The I Theory of Money*

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Abstract

A theory of money needs a proper place for financial intermediaries. Intermediaries create inside money and their ability to take risks determines the money multiplier. In downturns, intermediaries shrink their lending activity and fire-sell their assets. Moreover, they create less inside money. As the money multiplier shrinks, the value of money rises. This leads to a Fisher disinflation that hurts intermediaries and all other borrowers. The initial shock is amplified, volatility spikes up and risk premia rise. An accommodative monetary policy in downturns, focused on the assets held by constrained agents, recapitalizes intermediaries and hence mitigates these destabilizing adverse feedback effects. A monetary policy rule that accommodates negative shocks and tightens after positive shocks, provides an ex-ante insurance, mitigates financial frictions, reduces endogenous risk and risk premia but it also creates moral hazard.

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1 Introduction

A theory of money needs a proper place for financial intermediaries. Financial institutions are able to create money, for example by accepting deposits backed by loans to businesses and home buyers. The amount of money created by financial intermediaries depends crucially on the health of the banking system and the presence of profitable investment opportunities. This paper proposes a theory of money and provides a framework for analyzing the interaction between price stability and financial stability. It therefore provides a unified way of thinking about monetary and macroprudential policy.

Intermediaries serve three roles. First, intermediaries monitor end-borrowers. Second, they diversify by extending loans to and investing in many businesses projects and home buyers. Third, they are active in maturity transformation as they issue short-term (inside) money and invest in long-term assets. Intermediation involves taking on some risk. Hence, a negative shock to end borrowers also hits intermediary levered balance sheets. Intermediaries’ individually optimal response to an adverse shock is to lend less and accept fewer deposits. As a consequence, the amount of inside money in the economy shrinks. As the total demand for money as a store of value changes little, the value of outside money increases, i.e. disinflation occurs.

The disinflationary spiral in our model can be understood through two extreme polar cases. In one polar case the financial sector is undercapitalized and cannot perform its functions. As the intermediation sector does not create any inside money, money supply is scarce and the value of money is high. Savers hold only outside money and risky projects. Savers are not equipped with an effective monitoring technology and cannot diversify. The value of safe money is high. In the opposite polar case, intermediaries are well capitalized. Intermediaries mitigate financial frictions and channel funds from savers to productive projects. They lend and invest across in many loans and projects, exploiting diversification benefits and their superior monitoring technology. Intermediaries also create short-term (inside) money and hence the money multiplier is high. In this polar case the value of money is low as inside money supply supplements outside money.

As intermediaries are exposed to end-borrowers’ risk, an adverse shock also lowers the financial sector’s risk bearing capacity. It moves the economy closer to the first polar regime with high value of money. In other words, a negative productivity shock leads to deflation of Fisher (1933). Financial institutions are hit on both sides of their balance sheets. On the asset side, they are exposed to productivity shocks of end-borrowers. End-borrowers’ fire
sales depress the price of physical capital and liquidity spirals further erode intermediaries’ net worth (as shown in Brunnermeier and Sannikov (2014)). On the liabilities side, they are hurt by the Fisher disinflation. As intermediaries cut their lending and create less inside money, the money multiplier collapses and the real value of their nominal liabilities expands. The Fisher disinflation spiral amplifies the initial shock and the asset liquidity spiral even further.

Monetary policy can work against the adverse feedback loops that precipitate crises, by affecting the prices of assets held by constrained agents and redistributing wealth. Since monetary policy softens the blow of negative shocks and helps the reallocation of capital to productive uses, this wealth redistribution is not a zero-sum game. It can actually improve welfare. It can reduce endogenous (self-generated) risk and overall risk premia.

Simple interest rate cuts in downturns improve economic outcomes only if they boost prices of assets, such as long-term government bonds, that are held by constrained sectors. Wealth redistribution towards the constrained sector leads to a rise in economic activity and an increase in the price of physical capital. As the constrained intermediary sector recovers, it creates more (inside) money and reverses the disinflationary pressure. The appreciation of long-term bonds also mitigates money demand, as long-term bonds can be used as a store of value as well. As interest rate cuts affect the equilibrium allocations, they also affect the long-term real interest rate as documented by Hanson and Stein (2014) and term premia and credit spread as documented by Gertler and Karadi (2014). From an ex-ante perspective long-term bonds provide intermediaries with a hedge against losses due to negative macro shocks, appropriate monetary policy rule can serve as an insurance mechanism against adverse events.

Like any insurance, “stealth recapitalization” of the financial system through monetary policy creates a moral hazard problem. However, moral hazard problems are less severe as the moral hazard associated with explicit bailouts of failing institutions. The reason is that monetary policy is a crude redistributive tool that helps the strong institutions more than the weak. The cautious institutions that bought long-term bonds as a hedge against downturns benefit more from interest rate cuts than the opportunistic institutions that increased leverage to take on more risk. In contrast, ex-post bailouts of the weakest institutions create strong risk-taking incentives ex-ante.

**Related Literature.** Our approach differs in important ways from both the prominent New Keynesian approach but also from the monetarist approach. The New Keynesian approach emphasizes the interest rate channel. It stresses role of money as unit of account
and price and wage rigidities are the key frictions. Price stickiness implies that a lowering nominal interest rates also lowers the real interest rate. Households bring consumption forward and investment projects become more profitable. Within the class of New Keynesian models Christiano, Moto and Rostagno (2003) is closest to our analysis as it studies the disinflationary spiral during the Great Depression.

In contrast, our I Theory stresses the role of money as store of value and a redistributional channel of monetary policy. Financial frictions are the key friction. Prices are fully flexible. This monetary transmission channel works primarily through capital gains, as in the asset pricing channel promoted by Tobin (1969) and Brunner and Meltzer (1972). As assets are not held symmetrically in our setting, monetary policy redistributes wealth and thereby mitigate debt overhang problems. In other words, instead of emphasizing the substitution effect of interest rate changes, in the I Theory wealth/income effects of interest rate changes are the driving force.

Like in monetarism (see e.g. Friedman and Schwartz (1963)), an endogenous reduction of money multiplier (given a fixed monetary base) leads to disinflation in our setting. However, in our setting outside money is only an imperfect substitute for inside money. Intermediaries, either by channeling funds through or by underwriting and thereby enabling firms to approach capital markets directly, enable a better capital allocation and more economic growth. Hence, in our setting monetary intervention should aim to recapitalize undercapitalized borrowers rather than simply increase the money supply across the board. A key difference to our approach is that we focus more on the role of money as a store of value instead of the transaction role of money. The latter plays an important role in the “new monetarists economics” as outlined in Williamson and Wright (2011) and references therein. Instead of the “money view” our approach is closer in spirit to the “credit view” à la Gurley and Shaw (1955), Patinkin (1965), Tobin (1969, 1970), Bernanke (1983) Bernanke and Blinder (1988) and Bernanke, Gertler and Gilchrist (1999).1

As in Samuelson (1958) and Bewley (1980), money is essential in our model in the sense of Hahn (1973). In Samuelson households cannot borrow from future not yet born generations. In Bewley and Scheinkman and Weiss (1986) households face explicit borrowing limits and cannot insure themselves against idiosyncratic shocks. Agent’s desire to self-insure through precautionary savings creates a demand for the single asset, money. In our model households

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1The literature on credit channels distinguishes between the bank lending channel and the balance sheet channel (financial accelerator), depending on whether banks or corporates/households are capital constrained. Strictly speaking our setting refers to the former, but we are agnostic about it and prefer the broader credit channel interpretation.
can hold money and physical capital. The return on capital is risky and its risk profile differs from the endogenous risk profile of money. Financial institutions create inside money and mitigate financial frictions. In Kiyotaki and Moore (2008) money and capital coexist. Money is desirable as it does not suffer from a resellability constraint as physical capital does. Lippi and Trachter (2012) characterize the trade-off between insurance and production incentives of liquidity provision. Levin (1991) shows that monetary policy is more effective than fiscal policy if the government does not know which agents are productive. More recently, Cordia and Woodford (2010) introduced financial frictions in the new Keynesian framework. The finance papers by Diamond and Rajan (2006) and Stein (2012) also address the role of monetary policy as a tool to achieve financial stability.

