta)\}_m} \quad & (1 + r(\omega,\tau;\theta)) \left(\frac{\rho_\tau}{\xi} - 1\right) y(\omega,\tau;\theta) \\ s.t. \quad & \\ & y(\omega,\tau;\theta) = \int_M \sigma(m,\omega,\tau;\theta) d\eta^n(m,\omega,\tau;\theta) \\ & 0 \le \sigma(m,\omega,\tau;\theta) \le \bar{L} \\ & y(\omega,\tau;\theta) \le \hat{y}(\omega,\tau;\theta) \end{aligned} \tag{A.3}$$

(ii) $z_n \to 0$

(iii) $\sigma^n \to \sigma$, $y^n \to y$; $\forall(\omega,\tau), m$

Why don't that cross-sectional differences across $\sigma(m,\omega,\tau;\theta)$, at the same market m_0, reveal the firm $j = (\omega,\tau)$? We assume that $\sigma(m,\omega,\tau;\theta)$ is divisible, and firms submit unit by unit. The investment that can potentially serve as collateral, if $\tau = g$, is verified and "marked", to avoid double promising.

A.3 Expert Problem

There is a distribution $w(s)$ of investors of expertise s, each endowed with one unit of wealth. Experts consume at dates $t = 1, 2$ and participate in international markets at $t = 1$. Since this is after realization of aggregate shock θ, we will suppress the dependence of their decisions on θ.

Definition A.7 [Acceptance Rule] *An acceptance rule is a function* $\chi : \{R, U\} \times [0,1] \to \{0,1\}$.

Definition A.8 [Feasibility] *An acceptance rule* χ *is feasible for investor* s *if it is measurable with respect to his information set, i.e. if*

$$\chi(\omega,\tau) = \chi(\omega',\tau') \quad \textit{whenever} \quad x(\omega,\tau,s) = x(\omega',\tau',s).$$

Let X denote the set of all possible acceptance rules, and X_s the set of acceptance rules that are feasible for investor s.

Definition A.9 [Allocation Function] *An allocation function* A *assigns a measure* $A(.;\chi,m,\theta)$ *on* $[0,1]$ *to each acceptance rule-market pair* $(\chi,m) \in X \times M$.

If $I_0 \subseteq \{R,U\} \times [0,1]$, $A(I_0;\chi,m,\theta)$ represents the fraction of bonds issued by firms j with $(\omega,\tau) \in I_0$ that an investor will obtain if he demands to buy one unit in market m and imposes acceptance rule χ.

In each aggregate state θ, investor s chooses the market he participates in, m, how many bonds he intends to finance δ, and a feasible acceptance rule χ to maximize

$$\max_{m,\chi,\delta} \sum_{t=1}^{2} c_t$$

s.t.

$$\chi \in X_s \tag{A.4}$$

$$\delta \int_{(\omega,\tau)} dA(\omega,\tau;\chi,m,\theta) \leq 1 \tag{A.5}$$

$$c_1 = 1 - \delta \int_{(\omega,\tau)} dA(\omega,\tau;\chi,m;\theta) \tag{A.6}$$

$$c_2 = \big(1 + \tilde{r}(m;\theta)\big)\delta \int_{\omega} dA(\omega,g;\chi,m,\theta) \tag{A.7}$$

Constraint (A.4) restrict the investor to using feasible rules. Constraint (A.5) says that each investor can only provide credit from her own wealth. Constraint (A.6) says the investor consumes her leftover endowment at $t = 1$, while (A.7) says that at $t = 2$ she is paid back by good firms and consume. Substitute the consumption into investor utility function and simplify to get the objective function (10) in the text.

B Construction of Equilibrium

We proceed by backward induction. At $t = 1$, $\forall\theta, (\omega, \tau)$ we take function $\hat{y}(\omega, \tau; \theta)$, satisfying the two following properties as given

(i) $\hat{y}(.)$ is continuous in r,

(ii) $\frac{d\hat{y}}{dr} \leq 0$.

We construct a more general version of the equilibrium compared to the one used in the main text. Throughout, we point out how the simplified version of certain expressions look like, given the date $t = 0$ structure of the problem. In section B.2 we connect $\hat{y}(\omega, \tau; \theta)$ to $\ell(\omega, \tau; \theta)$, and show that the above two properties are satisfied in equilibrium.

B.1 t = 1: International Credit Market Equilibrium

Step 1. At $t = 1$, given the maximum level of liquidity that a firm can raise on the international markets,$\hat{y}(\omega, \tau, \theta)$, and under certain parametric assumptions, the equilibrium in international markets is such that firms maximize problem (A.1), international investors maximize problem (10), and active markets clear. The equilibrium in international markets is as follows.

We start with the firm problem at $t = 1$ in aggregate state θ. For simplicity, we sometimes suppress the dependence on aggregate state θ as the problem is solved state-by state. So $\hat{y}(\omega, \tau) \equiv \hat{y}(\omega, \tau, \theta)$, $\sigma(m, \omega, \tau) \equiv \sigma(m, \omega, \tau; \theta)$, $y(\omega, \tau) \equiv y(\omega, \tau, \theta)$, and $\eta(m, \omega, \tau) \equiv \eta(m, \omega, \tau; \theta)$. We will also use, whenever helpful to ease the notation, $\eta_H(\omega) = \eta(m_H, \omega, b; H)$, and $\eta_L(\omega) = \eta(\bar{m}, \omega, g; L)$.

B.1.1 t = 1; Bold Experts, θ = H.

Equilibrium Description. The equilibrium consists of a pair (r_H, s_H), firm and investor optimization, an allocation function, and a rationing function. Market m_H is the market defined by interest rate r_H and an LRF algorithm. The equilibrium is described as follows.

(i) Pair (r_H, s_H) is the solution to the pair of equations

$$r = \frac{\lambda(1-s)\bar{L}}{(1-\lambda)\int_0^1 \ell(\omega, g; H)d\omega} \tag{B.1}$$

$$\phi = \int_s^1 \frac{1}{\lambda(1-s')\bar{L} + (1-\lambda)\int_0^1 \ell(\omega, g; H)d\omega} w(s')ds' \tag{B.2}$$

(ii) Firm decision

- Good firm

$$\sigma(m,\omega,g;H)=\begin{cases}\min\{\bar{L},\hat{y}(\omega,g;H)\}=\hat{y}(\omega,g;H) & \text{if } \tilde{r}(m)=r_H\\ \bar{L} & \text{if } \tilde{r}(m)<r_H\\ 0 & \text{otherwise}\end{cases}$$

where the first line in σ follows from definition (A.3) along with construction of $\hat{y}(\omega,g)$.

- Bad firm

$$\sigma(m,\omega,b;H)=\min\{\bar{L},\hat{y}(\omega,b;H)\}=\bar{L} \qquad \forall\, m$$

which follows from construction of $\hat{y}(\omega,b;H)$.
$y(\omega,\tau;H)=\hat{y}(\omega,\tau;H)\eta(m_H,\omega,\tau;H)$, $\tau=g,b$, where the rationing function $\eta(m_H,\omega,\tau;H)$ is defined by (B.6).

(iii) Expert decision

- $s<s_H$

$$\begin{aligned}\delta_s&=0\\ m_s&=m_H\\ \chi_s(\omega,\tau)&=\mathbb{I}\left(\tau=g\ ||\ (\tau=b\ \&\ \omega\leq 1-s)\right)\end{aligned}$$

- $s\geq s_H$

$$\begin{aligned}\delta_s&=1\\ m_s&=m_H\\ \chi_s(\omega,\tau)&=\mathbb{I}\left(\tau=g\ ||\ (\tau=b\ \&\ \omega\leq 1-s)\right)\end{aligned}$$

(iv) Allocation function

- For market m_H and $\chi(\omega,\tau)=\mathbb{1}\left(\tau=g\ ||\ (\tau=b\ \&\ \omega\leq 1-h)\right)$ for some $h\in[0,1]$

$$\begin{aligned}a(\omega,\tau;\chi,m_H)&=\frac{1}{\hat{y}(\omega,\tau;H)}\frac{\left(\mathbb{I}(\tau=g)+\mathbb{I}(\tau=b\ \&\ \omega\leq 1-h)\right)\sigma(m,\omega,\tau;H)}{(1-\lambda)\int_0^1\sigma(m_H,\omega',g;H)d\omega'+\lambda\int_0^{1-h}\sigma(m_H,\omega',b;H)d\omega'}\\ &=\frac{\mathbb{I}(\tau=g)+\mathbb{I}(\tau=b\ \&\ \omega\leq 1-h)}{(1-\lambda)\int_0^1\sigma(m_H,\omega',g;H)d\omega'+\lambda(1-h)\bar{L}}\end{aligned} \tag{B.3}$$

- For market m_H and any other acceptance rule

$$a(\omega,\tau;\chi,m_H)=\begin{cases}\frac{\chi(\omega,\tau)[1-\eta(m_H,\omega,\tau;H)]}{\sum_{\tau'}\int_{\omega'}\chi(\omega',\tau')\hat{\sigma}(m_H,\omega',\tau';H)[1-\eta(m_H,\omega',\tau';H)]d\omega'} & \text{if } \chi(\omega',\tau')\notin X_s \ \& \ \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau')\sigma(m_H,\omega',\tau')[1-\eta(m_H,\omega',\tau')]d\omega'>0\\ \frac{\chi(\omega,\tau)[1-\eta(m_H,\omega,\tau;H)]}{\sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau')\sigma(m_H,\omega',\tau';H)[1-\eta(m_H,\omega',\tau';H)]} & \text{if } \chi(\omega',\tau')\notin X_s \ \& \ \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau')\sigma(m_H,\omega',\tau')[1-\eta(m_H,\omega',\tau')]d\omega'=0,\\ & \text{but } \sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau')\sigma(m_H,\omega',\tau')[1-\eta(m_H,\omega',\tau')]d\omega'>0\\ 0 & \text{otherwise}\end{cases} \tag{B.4}$$

where $\eta(m_H,\omega,\tau;H)$ is defined below.

- For any other market

$$a(\omega,\tau;\chi,m)=\begin{cases}\frac{\chi(\omega,\tau)S(m,\omega,\tau)}{\sum_{\tau'}\int_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau';H)S(m,\omega',\tau')d\omega'} & \text{if } \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau')S(m,\omega',\tau')d\omega'>0\\ \frac{\chi(\omega,\tau)S(m,\omega,\tau)}{\sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau';H)S(m,\omega',\tau')} & \text{if } \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau')S(m,\omega',\tau')d\omega'=0,\\ & \text{but } \sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau';H)S(m,\omega',\tau')d\omega'>0\\ 0 & \text{otherwise}\end{cases} \tag{B.5}$$

where

$$S(m,\omega,\tau)=\begin{cases}1 \quad \text{if} \begin{cases}\tau=b\\ \text{or}\\ \tau=g \ \& \ \tilde{r}(m)\in(0,r_H]\\ \text{or}\\ \tilde{r}(m)\le 0\end{cases}\\ 0 \quad \text{if } \tau=g,\ \tilde{r}(m)>r_H\end{cases}$$

(v) Rationing function

$$\eta(M_0,\omega,\tau;H)=\begin{cases}1 & \text{if } m_H\in M_0 \text{ and } \tau=g\\ \int_{s_H}^{1-\omega}\frac{1}{\phi\lambda(1-s)\bar{L}+\phi(1-\lambda)\int_0^1\ell(\omega',g;H)d\omega'}\,w(s)ds & \text{if } m_H\in M_0 \text{ and } \tau=b \text{ and } \omega\le 1-s_H\\ 0 & \text{otherwise}\end{cases} \tag{B.6}$$

Proof.

(i) $(\mathbf{r_H},\mathbf{s_H})$. There is a single market m_H, with $\tilde{r}(m_H)=r_H$, where all trades take place. In this market, good firms try to issue as many bonds as their liquidity needs, while bad firms supply the maximum possible number of bonds, $\bar{L}$. Total supply is therefore $(1-\lambda)\int_0^1\ell(\omega',g;H)d\omega'$ good bonds and $\lambda\bar{L}$ bad bonds. Supply decisions in markets $m\neq m_H$ have no effect on firm utility since $\eta(m,\omega,\tau;H)=0$, so they are determined in equilibrium by the robustness requirement.

