# A macroeconomic foundation for the equilibrium term structure of interest rates

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#### Abstract

This paper explores the term structure of interest rates implied by a stochastic endogenous growth model with imperfect price adjustment. The production and price-setting decisions of firms drive low-frequency movements in growth and inflation rates that are negatively related. With recursive preferences, these growth and inflation dynamics are crucial for rationalizing key stylized facts in bond markets. When calibrated to macroeconomic data, the model quantitatively explains the means and volatilities of nominal bond yields and the failure of the expectations hypothesis.

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# 1 Introduction

Explaining the term structure of interest rates is a challenge for standard macroeconomic models. Notably, Dunn and Singleton (1986) and Backus, Gregory, and Zin (1989) highlight the difficulty of consumption-based models with standard preferences in explaining the sign, magnitude, and volatility of the term spread. Consumption-based models with richer preference specifications and model dynamics, such as Wachter (2006), Piazzesi and Schneider (2007), Gallmeyer, Hollifield, Palomino, and Zin (2007), and Bansal and Shaliastovich (2012) find more success. From the other side, Jermann (2013) proposes a pure production-based framework for explaining the term structure. However, Donaldson, Johnsen, and Mehra (1990), den Haan (1995), Rudebusch and Swanson (2008), Li and Palomino (2012), van Binsbergen, Fernandez-Villaverde, Koijen, and Rubio-Ramirez (2012), and Rudebusch and Swanson (2012) show that integrating consumption- and production-based frameworks in a general equilibrium setting have trouble in explaining basic term structure facts jointly with macroeconomic aggregates.

This paper shows that endogenizing long-run growth and inflation rates in a general equilibrium model and assuming agents have recursive preferences can jointly explain term structure and macroeconomic dynamics. General equilibrium production-based asset pricing models have found success in linking equity returns to consumption, investment, labor and output decisions. This literature can be broadly divided into three categories. Jermann (1998), Lettau and Uhlig (2000), and Boldrin, Christiano, and Fisher (2001) analyze production-based asset models with habit preferences. Ai (2009), Croce (2012), Kuehn (2008), Kaltenbrunner and Lochstoer (2008), Favilukis and Lin (2012), and Kung and Schmid (2013) show how long-run risks arise in production economies. Barro (2006), Gourio (2012), and Kuehn, Petrosky-Nadeau, and Zhang (2013) consider rare disasters. Given the positive results of this literature, it seems encouraging to extend this paradigm to study the term structure of interest rates.

To link bond yields to macroeconomic fundamentals, I build a stochastic endogenous growth model with imperfect price adjustment. This framework has two distinguishing features. First, I embed an endogenous growth model of vertical innovations [e.g., Grossman and Helpman (1991),

Aghion and Howitt (1992), and Peretto (1999)] into a standard New Keynesian DSGE model.<sup>1</sup> Second, households are assumed to have recursive preferences so that they are sensitive towards uncertainty about long-term growth prospects [e.g., Epstein and Zin (1989) and Bansal and Yaron (2004)]. In this environment, the key determinants of the nominal yield curve, expected inflation and growth prospects, are driven by firms' production decisions and monetary policy. Methodologically, linking the term structure explicitly to production relates to Jermann (2013).

When calibrated to match macroeconomic data, such as consumption, output, investment, labor, inflation, and wage dynamics, the model can quantitatively explain the means, volatilities, and autocorrelations of nominal bond yields. The model also captures the empirical failure of the expectations hypothesis. Namely, excess bond returns can be forecasted by the forward spread [e.g., Fama and Bliss (1987)] and by a linear combination of forward rates [e.g., Cochrane and Piazzesi (2005)].

Three key ingredients allow the model to rationalize these bond market facts. First, the endogenous growth channel generates long-run risks through firms' innovation decisions as in Kung and Schmid (2013). Second, the presence of nominal rigidities helps to generate a negative relationship between expected growth and inflation. In particular, imperfect nominal price adjustment implies that equilibrium inflation is equal to the present discounted value of current and future real marginal costs. A positive productivity shock lowers marginal costs and therefore inflation. Also, firms invest more after an increase in productivity which raises expected growth prospects. With recursive preferences, a negative growth-inflation link leads to a positive and sizeable nominal term premium. Third, fluctuating productivity uncertainty leads to time-varying bond risk premiums.

An interesting dimension of the model is the link it produces between monetary policy and asset prices. Specifically, in the model, the monetary authority follows a short-term nominal interest rate rule that responds to current inflation and output deviations (i.e., a Taylor rule). In counterfactual policy experiments, more aggressive inflation stabilization increases the equity premium and decreases the average nominal yield spread while more aggressive output stabilization decreases the equity premium and increases the average nominal yield spread.

The paper is organized as follows. Section 2 outlines the benchmark model. Section 3 explores

<sup>&</sup>lt;sup>1</sup>See Woodford (2003) and Galí (2008) for textbook treatments of New Keynesian models.

the quantitative implications of the model. Section 4 concludes.