More generally, there is a large macro literature which also investigated how macro shocks that affect the balance sheets of intermediaries become amplified and affect the amount of lending and the real economy. These papers include Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999), who study financial frictions using a log-linearized model near steady state. In these models shocks to intermediary net worths affect the efficiency of capital allocation and asset prices. However, log-linearized solutions preclude volatility effects and lead to stable system dynamics. Brunnermeier and Sannikov (2014) also study full equilibrium dynamics, focusing on the differences in system behavior near the steady state, and away from it. They find that the system is stable to small shocks near the steady state, but large shocks make the system unstable and generate systemic endogenous risk. Thus, system dynamics are highly nonlinear. Large shocks have much more serious effects on the real economy than small shocks. He and Krishnamurthy (2013) also study the full equilibrium dynamics and focus in particular on credit spreads. In Mendoza and Smith’s (2006) international setting the initial shock is also amplified through a Fisher debt-disinflation that arises from the interaction between domestic agents and foreign traders in the equity market. In our paper debt disinflation is due to the appreciation of inside money. For a more detailed review of the literature we refer to Brunnermeier et al. (2013).

This paper is organized as follows. Section 2 sets up the model and derives first the solutions for two polar cases. Section 3 presents computed examples and discusses equilibrium properties, including capital and money value dynamics, the amount of lending through intermediaries, and the money multiplier for various parameter values. Section 4 introduces long-term bonds and studies the effect of interest-rate policies as well as open-market operations. Section 5 showcases a numerical example of monetary policy. Section 6 concludes.
The economy is populated by two types of agents: households and intermediaries. There are $I + J$ production technologies. A household can employ only a single technology at any time point $t \in [0, \infty)$. That is, its monitoring capacity is limited to only one project at any moment. However, over time, households can freely switch from one technology to another, i.e. there is free entry among household sectors.

Intermediaries are specialized: they can employ only the first $I$ production technologies. However, each intermediary can monitor and diversify among all of these technologies. Intermediaries are separate from households and they cannot switch, i.e. we assume an extreme mobility friction between intermediary and household sectors.

Technologies. Physical capital $k_t$ can be employed in any of $I + J$ technologies. If employed in technology $i \in \{1, 2, \ldots I + J\}$, capital produces the amount $ak_t$ of output $i$. Total amount of capital in the economy is denoted by $K_t$, and total amount of output $i$ produced, $Y_t^i$. Output goods are combined to make the total quantity of

$$Y_t = \left[ \frac{1}{I + J}(Y_t^1)^\frac{s+1}{s} + \ldots + \frac{1}{I + J}(Y_t^{I+J})^\frac{s+1}{s} \right]^{\frac{s}{s-1}},$$

of the aggregate good, where $s$ is the elasticity of substitution.\(^2\) With competitive goods markets, the market price of output $i$ in terms of the aggregate good is

$$P_t^i = \frac{1}{I + J} \left( \frac{Y_t^i}{Y_t} \right)^{1/s}.$$

Physical capital $k_t$ is subject to shocks that depend on the technology in which it is employed. If used in technology $i \in \{1, 2, \ldots I + J\}$, capital follows

$$\frac{dk_t}{k_t} = (\Phi(t_t) - \delta) dt + \sigma^i dZ_t^i,$$

where the Brownian motions $i = 1, \ldots I + J$ are independent. The investment function $\Phi$ has the standard properties $\Phi' > 0$ and $\Phi'' \leq 0$, i.e. there may be adjustment costs, and the input for investment $t_t$ is the aggregate good. We assume that risks $\sigma^i$ are all the same and equal to $\sigma^I$ for $i = 1, \ldots I$, and to $\sigma^J$ for $i = I + 1, \ldots J$. Typically we take $\sigma^I \geq \sigma^J$, i.e.

\(^2\)For $s = \infty$ the outputs are perfect substitutes, for $s = 0$ there is no substitutability at all, while for $s = 1$ the substitutability corresponds to that of a Cobb-Douglas production function.
intermediaries specialize in riskier projects. The vector of Brownian motions \(1, \ldots, I + J\) is denoted by \(Z_t\).

**Preferences.** All agents have symmetric preferences: they have logarithmic utility with a common discount rate \(\rho\). That is, any agent maximizes the expected utility

\[
E \left[ \int_0^\infty e^{-\rho t} \log c_t \, dt \right],
\]

subject to individual budget constraints, where \(c_t\) is the consumption of the aggregate good at time \(t\).

**Assets and Returns.** Agents can hold capital employed in risky production technologies. Households can allocate only to a single technology each, while intermediaries can diversify among technologies \(1\) through \(I\). Agents can also hold money: physical outside money or inside money, i.e. monetary IOUs issued by other agents. Agents can also create inside money by borrowing money from other agents. In the baseline model, there is a fixed amount of fiat money in the economy that pays zero interest.

We assume that the price of capital per unit follows a Brownian process of the form

\[
\frac{dq_t}{q_t} = \mu_t^q \, dt + \sigma_t^q \, dZ_t.
\]  

(2.4)

If so, then the return on capital employed in any production technology \(i\) can be found using Ito’s lemma. The capital gains rate is given by \(d(k_t q_t)/(k_t q_t)\). The dividend yield is \((a P^i_t - \iota)/q_t\), so that the total return is

\[
dr_t^i = \frac{P^i_t a - \iota_t}{q_t} \, dt + \left( \Phi(\iota_t) - \delta + \mu_t^q + \sigma_t^q (\sigma^i 1^i)^T \right) \, dt + \left( \sigma_t^q + \sigma^i 1^i \right) \, dZ_t,
\]

where \(1^i\) is the row coordinate vector with a single 1 in the \(i\)-th position. The optimal investment rate \(\iota_t\), which maximizes the return of any technology, is given by the first-order condition \(1/q_t = \Phi'(\iota_t).\) We denote the investment rate that satisfies this condition by \(\iota(q_t)\).

Intermediaries choose to employ technologies \(i = 1, \ldots, I\) in equal amounts since the prices of the first \(I\) goods are the same by symmetry (and equal to \(P^I_t\)). Thus, the return that they get from capital is

\[
dr_t^I = \frac{P^I_t a - \iota}{q} \, dt + \left( \Phi(\iota) - \delta + \mu_t^q + \sigma_t^q \left( \sigma^I \frac{1^I}{I} \right)^T \right) \, dt + \left( \sigma_t^q + \sigma^I \frac{1^I}{I} \right) \, dZ_t,
\]
where $I^I$ is a vector with ones in positions 1 through $I$ and zeros everywhere else.

Recall that in the baseline model the total money supply is fixed. The value of all money depends on the size of the economy. Denote the value of money by $p_t K_t$, and assume that $p_t$ follows a Brownian process of the form

$$
\frac{dp_t}{p_t} = \mu^p_t \, dt + \sigma^p_t \, dZ_t. \quad (2.5)
$$

The law of motion of aggregate capital $K_t$ depends on its allocation to various technologies. If the fraction of world capital $\psi^J$ is dedicated by households to technologies $I + 1$ through $J$, then the risk of aggregate capital is

$$
\sigma^K_t = (1 - \psi^I_t) \frac{\sigma^I}{I} + \psi^J_t \frac{\sigma^J}{J}.
$$

Thus, the return on money is

$$
dr^M_t = (\Phi(\nu) - \delta + \mu^p_t + \sigma^p_t (\sigma^K_t)^T) \, dt + (\sigma^K_t + \sigma^p_t) \, dZ_t.
$$

It is useful to denote by

$$
\pi_t = \frac{p_t}{q_t + p_t}
$$

the fraction of the world wealth that is in the form of money, and the law of motion of $\pi_t$ by

$$
\frac{d\pi_t}{\pi_t} = \frac{\mu^\pi_t}{\mu^\pi_t} \, dt + \sigma^\pi_t \, dZ_t.
$$

**Equilibrium Definition.** The agents initially start with some endowments of capital and money. Over time, they trade - they choose how to allocate their wealth between capital and money and which output goods to produce. That is, they solve their individual optimal consumption and portfolio choice problems to maximize utility. Individual agents take prices as given. Given prices, markets for capital, money and consumption goods have to clear.

At any time $t \in [0, \infty)$, we denote by $\psi$ the fraction of world capital held by intermediaries and employed in technologies 1 through $I$, and by $\psi^I_t$ and $\psi^J_t$ those employed by households in technologies 1 through $I$ and $I + 1$ through $I + J$ respectively. If the net worth of intermediaries is $N_t$ and the world wealth is $(q_t + p_t)K_t$, then the intermediaries’ net worth share is denoted by

$$
\eta_t = \frac{N_t}{(q_t + p_t)K_t}. \quad (2.6)
$$
Let \( \eta^I_t \) be the net worth share of households who employ technologies 1 through \( I \), and \( \eta^J_t \), the net worth share of those who employ technologies \( I + 1 \) through \( I + J \). Then \( \eta^I_t + \eta^J_t = 1 - \eta \). Denote by \( \zeta_t, \zeta^I_t \) and \( \zeta^J_t \) the consumption rates, per unit of net worth, of intermediaries and the two types of households respectively.