Buying from markets with interest rate other than r_H is not optimal for investors. At interest rates above r_H, the supply includes only bad firms, so investors prefer to stay away, whereas at interest rate below r_H, the supply of bonds is exactly the same as at interest rate r_H but the interest rate is lower. This does not settle the question of whether an investor chooses to buy at all. Expert optimization below then shows that investor with $s = s^*$ faces terms of trace of $\tau(s^*) = 1$ in market m_H, and is indifferent between buying and not buying. This results in equation (B.1).

All investors with $s > s^*$ thus spend all of their wealth buying in market m_H and those with $s < s^*$ choose not to buy at all. The fraction of bonds by firm $j = (\omega, \tau)$ that can be issued in market m_H is given by the ratio of the total allocation of that bond across investors, to the supply of that bond. Noticing that only firms hit by liquidity shock issue bonds, and adding across investors and imposing that all good bonds are issued results in (B.2).

Next define the monotone transformation $q_H = \frac{r_H}{1+r_H}$. (B.1) can be represented as

$$q = \frac{\lambda(1-s)\bar{L}}{\lambda(1-s)\bar{L} + (1-\lambda)\int_0^1 \ell(\omega, g; H)d\omega} \tag{B.7}$$

Let $H(r_H) = \bar{L}$ and $G(r_H) = \int_0^1 \ell(\omega, g; H)d\omega$, noting that $\ell(\omega, g; H)$ implicitly depends on q_H (or equivalently r_H). From lemma C.2, there exists a solution to pair of equations (B.7) and (B.2) such that $q_H > 0$ and $0 < s_H < 1$, which in tern implies there exists a solution to pair of equations (B.1) and (B.2) such that $r_H > 0$ and $0 < s_H < 1$.

(ii) **Firm optimization.** Taking the equilibrium market structure, rationing function and allocation function as given, $y(\omega, \tau) = \sigma(m_H, \omega, \tau)\eta(m_H, \omega, \tau)$. Since $\frac{\rho_\tau}{\xi} - 1 > 0$, firm j's optimal choice of $\sigma(m_H, \omega, \tau)$ is determined by the constraints. For a good firm, $\eta(m_H, \omega, g) = 1$ from rationing function (B.6), which implies $y(\omega, g) = \sigma(m_H, \omega, g)$. As such, condition (A.2) is the binding constraint which in turn implies $y(\omega, g) = \sigma(m_H, \omega, g) = \hat{y}(\omega, g)$, when $\theta = H$.

For a bad firm $\eta(m_H, \omega, b) = \int_{s_H}^{1-\omega} \frac{1}{\lambda(1-s)\bar{L}+(1-\lambda)\int_0^1 \ell(\omega', g; H)d\omega'} \frac{w(s)}{\phi} ds$ from rationing function (B.6). First note that from equation B.2, $\eta(m_H, L, 0) = 1$, so $\eta(m_H, \omega, b) < 1$, $\forall\ 1 - s_H < \omega \leq 1$. This implies $y(\omega, b) \leq \sigma(m_H, \omega, b)$. Since $\hat{y}(\omega, b) = \bar{L}$, constrain (A.2) is slack, and $\sigma(m_H, \omega, b) = \bar{L}$, which in turn implies $y(\omega, b) = \eta(m_H, \omega, b)\bar{L}$.

Put together, the rationing function (B.6) implies that in market m^H, all good firms will be able to issue as many bonds as they demand, so they submit exactly what they want to get, $\hat{y}$, and not $\bar{L}$. An bad firm from country ω will be able to sell a fraction $\eta(m_H, \omega, b) < 1$ of bonds he demands, so he submits $\bar{L}$. No other bond can be issued. Put together this implies

$$y(\omega, \tau) = \begin{cases} \hat{y}\,(\omega, \tau; \theta) & \text{if } \tau = g \\ \bar{L}\eta(m_H, \omega, \tau) & \text{if } \tau = b \end{cases} \tag{B.8}$$

Off equilibrium, in all cheaper markets (lower interest rate), all good firms submit $\bar{L}$. In all more expensive markets, they submit zero demand. All bad firms submit the maximum that they can submit on every market, $\bar{L}$. These decisions satisfy the robust program (A.3). Note that the equilibrium $\sigma(m, \omega, \tau)$ satisfy the form of lemma C.1.

Finally, we verify that what they raise, $y(\omega, b) = \ell(\omega, b)$, which is immediate from comparing equation (B.8) and equations (B.29) and (B.30).

(iii) **Expert optimization.**

Choosing any feasible acceptance rule other than $\chi(\omega, \tau) = x(\omega, \tau, s)$ in market m_H would, according to (B.3) and (B.4), result in a lower fraction of good assets, so choosing $\chi(\omega, \tau) = \mathbb{1}\left(\tau = g \;||\; (\tau = b \;\&\; \omega \leq 1 - s)\right)$ is optimal.

Define the terms of trade that an investor obtains in market m with acceptance rule χ as

$$\tau(m, \chi) = \begin{cases} \frac{(1+\tilde{r}(m)) \int_{\omega} \mathbb{1}[\tau=g]\sigma(m,\omega,g;\theta)dA(\omega,g;\chi,m,\theta)}{\int_{\omega,\tau} \sigma(m,\omega,\tau;\theta)dA(\omega,\tau;\chi,m,\theta)} & \text{if } A(\{g,b\},[0,1];\chi,m) > 0 \\ 0 & \text{otherwise} \end{cases}$$

which is his expected repayment per unit of bond he finances, i.e. the principal and interest rate he receives at $t = 2$. Let

$$\tau^{max}(s) \equiv \max_{m \in M, \chi \in X_s} \tau(m, \chi)$$

be the best term of trade that investor s can achieve, and let $M^{max}(s)$ be the set of markets where investor s can obtain terms of trade τ^{max} with a feasible acceptance rule.

Necessary and sufficient condition for investor optimization are that investors for whom $\tau^{max} < 1$ choose not to finance any bonds, investors for whom $\tau^{max} > 1$ spend their entire endowment in a market $m \in M^{max}(s)$, and investors for whom $\tau^{max} = 1$ choose a market $m \in M^{max}(s)$. Using equation (B.3), an investor s that uses acceptance rule $\chi(\omega, \tau) = \mathbb{1}\left(\tau = g \;||\; (\tau = b \;\&\; \omega < 1 - s)\right)$ in market m obtains terms of trade

$$\tau(m, \chi) = \begin{cases} \frac{(1+\tilde{r}(m))(1-\lambda)\int_0^1 \ell(\omega,g;H)d\omega}{\lambda(1-s)\bar{L}+(1-\lambda)\int_0^1 \ell(\omega,g;H)d\omega} & \tilde{r}(m) \leq r_H \\ 0 & \text{otherwise} \end{cases}$$

Thus for all investors

$$\tau^{max}(s) = \frac{(1 + r_H)(1 - \lambda)\int_0^1 \ell(\omega, g; H)d\omega}{\lambda(1 - s)\bar{L} + (1 - \lambda)\int_0^1 \ell(\omega, g; H)d\omega}$$

and the maximum is attained in any market where the interest rate is r_H, including m_H.

Rewrite

$$\tau^{max}(s) = (1 + r_H)J(s)$$

$$J(s) = \frac{(1-\lambda)\int_0^1 \ell(\omega, g; H)d\omega}{\lambda(1-s)\bar{L} + (1-\lambda)\int_0^1 \ell(\omega, g; H)d\omega},$$

In the special case where $\hat{y}(\omega, \tau; L)$ is determined given the specific structure of $t = 0$, the above equations simplify to

$$\tau(m, \chi) = \begin{cases} \frac{(1+\tilde{r}(m))(1-\lambda)\bar{D}(q)}{\lambda(1-s)\bar{L}+(1-\lambda)\bar{D}(q)} & \tilde{r}(m) \leq r_H \\ 0 & \text{otherwise} \end{cases}$$

$$\tau^{max}(s) = \frac{(1+r_H)(1-\lambda)\bar{D}(r_H)}{\lambda(1-s)\bar{L} + (1-\lambda)\bar{D}(r_H)}$$

$$J(s) = \frac{(1-\lambda)\bar{D}(r_H)}{\lambda(1-s)\bar{L} + (1-\lambda)\bar{D}(r_H)},$$

Note that from equation (B.1), $J(s_H) = \frac{1}{1+r}$, so $\tau^{max}(s_H) = 1$. Moreover, $J'(s) > 0$. This implies that investors $s < s_H$ have $\tau^{max}(s) < 1$, so not financing any bonds is optimal for them. Experts of types $s \geq s_H$ have $\tau^{max}(s) \geq 1$, so financing bonds such that they spend their entire wealth in market m_H at $t = 2$ is optimal for them too.

(iv) **Allocation function.**

In all markets except m_H (off equilibrium path), there are no investors, so for any clearing algorithm the residual set of bonds any investor faces is just the original set of bonds demanded by firm on that market. In this case, (B.5) follows from Appendix B of Kurlat (2016), equation (65).

For market m_H, the LRF algorithm implies that an investor who imposes
$\chi(\omega, \tau) = \mathbb{1}(\tau = g \,||\, (\tau = b \,\&\, \omega \leq 1-h))$ faces a residual firm demand of acceptable bonds that is proportional to the original firm demand. Therefore, the measure of assets he will obtain is the same as if he traded first. Therefore (B.3) follows from Appendix B of Kurlat (2016), equation (65).

For market m_H and rules that are not of the form $\chi(\omega, \tau) = \mathbb{1}(\tau = g \,||\, (\tau = b \,\&\, \omega \leq 1-h))$, (off equilibrium path), their trades clear after all other investors, so the bond financing demand they face only includes bonds demanded by bad firms. Therefor (B.4) follows from Appendix B of Kurlat (2016), equation (65).

(v) **Rationing function.**

(B.6) follows from B.2 using Appendix B of Kurlat (2016), equation (67). It is the fraction of bonds that the firm is able to issue , out of the total bonds offered (i.e. a number between zero and one).

B.1.2 t = 1; Cautious Experts, $\theta = \mathbf{L}$.

Equilibrium Description. The equilibrium consists of an interest rate schedule $0 \le r_L(\omega) \le \bar{r}$, cut-offs $\omega_1 < \omega_2 < \omega_3$, firm and investor optimization, an allocation function, and a rationing function. For any $\omega \in [0,1]$, let $m(\omega)$ denote the market where the price is $r_L(\omega)$, where $r_L(\omega)$ is found by the procedure described in the proof below, and the clearing algorithm is NMR. Because of bunching, $m(\omega)$ could mean the same market for different ω. For any $\Omega_0 \subseteq [0,1]$, let the set of markets $M(\Omega_0)$ be $M(\Omega_0) = \{m(\omega) : \omega \in \Omega_0\}$. The set of active markets is $M([0,1])$.

The equilibrium is described as follows.

(i) Premium schedule $0 \le r_L(\omega) \le \bar{r}$ such that the interest rate falls into one of the cash-in-the-market, bunching, bunching-with-scarcity, or non-selective regions as described below.

(ii) Firm decision

- Good firm

$$\sigma(m,\omega,g;L) = \begin{cases} \min\{\bar{L}, \hat{y}(\omega,g;L)\} = \hat{y}(\omega,g;L) & \text{if } \tilde{r}(m) = r_L(\omega), \omega \ge \omega_2 \\ \hat{y}(\omega_2,g;L) & \text{if } \tilde{r}(m) = r_L(\omega), \omega < \omega_2 \\ \bar{L} & \text{if } \tilde{r}(m) < r_L(\omega) \\ 0 & \text{otherwise} \end{cases}$$

$$y(\omega,g;L) = \int_{M([\omega,1])} \hat{y}(\omega,g;L) d\eta(m,\omega,g;L)$$

where the first line in σ follows from definition (A.3) along with construction of $\hat{y}(\omega,g;L)$.

- Bad firm

$$\sigma(m,\omega,b;L) = \min\{\bar{L}, \hat{y}(\omega,b;L)\} = \bar{L} \qquad \forall\, m$$

$$y(\omega,b;L) = \bar{L} \int_{M([0,1])} d\eta(m,0,b;L)$$

where σ follows from construction of $\hat{y}(\omega,b;L)$.

The rationing functions $\eta(m,\omega,\tau;L)$ are defined in (B.14) and (B.15).

Selling decisions follow the reservation interest rate strategy. A good firm (ω,g) raises total liquidity equal to all the bonds they are able to sell on all $M([\omega,1])$ markets. A bad firm (ω,b) tries to sell in all markets $M([0,1])$. Since in equilibrium all bad assets sell at the same ratio, $\eta(m,\omega,b;L) = \eta(m,0,b;L)$, $\forall\, m, \forall\, \omega \in [0,1]$.