# 2 Model

### 2.1 Households

Assume a representative household that has recursive utility over streams of consumption  $C_t$  and leisure  $\overline{L} - L_t$ :

$$U_t = \left\{ (1 - \beta) (C_t^{\star})^{\frac{1 - \gamma}{\theta}} + \beta \left( E_t \left[ U_{t+1}^{1 - \gamma} \right] \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1 - \gamma}}, \tag{1}$$

$$C_t^{\star} \equiv C_t (\overline{L} - L_t)^{\tau}, \tag{2}$$

where  $\gamma$  is the coefficient of risk aversion,  $\psi$  is the elasticity of intertemporal substitution,  $\theta \equiv \frac{1-\gamma}{1-1/\psi}$  is a parameter defined for convenience,  $\beta$  is the subjective discount rate, and  $\overline{L}$  is the agent's time endowment. The time t budget constraint of the household is

$$\mathcal{P}_t C_t + \frac{\mathcal{B}_{t+1}}{\mathcal{R}_{t+1}} = \mathcal{D}_t + \mathcal{W}_t L_t + \mathcal{B}_t, \tag{3}$$

where  $\mathcal{P}_t$  is the nominal price of the final goods,  $\mathcal{B}_{t+1}$  is the quantity of nominal one-period bonds,  $\mathcal{R}_{t+1}$  is the gross one-period nominal interest rate set at time t by the monetary authority,  $\mathcal{D}_t$  is nominal dividend income received from the intermediate firms,  $\mathcal{W}_t$  is the nominal wage rate, and  $L_t$  is labor hours supplied by the household.

The household's intertemporal condition is

$$1 = E_t \left[ M_{t+1} \frac{\mathcal{P}_t}{\mathcal{P}_{t+1}} \right] \mathcal{R}_{t+1}, \tag{4}$$

where

$$M_{t+1} = \beta \left(\frac{C_{t+1}^{\star}}{C_t^{\star}}\right)^{\frac{1-\gamma}{\theta}} \left(\frac{C_{t+1}}{C_t}\right)^{-1} \left(\frac{U_{t+1}^{1-\gamma}}{E_t[U_{t+1}^{1-\gamma}]}\right)^{1-\frac{1}{\theta}}$$
(5)

<sup>&</sup>lt;sup>2</sup>The parameters  $\gamma$  and  $\psi$  are defined over the composite good  $C_t^{\star}$ .

is the real stochastic discount factor. The intratemporal condition is

$$\frac{\mathcal{W}_t}{\mathcal{P}_t} = \frac{\tau C_t}{\overline{L} - L_t}.$$
 (6)

### 2.2 Firms

Production is comprised of a final goods and an intermediate goods sector.

#### 2.2.1 Final Goods

A representative firm produces the final (consumption) goods  $Y_t$  in a perfectly competitive market. The firm uses a continuum of differentiated intermediate goods  $X_{i,t}$  as input in a constant elasticity of substitution (CES) production technology:

$$Y_t = \left(\int_0^1 X_{i,t}^{\frac{\nu-1}{\nu}} di\right)^{\frac{\nu}{\nu-1}},\tag{7}$$

where  $\nu$  is the elasticity of substitution between intermediate goods. The profit maximization problem of the firm yields the following isoelastic demand schedule with price elasticity  $\nu$ :

$$X_{i,t} = Y_t \left(\frac{\mathcal{P}_{i,t}}{\mathcal{P}_t}\right)^{-\nu},\tag{8}$$

where  $\mathcal{P}_t$  is the nominal price of the final goods and  $\mathcal{P}_{i,t}$  is the nominal price of intermediate goods i. The inverse demand schedule is

$$\mathcal{P}_{i,t} = \mathcal{P}_t Y_t^{\frac{1}{\nu}} X_{i,t}^{-\frac{1}{\nu}}.$$

#### 2.2.2 Intermediate Goods

The intermediate goods sector is characterized by a continuum of monopolistic firms. Each intermediate goods firm produces  $X_{i,t}$  with physical capital  $K_{i,t}$ , R&D capital  $N_{i,t}$ , and labor  $L_{i,t}$  inputs using the following technology, similar to Peretto (1999),

$$X_{i,t} = K_{i,t}^{\alpha} (Z_{i,t} L_{i,t})^{1-\alpha},$$
 (9)

and measured total factor productivity (TFP) is

$$Z_{i,t} \equiv A_t N_{i,t}^{\eta} N_t^{1-\eta},\tag{10}$$

where  $A_t$  represents a stationary aggregate productivity shock,  $N_t \equiv \int_0^1 N_j dj$  is the aggregate stock of R&D and  $(1-\eta) \in [0,1]$  captures the degree of technological spillovers. Thus, firm-level TFP is comprised of two aggregate components,  $A_t$  and  $N_t$ , and a firm-specific component  $N_{i,t}$ . Notably, the firm can upgrade its technology directly by investing in R&D. Furthermore, there are spillover effects from innovating: Firm-level investments in R&D also improve aggregate technology. These spillover effects are crucial for generating sustained growth in the economy and are a standard feature in endogenous growth models.<sup>3</sup>