**Definition.** Given any initial allocation of capital and money among the agents, an equilibrium is a map from histories \( \{ Z_s, s \in [0, t] \} \) to prices \( p_t \) and \( q_t \), allocations \( (\psi_t, \psi^I_t, \psi^J_t) \) and \( (\zeta_t, \zeta^I_t, \zeta^J_t) \), such that

(i) *all markets, for capital, money and consumption goods, clear*

(ii) *all agents choose technologies, portfolios and consumption rates to maximize utility*

One important choice here is that of households: they can employ only one technology and must choose which technology to use. Utility maximization implies that households must be indifferent among all technologies, which a positive fraction of households employ, and weakly prefer those technologies to all other technologies. Typically, households will employ only technologies \( I + 1 \) through \( I + J \), so that \( \psi^I_t = \eta^I_t = 0 \), except when intermediaries are undercapitalized. When intermediaries do not have sufficient funds and produce too little of goods 1 through \( I \), so that the price of these goods rises, then some households may switch to employing these technologies even though they cannot diversify.

### 2.1 Equilibrium Conditions.

Logarithmic utility has two well-known tractability properties. First, an agent with logarithmic utility and discount rate \( \rho \) consumes at the rate given by \( \rho \) times net worth. Thus, \( \zeta_t = \zeta^I_t = \zeta^J_t = \rho \) and the market-clearing condition for the consumption good is

\[
\rho (q_t + p_t) K_t = Y_t - \iota_t K_t. \tag{2.7}
\]

Second, the excess return of any risky asset over any other risky asset is explained by the covariance between the difference in returns and the agent’s net worth.

The law of motion of net worth of a representative intermediary is given by

\[
\frac{dN_t}{N_t} = x_t \, dr^I_t + (1 - x_t) \, dr^M_t - \rho \, dt,
\]
\[ x_t = \frac{\psi_t q_t}{\eta_t (q_t + p_t)} = \frac{\psi_t (1 - \pi_t)}{\eta_t} \]
is the portfolio weight on capital and \(1 - x_t\) is the portfolio weight on money. Thus, the volatility of the intermediary’s net worth is

\[ \sigma^N_t = x_t \left( \frac{\sigma^I}{I} + \sigma^q_t - \sigma^K_t - \sigma^P_t \right) + \sigma^K_t + \sigma^P_t, \]

where \(\nu_t\) is the notation we introduce for the difference in the average risk of capital held by intermediaries and the risk of money. The condition that determines the portfolio weight \(x_t\) is then

\[ \frac{E_t [d r^I_t - d r^M_t]}{d t} = \nu_t (\sigma^N_t)^T. \] (2.8)

Given (2.8), the law of motion of the intermediaries’ net worth has a simple representation in terms of just the risks taken and the return on money, i.e.

\[ \frac{dN_t}{N_t} = d r^M_t - \rho \, dt + x_t \nu_t (\sigma^N_t)^T \, dt + x_t \nu_t \, dZ_t. \] (2.9)

Likewise, the risk of net worth of households who employ technology \(j \in \{I+1, \ldots I+J\}\) is

\[ \sigma^{Nj}_t = x^j_t \left( \frac{\sigma^I}{I} + \sigma^q_t - \sigma^K_t - \sigma^P_t \right) + \sigma^K_t + \sigma^P_t, \]

where \(x^j_t = \psi^j_t (1 - \pi_t)/\eta^j_t\) is the portfolio weight on capital. The asset-pricing condition is then

\[ \frac{E_t [d r^j_t - d r^M_t]}{d t} = \nu^j_t (\sigma^{Nj}_t)^T. \] (2.10)

For \(i \in \{1, \ldots I\}\), we also have

\[ \frac{E_t [d r^i_t - d r^M_t]}{d t} = x^i_t \left( \frac{\sigma^I}{I} + \sigma^q_t - \sigma^K_t - \sigma^P_t \right) + \sigma^K_t + \sigma^P_t \right)^T, \] (2.11)

where \(x^i_t\) is the optimal allocation to capital of any household that chooses to employ technology \(i\). If \(\eta^i_t > 0\), then \(x^i_t = \psi^i_t (1 - \pi_t)/\eta^i_t\), of course. However, \(x^i_t\) is well-defined by (2.10) even in the case when \(\eta^i_t = 0\).
To determine whether $\eta^I_t > 0$, we have to evaluate the households’ utilities from using technologies 1 through $I$ and $I+1$ through $I+J$. The following proposition summarizes the relevant condition here.

**Proposition 1.** In equilibrium

$$
(x^I_t)^T \nu^I_t (\nu^I_t)^T \geq (x^I_{I+1})^2 \nu^I_t (\nu^I_t)^T.
$$

(2.12)

and if some households use technologies 1 through $I$, then (2.12) must hold with equality.

**Proof.** See Appendix.

### 2.2 Two Benchmark Cases.

To understand what drives the value of money in this economy, we identify two important benchmarks. One extreme is the money regime benchmark that arises when intermediaries have zero net worth and cannot function. With zero loss absorption capacity intermediaries cannot create any inside money. As a consequence, any household holds only outside money and capital employed to produce a single good $i$. The other extreme is the frictionless benchmark, in which intermediaries always function. To capture this benchmark, we explore what happens in the hypothetical scenario that households can switch to become intermediaries, and vice versa. In both cases the economy is stationary, so the prices of capital and money $q_t$ and $p_t$ stay constant.

**The Money Regime.** To gain intuition, consider the case in which all technologies have the same risk, i.e. $\sigma^I = \sigma^J = \sigma$. Then, since intermediaries have zero net worth, capital is devoted equally to all production technologies, so $Y_i = Y^i_t = aK_t/(I+J)$ and $P^i_t = a/(I+J)$ for all $i$. The market-clearing condition for output implies that

$$
\frac{aK_t}{I+J} - \iota K_t = \rho(p_t + q_t)K_t \quad \Rightarrow \quad \frac{aP^i_t - \iota}{q_t} = \frac{\rho}{1 - \pi}
$$

and so the return from any technology $i$ is given by

$$
dr^i_t = \frac{\rho}{1 - \pi} \, dt + (\Phi(\iota_t) - \delta) \, dt + \sigma 1^i \, dZ_t.
$$
The return on money is
\[ dr^M_t = (\Phi(t_i) - \delta) dt + \sigma \frac{\vec{1}}{I + J} dZ_t, \]
where \( \vec{1} \) is a vector of ones. Money has lower risk than any individual non-diversifiable technology, but it also lacks the dividend yield.

The value of money is determined by the optimal portfolio weight on capital. We have
\[
\left[ E_t[dr^i_t - dr^M_t] \right]/\rho/(1-\pi) = \left( \sigma_1^i - \sigma \frac{\vec{1}}{I + J} \right) \left( x^i_t \sigma_1^i + (1 - x^i_t) \sigma \frac{\vec{1}}{I + J} \right)^T \\
\]

There is an equilibrium in which fiat money has positive value if \( \rho < \sigma_2^i \frac{I + J - 1}{I + J} \). If so, then \( x^i_t = 1 - \pi \) and the value of money in the stationary equilibrium is given by
\[
\pi = 1 - \sqrt{\frac{\rho}{\sigma_2^2 \frac{I + J - 1}{I + J}}}. \tag{2.13} \]

Money can have positive value only if there is sufficient demand for a safe asset, i.e. the productive technologies available to individual households are sufficiently risky.\(^3\)

The following proposition characterizes equilibrium when \( \sigma^I \neq \sigma^J \).