(iii) Expert decision:

Let ω_3 denote the the lowest-ω country whose firm face a zero interest rate when investors are cautious, $r_L(\omega) = 0$. Define s_N by

$$\int_{s_N}^{\hat{s}(\omega_3)} w(s)ds = \phi(1-\lambda)\int_{\omega_3}^{1} \hat{y}(\omega, g; L)d\omega$$

In the special case where $\hat{y}(\omega, \tau; L)$ is determined given the specific structure of $t = 0$, the right hand side of the above equation simplifies to

$$\phi(1-\lambda)\int_{\omega_3}^{1} D(0; r_H)d\omega = \phi(1-\lambda)(1-\omega_3)D(0; r_H)$$

so that the aggregate wealth of investors in the interval $[s_N, \hat{s}(\omega_1)]$ is just sufficient to finance all the bonds offered by good firms from countries $\omega > \omega_3$ at interest rate 0, and each of these investors can identify some good bond in this interval. Investors with lower degree of expertise than s_N either buy non-selectively or do not buy at all.

Define the function $\tilde{s}(\omega)$ as the solution to the following differential equation

$$\tilde{s}'(\omega) = -\frac{1}{w(\tilde{s}(\omega))}\phi\left[\lambda\bar{L} + (1-\lambda)\int_0^{\omega} \hat{y}(\omega', g; L)d\omega'\right]\varepsilon'(\omega) \tag{B.9}$$

In the special case where $\hat{y}(\omega, g; L)$ is determined given the specific structure of $t = 0$, the above equation simplifies to

$$\tilde{s}'(\omega) = -\frac{1}{w(\tilde{s}(\omega))}\phi\left[\lambda\bar{L} + (1-\lambda)\int_0^{\omega} D(r_L(\omega'); r_H)d\omega'\right]\varepsilon'(\omega)$$

with boundary condition $\tilde{s}(1) = s_N$. Finally, let $s_0 = \tilde{s}(0)$ and define $\tilde{\omega}(s)$ for $s \in [s_0, s_N]$ by

$$\tilde{\omega}(s) = \min\{\omega : \tilde{s}(\omega) = b\}$$

(a) for $s \geq s_N$

$$\begin{aligned} \delta_s &= 1 \\ m_s &= m(1-s) \\ \chi_s(\omega, \tau) &= \mathbb{I}(\tau = g \ \&\ \omega \geq 1-s) \end{aligned}$$

(b) $s \in [s_0, s_N)$

$$\begin{aligned} \delta_s &= 1 \\ m_s &= m(\tilde{\omega}(s)) \\ \chi_s(\omega, \tau) &= 1 \end{aligned}$$

(c) $s < s_0$

$$\delta_s = 0$$
$$m_s = m(1)$$
$$\chi_s(\omega, \tau) = 1$$

Experts $s \geq s_N$ spend their entire endowment financing bonds in market $m(1-s)$, i.e. in the market for the lowest transparency (most opaque) country ω for which they can observe a good signal, and they use the selective acceptance rule $\mathbb{I}(\tau = g \;\&\; \omega \geq 1-s)$, which only accepts good assets. Some of these investor are in cash-in-the-market region, some in bunching, and some in bunching-with-scarcity. Experts $s \in [s_0, s_N)$ are nonselective. The function $\tilde{\omega}(s)$ assigns each one to a market: in market $m(\omega)$, nonselective investors bring down the un-financed reminder fraction by $\varepsilon'(\omega)$, which requires buying $\varepsilon'(\omega)\phi(1-\lambda)\int_0^\omega \hat{y}(\omega', g; L)d\omega'$ good assets and $\varepsilon'(\omega)\phi\lambda\bar{L}$ bad assets. If investor $\tilde{s}(\omega)$ is the nonselective investor that buys in market $m(\omega)$ then the total nonselective wealth available in that market is $-w\left(\tilde{s}(\omega)\right)\tilde{s}'(\omega)$, so market clearing implies (B.9). Inverting this function results in investor s choosing market $m\left(\tilde{\omega}(s)\right)$. Experts $s < s_0$ don't finance (buy) anything. Since they are indifferent between buying and not buying, many other patterns of demand among non-selective investors are possible.

(iv) Allocation function

- For markets $m(\omega) \in M([0,1])$ where ω falls in either a cash-in-the-market or a nonselective region

$$a(\omega,\tau;\chi,m) = \begin{cases} \frac{\chi(\omega,\tau)S(m,\omega,\tau)}{\sum_{\tau'}\int_{\omega'}\sigma(m,\omega',\tau';L)\chi(\omega',\tau')S(m,\omega',\tau')d\omega'} & \text{if } \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau';L)S(m,\omega',\tau')d\omega' > 0 \\ \frac{\chi(\omega,\tau)S(m,\omega,\tau)}{\sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau';L)S(m,\omega',\tau')} & \text{if } \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau';L)S(m,\omega',\tau')d\omega' = 0, \\ & \text{but } \sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau';L)S(m,\omega',\tau')d\omega' > 0 \\ 0 & \text{otherwise} \end{cases} \tag{B.10}$$

where

$$S(m,\omega,\tau) = \begin{cases} 1 \quad \text{if} \begin{cases} \tau = b \\ \text{or} \\ \tau = g \;\&\; \tilde{r}(m) \in (0, r_L(\omega)] \\ \text{or} \\ \tilde{r}(m) \leq 0 \end{cases} \\ 0 \quad \text{if } \tau = g,\ \tilde{r}(m) > r_L(\omega) \end{cases}$$

- For market $m(\omega)$ where ω falls in $[\omega^L, \omega^H]$ which is either a bunching, or bunching-with-scarcity region ($[\omega^L, \omega^H] = [0, \bar{\omega}]$); and χ is of the form $\mathbb{I}(\tau = g \ \& \ \omega \geq 1 - h)$:

$$a(\omega, \tau; \chi, m) = \begin{cases} \frac{\chi(\omega,\tau)S^h(m,\omega,\tau)}{\sum_{\tau'}\int_{\omega'}\sigma(m,\omega',\tau';L)\chi(\omega',\tau')S^h(m,\omega',\tau')d\omega'} & \text{if } \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau';L)S^h(m,\omega',\tau')d\omega' > 0 \\ \frac{\chi(\omega,\tau)S^h(m,\omega,\tau)}{\sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau';L)S^h(m,\omega',\tau')} & \text{if } \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau';L)S^h(m,\omega',\tau')d\omega' = 0, \\ & \text{but } \sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau')\sigma(m,\omega',\tau';L)S^h(m,\omega',\tau')d\omega' > 0 \\ 0 & \text{otherwise} \end{cases} \tag{B.11}$$

where $S^h(m, \omega, \tau)$ is the solution to differential equation

$$\frac{dS^h(m,\omega,\tau)}{dh} = \begin{cases} -w(h)\frac{S^h(m,\omega,\tau)\mathbb{I}[1-h\leq s\leq\omega^H]}{\int_g^{\omega^H}\sigma(m,\omega',\tau';L)S^h(m,\omega,g')d\omega'} & \text{if } \tau = g \text{ and } 1-h \in [\omega^L, \omega^H] \\ 0 & \text{otherwise} \end{cases} \tag{B.12}$$

If $m(\omega)$ is in a bunching-with-scarcity region, $\sigma(m, \omega, g; L) = \sigma(m, \bar{\omega}, g; L)$, $\forall\ \omega$. The terminal condition is

$$S^0(m, \omega, \tau) = \begin{cases} 1 & \text{if } \tau = b \text{ or } (\tau = g \text{ and } \omega \in [0, \omega^H]) \\ 0 & \text{otherwise} \end{cases} \tag{B.13}$$

Except for bunching and bunching-with-scarcity markets, the clearing algorithm implies that all investors draw bonds from a sample that is proportional to the original supply. This results in (B.10). In bunching markets, investor s imposes acceptance rule of the form $\chi_s(\omega, \tau) = \mathbb{I}(\tau = g \ \& \ \omega \geq 1 - h)$ with $h = s$; therefore when he buys his bond portfolio, the demand for bonds from good firms in country ω falls in proportion to his wealth, $w(s)$, times the ratio between the the demand for bonds by good firms from country ω and all the other bonds acceptable by investor s. This results in differential equation (B.12) which characterizes how the demand for bonds fall as the clearing algorithm progresses.

(v) Rationing function

- Firm (ω, τ), $\omega \geq \omega_1$

$$\eta(M([l, 1]), \omega, \tau; L) = \begin{cases} 1 - \varepsilon(l) & \omega < l \text{ or } (\omega = l \text{ and } \tau = b) \\ 1 & \omega \geq l \text{ and } \tau = g \\ 0 & \text{otherwise} \end{cases} \tag{B.14}$$

- Firm (ω, τ), $\omega < \omega_1$

$$\eta(M([l,1]),\omega,\tau;L) = \begin{cases} 1-\varepsilon(l) & \omega < l \\ \int_{1-\omega}^{1} \frac{1}{R_D(\tilde{\omega},\bar{\omega},\bar{r},1;x)+(\tilde{\omega}-(1-s))\phi(1-\lambda)\hat{y}(\bar{\omega},g;L)} w(s)ds & \omega \geq l \text{ and } \tau = g \\ 0 & \text{otherwise} \end{cases} \tag{B.15}$$

where $R_D(.)$ and $\tilde{\omega}$ are defined in equations (B.23) and (B.24), respectively. The rationing function is separately defined for firms from countries $\omega \geq \omega_1$ and $\omega < \omega_1$. It says that when $\omega \geq \omega_1$, a good firm from country $\omega < l$ who offers a bond at every market with interest rate $r(m) \in [0, r_L(l)]$, $(r_L(l) < \bar{r})$ will be able to sell a fraction $1-\varepsilon(l)$ (so the unsold *fraction* of the good assets is $\varepsilon(l)$), and $\varepsilon(\omega)$ fraction can be sold in market $m(\omega)$.

When $\omega < \omega_1$, a good firm from country $\omega < l$ who offers a bond at every market with interest rate $r(m) \in [0, r_L(l)]$, $(r_L(l) < \bar{r})$ will be able to sell a fraction $1-\varepsilon(l)$, and then $\varepsilon(\omega)\eta_L(\omega)$ fraction can be sold in market $m(\omega)$ which has interest rate $\bar{r}$, where $\eta_L(\omega) = \eta(m(\omega), \omega, g; L)$ is defined in (B.36).

Condition ($l = \omega$ and $\tau = b$) in (B.14) handles bad firms from country ω who sells on the same non-selective pricing market where the good firms from the same country sell all the reminder of their bonds.

Proof. From lemma C.1, each firm decision is expressed in terms of a reservation interest rate $r^L(\omega, \tau)$. The idea is to show the following statements: all bad firms are identified under equilibrium acceptance rule, so $r^L(\omega, b) = 0$. However, unlike $\theta = H$, $r^L(\omega, g)$ is different for good firms of different countries. Finding $r^L(\omega, g)$ is equivalent to finding the highest interest rate at which bonds of a good firm from country ω trades.

Moreover, unlike $\theta = H$, firm (ω, g) might be able to sell some bonds at interest rate below $r^L(\omega, g)$, so the equilibrium must characterize $r^L(\omega, g)$ and any other prices at which bonds of firm (ω, g) are sold.

(i) **Premium schedule $\mathbf{0 \leq r_L(\omega) \leq \bar{r}}$**
Let $r_L(\omega) = r^L(\omega, g)$. $r_L(\omega)$ falls into three possible classes: a "cash-in-the-market" interest rate, a"bunching" interest rate, a"bunching-with-scarcity" interest rate, or a "non-selective" interest rate.

Cash-in-the-market. The cash in the market interest rate $r^C(\omega)$ for the bond issued by the good firm of country ω is determined by equating demand and supply in the corresponding market.[18] The total amount of liquidity demanded by firm $j = (\omega, g)$ at interest rate $r^C(\omega)$

[18]The full notation would be $r^C(\omega, \tau)$, but since when $\theta = L$ only $\tau = g$ firms have a reservation interest rate, we suppress the dependence on τ.

should be equal to total wealth of investor $\hat{s}(\omega)$ which is the financier in that market.

$$\varepsilon(\omega)\phi(1-\lambda)\hat{y}(\omega, g; L; r^C(\omega)) = w\left(\hat{s}(\omega)\right) \tag{B.16}$$

In the special case where $\hat{y}(\omega, \tau; L; r^C(\omega))$ is determined given the specific structure of $t = 0$, use equation (12) to derive the total demand for bond in the corresponding market and simplify the above equation to

$$\frac{\varepsilon(\omega)\phi(1-\lambda)\xi}{1 + \phi\xi(\pi_H \frac{r_H}{1+r_H} + \pi_L \frac{r^C(\omega)}{1+r^C(\omega)})} = w(1-\omega) \tag{B.17}$$

As long as $r^C(\omega)$ is a strictly decreasing function and in the correct range, the equilibrium would be a cash-in-the-market pricing equilibrium. Each good firm of country ω demands bonds in all markets where $r(m) \leq r^C(\omega)$, and no market with a higher interest rate, while bad firms demand maximum bonds on every (active) market. Given the prudence shocks , each investor imposes $\chi_s(\omega, \tau) = \mathbb{I}(\tau = g \ \& \ \omega \geq 1 - s)$, i.e. he finances bond in the most profitable (highest interest rate) market for which he observes $x(g; \omega, s, L) = g$. Now consider a market with $r = r^C(\omega)$. Firms from countries with $\omega' \leq \omega$ demand bonds in that market, but no firm from countries $\omega' > \omega$ demand in this market because they have been able to issue all the bonds that they want at lower interest rate. Investor $s = \hat{s}(\omega)$ is able to recognize good assets in this markets, but investors $s < \hat{s}(\omega)$ are not. Moreover, if $r^C(\omega)$ is strictly decreasing, this is the highest interest rate where $\hat{s}(\omega)$ can detect good firms, so he will spend his entire wealth financing bonds demanded on this market. Then equation (B.16) implies all the bonds demanded by firm $j = (\omega, g)$ are financed at this market, and there will be non of them for sale at interest rate higher than $r^C(\omega)$.

Bunching. If $r^C(\omega)$ turns out to be upward sloping in any range, the logic of cash-in-the-market pricing breaks down because it implies the good firm from a higher transparency country is paying a higher interest rate to issue bonds, $\omega > \omega'$ and $r^C(\omega) > r^C(\omega')$. The investor who is financing the firm from lower transparency country, ω', can also identify the firm from a higher transparency country, ω, so he is better off financing the more transparent firm and collect a higher interest rate $r^C(\omega) > r^C(\omega')$, so there will be no financier for the less transparent firm ω'; a contradiction. In this region, there will be "bunching" of all the firms $[\omega', \omega]$ at a single price, i.e. an ironing procedure that restores a weakly monotone function. The clearing algorithm is such that the lower s investor picks the bonds that he finances first in a bunching market.

Since $w(.)$ function is decreasing, for small enough s, high enough ω, and appropriate set of parameters, equation (B.17) requires $r^C(\omega) < 0$. Let $\hat{\omega} = \min\ \omega$ such that $r^C(\omega) \leq 0$, then $\forall\ \omega'$ s.t. $\hat{\omega} < \omega' \leq 1$, $r^C(\omega') < 0$. Thus the requirement that there is a zero lower bound on the interest rate (no negative interest rate), implies there is a range of countries at the top, $\omega \geq \hat{\omega}$, whose good firms face zero interest rate in issuing bonds. Investors with $s \leq 1 - \hat{\omega}$ have idle wealth that is not financing any bonds, as there is not enough credit demand from good firms that they can recognize. In order for $\hat{\omega} < 1$ it must be that

$$w(0) > \phi(1-\lambda)\ell(1, g; L), \tag{B.18}$$

where we have used that $\hat{y}(\omega, g; L) = \ell(\omega, g; L)$ if $r_L(\omega) < \bar{r}$, and that $r_L(1) = 0$. In the special case where $\hat{y}(\omega, \tau; L)$ is determined given the specific structure of $t = 0$, the above condition simplifies to

$$w(0) > \phi(1-\lambda)D(0; r_H). \tag{B.19}$$

Use equation (18) to observe that assumption 2.(ii) ensures that (B.19) holds. Moreover, in equilibrium we have $\omega_3 = \hat{\omega}$.

Nonselective pricing. Consider a market m with interest rate $r = \tilde{r}(m)$, where good firms from country ω submit credit demand in that market. That implies all the good firms from countries $\omega' < \omega$ also submit demand in market m, as well as all the bad firms from all countries. An investor can choose to impose $\chi_s(\omega, \tau) = 1$ in market m and buy a representative sample of the pool.

The terms of trade that he will get is

$$\begin{aligned}\tau^N(r) &= \frac{(1+r)(1-\lambda)[\text{R Supply at interest rate q in FN}]}{\left(\lambda\bar{L} + (1-\lambda)[\text{R Supply at interest rate q in FN}]\right)} \\ &= \frac{(1+r)(1-\lambda)\int_0^\omega \hat{y}(\omega, g')d\omega'}{\left(\lambda\bar{L} + (1-\lambda)\int_0^\omega \hat{y}(\omega, g')d\omega'\right)}\end{aligned}$$

As long as $\omega_3 < 1$, there are (low expertise) international investors who finance bonds issued by good firms from countries $\omega > \omega_3$. The interest rate for these bonds is zero, so these investors make zero profits and are indifferent between financing and not financing bonds. Alternatively, if they trade non-selectively at a market at interest rate r, they can get the above terms of trade. As a result if $\tau^N(r) > 1$ these investors are better off trading at interest rate r nonselectively, which in turn implies no good bond from country ω can be offered at a interest rate above $r^{NS}(\omega)$. In other words, $\tau_N(r) \leq 1$ implies $r \leq r^{NS}(\omega)$.

$$\frac{(1+r)(1-\lambda)\int_0^\omega \hat{y}(\omega', g; L)d\omega'}{\left(\lambda\bar{L} + (1-\lambda)\int_0^\omega \hat{y}(\omega', g; L)d\omega'\right)} \leq 1$$
$$\frac{1}{1+r} \geq \frac{(1-\lambda)\int_0^\omega \hat{y}(\omega', g; L)d\omega'}{\left(\lambda\bar{L} + (1-\lambda)\int_0^\omega \hat{y}(\omega', g; L)d\omega'\right)} = \frac{1}{1 + \frac{\lambda\bar{L}}{(1-\lambda)\int_0^\omega \hat{y}(\omega', g; L)d\omega'}}$$
$$r \leq \frac{\lambda\bar{L}}{(1-\lambda)\int_0^\omega \hat{y}(\omega', g; L)d\omega'}$$

so

$$r^{NS}(\omega) \equiv \frac{\lambda\bar{L}}{(1-\lambda)\int_0^\omega \hat{y}(\omega', g; L)d\omega'} \tag{B.20}$$

When this upper bound interest rate is operative, bonds are finances in markets where both selective and non-selective buyers are active. In the markets where the interest rate is $r^{NS}(\omega)$, non-selective buyers will buy just enough assets (distributed pro-rate among the assets offered) such that the interest rate $r^C(\omega)$ is pushed down such that marginal investor $\hat{s}(\omega)$ can charge exactly interest rate $r^{NS}(\omega)$:

$$\varepsilon(\omega)\phi(1-\lambda)\hat{y}(\omega, g; L; r^{NS}(\omega)) = w\left(\hat{s}(\omega)\right) \tag{B.21}$$

In the special case where $\hat{y}(\omega, \tau; L; r^{NS}(\omega))$ is determined given the specific structure of $t = 0$, using (12) the above equation simplifies to

$$\frac{\varepsilon(\omega)\phi(1-\lambda)\xi}{1 + \phi\xi(\pi_H \frac{r_H}{1+r_H} + \pi_L \frac{r^{NS}(\omega)}{1+r^{NS}(\omega)})} = w(1-\omega) \tag{B.22}$$

In other words, if $\hat{s}(\omega) = 1 - s$ international investors are poor, that requires a high interest rate to push the demand of firms (ω, g) down so that equation (B.21) is satisfied. At this high interest rate, investors financing high ω good firms will enter this market and be non-selective financiers. This takes some bonds off of the market, which in turn implies a lower interest rate.

Bunching-with-scarcity. If there is a maximum interest rate $\bar{r}$ that firms are willing to pay to get bonds from investors, and if the wealth of smart investors, in the sense precisely defined below, is in short supply, then there will be a bunching region where some good firms will be rationed.

At any interest rate $r > \bar{r}$, good firms have zero demand for bonds, and (with linear objective function) at $r = \bar{r}$ they are indifferent between all levels of bond issued. So if the interest rate hits $\bar{r}$ in any market, it cannot increase any further than that.

Let $\bar{m}$ denote the market with interest rate $\bar{r}$, $\tilde{r}(\bar{m}) = \bar{r}$, and let $\bar{\omega}$ denote the index of highest type country whose good firms demand to issue bonds on market $\bar{m}$. Firms $(\bar{\omega}, g)$ submit $\sigma(\bar{m}, \bar{\omega}, g; L) = \hat{y}(\bar{\omega}, g; L)$ on market $\bar{m}$ and by definition their demand is exactly fully satisfied at interest rate $\bar{r}$. Good firms from all countries $\omega < \bar{\omega}$ also demand bonds on this market. Since these firms are indifferent about how many bonds they raise on market $\bar{m}$ (given the linearity of $t = 0$ objective function), we assume that all of them submit $\hat{y}(\bar{\omega}, g; L)$: $\forall\ \omega < \bar{\omega},\ \sigma(\bar{m}, \bar{\omega}, g; L) = \hat{y}(\bar{\omega}, g; L)$; and how many bond they raise is determined by rationing explained next.[19]

Bad firms from all countries also demand bonds on market $\bar{m}$, but none is able to issue any bonds in this market. Thus the demand submitted on market $\bar{m}$ is given by

$$\sigma(\bar{m}, \omega, \tau; L) = \begin{cases} \bar{L} & \text{if } \tau = b \\ \hat{y}(\bar{\omega}, g; L) & \text{if } \tau = g \text{ and } \omega < \bar{\omega} \\ 0 & \text{otherwise} \end{cases}$$

As such, if

$$\bar{\omega} \times \hat{y}(\bar{\omega}, g; L) > \int_{1-\bar{\omega}}^{1} w(s)ds,$$

then the wealth of investor who are able to recognize good firms from some country in $(0, \bar{\omega})$ is collectively in short supply, and some of the good firm demand is rationed at maximum interest rate $\bar{r}$. Next we determine the range of countries whose good firm demand for bonds is fully satisfied at interest rate $\bar{r}$. In order to do so, introduce the following function.

$\mathbf{R_D}(\omega', \omega, \mathbf{r}, \varepsilon; \mathbf{x})$**.** For $\omega' < \omega$, and interest rate r, let

$$R_D(\omega', \omega, r, \varepsilon; x) \equiv \varepsilon\phi(1-\lambda) \int_{\omega'}^{\omega} \hat{y}(z, g; L; r)dz - \int_{\hat{s}(\omega)}^{\hat{s}(\omega')} w(s)ds. \tag{B.23}$$

where x is a parameter.

$R_D(\omega', \omega, r, \varepsilon; x)$ Measures the excess residual demand (ε) by good firms of countries (ω', ω) at interest rate r which is not met by the cumulative wealth of the investors who are able to identify some good firm in this interval but no good firms from countries $\omega'' < \omega'$, i.e. $2 - \omega \leq s \leq 1 - \omega'$.

For $\omega = \bar{\omega}$ and $\varepsilon(\bar{\omega}) = 1$, we have

[19] This is slightly stronger than what we actually need to simplify the equilibrium derivation. The precise assumption we need is that when $\theta = L$, on the market where interest rate is $\bar{r}$, no good firm submits a total international demand which is higher than the total international demand submitted by the highest-transparency good firm. The latter firm is $j = (\omega_2, g)$, and even absent this assumption, $\hat{y}(\omega, g; L) = \hat{y}(\omega_2, g; L)$ for $\omega_1 \leq \omega < \omega_2$. So what we need is $\hat{y}(\omega, g; L) = \hat{y}(\omega_1, g; L)$ for $\omega_1 \leq \omega < \omega_1$, weaker than what specified here.

$$R_D(\omega', \bar{\omega}, \bar{r}, 1; x) = \phi(1-\lambda)(\bar{\omega} - \omega')\hat{y}(\bar{\omega}, g; L; \bar{r}) - \int_{1-\bar{\omega}}^{1-\omega'} w(s)ds$$

In the special case where $\hat{y}(\omega, \tau; L; \bar{r})$ is determined given the specific structure of $t = 0$, the above equation simplifies to

$$R_D(\omega', \bar{\omega}, \bar{r}, 1; x) = \phi(1-\lambda)(\bar{\omega} - \omega')\frac{D(\bar{r}; x)}{1+\bar{r}} - \int_{1-\bar{\omega}}^{1-\omega'} w(s)ds$$

which is the excess residual demand of the good firms in $(\omega', \bar{\omega})$ which should be absorbed by investors with expertise $s > 1 - \omega'$. Recall that in markets where there is bunching, the clearing algorithm used lets lower-s investors, who impose more restrictive acceptance rules, trade before higher-s investors.