\(^3\)Of course, there is always an equilibrium in which money has no value and each household allocates all of its wealth in a single productive technology. (There are also many non-stationary equilibria between these two extremes, in which \( \pi_t \) fluctuates between the level given by (2.13) and 0). The equilibrium with money is more efficient than one without money, as the utility of all households is strictly higher in the former equilibrium. To see this, note that in the former equilibrium, the consumption stream from the latter equilibrium is still available to any household, if it chooses to use the former technology and invest as much as in the equilibrium without money. Note also that in the equilibrium without money, the price of capital satisfies
\[
aK_t/(I + J) - \iota K_t = \rho q_t K_t, \]
and so it is greater than the price in the equilibrium with money. Thus, the equilibrium without money exhibits inefficient over-investment. Had an asset with risk \( \sigma_1^i/(I + J) dZ_t \) been available to households in this equilibrium, they would demand a positive amount of this asset as long as its expected return \( \tilde{r} \) satisfied
\[
\tilde{r} - \rho - (\Phi(\iota) - \delta) \geq \left( \sigma \frac{\vec{1}}{I + J} - \sigma^i_1 \right) \sigma^i_1 \quad \Leftrightarrow \quad \tilde{r} \geq \Phi(\iota) - \delta + \rho - \sigma^2 \frac{I + J - 1}{I + J} \text{ if } \sigma^2 \frac{I + J - 1}{I + J} < 0.
\]

Because the required return on an asset whose risk equals to that of the economy overall is less than the rate of growth of the economy, there economy is prone to bubbles. Money is such a bubble, and existence of money makes the economy more dynamically efficient.
Proposition 2. If $\sigma^I \neq \sigma^J$ then the equilibrium can be found by solving the following system of simultaneous equations

\[
\begin{align*}
\sigma^K &= (1 - \psi^J)\frac{\nu^I}{I} + \psi^J \sigma^J \frac{\nu^J}{J}, \\
\nu^I &= 1^i \sigma^I - \sigma^K, \\
\nu^J &= 1^j \sigma^J - \sigma^K,
\end{align*}
\]

\[x^I = \sqrt{\frac{\rho}{\nu^I (\nu^I)^T}}, \quad x^J = \sqrt{\frac{\rho}{\nu^J (\nu^J)^T}}, \quad \frac{1 - \psi^J}{x^I} + \frac{\psi^J}{x^J} = \frac{1}{1 - \pi}, \quad \frac{Y - \iota(q) K}{qK} = \frac{\rho}{1 - \pi}.
\]

where $Y$, $P^I$, and $P^J$ are determined by (2.1) and (2.2).

The net worth share of the agents who employ technologies 1 through $I$ is $(1 - \psi^J)(1 - \pi)/x^I$, and of the remaining agents, $\psi^J(1 - \pi)/x^J$.

Proof. See Appendix.\footnote{The equilibrium can be found numerically by guessing $\psi^J$, solving the equations sequentially, and adjusting the initial guess of $\psi^J$ based on (2.14).}

Proposition 2 characterizes equilibrium in an economy, in which agents can freely switch between production technologies. The indifference condition of Proposition 1 holds, i.e. any agent is indifferent between choosing one of technologies 1 through $I$ or any of the remaining technologies. The formulas of Proposition 2 can be used, with one minor adjustment, to find what happens in the frictionless economy. In that case, agents can switch across the technologies, and can also diversify among technologies 1 through $I$.

The Frictionless Economy. Suppose that agents can freely switch between becoming intermediaries and households. In this case, we want to be able to find the prices of capital and money as well as the net worth of agents who employ technologies 1 through $I$ as well as those who employ technologies $I + 1$ through $I + J$. These quantities can be found from the following corollary of Proposition 2.

Corollary 1. The equilibrium in the frictionless economy can be found through the same set of equations as in Proposition 2, except replacing the expression for $\nu^I$ with

\[\nu^I = \sigma^I \frac{\nu^I}{I} - \sigma^K. \quad (2.15)\]
Proof. All equilibrium conditions are identical to those of Proposition 2, except that agents who use technologies 1 through \( I \) can diversify. Therefore, the risk of allocating wealth to capital instead of money for these agents is given by (2.15).

Figure 1 illustrates the two benchmarks for parameter values \( \rho = 5\% \), \( a = 2 \), \( I = 8 \), \( J = 12 \), \( \sigma^I = 0.7 \), \( \sigma^J = 0.5 \), \( s = 0.6 \) and \( \Phi(\iota) = \log(\kappa \iota + 1)/\kappa \) with \( \kappa = 2 \). The horizontal axis marks the net worth share of agents who use technologies 1 through \( I \) and diversify. The vertical axis marks the prices \( q \) and \( p \) of capital and money.

The value of the safe asset \( p \), i.e. money, is high when the money regime in which agents who use technologies 1 through \( I \) cannot diversify. These agents choose to put only about 33\% of their wealth in capital and the rest, in money, due to risk. Denote by \( p \) and \( q \) the prices of money and capital in the money regime.

In contrast, under the frictionless benchmark the value of money \( p \) is low. Only agents who use technologies \( I + 1 \) through \( I + J \) hold money, as they cannot diversify. In contrast, agents who become intermediaries and diversify among technologies 1 through \( J \) use leverage: they put 155\% of their wealth in these technologies. These agents supply the safe asset and
create *inside money*. Because intermediaries can diversify and use leverage, they do not need large net worth: their net worth share in this example is only 22%.

In the frictionless benchmark, entry into and exit from any sector is completely free. The choice of whether to become an intermediary or a regular household is determined by the indifference condition: the agents get the same expected utility regardless of what they become. However, this is just a thought experiment to gain intuition.

In contrast, in our actual model agents cannot switch types between intermediaries and households. Households remain households forever. They can engage in all technologies, including 1 through $I$, but can never diversify. Therefore, the net worth of intermediaries fluctuates due to shocks and intermediaries may become undercapitalized. The money regime provides the boundary condition for what happens if the intermediary net worth drops to 0. In contrast, the frictionless benchmark is not an actual state of the dynamic equilibrium, but it captures well the forces that are at play when intermediaries are well-capitalized. The steady state of the full system resembles the frictionless benchmark. The steady-state wealth distribution of the full system is attained when intermediaries and households have commensurate earnings rate, i.e. neither group has the advantage because the output they produce is sufficiently scarce and expensive. This condition is related to the indifference condition in the frictionless benchmark, where agents can switch their types freely.

3 Equilibrium in the Dynamic Model.

The full model fluctuates around the steady state, at which the earnings rates of intermediaries and households balance each other out. If intermediaries experience a negative shock, then the economy moves towards the money regime. As intermediaries’ net worth declines, their capacity to mitigate financial frictions declines and the price of capital falls towards $q$. The creation of inside money by intermediaries collapses, and the price of money rises towards $p$. As the economy recovers, intermediaries create more money, the money multiplier expands and the value of outside money falls. Upon recovery, the economy gets closer to the frictionless benchmark described above, although that benchmark is never reached. In booms, intermediaries need net worth buffers to manage risks, and there is always a strictly positive chance of lapsing back into a crisis.

To characterize the equilibrium formally, we use the equilibrium conditions from Section 2.1 to derive the law of motion of the intermediaries’ net worth share $\eta_t$, and to express all equilibrium quantities - prices and allocations - as functions of $\eta_t$. This leads us to the
Proposition 3. The equilibrium law of motion of \( \eta_t \) is given by

\[
\frac{d\eta_t}{\eta_t} = (1 - \eta_t) \left( x_t^2 \nu_t \nu_t^T - (x_t^I)^2 \nu_t^I (\nu_t^I)^T \right) \, dt + \left( x_t \nu_t + \sigma_t^\pi \right) ((\sigma_t^\pi)^T \, dt + dZ_t). \tag{3.1}
\]

The law of motion of \( \eta_t \) is so simple because the earnings of intermediaries and households can be expressed in terms of risks they take on and the required equilibrium risk premia. The first term on the right-hand side reflects the relative earnings of intermediaries and households determined by the risks they take on. The condition that the earnings rate balance each other out - which happens at the steady state of the system - is related to the indifference condition of the frictionless economy, i.e. \( x_t^2 \nu_t \nu_t^T = (x_t^I)^2 \nu_t^I (\nu_t^I)^T \). The second term on the right-hand side of (3.1) reflects mainly the volatility of \( \eta_t \), due to the imperfect risk sharing between intermediaries and households.