Moreover, note that $R_D(\omega', \bar{\omega}, \bar{r}, 1; x) > 0$, $\forall\ \omega' < \bar{\omega}$. The reason is the following. By the logic of cash-in-the-market pricing, $\bar{r}$ is the interest rate at which demand of good firms of country $\bar{\omega}$ is exactly absorbed by wealth of the marginal investor $s(\bar{\omega})$. Consider a good firm from country ω' right below $\bar{\omega}$. Let $\tilde{r}'$ denote the hypothetical interest rate which clears the market for such good firm $\omega' < \bar{\omega}$, if this firm was still in a cash-in-the-market pricing. Again, using the logic of cash-in-the-market pricing, and the downward sloping wealth distribution of investors, it must be that $\tilde{r}' > \bar{r}$ as $\omega' < \bar{\omega}$. However, since $\bar{r}$ is the maximum interest rate any good firm accept, good firm $\omega' < \bar{\omega}$ faces a lower interest rate compared to what would clear his demand using only the wealth of his marginal investors, $s(\omega')$. Applying the same logic backward which would lead to a positive excess demand by good firms from countries in $(\omega', \bar{\omega})$ compared to what can be absorbed by their marginal investors collectively.

Let $\tilde{\omega} \in (0, \bar{\omega})$ be the index of the lowest country where the demand of good firms is fully absorbed by all the investors active in market $\bar{m}$.

$$R_D(\tilde{\omega}, \bar{\omega}, \bar{r}, 1; x) = \int_{1-\tilde{\omega}}^{1} \frac{R_D(\tilde{\omega}, \bar{\omega}, \bar{r}, 1; x)}{R_D(\tilde{\omega}, \bar{\omega}, \bar{r}, 1; x) + (\tilde{\omega} - (1-s))\,\phi(1-\lambda)\hat{y}(\bar{\omega}, g; L; \bar{r})} w(s)ds$$

which implies $\tilde{\omega}$ is the solution to

$$1 = \int_{1-\tilde{\omega}}^{1} \frac{1}{R_D(\tilde{\omega}, \bar{\omega}, \bar{r}, 1; x) + (\tilde{\omega} - (1-s))\,\phi(1-\lambda)\hat{y}(\bar{\omega}, g; L; \bar{r})} w(s)ds \tag{B.24}$$

In the special case where $\hat{y}(\omega, \tau; L; \bar{r})$ is determined given the specific structure of $t = 0$, the above equation simplifies to

$$1 = \int_{1-\tilde{\omega}}^{1} \frac{1}{\phi(1-\lambda)(\bar{\omega} - (1-s))\frac{D(\bar{r}; x)}{1+\bar{r}} - \int_{1-\bar{\omega}}^{1-\tilde{\omega}} w(s)ds} w(s)ds$$

Assumption 2.(ii), $\lim_{s\to 0} w(s) = 0$, insures that $\tilde{\omega} > 0$.

For a good firm from any country $\omega < \tilde{\omega}$, none of his offered bonds can be bought by investors of expertise $s < 1-\tilde{\omega}$, since those investors cannot identify him as good. Thus he can only sell what can be absorbed by the residual wealth of the subset of investors $s > 1-\tilde{\omega}$ who can identify him, $s > 1-\omega > 1-\tilde{\omega}$.

For $s > 1-\tilde{\omega}$, let

$$\zeta(s) = \frac{(\tilde{\omega} - (1-s))\,\phi(1-\lambda)\hat{y}(\bar{\omega}, g; L; \bar{r})}{R_D(\tilde{\omega}, \bar{\omega}, \bar{r}, 1; x) + (\tilde{\omega} - (1-s))\,\phi(1-\lambda)\hat{y}(\bar{\omega}, g; L; \bar{r})}$$

$\zeta(s)$ captures how much of the portfolio held by investor $s > 1-\tilde{\omega}$ is bonds issued "collectively" by good firms from countries $\omega < \tilde{\omega}$ that s can identify. The measure of those good firms is $(\tilde{\omega} - (1-s))\,\phi(1-\lambda)$. Thus for an individual firm of country $\omega < \tilde{\omega}$, aggregating over holdings of his bonds, by all the investors $s > 1-\omega$, we find how much $j = (\omega, g)$ can issue.

let $\eta_L(\omega) = \eta(\bar{m}, \omega, g; L)$ denote the rationing function in this market. The above argument implies

$$\begin{aligned}
\eta(\bar{m}, \omega, g; L) = \eta_L(\omega) &= \frac{1}{\hat{y}(\bar{\omega}, g; L; \bar{r})} \int_{1-\omega}^{1} \frac{1}{(\tilde{\omega} - (1-s))\,\phi(1-\lambda)} \zeta(s) w(s) ds \\
&= \int_{1-\omega}^{1} \frac{1}{R_D(\tilde{\omega}, \bar{\omega}, \bar{r}, 1; x) + (\tilde{\omega} - (1-s))\,\phi(1-\lambda)\hat{y}(\bar{\omega}, g; L; \bar{r})} ds
\end{aligned}$$

and for good firms from countries $\tilde{\omega} \le \omega < \bar{\omega}$, $\eta_L(\omega) = 1$.

In the special case where $\hat{y}(\omega, \tau; L; \bar{r})$ is determined given the specific structure of $t = 0$, the above equation simplifies to

$$\eta(\bar{m}, \omega, g; L) = \int_{1-\omega}^{1} \frac{1}{\phi(1-\lambda)(\bar{\omega} - (1-s))\frac{D(\bar{r}; x)}{1+\bar{r}} - \int_{1-\bar{\omega}}^{1-\tilde{\omega}} w(s) ds} ds$$

Moreover, we will have that in equilibrium, $\omega_1 = \tilde{\omega}$ and $\omega_2 = \bar{\omega}$. This leads to the rationing function in equation (B.36).

Interest Rate Regimes. Next, we need to determine what range of bonds has each kind of interest rate. In order to do so, introduce the following function.

$\mathbf{E}(\omega, \mathbf{r}, \varepsilon; \mathbf{x})$. Define

$$E(\omega, r, \varepsilon; x) \equiv \max_{\omega' \in [0,\omega]} \int_{\hat{s}(\omega)}^{\hat{s}(\omega')} w(s) ds - \varepsilon\phi\left(\lambda \int_{\omega'}^{\omega} \hat{y}(z, b; L; r) dz + (1-\lambda) \int_{\omega'}^{\omega} \hat{y}(z, g; L; r) dz \kappa\right)$$

In the special case where $\hat{y}(\omega, \tau; L; r)$ is determined given the specific structure of $t = 0$, the above equation simplifies to

$$\begin{aligned} E(\omega, r, \varepsilon; x) &= \max_{\omega' \in [0,\omega]} \int_{\hat{s}(\omega)}^{\hat{s}(\omega')} w(s)ds - \varepsilon\phi\left(\lambda(\omega - \omega')\bar{L} + (1-\lambda)\int_{\omega'}^{\omega} D(r; x)d\kappa\right) \\ &= \max_{\omega' \in [0,\omega]} \int_{\hat{s}(\omega)}^{\hat{s}(\omega')} w(s)ds - \varepsilon\phi(\omega - \omega')\Big(\lambda\bar{L} + (1-\lambda)D(r; x)\Big) \end{aligned}$$

where again x is a parameter here.

For a bond issued by good firm of country ω, interest rate r and remaining firm demand for bonds issuance ε, $E(\omega, r, \varepsilon; x)$ measures the maximum over $\omega' < \omega$ of the difference between the endowment of all investors who can recognize ω is good but cannot recognize that ω' is good, and how much is needed to finance ε units of all the bonds in $[\omega', \omega]$ which firms demand if they all face interest rate r. A bond interest rate can only be determined by cash-in-the-market if $E(\omega, r^C(\omega), \varepsilon(\omega); x) = 0$. A strictly positive value would mean that there exists a range of investors $[\hat{s}(\omega), \hat{s}(\omega')]$ for some $\omega' < \omega$, all of whom can identify some bond in the range $[\omega', \omega]$ as a good bond (but not any bonds offered by firms from countries lower than ω') and whose collective endowment exceeds what is necessary to finance all the bonds demanded by firms in $[\omega', \omega]$ facing a interest rate $r^C(\omega)$. Since these investors will want to spend their entire endowment financing bonds, it must be that some bond in the range $[\omega', \omega]$ must face a interest rate lower than $r^C(\omega)$. This is because $\frac{\partial D}{\partial r} < 0$, and a lower interest rate would push the firm demand up and bring demand closer to supply. But then monotonicity implies that the interest rate faced by a good firm from country ω must be lower than $r^C(\omega)$, a contradiction.

Next, suppose one knows that $\breve{\omega}$ is the upper limit of one type of region. In a similar manner to (Kurlat, 2016), the following procedure finds the lower end of that region, the type of region immediately below and the prices within the region.

1. For a cash-in-the-market region, the lower end is

$$\sup\{\omega < \breve{\omega} : r^{NS}(\omega) < r^C(\omega) \text{ or } E(\omega, r^C(\omega), \varepsilon(\omega); r_H) > 0 \text{ or } r^C(\omega) > \bar{r}\} \tag{B.25}$$

   and the region to the left is a nonselective region (first condition) or a bunching region (second condition), and bunching-with-scarcity (third condition), respectively. Within the region, $r_L(\omega) = r^C(\omega)$ and $\varepsilon(\omega) = \varepsilon(\breve{\omega})$.

2. For a bunching region, the lower end is

$$\max\{\omega < \breve{\omega} : E(\omega, r^C(\bar{\omega}), \varepsilon(\bar{\omega}); r_H) = 0\} \tag{B.26}$$

   and the region to the left is a cash-in-the-market region. Within the region, $r_L(\omega) =$

$r_L(\breve{\omega})$ and $r(\omega) = r(\breve{\omega})$.

3. For a non-selective region, the lower end is

$$\sup \left\{ \omega < \breve{\omega} : \frac{w\left(\hat{s}(\omega)\right)}{\phi(1-\lambda)\hat{y}(\omega, g; L, r^{NS}(\omega))} > r(\omega') \text{ for some } \omega' \in (\omega, \breve{\omega}) \right.$$
$$\left. \text{or } E(\omega, r^C(\breve{\omega}), \varepsilon(\breve{\omega}); r_H) > 0 \text{ or } r^{NS}(\omega) \geq \bar{r} \right\} \tag{B.27}$$

and the region to the left is a cash-in-the-market region (former condition), (second condition), and bunching-with-scarcity (third condition), respectively. Within the region, $r_L(\omega) = r^{NS}(\omega)$ and $\varepsilon(\omega) = \frac{w(\hat{s}(\omega))}{\phi(1-\lambda)\hat{y}(\omega,g;L,r^{NS}(\omega))}$.

(a) For a bunching-with-scarcity-region, the lower end is 0. Within the region, $r_L(\omega) = \bar{r}$ and $\varepsilon(\omega) = \varepsilon(\breve{\omega})$.

The first region is a bunching region with $\breve{\omega} = 1$, $r_L(\breve{\omega}) = 0$, and $\varepsilon(\breve{\omega}) = \phi(1-\lambda)\bar{L}$. Either one of the sets defined by B.25, B.26, B.27 is empty, in which case that region extends up to 0; or the bunching-with-scarcity region is hit and it extends all the way to zero.

(ii) **Firm optimization.**

Let ω_2 denote the index of the highest ω country who faces a interest rate $\bar{r}$

$$w(1-\omega_2) = \phi(1-\lambda)\ell(\omega_2, g; L),$$

and let ω_1 denote the index of the lowest ω country whose good firms do not face rationing in the bunching-with-scarcity region, defined as the solution to equation (B.24).

In the special case where $\hat{y}(\omega, \tau; L)$ is determined given the specific structure of $t = 0$, the above equation simplifies to $w(1-\omega_2) = \phi(1-\lambda)D(\bar{r}; r_H)$.

For any good firm from countries $\omega > \omega_2$, since $r(\omega) > 0$, the rationing function (B.14) implies that in order to issue all of the firm bonds, the reservation interest rate should be $r_L(\omega)$. Good firm from countries $\omega < \omega_2$ are indifferent between raising any number of bonds, so the issuance decision is optimal. For any bad firm, the rationing function implies that reservation interest rate is $\bar{r}$. Therefor credit issuance decisions are optimal for all firms and the total number of bonds they issue follows directly.

(iii) **Expert optimization.**

For $s \in [s_N, 1]$, each investor chooses the highest interest rate market on which there is a country ω such that $x(g; \omega, s, L) = g$ and $S(m, \omega, g) > 0$. Since $\tilde{r}(m) \geq 0$, this is optimal. For $s \in [s_0, s_N)$, investors only place weight on markets where nonselective pricing prevails.

Equation (B.20) implies they are indifferent between financing bonds and staying out, since the highest interest rate market where there is a ω such that $x(b, \omega, g) = 1$ and $S(m, \omega, g) > 0$ has $\tilde{r}(m) = 0$, there is no other market in which they would strictly prefer to trade. For $s < s_0$, the same logic implies that no trading is optimal.

(iv) **Allocation function.**

For any market $m(\omega)$ where ω falls in either a cash-in-the-market or a nonselective range, the NMR algorithm implies that all the investors face a residual supply proportional to the original supply, so equation (B.10) follows from Appendix B of Kurlat (2016), equation (65).

For markets $m(\omega)$ where ω falls in a bunching or bunching-with-scarcity region as described above and χ is of the form $\chi(\omega, \tau) = \mathbb{I}(\tau = g \;\&\; \omega > 1 - h)$, then the differential equations (B.12) follows from Appendix B of Kurlat (2016), equation (66), along with equation (19). Then (B.11) follows from applying the NMR algorithm.

(v) **Rationing function.**

Follows from applying equation (67) in Appendix B of Kurlat (2016).

■

B.1.3 Specializing the Wealth Function

Consider $r^C(\omega)$ and $r^{NS}(\omega)$ defined in equation (B.16) and (B.20), respectively. We make two assumptions to restrict the wealth function.

First, we have assumed that wealth function is monotonically decreasing, $w'(s) < 0$, so $r^C(\omega)$ does not become non-monotone. Thus bunching region can only emerge above some threshold, $\underline{\omega} < \omega \leq 1$, and bunching-with-scarcity only below some threshold, $0 \leq \omega < \bar{\omega}$.

Second, in what follows, we derive a parametric assumption to ensure that non-selective region does not emerge. Non-selective interest rate schedule is an upper bound on the prevailing interest rate in each market. Thus a sufficient condition for this upper bound to never be active, i.e. for the non-selective pricing region not to emerge, is to have $r^C(\omega) \leq r^{NS}(\omega)$ for markets where $0 < \tilde{r}(m) < \bar{r}$.

$$r^C(\omega) = \ell^{-1}\left(\frac{w\left(\hat{s}(\omega)\right)}{\phi(1-\lambda)}\right) \leq \frac{\lambda \bar{L}}{(1-\lambda)\int_0^{\omega} \hat{y}(\omega, g; L; \{r_H, r^C(\omega)\}) d\omega'},$$

where $\ell^{-1}(.)$ denotes the inverse of function $\ell(\omega, g; L; \{r_H, r^C(\omega)\})$ with respect to $r^C(\omega)$, and $\{r_H, r^C(\omega)\}$ indicates the dependence of demand function on (H, L) interest rate explicitly.

Moreover, we have used that $\hat{y}(\omega, g; L; \{r_H, r^C(\omega)\}) = \ell(\omega, g; L; \{r_H, r^C(\omega)\})$ for $r^C(\omega) < \bar{r}$, and that $\varepsilon(\omega) = 1$ when there is no non-selective region in equilibrium. Note that $\hat{y}(\omega', g; L) = \bar{L}$ ($\forall\ \omega'$) minimizes the right hand side on the above equation, which yields the following sufficient condition

$$r^C(\omega) = \ell^{-1}\left(\frac{w\left(\hat{s}(\omega)\right)}{\phi(1-\lambda)}\right) \leq \frac{\lambda}{(1-\lambda)\omega}. \tag{B.28}$$

In the proof of proposition 5 we derive a sufficient condition on primitives to ensure that (B.28) holds.

Under assumption 2, there is no non-selective region when $\theta = L$, $\omega_1 > 0$ and $\omega_3 < 1$. The equilibrium pricing regions are thus characterized by three thresholds $\omega_1 < \omega_2 < \omega_3$ such that

(i) Good firms of countries $0 \leq \omega < \omega_1$ are in bunching-with-scarcity market $\bar{m}$ at interest rate $\bar{r}(r_H)$, defined in (17), and $\eta(\bar{m}, \omega, g; L) < 1$.

(ii) Good firms of countries $\omega_1 \leq \omega < \omega_2$ are in bunching-with-scarcity market $\bar{m}$ at interest rate $\bar{r}(r_H)$, defined in (17), and $\eta(\bar{m}, \omega, g; L) = 1$.

(iii) Good firms of countries $\omega_2 \leq \omega < \omega_3$ are in cash-in-the-market pricing region.

(iv) Good firms of countries $\omega_3 \leq \omega < 1$ are in bunching region and face zero interest rate.

(v) No bad firm issues any bonds in any market.

B.2 t = 0: Optimal Initial and Maintained Investment Problem

Throughout this section, for brevity we will use $q_H = \frac{r_H}{1+r_H}$, $r_L(\omega) = r(\omega, g; L)$, $q_L(\omega) = \frac{r_L(\omega)}{1+r_L(\omega)}$, and $\bar{q} = \frac{\bar{r}}{1+\bar{r}}$.

Step 2. At $t = 0$, firms anticipate the date $t = 1$ continuation value and choose the initial and maintained investment levels, $I(\omega, \tau), \{i(\omega, \tau; \theta)\}_\theta$, to maximize their expected utility as defined in program (9).

We start by constructing $\hat{y}(\omega, \tau; \theta)$, i.e. the maximum liquidity that a firm can raise on the international markets. Maintaining $i(\omega, \tau; \theta)$ units allows a good firm to issue up to $\ell(\omega, \tau; \theta) = \frac{1}{1+r(\omega,\tau;\theta)}\xi i(\omega, \tau; \theta)$ bonds, with unit face value each, without violating the pledgeability constraint. Bad firms are not subject to pledgeability constraint since investors cannot seize anything from their output, thus they can issue up to $\bar{L}$. Thus we have

(i) Good firms, $\tau = g$:[20]

$$\hat{y}\left(\omega, \tau; \theta\right) = \ell\left(\omega, \tau; \theta\right) \qquad \forall\, \theta, \forall\, \omega \tag{B.29}$$

[20] We will show that when $\theta = L$, good firms from countries $\omega < \omega_2$ are indifferent in the scale at which they continue, Thus the $\ell\left(\omega, \tau; \theta\right) = \ell\left(\omega_2, \tau; \theta\right)$ for $\omega \leq \omega_2$. We pick this tie-breaking rule because it simplifies the exposition. For more detail see section B.1.2, Bunching-with-scarcity.

(ii) Bad firms, $\tau = b$:

$$\hat{y}\left(\omega, \tau; \theta\right) = \bar{L} \qquad \forall\, \theta, \forall\, \omega. \tag{B.30}$$

Remark. Recall that in section B.2 we assumed $\hat{y}(\omega, \tau; \theta)$ is decreasing in the (common) interest rate when $\theta = H$. With the above mapping, we need to verify that the equilibrium $\ell(\omega, \tau; \theta)$ is in fact downward sloping in r_H, which we will do in this section.

We specialize the investor wealth function to satisfy the sufficient condition 2.(iv). Thus the equilibrium in the international markets for $\theta = L$ is described in section B.1.3.

Consider the firm problem (9). Each firm j takes his optimal behavior at $t = 1$ as given, which along with $t = 1$ prices in different prudence shocks , the allocation function and the rationing function fully describes firm j continuation payoff. Firm j then chooses his business plan to maximize his expected utility given this continuation payoff.

Derivation of firm optimal choice of bond issuance, equations 6 and 8. A firm hit by liquidity shock has three possible options, at $t = 0$, in how to manage a liquidity shock in each aggregate state at $t = 1$. First, the firm can choose not to insure against the liquidity risk and abandon investment if a liquidity shock happen. This would lead to the highest ex-ante level of investment, $I(\omega, \tau)$. Second, the firm can choose to save enough out of his own endowment, through the banker, such that he has sufficient liquidity at $t = 1$ and does not need to raise any extra financing on the international markets. This option leads to the lowest ex-ante level of investment. Third, the firm can choose to save a lower amount from his initial endowment and borrow the rest from international investors. This leads to an intermediate level of ex-ante investment.

From the linearity of the firm problem, the firm chooses one option and does the same thing for all units of investment. Moreover, assumption 2.(i) implies the first option dominates second. Then assumption 2.(iii) implies that borrowing on the international markets are sufficiently cheap that the third option dominates the first one, which in turn leads to firm's optimal liquidity choice, equation 6. Conditions (i) and (iii) of assumption (2) are derived in proof of proposition 5.

Alternatively, a good firm who is not hit by a liquidity shock is indifferent between issuing bonds or not if $r(\omega, \tau; \theta) = 0$, and otherwise prefers not to issue. Thus these firms do not participate in the international markets. It follows that, if a bad firm not hit by a liquidity shock tries to issue bonds, his type is revealed and he does not succeed in raising funding, and it will not participate either.

As such, only firms hit by liquidity shock attempt to raise funding from international investors at $t = 1$, which in turn implies the ex-ante budget constraint 8.

Firm problem given the optimal choice of issuance. Since problem (9) is linear, equations (3)-(8) determine the optimal firm choices, $i(\omega, \tau; \theta)\ \forall\, \theta$ whenever they are non-zero. Plugging these solutions into (8) determines $I(\omega, \tau)$.

The rest of the argument follows from a parallel logic to (Holmström and Tirole, 1998), (Holmström and Tirole, 2011). Conjecture $i(\omega,\tau;H) = I(\omega,\tau)$, and let $0 \leq x \leq 1$ denote the scale of investment for firm $j = (\omega,\tau)$ when $\theta = L$.

Use the $t = 2$ interest rate along with equation (8) to get $I(\omega,\tau)$. Substitute $I(\omega,\tau)$ in the objective function (9).

First consider a good firm $j = (\omega, g)$. The objective function of the good firm boils down to

$$\Pi(x) = \frac{\phi(\rho_g - \xi)(\pi_H + \pi_L x) + (1-\phi)\rho_g}{1 + \phi\xi(\pi_H q_H + \pi_L q_L(\omega)x)} - 1$$

Thus he optimal investment is determined by

$$\Pi'(x) = \frac{\pi_L\phi\Big(\rho_g - \xi - \pi_H\phi\xi^2(q_H - q_L(\omega)) - \rho_g\xi\big(q_L(\omega)\,(1-\pi_L\phi) - \pi_H q_H\phi\big)\Big)}{(1 + \phi\xi(\pi_H q_H + \pi_L q_L(\omega)x))^2}$$

As such, $\Pi'(x) > 0 (< 0)$ implies $x = 1 (x = 0)$, and if $\Pi'(x) = 0$ good firm j is indifferent between any level of continuation when $\theta = L$ and the firm has a liquidity shock . This implies

$$q_L(\omega) < \bar{q} = \frac{(\rho_g - \xi)(1 + \phi\pi_H r_H\xi)}{\xi((1-\phi)\rho_g + \phi\pi_H(\rho_g - \xi))}. \tag{B.31}$$

Substitute $\frac{r_L(\omega)}{1+r_L(\omega)}$ for $q_L(\omega)$ to get equation (17).

Next, we need to make sure that our conjecture for continuation at full scale in high state regime, $i(\omega,\tau;H) = I(\omega,\tau)$, is correct for a good firm. For this conjecture to hold, it must be that $r_H < \bar{r}_H$ such that every good firm j prefers to submit liquidity demand to international markets when $\theta = H$. Using assumption 2.(i), the alternative is to set $i(\omega,\tau;H) = 0$, do not do any liquidity risk management and abandon production if hit by a liquidity shock in state $\theta = H$, and instead increase $I(\omega,\tau)$. Since firms from country $\omega = 1$ are those who face the lowest interest rate in $\theta = L$, such deviation is most profitable for them. Thus it is sufficient to ensure that they do not want to deviate. Thus $\bar{r}_H$ solves

$$\rho_g(1-\phi) + (\rho_\tau - \xi)\phi\pi_L = \frac{\rho_g(1-\phi) + (\rho_g - \xi)\phi}{1 + \phi\pi_H\xi\frac{\bar{r}_H}{1+\bar{r}_H}}$$

Thus if

$$r_H < \bar{r}_H = \frac{(\rho_g - \xi)}{\rho_g\xi(1-\phi) + (\rho_g - \xi)(\phi\pi_L\xi - 1)} \tag{B.32}$$

all good firms prefer to do liquidity management using a combination of own saving and international markets.