Proof. The law of motion of the intermediaries’ net worths, for the risks that they take, is given by (2.9). The law of motion of world wealth \( (q_t + p_t)K_t \), the denominator of (2.6), can be found from the total return on world wealth, after subtracting the dividend yield (or aggregate consumption). To find the returns, we take into account the risk premia that various agents have to earn. We have

\[
\frac{d((q_t + p_t)K_t)}{(q_t + p_t)K_t} = dt^M - \frac{Y_t - \iota(q_t)K_t}{(q_t + p_t)K_t} \, dt + \eta_t \, x_t \nu_t \, \left( \left( x_t \nu_t + \sigma_t^K + \sigma_t^p T \right) \, dt + dZ_t \right) + \frac{dN_t}{\rho \, dt - dr_t^M} + \eta_t^I \, x_t^I \nu_t^I \left( \left( x_t^I \nu_t^I + \sigma_t^K + \sigma_t^p T \right) \, dt + dZ_t \right) + \eta_t^J \, x_t^J \nu_t^J \left( \left( x_t^J \nu_t^J + \sigma_t^K + \sigma_t^p T \right) \, dt + dZ_t \right).
\]

Here, the aggregate dividend yield can be found from the market-clearing condition for consumption goods

\[
Y_t - \iota_t = \rho(p_t + q_t). \tag{3.2}
\]

Since the total risk of world wealth is\(^5\)

\[
\eta_t x_t \nu_t + \eta_t^I x_t^I \nu_t^I + \eta_t^J x_t^J \nu_t^J + \sigma_t^K + \sigma_t^p = \sigma_t^K + \sigma_t^p + (1 - \pi)(\sigma_t^q - \sigma_t^p) \frac{1}{-\sigma_t^p}.
\]

\(^5\)Ito’s lemma implies that \( \sigma_t^\pi = (1 - \pi)(\sigma_t^p - \sigma_t^q) \) and \( \mu_t^\pi = (1 - \pi)(\mu_t^p - \mu_t^q) - \sigma_t^\pi \sigma_t^p + (\sigma_t^\pi)^2. \)
the law of motion of aggregate wealth can be written as

\[
dt^M - \rho dt - \sigma^\pi((\sigma^K_t + \sigma^p_t) \ dt + dZ_t) + (\eta_t x^2 \nu_t \nu_t^T + \eta_t^I (x^I_t)^2 \nu_t^I (\nu_t^I)^T + \eta_t^J (x^J_t)^2 \nu_t^J (\nu_t^J)^T) dt.
\]

Thus, using Ito's lemma, (3.1) follows. \( \square \)

The equilibrium conditions of Section 2.1, together with the law of motion of the intermediaries' net worth share \( \eta_t \), allow us to compute the equilibrium. We obtain a second-order differential equation for \( \pi(\eta) \) through the following procedure:

**Procedure.** The second-order differential equation that the function \( \pi(\eta) \) has to satisfy in equilibrium can be found through the following sequence of steps. First, given \( \eta \) and \( (\pi(\eta), \pi'(\eta)) \) find the allocation of capital \( (\psi_t, \psi_t^j) \) that satisfies the conditions

\[
x_t = \psi_t(1 - \pi(\eta)), \quad \eta^\pi = \frac{x_t \left( \sigma^I_t I - \sigma^K_t \right)}{1 + (\psi_t - \eta_t) \frac{\pi'(\eta)}{\pi(\eta)}}, \quad \pi^\pi = \frac{\pi'(\eta)}{\pi(\eta)} \eta^\pi, \quad Y - \ell(q) = \frac{\rho q}{1 - \pi(\eta)},
\]

\[
x_t \nu_t \nu_t^T = x_t^I \nu_t^I (\nu_t^I)^T, \quad \frac{(p_t^I - p_t^J) \ a}{q} = x_t^I \nu_t^I (\nu_t^I)^T - x_t^J \nu_t^J (\nu_t^J)^T + (\sigma^I_j I - \sigma^J_j j) \left( \sigma^K_t + \frac{\sigma^\pi}{1 - \pi} \right)^T,
\]

\[
\eta + \psi^J (1 - \pi(\eta)) + (1 - \psi^J - \psi)(1 - \pi(\eta)) = 1 \quad \text{and} \quad (x_t^I)^2 \nu_t^I (\nu_t^I)^T \leq (x_t^J)^2 \nu_t^J (\nu_t^J)^T,
\]

with equality of \( \psi + \psi^J < 1 \). Then compute

\[
\mu^\pi = (\sigma^\pi)^2 + (1 - \pi(\eta)) \left( \frac{p_t^I a - \ell}{q} - x_t \nu_t \nu_t^T - \left( \frac{\sigma^I_t I - \sigma^K_t}{\sigma_t^I} \right) \left( \sigma^K_t + \frac{\sigma^\pi}{1 - \pi} \right)^T \right),
\]

and \( \pi'^\pi(\eta) = \frac{\mu^\pi \pi(\eta) - \pi'(\eta) \mu^\pi \eta}{\eta^2 \sigma_t^I (\sigma_t^\eta)^T / 2} \),

where the drift and volatility of \( \eta, d\eta_t/\eta_t = \mu_t^\eta dt + \sigma_t^\eta dZ_t \), are taken from (3.1).

---

\( ^6 \)If processes \( X_t \) and \( Y_t \) follow

\[
dX_t/X_t = \mu_t^X dt + \sigma_t^X dZ_t \quad \text{and} \quad dY_t/Y_t = \mu_t^Y dt + \sigma_t^Y dZ_t,
\]

then

\[
\frac{d(X_t/Y_t)}{X_t/Y_t} = (\mu_t^X - \mu_t^Y) dt + (\sigma_t^X - \sigma_t^Y)(dZ_t - (\sigma_t^X)^T dt).
\]
Next, we provide several examples to illustrate equilibrium dynamics, and to explore how
the equilibrium depends on model parameters.

3.1 An Example.

First, let us discuss the general properties of equilibria. Figure 2 provides the result of
computation for the parameters $\rho = 5\%, \ a = 2, \ I = 8, \ J = 12, \ \sigma^I = 0.7, \ \sigma^J = 0.5, \ s = 0.6$
and $\Phi(\iota) = \log(\kappa \iota + 1)/\kappa$ with $\kappa = 2$, the same values that we used in Figure 1.

![Figure 2: An example: equilibrium prices, allocations, drift and volatility.](image)

The top left panel shows the equilibrium prices and compares them with those of the two
benchmarks. The money regime, in which the value of money is high, serves as the boundary
condition at $\eta = 0$ where the intermediary sector disappears. The frictionless benchmark
provides an estimate for what happens at the steady state, the point at which the drift of $\eta_t$ becomes zero (see the bottom left panel). The estimate is not exact, since it relies on the
assumption that allocation of funds across sectors remains optimal forever. In particular, the price of money in equilibrium at the steady state is higher than it is at the frictionless benchmark, since agents take into account the possibility that intermediary net worth drops and the economy enters the crisis region where the value of money is much higher.

Crisis manifests itself, in particular, in the misallocation of capital. The top right panel illustrates the allocation of capital by various sectors to different technologies. When \( \eta \) becomes low, households start employing technologies 1 through \( I \) even though they are less efficient at using them, as they cannot diversify. Likewise, when \( \eta \) becomes excessively high and the household sector becomes undercapitalized, the economy produces too little of goods \( I + 1 \) through \( I + J \). However, in this example the states where intermediaries are undercapitalized are much more likely to realize than those where households are undercapitalized, since the drift of \( \eta_t \) becomes much more negative above the steady state than it is below the steady state.

As \( \eta_t \) falls below the steady state, there are two amplification channels. First, the traditional amplification channel works on the asset sides of the intermediary balance sheets: as the price of physical capital \( q(\eta) \), shown on the top left panel of Figure 2 in blue, drops following a shock. In addition, shocks hurt intermediaries on the liability sides of the balance sheets through the Fisher disinflationary spiral. As we can observe from red money price curve \( p(\eta) \) on the top left panel Figure 2, money appreciates following a negative shock.

The reason for the disinflationary pressure when intermediaries are undercapitalized is as follows. As intermediaries suffer losses, they contract their balance sheets. Thus, they take fewer deposits and create less inside money.\(^7\) The total supply of money (inside and outside) shrinks and the money multiplier collapses, but the demand for money does not change significantly since saving households still want to allocate a portion of their savings to safe money. As a result, the value of money goes up.

Amplification can be isolated by expressing the risk that the intermediaries face, with capital funded by money on the liability side, in the form

\[
\nu_t = \left( 1 - \frac{\pi'(\eta)}{\pi(\eta)(1 - \pi(\eta))} \right) \frac{\sigma^I 1^I - \sigma^K}{T}.
\]

Figure 3 shows the amount \( \frac{-\pi'(\eta)}{\pi(\eta)(1 - \pi(\eta))} \) by which fundamental risk is amplified. We observe

\(^7\)In reality, rather than turning savers away, financial intermediaries might still issue demand deposits and simply park the proceeds with the central bank as excess reserves.
that amplification is significant, particularly below the steady state when intermediaries are undercapitalized.

\[ \frac{-\pi'(\eta)}{\pi(\eta)(1-\pi(\eta))} \]

Figure 3: Amplification on the intermediaries’ balance sheets.

3.2 Inefficiencies and Welfare.

There are several inefficiencies in equilibrium connected to production, investment and risk sharing. First, because intermediate goods are imperfect substitutes in the production of the aggregate good, there is inefficient underproduction of some of the goods when either intermediaries or households are undercapitalized. Second, investment is inefficient, as it fluctuates with the price of capital \( q_t \). Third, perhaps most importantly, risk sharing is inefficient in many ways.