Now consider a bad firm, $\tau = b$. Assumption 2.(i) implies firms either do liquidity management using international markets, or do not do any liquidity management. Moreover, if $\theta = L$ a bad firms hit by a liquidity shock is not able to raise any international financing, so he has to liquidate investment, which means for a bad firm $x = 0$. Next, note that when $\theta = L$, a good firm from the least transparent country, $j = (0, g)$, faces the exact same funding condition as all bad firms, and he is not able to raise any financing, $x = 0$. Thus at $t = 0$, similar to all bad firms, firm j does not save any precautionary liquidity for $\theta = L$ state. Moreover, as long as $r_H < \bar{r}_H$, firm j (along with all other good firms) prefers to do liquidity management against liquidity shock in $\theta = H$ state. Bad firms also face the same interest rate r_H when $\theta = H$, and furthermore they do not pay back, so if firm j participates in the international markets when $\theta = H$, all bad firms will do so as well.

Firm investment at $\mathbf{t = 0}$. Next we characterize the $t = 0$ firm investment. Substitute back the optimal continuation decision, and corresponding date $t = 2$ prices into equation (8) to get the optimal investment decision

$$I(\omega, g) = \begin{cases} \frac{1}{1+\xi(\pi_H q_H + \pi_L q_L(\omega))} & \text{if } \omega > \omega_2 \\ \frac{1}{1+\xi(\pi_H q_H + \pi_L \bar{q})} & \text{if } \omega_1 < \omega \leq \omega_2 \\ \frac{1+(1-\eta_L(\omega))\frac{\phi\xi\pi_L\bar{q}}{1+\phi\xi\pi_H q_H}}{1+\phi\xi(\pi_H q_H + \pi_L \bar{q})} & \text{if } \omega \leq \omega_1 \end{cases} \tag{B.33}$$

where $\eta_L(\omega) = \eta\,(m, \omega, g; L)$; and the investment for $\omega \leq \omega_1$ follows from substituting the continuation decision corresponding to each aggregate state in the date $t = 0$ budget constraint:

$$\begin{aligned} I(\omega, g) &= \frac{1 - \phi\pi_L\bar{q}D(\bar{r}; r_H)\eta_L(\omega)}{1 + \phi\xi\pi_H q_H} = \frac{1 - \frac{\phi\xi\pi_L\bar{q}}{1+\phi\xi(\pi_H q_H + \pi_L\bar{q})}\eta_L(\omega)}{1 + \phi\xi\pi_H q_H} \\ &= \frac{\frac{1+\phi\xi\pi_H q_H + \phi\xi\pi_L\bar{q}(1-\eta_L(\omega))}{1+\xi(\pi_H q_H + \pi_L\bar{q})}}{1 + \phi\xi\pi_H q_H} = \frac{1 + (1-\eta_L(\omega))\,\frac{\phi\xi\pi_L\bar{r}}{1+\phi\xi\pi_H q_H}}{1 + \phi\xi\,(\pi_H q_H + \pi_L\bar{q})}. \end{aligned}$$

Moreover,

$$I(\omega, b) = 1 - q_H\xi\phi\pi_H\eta_H(\omega)\bar{L} \tag{B.34}$$

where $\eta_H(\omega) = \eta(m_H, \omega, b; H)$.

Finally, firm j realized issuance of bonds on the international market, $\ell(\omega, \tau; \theta)$, is given by:

(i) Good firms, $\tau = g$

$$\ell\,(\omega, g; \theta) = \int_{M_j} \xi I(\omega, \tau) d\eta\,(m, \omega, g; \theta) \tag{B.35}$$

where $\eta(m, \omega, g; \theta)$ is given by

$$\eta(m,\omega,g;\theta) = \begin{cases} \int_{1-\omega}^{1} \frac{1+\bar{r}}{\phi(1-\lambda)D(\bar{r};r_H)(\omega_2-(1-s))-\int_{1-\omega_2}^{1-\omega_1} w(s)ds} w(s)ds, & \tilde{r}(m)=\bar{r} \ \& \ \omega<\omega_1 \ \& \ \theta=L \\ 1 & \left(\tilde{r}(m)<\bar{r} \text{ or } \left(\tilde{r}(m)=\bar{r} \ \& \ \omega \geq \omega_1\right)\right) \ \& \ \theta=L \\ & \text{or } \tilde{r}(m)=r_H \ \& \ \theta=H \\ 0 & \text{otherwise} \end{cases} \tag{B.36}$$

(ii) Bad firms, $\tau = b$

$$\ell(\omega,\tau;\theta) = \bar{L}\int_{m\in M_j} \eta(m,\omega,\tau;\theta)dm \tag{B.37}$$

where M_j, $j = (\omega, b)$ is the set of markets firm $j = (\omega, b)$ can sell bonds in, and

$$\eta(m,\omega,b;\theta) = \begin{cases} \int_{s_H}^{1-\omega} \frac{1}{\lambda(1-s)D(0;r_H)+(1-\lambda)\bar{D}(r_H)} \frac{w(s)}{\phi(1-r_H)}ds & \tilde{r}(m)=r_H \ \& \ \omega \leq 1-s_H \ \& \ \theta=H \\ 0 & \text{otherwise} \end{cases} \tag{B.38}$$

To complete the proof we need to verify that there is a fixed point to the joint $t = 0, 1$ problem, i.e. date $t = 0$ optimal outcomes do constitute an equilibrium in the international markets at $t = 1$. We do this in proposition 5.

Maximum liquidity demand on international markets. By construction of the model, firms can always submit excess demand $\hat{y}$ on (different) markets at $t = 1$ to undo the rationing by investors. To avoid this, we need to impose an exogenous upper bound on how much demand for bond issuance firm can submit. We choose $\bar{L} = D(0; r_H)$ for convenience as in this case good firms are not constrained by this limit in equilibrium, while no bad firms can undo rationing by submitting more than what they need.

C Proofs

Proof of Lemma 1.

$$\begin{aligned} Y^{A}(\omega,\tau;\theta) &\equiv \rho_\tau \left[(1-\lambda)\left((1-\phi)I^{A}(\omega,g)+\phi i^{A}(\omega,g;\theta)\right)+\lambda\left((1-\phi)I^{A}(\omega,b)+\phi i^{A}(\omega,b;\theta)\right)\right] \\ &= \rho_\tau \max\left(1-\phi, \frac{1}{1+\phi\xi}\right). \end{aligned}$$

■

Lemma C.1 *Every solution to robust program A.3 satisfies*

$$\sigma(m,j,k;\theta)=\begin{cases} \geq \hat{y}\left(\omega,\tau;\theta;\{r_H(\omega),r_L(\omega)\}\right) & \text{if } \tilde{r}(m;\theta)<r^{L}(\omega,\tau;\theta) \\ 0 & \text{if } \tilde{r}(m;\theta)>r^{L}(\omega,\tau;\theta) \end{cases}$$

Proof. For simplicity, let j denote the firm (ω,τ), $\hat{y}(\omega,\tau)\equiv\hat{y}\left(\omega,\tau;\theta;\{r_H,r_L(\omega)\}\right)$, $\sigma(m,j)\equiv\sigma(m,\omega,\tau;\theta)$, and $\eta(m,j)\equiv\eta(m,\omega,\tau;\theta)$. Also, we suppress the dependence of interest rate on prudence shock $\theta=H,L$ and write $\tilde{r}(m)$. Each individual firm is small and takes the prices as given, and does not affect the schedule of prices either.

Assume the contrary. This implies that there are two markets, m and m' with $\tilde{r}(m')<\tilde{r}(m)$ such that, for some j, the firm chooses $\sigma(m,j)>0$ and $\sigma(m',j)<\hat{y}(\omega,\tau)$. There are four possible cases:

(i) $\eta(m;i)>0$ and $\eta(m',j)>0$. Then the firm can increase his utility by choosing demand $\tilde{\sigma}$ with $\tilde{\sigma}(m',j)=\sigma(i,m')+\epsilon$ and $\tilde{\sigma}(m,j)=\sigma(m',j)-\epsilon\frac{\eta(m',j)}{\eta(m,j)})$ for some positive ϵ.

(ii) $\eta(m,j)>0$ and $\eta(m',j)=0$. Consider a sequence such that $\eta^{n}(m',j)>0$. By the argument in part 1, for any n the solution to robust firm problem must have either $\sigma^{n}(m,j)=0$ or $\sigma^{n}(m',j)\geq\hat{y}(\omega,\tau)$ (or both). Therefore either the condition that $\sigma^{n}(m,j)\to\sigma(m,j)$ or the condition that $\sigma^{n}(m',j)\to\sigma(j,m')$ in a robust solution is violated.

(iii) $\eta(m,j)=0$ and $\eta(m',j)>0$. Consider a sequence such that $\eta^{n}(m',j)>0$. By the argument in part 1, for any n the solution to robust firm problem must have either $\sigma^{n}(m,j)=0$ or $\sigma^{n}(m',j)\geq\hat{y}(\omega,\tau)$ (or both). Therefore either the condition that $\sigma^{n}(m,j)\to\sigma(m,j)$ or the condition that $\sigma^{n}(m',j)\to\sigma(m',j)$ in a robust solution is violated.

(iv) $\eta(m,j)=\eta(m',j)=0$. Consider a sequence such that $\eta^{n}(m',j)>0$ and suppose that there is a sequence of solutions to robust firm problem which satisfies $\sigma^{n}(m',j)\to\sigma(m',j)<\hat{y}(\omega,\tau)$. This implies that for any sequence such that $\eta^{n}(m,j)>0$ and for any n, the solution to robust firm problem must have $\sigma^{n}(m,j)=0$. Therefore the condition that $\sigma^{n}(m,j)\to\sigma(m,j)$ in a robust solution is violated.

■

Lemma C.2 *Assume $G(x)$ and $H(x)$ are continuous. Equation* (C.1) *has a fixed point $x \in [0,1]$,*

$$F(x) = \frac{\lambda(1 - s_H(x))H(x)}{\lambda(1 - s_H(x))H(x) + (1-\lambda)G(x)} \tag{C.1}$$

where $s_H(x)$ solves

$$\int_{s_H(x)}^{1} \frac{1}{\lambda(1-s)H(x) + (1-\lambda)G(x)} w(s)ds = \phi(1-x), \tag{C.2}$$

if equation (C.2) *has a solution, and $s_H(x) = 0$ otherwise.*

Proof of Lemma C.2.

Case 1 [$s_H \in (0,1)$]. Consider the case where s_H is interior. Consider the self-map on $F : [0,1] \mapsto [0,1]$. We use Brouwer's fixed-point theorem to prove existence of a fixed point. $[0,1]$ is a compact convex set. We need to show is that $F(x)$ is a continuous function, and maps $[0,1]$ to itself, which is immediate since the ratio in $F(x)$ is positive and (weakly) smaller than one.

Next we move to proving continuity. $G(x)$ is continuous and so is $H(x)$. Thus $s_H(x)$ which is the solution to equation C.2 is also continuous as long as it is interior, $s_H \in (0,1)$.

This implies that everything on the right hand side of equation C.1 is continuous, so $F(x)$ is a continuous map from $[0,1]$ to $[0,1]$[21], which implies by Brouwer's theorem a fix point exists.

Case 2 [$s_H = 0$]. Then equation (C.2) only holds with inequality. Equation (C.1) becomes one equation in one unknown in x, which with the same argument as the previous case has a fixed point. ■

Proof of Proposition 5.

We start with explaining the mapping between the equations in the statement of the proposition to the solution developed in sections B.2 and B.1. To simplify the formulas, the proposition is stated in terms of premia rather than interest rates, using the monotone transformation

$$q = \frac{r}{1+1}. \tag{C.3}$$

Equation (18) writes the general form of required maintenance cost of a firm who faces premia x when $\theta = H$, and y when $\theta = L$, or interest rates $\frac{x}{1-x}$ and $\frac{y}{1-y}$, respectively. It uses the equilibrium firm ex-ante investment, defined by equations (B.33) and (B.34), and optimal continuation scale. Substitute in equation (6) to get firm liquidity demand in international markets.