There are many inefficient risk exposures, some of which lead to impaired balance sheets and inefficient production. Households who invest in a single technology have no way of passing on idiosyncratic risk. Their risk exposure is mitigated somewhat by the existence of money as a safe investable asset. Fortunately, the households’ idiosyncratic risk exposures do not lead to inefficient production due to free entry among the households in technologies \( I+1 \) through \( I+J \). Intermediaries can diversify, but cannot passing the risk of technologies 1 through \( I \) to households. They are exposed to a mismatch in risks on the asset and liability sides of their balance sheets, and the risks of the mismatch are amplified, as seen in (3.3).

If intermediaries become undercapitalized, barriers to entry into the intermediary sector
help the intermediaries: the prices of goods 1 through I rise when \( \eta_t \), mitigating the intermediaries risk exposures and allowing the intermediaries to recapitalize themselves. Thus, the limited competition in the intermediary activities creates a terms-of-trade hedge, which depends on the extent to which intermediaries cut back production in downturns, the extent to which households enter, and the substitutability \( s \) among the intermediate goods.

It is good to understand the inefficiencies, as well as factors that can potentially affect them - these include the amplification of risks, the degree of competition, barriers to entry and natural hedges. However, to understand the cumulative effect of all these factors, one needs a proper welfare measure. We finish this section by developing a way to evaluate the utilities of all agents in equilibrium.

To evaluate welfare, one complicating factor is heterogeneity. We cannot focus on a representative household, since different households are exposed to different idiosyncratic risks. Some households become richer, while others become poorer. Our welfare measure has to take this into account. The following proposition evaluates the welfare of any agent as a function of his/her investment opportunities.

**Proposition 4.** The welfare of an agent with wealth \( n_t \) who can invest only in money takes the form \( \omega(\eta_t) + \log(n_t/p_t)/\rho \), where \( \omega(\eta) \) satisfies the equation

\[
\rho \omega(\eta) = \log(\rho p(\eta)) + \frac{\Phi(\iota(\eta)) - \delta - \rho - \sigma^2/2}{\rho} + \omega'(\eta) \mu^n_t \eta + \frac{\omega''(\eta) \eta^2 \sigma^n_t (\sigma^n_t)^T}{2}. \tag{3.4}
\]

The welfare of an intermediary with net worth \( n_t \) is \( h(\eta_t) + \omega(\eta_t) + \log(n_t/p_t)/\rho \), where

\[
\rho h(\eta) = \frac{(x_t)^2 \nu_t \nu_T^T}{2 \rho} + h'(\eta) \mu^n_t \eta + \frac{h''(\eta) \eta^2 \sigma^n_t (\sigma^n_t)^T}{2}. \tag{3.5}
\]

The welfare of a household is \( h^J(\eta_t) + \omega(\eta_t) + \log(n_t/p_t)/\rho \), where \( h^J \) satisfies equation (3.5) with the term \( (x_t)^2 \nu_t \nu_T^T \) replaced by \( (x_t)^2 \nu_T^T \nu_T^T \).

Equation (3.5) takes into account the specific investment opportunities that an individual agent takes, and evaluates the benefit of these opportunities by measuring the risk \( (x_t)^2 \nu_t \nu_T^T \) that the agent chooses to take over the reference scenario of holding just money. Note that, given the form of equation (3.5), it makes sense why Proposition 1 gives the right condition for the household to be indifferent between technologies 1 through I and \( I + 1 \) through \( I + J \). Under condition (2.12), the household has the same welfare regardless of the technology it chooses to pursue.
Proof. Conjecture that the utility of an agent with wealth $n_t$ who can invest only in money takes the form $\omega(\eta_t) + \log(n_t/p_t)/\rho$. We can express the agent’s wealth in the form $n_s = p_s k_s$, where

$$\frac{dk_t}{k_t} = (\Phi(u_t) - \delta - \rho) \, dt + \sigma \, dZ_t,$$

since the agent consumes at rate $\rho$, and $k_s = n_s/p_s$. Then the agent’s utility has to satisfy the equation

$$\rho \left( \omega(\eta_t) + \frac{\log(k_t)}{\rho} \right) = \log(\rho n_t) + E[d \log(k_t)]/dt + E[d \omega(\eta_t)]/dt,$$

which expands to (3.4) using Ito’s lemma.

Let $H_0(\eta) = \omega(\eta) - \log(p_t)/\rho$. Then this agent’s utility can be also expressed as $H_0(\eta_t) + \log(n_t)/\rho$, where $H_0$ satisfies the HJB equation

$$\rho H_0(\eta) = \log(\rho) + \frac{1}{\rho} E \left[ \frac{d n_t}{n_t} \right] /dt - \frac{(\sigma_t^P + \sigma_t^K)^2}{2\rho} + H_0'(\eta) \mu^\eta_t \eta + \frac{H_0''(\eta)\eta^2 \sigma_t^\eta (\sigma_t^\eta)^T}{2}. \quad (3.6)$$

Now, consider another agent whose best investment opportunity over money has risk $\nu_t$ and who (given the returns) chooses to put portfolio weight $x_t$ on this opportunity. Then this agent’s wealth follows

$$\frac{d \tilde{n}_t}{\tilde{n}_t} = \frac{d n_t}{n_t} + x_t \nu_t (x_t \nu_t + \sigma_t^P + \sigma_t^K)^T \, dt + x_t \nu_t \, dZ_t.$$

The utility of this agent can be represented in the form $H(\eta_t) + \log(\tilde{n}_t)/\rho$, where

$$\rho H(\eta_t) = \log(\rho) + \frac{1}{\rho} E \left[ \frac{d n_t}{n_t} \right] /dt + \frac{x_t \nu_t (x_t \nu_t + \sigma_t^P + \sigma_t^K)^T}{\rho} - \frac{[\sigma_t^P + \sigma_t^K]^2}{2\rho} - \frac{x_t \nu_t (x_t \nu_t + \sigma_t^K)^T}{\rho} - \frac{x_t^2 \nu_t \nu_t^T}{2\rho} + H'(\eta) \mu^\eta_t \eta + \frac{H''(\eta)\eta^2 \sigma_t^\eta (\sigma_t^\eta)^T}{2}.$$

Subtracting (3.6), we find that the difference in the utilities of these two agents, $h(\eta_t) = H(\eta_t) - H_0(\eta_t)$, satisfies the ordinary differential equation

$$\rho h(\eta) = \frac{x_t^2 \nu_t \nu_t^T}{2\rho} + h'(\eta) \mu^\eta_t \eta + \frac{H''(\eta)\eta^2 \sigma_t^\eta (\sigma_t^\eta)^T}{2}. \quad (3.7)$$

\[\square\]
We finish this section by explaining the use of equations (3.4) and (3.5) to evaluate welfare. The left panel of Figure 4 shows the functions $h(\eta) + \omega(\eta) - \log(p_t)/\rho$ and $h^J(\eta) + \omega(\eta) - \log(p_t)/\rho$ for the example in Figure 2, the welfare of intermediaries and households when they have unit net worth. Interestingly, these welfare measures improve when agents in the same group are undercapitalized, since then the investment opportunities of an individual agent with unit net worth improve.

These measures, however, do not adequately represent welfare in equilibrium. If we normalize the amount of capital in the economy to 1, then total wealth is $p_t + q_t = p_t/\pi_t$. Splitting total wealth proportionately to net worth shares, we can evaluate the welfare of experts as $h(\eta) + \omega(\eta) + \log(\eta/\pi(\eta))/\rho$ and that of households according to $h^J(\eta) + \omega(\eta) + \log((1 - \eta)/\pi(\eta))/\rho$. The right panel of Figure 4 shows the sum of these two measures. We have put equal weights intermediaries and households, but any other split makes sense as well. Total welfare is maximized close to steady state, at a point slightly lower than the steady state.
4 Monetary and Macro-prudential Policy

Policy has the potential to mitigate some of the inefficiencies that arise in equilibrium. It can undo some of the endogenous risk by redistributing wealth towards compromised sectors. It can control the creation of endogenous risk by affecting the path of deleveraging. It can also work to prevent the build-up of systemic risk in booms.

Policies affect the equilibrium in a number of ways, and can have unintended consequences. Interesting questions include: How does a policy affect equilibrium leverage? Does the policy create moral hazard? Does the policy lead to inflated asset prices in booms? What happens to endogenous risk? How does the policy affect the frequency of crises, i.e. episodes characterized by resource misallocation and loss of productivity?

We focus on several monetary policies in this section. These policies can be divided in several categories. Traditional monetary policy sets the short-term interest rate. It affects the yield curve through the expectation of future interest rates, as well as through the expected path of the economy, accounting for the supply and demand of credit. When the zero lower bound for the short-term policy rate becomes a constraint, forward guidance is an additional policy tool that is often employed in practice. The use of this tool depends on central bank’s credibility, as it ties the central bank’s hands in the future and leaves it less room for discretion. In this paper we assume that the central bank can perfectly commit to contingent future monetary policy and hence the interest rate policy incorporates some state-contingent forward guidance.

Several non-conventional policies have also been employed. The central bank can directly purchases assets to support prices or affect the shape of the yield curve. The central bank can lend to financial institutions, and choose acceptable collateral as well as margin requirements and interest rates. Some of these programs work by transferring tail risk to the central bank, as it suffers losses (and consequently redistributes them to other agents) in the event that the value of collateral becomes insufficient and the counterparty defaults. Other policies include direct equity infusions into troubled institutions. Monetary policy tools are closely linked to macroprudential tools, which involve capital requirements and loan-to-value ratios.

The classic “helicopter drop of money” has in reality a strong fiscal component as money is typically paid out via a tax rebate. Importantly, the helicopter drop also has redistributive effects. As the money supply expands, the nominal liability of financial intermediaries and hence the household’s nominal savings are diluted. The redistributive effects are even stronger if the additional money supply is not equally distributed among the population but
targeted to specific impaired (sub)sectors in the economy.

Instead of analyzing fiscal policy, we focus this paper on conventional and non-conventional monetary policy. For example, a change in the short-term policy interest rate redistributes wealth through the prices of nominal long-term assets. The redistributive effects of monetary policy depend on who holds these assets.\(^8\) In turn, asset allocation depends on the anticipation of future policy, as well as the demand for insurance. Specifically, we introduce a perpetual long-term bond, and allow the monetary authority to both set the interest rate on short-term money, and affect the composition of outstanding government liabilities (money and long-term bonds) through open-market operations.

### 4.1 Extended Model with Long-term Bonds

**Money and Long-Term Bonds.** We extend our baseline model in two ways: we allow money to pay the floating rate interest and we introduce perpetual bonds, which pay interest at a fixed rate in money. Monetary policy sets interest \( r_t \geq 0 \) on money and controls the value \( b_t K_t \) of all perpetual bonds outstanding. These policies are independent of fiscal policy - the monetary authority pays interest and performs open-market operations by printing money and not by using taxes.

We now denote by \( p_t K_t \) the supply of all outstanding nominal assets: outside money and perpetual bonds. Also, let \( B_t \) be the endogenous equilibrium price of a single perpetual bond, which follows

\[
\frac{dB_t}{B_t} = \mu_t^B \, dt + \sigma_t^B \, dZ_t. \tag{4.1}
\]

Note that \( r_t \) and \( b_t \) are policy instruments, while \( B_t \) is an endogenous equilibrium process.

**Returns.** The expressions for the return on capital from Section 2 do not change, but money earns the return that depends on policy. If an agent holds all nominal assets in the economy - bonds and outside money - the return is

\[
\left( \Phi(t) - \delta + \mu_t^P + \sigma_t^P (\sigma_t^K)^T \right) \, dt + (\sigma_t^K + \sigma_t^P) \, dZ_t.
\]

This is the return on a portfolio with weights \( b_t/p_t \) on bonds and \( 1 - b_t/p_t \) on money. To isolate the returns on money and bonds, consider a strategy that buys bonds to earn \( dr_t^B \)

\(^8\)Brunnermeier and Sannikov (2012) discuss the redistributive effects in a setting in which several sectors’ balance sheets can be impaired. Forward guidance not to increase the policy interest rate in the near future has different implications than a further interest rate cut, since the former narrows the term spread while the latter widens it.
by borrowing money, paying \( dr_t^M \). We can find the payoff of this strategy by focusing on the value of bonds in money. Using Ito’s lemma,

\[
dr_t^B - dr_t^M = \left( \frac{1}{B_t} - r_t + \mu_t^B + \sigma_t^B (\sigma_t^M)^T \right) dt + \sigma_t^B dZ_t,
\]

where \( \sigma_t^M \) is the risk of money, which satisfies

\[
\sigma_t^K + \sigma_t^p = \sigma_t^M + \frac{b_t}{p_t} \sigma_t^B \quad \Rightarrow \quad \sigma_t^M = \sigma_t^K + \sigma_t^p - \frac{b_t}{p_t} \sigma_t^B.
\]

Thus, money earns the return of

\[
dr_t^M = (\Phi(\iota) - \delta + \mu_t^p + \sigma_t^p (\sigma_t^K)^T) dt - \frac{b_t}{p_t} \left( \frac{1}{B_t} - r_t + \mu_t^B + \sigma_t^B (\sigma_t^M)^T \right) dt + \sigma_t^M dZ_t.
\]

**Equilibrium Conditions.** For expositional purposes, we focus on policies that set the short-term interest rate \( r_t \) and the value of bonds \( b_t \) as functions of the state variable \( \eta_t \). Then the price of bonds \( B_t \) will also be a function of \( \eta_t \). Consider for concreteness policies that lead to a decreasing function \( B(\eta) \), which follows from policies that cut the short-term interest rate when \( \eta_t \) is low, making bonds appreciate. Such a policy is designed to help intermediaries transfer some of the risk to households - by borrowing money and buying long-term bonds, intermediaries get a natural hedge that gives them insurance in the event that \( \eta_t \) drops and the entire intermediary sector suffer losses. The appreciation in bonds can offset partially other risks that the intermediaries face, including endogenous risks driven by amplification. In the equations below, we assume that intermediaries hold all long-term bonds as a hedge, and letter verify that this is indeed the case.

To write down the asset-pricing equations, we must first isolate the risk of buying capital (or bonds) by borrowing money. Since the return on money depends on monetary policy, we need to adjust our formulas. Thus, we have

\[
\nu_t = \sigma_t^{1} - \sigma_t^K - \frac{\sigma_t^\pi}{1 - \pi} + \frac{b_t}{p_t} \sigma_t^B \quad \text{and} \quad \nu_t^i = \sigma_t^{1i} - \sigma_t^K - \frac{\sigma_t^\pi}{1 - \pi} + \frac{b_t}{p_t} \sigma_t^B.
\]

Intermediaries hold money, \( \psi_t \) of the world capital, and all of the world bonds. Their portfolio weights on capital and bonds are \( x_t = \psi_t (1 - \pi_t)/\eta_t \) and \( 1/\eta_t b_t/(p_t + q_t) = \pi_t/\eta_t b_t/p_t \). Thus,
the volatility of their net worth is

\[ \sigma_t^N = \sigma_t^M + x_t \nu_t + \frac{\pi_t b_t}{\eta_t p_t} \sigma_t^B. \]

The pricing conditions for capital and money on the intermediaries balance sheets are

\[ E_t[dr_t^I - dr_t^M] = \nu_t(\sigma_t^N)^T \quad \text{and} \quad \frac{1}{B_t} - r_t + \mu_B^t + \sigma_t^B(\sigma_t^N)^T = \sigma_t^B(\sigma_t^N)^T. \]

Likewise, the households optimal allocations to capital \( x_t^I \) and \( x_t^J \) must satisfy

\[ E_t[dr_t^I - dr_t^M] = \nu_t^I(\sigma_t^M + x_t^J \nu_t^J)^T \quad \text{and} \quad E_t[dr_t^I - dr_t^M] = \nu_t^J(\sigma_t^M + x_t^J \nu_t^J)^T. \]

We must also verify that

\[ \frac{1}{B_t} - r_t + \mu_B^t + \sigma_t^B(\sigma_t^M)^T \leq \sigma_t^B(\sigma_t^M + x_t^J \nu_t^J)^T, \quad \sigma_t^B(\sigma_t^M + x_t^J \nu_t^J)^T \]

to ensure that none of the households choose to hold bonds (or else we have to expand the equations to solve for the optimal equilibrium allocation of bonds).

The following proposition provides the law of motion of \( \eta_t \) with such a policy.