At the end of this proposition we state sufficient conditions on the parameters to ensure that firms choose to participate in international markets when $\theta = H$, and the equilibrium when $\theta = L$ is described in section B.1.3. Under this equilibrium structure, equation (19) aggregates the total required maintenance across the pricing regions when $\theta = L$, described in section B.1.3.

[21]whenever $s_H(x) \in (0,1)$ at the fixed point

Equation (17) rewrites the maximum premium $\bar{q}$ in $\theta = L$, defined in equation (B.31), when the common premium in high state is x. The threshold countries are defined in equations (21), (22), and (25). Equation (22) determines the threshold where bunching region ends, at zero interest rate, given liquidity demand function (18). Equation (21) determines the threshold where bunching-with-scarcity region starts, at premium $\bar{q}$ (interest rate $\bar{r}$). Equation (25) determines the threshold where rationing starts in bunching-with-scarcity region, given the liquidity demand

Finally, equations (15) and (16) jointly determine the pooling premium and marginal investor when $\theta = H$, at the above liquidity demand levels.

We use lemma C.2 to prove existence of equilibrium. Let $G(x) = \bar{D}(x)$ and $H(x) = D(0; x)$. As such we need to show both functions are continuous.

$\bar{y}(x)$ is continuous. $D(y; x)$ is continuous in x for any $x, y > 0$ since $1 + \phi\xi(\pi_H x + \pi_L y) > 0$. Thus $D(0; x)$ and $D(\bar{y}(x); x)$ are also continuous.

Now turn to $\omega_1(x)$, $\omega_2(x)$ and $\omega_3(x)$. $w(.)$ and $D(y; x)$ are continuous in x. $D(y, x)$ is constant in ω and $w(.)$ is increasing in ω, so equations (21) and (22) have a unique solution in ω, so $\hat{\omega}_3(x)$ and $\hat{\omega}_2(x)$ exist, are unique, and continuous.

Next, $D(0, x)$ is decreasing in x. Moreover

$$D(\bar{y}(x); x) - (1 - \bar{y}(x))D(\bar{y}(x); x) = \bar{y}(x)D(\bar{y}(x); x) = \frac{\rho_H - \xi}{\rho_g - \phi\xi},$$

$$\frac{d\Big((1 - \bar{y}(x))D(\bar{y}(x); x)\Big)}{dx} = \frac{dD(\bar{y}(x); x)}{dx} = \frac{\xi^2 \pi_H \phi(\rho_g(1 - \phi) + \phi\pi_H(\rho_g - \xi))}{(\rho_g - \phi\xi)(1 + \phi\xi\pi_H x)^2} < 0.$$

Thus both $\hat{\omega}_2(x)$ and $\hat{\omega}_3(x)$ are monotonically decreasing in x. Since $\hat{\omega}_2(x)$ and $\hat{\omega}_3(x)$ are continuous, (23) and (24) imply that $\omega_2(x)$ and $\omega_3(x)$ are also continuous and weakly decreasing in x.

Next consider the right hand side of (25). $\omega_2(x)$ is continuous. Moreover, (25) is the simplified version of (B.24). We have already shown that $R_D(\omega_1, \omega_2(x), \bar{y}(x), 1; x) > 0$, and $\hat{y}(\omega_2(x), g; L) = D(\bar{y}(x); x)(1 - x) > 0$, thus the denominator is positive. Each term is also continuous in x, which in turn implies the right hand side is continuous in x. Thus $\hat{\omega}_1(x)$ is continuous as well, and using equation (26), $\omega_1(x)$ is also continuous.

Finally, continuity of $\omega_i(x)$ $i = 1, 2, 3$, along with continuity of $w(.)$, $\eta(.)$ and $D(y; x)$ (in x) implies $\bar{D}(x)$ is continuous. So by lemma C.2, the fixed point exists.

Parametric Assumptions.

Optimal Firm Decision without Access to International Market [Assumption 2.(i)]. Assume the firm does not have access to international investors. So the firm can do one of the two things. The first option is to nvest all of his initial endowment. Then the firm continues with a high scale, $I_I = 1$, if not hit by a liquidity shock, and terminate the project if hit. Thus the payoff is $\Pi_I = \rho_\tau(1-\phi)I_1 = \rho_\tau(1-\phi)$. Alternatively, the firm can save enough of his own endowment using bankers to insure against the liquidity shock in either or both aggregate states. Since the aggregate state is only relevant in the interaction with the international investors, if the firm choose to insure against liquidity shock from own endowment, it will be for both aggregate states. The firm investment scale is given by $I_S = \frac{1}{1+\phi\xi}$, and his expected payoff is $\Pi_I = \rho_\tau I_2$. Thus for $\Pi_I > \Pi_S$ we need

$$1-\phi > \frac{1}{1+\phi\xi} \Rightarrow \phi < \frac{\phi\xi}{1+\phi\xi} \Rightarrow \xi > \frac{1}{1-\phi},$$

which is assumption 2.(i). Under this assumptions when firms can access the international credit market, we only need to compare borrowing on the international markets with investing all of their endowment. This is the next parametric restriction that we consider.

Sufficient Condition for Inequality (B.32) [**Assumption 2.(ii)**]. Let $q_H = \frac{r_H}{1+r_H}$. From equation (15)

$$q_H = \frac{r_H}{1+r_H} = \frac{\lambda\,(1-s_H(r_H))}{\lambda\,(1-s_H(r_H)) + (1-\lambda)\,\frac{\bar{D}(r_H)}{D(0;r_H)}} \leq \frac{\lambda}{\lambda + (1-\lambda)\frac{\bar{D}(r_H)}{D(0;r_H)}},$$

which in turn implies

$$r_H \leq \frac{\lambda}{(1-\lambda)\frac{\bar{D}(r_H)}{D(0;r_H)}} = \frac{\lambda}{1-\lambda}\frac{D(0;r_H)}{\bar{D}(r_H)}.$$

So to find an upper bound on r_H, it is sufficient to find an upper bound on $\frac{D(0;r_H)}{\bar{D}(r_H)}$. Note that $D(0;r_H)$ is the maintenance cost of the firms from highest transparency country. Moreover, $\bar{L} = \frac{D(0;r_H)}{1+r_H}$ is by construction have the highest liquidity demand submitted by any firm to the international markets, which in turn implies $D(0;r_H)$ is the highest maintenance cost for any good firm from any country. Thus $\bar{D}(r_H) \leq D(0;r_H)$, which in turn implies

$$r_H \leq \frac{\lambda}{1-\lambda} \Rightarrow q_H \leq \lambda$$

In assumption 2.(ii) we assume $\frac{\lambda}{1-\lambda} \leq \bar{r}_H$, where $\bar{r}_H$ is defined in equation (B.32). This in turn insures that $r_H \leq \bar{r}_H$. Moreover, one can substitute λ for x in (17) to get an upper bound on $\bar{q}$.

Sufficient Condition for Inequality (B.19) [**Assumption 2.(iii)**]. A sufficient condition is

$$w(0) \geq \phi(1-\lambda)\xi, \tag{C.4}$$

which ensure that $\omega_3 < 1$, and constitutes assumption 2.(iii).

Sufficient Condition for Inequality (B.28) [**Assumption 2.(iv)**]. The only set of markets we need to consider are those with cash in the market pricing. Let $q^C(\omega) = \frac{r^C(\omega)}{1+r^C(\omega)}$ and $\bar{q}(r_H) = \frac{\bar{r}(r_H)}{1+\bar{r}(r_H)}$. From (B.28)

$$q^C(\omega) \leq \frac{\lambda}{\lambda + (1-\lambda)\omega}$$

Start by noting that $\bar{q}(r_H)$ is the maximum $q^C(\omega)$ can achieve, so a sufficient condition for inequality (B.28) is

$$\min\{\bar{q}(r_H), q^C(\omega)\} \leq \frac{\lambda}{\lambda + (1-\lambda)\omega}.$$

Next from (17)

$$\bar{q}(r_H) = \frac{(\rho_g - \xi)(1 + \phi\xi\pi_H q_H)}{(\rho_g(1-\phi) + \phi(\rho_g - \xi)\pi_H)\,\xi} \leq \frac{(\rho_g - \xi)\,(1 + \lambda\phi\xi\pi_H)}{(\rho_g\,(1-\phi) + \phi\,(\rho_g - \xi)\,\pi_H)\,\xi}$$

where the inequality used part (ii) to replace q_H with it's maximum, λ. Next, from (20)

$$q^C(\omega) = \frac{\xi\phi(1-\lambda) - w(1-\omega)(1 + \phi\xi\pi_H x)}{\xi\phi((1-\lambda) + w(1-\omega)\pi_L)} \leq \frac{\xi\phi(1-\lambda) - w(1-\omega)}{\xi\phi((1-\lambda) + w(1-\omega)\pi_L)}$$

where the inequality just uses $q_H \geq 0$. Substitute both back to get a sufficient condition

$$\min\left\{\frac{(\rho_g - \xi)\,(1 + \lambda\phi\xi\pi_H)}{(\rho_g\,(1-\phi) + \phi\,(\rho_g - \xi)\,\pi_H)\,\xi}, \frac{\xi\phi(1-\lambda) - w(1-\omega)}{\xi\phi((1-\lambda) + w(1-\omega)\pi_L)}\right\} \leq \frac{\lambda}{\lambda + (1-\lambda)\omega}.$$

which is assumption 2.(iv).

■

Lemma C.3 *In any equilibrium $r_L(\omega) = r^R(\omega, g)$ is nonincreasing in ω everywhere.*

Proof.

Assume the contrary. Then when $\theta = L$, there exists bonds offered by good firms from countries ω, ω' with $\omega' > \omega$ such that $r_L(\omega') > r_L(\omega)$.For this to be consistent with firm optimization, it must be that

$$\eta(M_0, \omega', g; L) < \eta(M_0, \omega, g; L) = 1,$$

where the inequality follows from firm optimization, and the equality from definition of $r_L(\omega)$ and M_0, where $M_0 = \{m : \tilde{r}(m) \leq r_L(\omega)\}$. But investor optimization and the signal structure when $\theta = L$ requires that investors only use rules of the form $\chi(\omega, \tau) = \mathbb{1}\,(\tau = g \,\&\, \omega \geq 1 - h)$. This implies that for any $M_0 \subset M$,

$$\eta(M_0, \omega', g; L) \geq \eta(M_0, \omega, g; L),$$

a contradiction. ■

D Aggregate productivity shock, state dependent fraction of good firms

In this part, we briefly discuss how the equilibrium objects change under the generalization that the fraction of good firms, λ, is state dependent.

As we assume that each firm knows its type already in period 0, in this version we modify the timing of the realization of the aggregate state. We assume that the aggregate state is realized in period 0, determining the fraction of bad firms, λ_θ, but firms do not observe this until period 1.

Then, going through the same derivation as before, we replace each λ in each expression by λ_H or λ_L depending on whether that expression is determined by the fraction of bad firms in the high or the low state. For instance, expressions 15-16 change to

$$F\left(x\right)=\frac{\lambda_H\left(1-s_H(x)\right)D\left(0;x\right)}{\lambda_H\left(1-s_H(x)\right)D\left(0;x\right)+\left(1-\lambda_H\right)\bar{D}(x)} \tag{D.5}$$

and

$$\int_{s_H(x)}^{1}\frac{1}{\lambda_H(1-s)D(0;x)+(1-\lambda_H)\bar{D}(x)}w(s)ds=(1-x)\phi, \tag{D.6}$$

as D.5 is determined by the fraction of good and bad firms in an investor portfolio with skill $s_H(x)$ in the high state, and the fraction in the integrand of the left hand side of D.6 is determined by the fraction of wealth an investor with skill s is allocated towards good firms in the high state.

In contrast, expression 20 is determined by the market clearing condition for firms in country $\omega\in[\omega_2(x),\omega_3(x)]$ in the low state. Therefore, it changes to

$$y^C(\omega)\equiv\frac{\xi\phi(1-\lambda_L)-w(1-\omega)(1+\phi\xi\pi_Hx)}{\xi\phi((1-\lambda_L)+w(1-\omega)\pi_L)}\qquad\qquad \omega\in\left[\omega_2(x),\omega_3(x)\right]. \tag{D.7}$$