**Proposition 5.** In equilibrium

\[ \frac{d\eta_t}{\eta_t} = (1 - \eta_t) \left( |x_t \nu_t + \frac{\pi_t b_t}{\eta_t p_t} \sigma_t^B|^2 dt - |x_t^J \nu_t^J|^2 \right) + \left( \sigma^\pi + x_t \nu_t + \frac{\pi_t b_t}{\eta_t p_t} \sigma_t^B \right) \left( dZ_t + \left( \sigma^\pi - \frac{b_t}{p_t} \sigma_t^B \right) dt \right). \]  

(4.2)

**Proof.** See Appendix.

Note that the effect of monetary policy on the asset-pricing conditions as well as the law of motion of \( \eta_t \) enters exclusively through the term \((b_t/p_t)\sigma_t^B\). If monetary policy sets the short-term interest rate \( r_t \) as well as the level of \( b_t \) as functions of \( \eta_t \), then the risk of the bond price \( \sigma_t^B \) is collinear with \( \sigma_t^1, \sigma_t^y, \sigma_t^\pi \). Thus, monetary policy can be used to work against endogenous risk.
In fact, note that (4.2) implies that
\[
\sigma_t^\eta = \frac{x_t \left( \sigma_{I1}^{\eta} - \sigma_t^K \right)}{1 + \left( \psi - \eta \right) \frac{\pi'(\eta)}{\pi(\eta)} - \left( x_t - 1 + \frac{\pi}{\eta} \right) \frac{b_t}{p_t} \frac{B'(\eta)}{B(\eta)}}.
\] (4.3)

Policy gives rise to the extra term \((x_t - 1 + \frac{\pi}{\eta}) \frac{b_t}{p_t} \frac{B'(\eta)}{B(\eta)}\) in this expression. This term reduces the amplification of fundamental risk, captured by the denominator of (4.3). Likewise, with policy (3.3) changes to
\[
\nu_t = \left( 1 - \frac{\pi'(\eta)}{\pi(\eta)(1 - \pi(\eta))} + \frac{b_t}{p_t} \frac{B'(\eta)}{B(\eta)} \right) \left( \sigma_{I1}^{\eta} - \sigma_t^K \right). \] (4.4)

Therefore, it makes sense to investigate the effect of policies by focusing directly on the risk transfer effect \((b_t/p_t)B'(\eta)/B(\eta)\). This one-dimensional function fully captured the effect of two policy tools \(r_t\) and \(b_t\), and of course a single function \((b_t/p_t)B'(\eta)/B(\eta)\) can be implemented in multiple ways. In the following section, we examine the effect of policy by focusing specifically on the amount of risk transfer \((b_t/p_t)B'(\eta)/B(\eta)\) that the policy attains.

4.2 Monetary Policy: An Example

This section provides an example of how monetary policy can affect equilibrium dynamics and welfare. We take the same parameters as in our example in Section 3. We then focus on the extent to which policy reduces the degree of amplification in (4.3). Specifically, we consider
\[
\left( x_t - 1 + \frac{\pi(\eta)}{\eta} \right) \frac{b_t}{p_t} \frac{B'(\eta)}{B(\eta)} = \alpha(\eta) \underbrace{\frac{x_t - 1 + \frac{\pi(\eta)}{1 - \pi(\eta)} }{\pi(\eta)}}_{\psi-\eta} \frac{\pi'(\eta)}{\pi(\eta)},
\]
where \(\alpha(\eta) \in [0, 1]\). In the following example, \(\alpha(\eta) = \max(0.5 - \eta, 0)\), i.e. monetary policy eliminates up to a half of endogenous risk for \(\eta < 0.5\). Figure 5 compares several equilibrium quantities with and without policy, and zooms around the steady state to make the comparison clearer.

The bottom right panel illustrates that policy reduces risk. As a result, intermediaries are able to employ their technologies to a greater extent even for lower levels of net worth, and households step in to inefficiently use the intermediaries’ technologies at a lower level.
of $\eta_t$ - see top right panel. At the same time, intermediaries are able to maintain higher leverage - their net worth at the steady state is reduced as shown in the bottom left panel. The top left panel shows that the value of money is somewhat lower with the policy - due to the fact that intermediaries are able to function more efficiently and create more inside money. The price of capital $q(\eta)$ does not change significantly (it becomes slightly higher with the policy).

Figure 6 confirms that this policy, though appropriate risk transfer, indeed improves welfare. It replicates Figure 4, and compares welfare measures with and without policy. Overall welfare clearly improves with policy, as we can see from the right panel of Figure 6. Moreover, total welfare is maximized at a lower level of $\eta$, so that the shift of the steady state to the left is not detrimental. We estimated that this improvement in welfare is equivalent to increased consumption by about 9% per year. The left panel shows the effect on individual
agents. Households get slightly lower utility given any wealth level with policy, but also at the steady state they have greater wealth.

4.3 Policy to undo Endogenous Risk.

Consider a policy that fully stabilizes the value of money relative to capital and sets

$$\frac{\sigma^\pi}{1-\pi} = \sigma_t^p - \sigma_t^q = \frac{b_t}{p_t} \sigma_t^B$$

Then

$$\nu_t = \sigma^I \frac{1}{I} - \sigma_t^K$$

and

$$\nu_t^i = \sigma^I \frac{1}{I} - \sigma_t^K.$$

$$\sigma_t^N = \sigma_t^K + \sigma_t^p - \frac{b_t}{p_t} \sigma_t^B + x_t \left( \sigma^I \frac{1}{I} - \sigma_t^K \right) + \frac{\pi_t}{\eta_t} \frac{b_t}{p_t} \sigma_t^B.$$

$$\sigma_t^M = \sigma_t^K + \sigma_t^q - \frac{b_t}{p_t} \sigma_t^B + x_t \left( \sigma^I \frac{1}{I} - \sigma_t^K \right) + \frac{\pi_t}{\eta_t} \frac{b_t}{p_t} \sigma_t^B.$$
The pricing conditions for capital and money on the intermediaries balance sheets are

$$
\frac{E_t[dr^I_t - dr^M_t]}{dt} = \nu_t(\sigma^N_t)^T \quad \text{and} \quad \frac{1}{B_t} - r_t + \mu^B_t + \sigma^B_t(\sigma^M_t)^T = \sigma^B_t(\sigma^N_t)^T.
$$

Likewise, the households optimal allocations to capital $x^I_t$ and $x^J_t$ must satisfy

$$
\frac{E_t[dr^I_t - dr^M_t]}{dt} = \nu^I_t(\sigma^M_t + x^I_t\nu^I_t)^T \quad \text{and} \quad \frac{E_t[dr^J_t - dr^M_t]}{dt} = \nu^J_t(\sigma^M_t + x^J_t\nu^J_t)^T.
$$

We must also verify that

$$
\frac{1}{B_t} - r_t + \mu^B_t + \sigma^B_t(\sigma^M_t)^T \leq \sigma^B_t(\sigma^M_t + x^I_t\nu^I_t)^T, \sigma^B_t(\sigma^M_t + x^J_t\nu^J_t)^T
$$

5 Conclusion

We consider an economy in which household entrepreneurs and intermediaries make investment decisions. Household entrepreneurs can invest only in a single real production technology at a time, while intermediaries have the expertise to invest in a number of projects. In equilibrium intermediaries take advantage of their expertise to diversify across several investment projects. They scale up their activity by issuing demand deposits, inside money. Households hold this inside money in addition to outside money provided by the government. Intermediaries are leveraged and assume liquidity mismatch. Intermediaries’ assets are long-dated and have low market liquidity - after an adverse shock the price can drop - while their debt financing is short-term. Endogenous risk emerges through amplification mechanism in form of two spirals. First, the liquidity spiral: a shock to intermediaries causes them to shrink balance sheets and “fire sale some of their assets”. This depresses the price of their assets which induces further fire-sales and so on. Second, the disinflationary spiral: as intermediaries shrink their balance sheet, they also create less inside money; such a shock leads to a rising demand for outside money, i.e. disinflation. This disinflationary spiral amplifies shocks, as it hurts borrowers who owe nominal debt. It works on the liabilities side of the intermediary balance sheets, while the liquidity spiral that hurts the price of capital works on the asset side. Importantly, in this economy the money multiplier, the ratio between inside and outside money, is endogenous: it depends on the health of the intermediary sector.

Monetary policy can mitigate the adverse effects of both spirals in the presence of default-free long-term government bonds. Conventional monetary policy changes the path of interest
rate earned on short-term money and consequently impacts the relative value of long-term government bond and short-term money. For example, interest rate cuts in downturns that are expected to persist for a while enable intermediaries to refinance their long-bond holding more cheaply. This recapitalizes institutions that hold these assets and and also increases the (nominal) supply of the safe asset. The resulting reduction in endogenous risk leads to welfare improvements. Of course, any policy that provides insurance against downturns could potentially create moral hazard. Indeed, intermediaries take on higher leverage, but more hazard is limited. The reason is that the “stealth recapitalization” through a persistent interest rate cut not only recapitalizes institutions with high leverage because they funded many real projects but also the ones which simply held long-term (default-free) Government bonds. The finding that moral hazard is limited might change if one were to include intermediaries with negative net worth. Including zombie banks is one fruitful direction to push this line of research further.

6 Bibliography


### A Numerical Procedure to find Equilibrium.

(to be completed)

### B Proofs.

(to be completed)