The law of motion for log productivity,  $a_t \equiv \log(A_t)$ , is

$$a_t = (1 - \rho)a^* + \rho a_{t-1} + \sigma_{t-1}\epsilon_t, \tag{11}$$

$$\sigma_t^2 = \overline{\sigma}^2 + \lambda(\sigma_{t-1}^2 - \overline{\sigma}^2) + \sigma_e e_t, \tag{12}$$

where  $\epsilon_t, e_t \sim N(0,1)$  are uncorrelated and iid. Following Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), and Malkhozov and Shamloo (2012), time-varying aggregate uncertainty  $\sigma_t$  is incorporated in the productivity process. Croce (2012) and Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) provide empirical support for conditional heteroscedasticity in aggregate productivity.

The law of motion for  $K_{i,t}$  is

$$K_{i,t+1} = (1 - \delta_k)K_{i,t} + \Phi_k\left(\frac{I_{i,t}}{K_{i,t}}\right)K_{i,t}, \tag{13}$$

$$\Phi_k\left(\frac{I_{i,t}}{K_{i,t}}\right) = \frac{\alpha_{1,k}}{1 - \frac{1}{\zeta_k}} \left(\frac{I_{i,t}}{K_{i,t}}\right)^{1 - \frac{1}{\zeta_k}} + \alpha_{2,k},\tag{14}$$

where  $I_{i,t}$  is capital investment (using the final goods) and the function  $\Phi_k(\cdot)$  captures capital adjustment costs as in Jermann (1998). The parameter  $\zeta_k$  represents the elasticity of new capital

<sup>&</sup>lt;sup>3</sup>See, for example, Romer (1990) and Aghion and Howitt (1992).

investments relative to the existing stock of capital. <sup>4</sup>

The law of motion for  $N_{i,t}$  is

$$N_{i,t+1} = (1 - \delta_n) N_{i,t} + \Phi_n \left( \frac{S_{i,t}}{N_{i,t}} \right) N_{i,t}, \tag{15}$$

$$\Phi_n\left(\frac{S_{i,t}}{N_{i,t}}\right) = \frac{\alpha_{1,n}}{1 - \frac{1}{\zeta_n}} \left(\frac{S_{i,t}}{N_{i,t}}\right)^{1 - \frac{1}{\zeta_n}} + \alpha_{2,n},\tag{16}$$

where  $S_{i,t}$  is R&D investment (using the final goods) and the function  $\Phi_n(\cdot)$  captures adjustment costs in R&D investments. The parameter  $\zeta_n$  represents the elasticity of new R&D investments relative to the existing stock of R&D. <sup>5</sup>

Substituting the production technology into the inverse demand function yields the following expression for the nominal price for intermediate goods i

$$\mathcal{P}_{i,t} = \mathcal{P}_t Y_t^{\frac{1}{\nu}} \left[ K_{i,t}^{\alpha} \left( A_t N_{i,t}^{\eta} N_t^{1-\eta} L_{i,t} \right)^{1-\alpha} \right]^{-\frac{1}{\nu}}$$

$$\equiv \mathcal{P}_t J(K_{i,t}, N_{i,t}, L_{i,t}; A_t, N_t, Y_t).$$

Further, nominal revenues for intermediate firm i can be expressed as

$$\mathcal{P}_{i,t}X_{i,t} = \mathcal{P}_tY_t^{\frac{1}{\nu}} \left[ K_{i,t}^{\alpha} \left( A_t N_{i,t}^{\eta} N_t^{1-\eta} L_{i,t} \right)^{1-\alpha} \right]^{1-\frac{1}{\nu}}$$

$$\equiv \mathcal{P}_tF(K_{i,t}, N_{i,t}, L_{i,t}; A_t, N_t, Y_t).$$

Each intermediate firm also faces a cost of adjusting its nominal price à la Rotemberg (1982), measured in terms of the final goods as

$$G(\mathcal{P}_{i,t}, \mathcal{P}_{i,t-1}; \mathcal{P}_t, Y_t) = \frac{\phi_R}{2} \left( \frac{\mathcal{P}_{i,t}}{\prod_{ss} \mathcal{P}_{i,t-1}} - 1 \right)^2 Y_t, \tag{17}$$

where  $\Pi_{ss} \ge 1$  is the gross steady-state inflation rate and  $\phi_R$  is the magnitude of the costs.

<sup>&</sup>lt;sup>4</sup>The parameters  $\alpha_{1,k}$  and  $\alpha_{2,k}$  are set to values so that there are no adjustment costs in the deterministic steady state. Specifically,  $\alpha_{1,k} = (\Delta N_{ss} - 1 + \delta_k)^{\frac{1}{\zeta_k}}$  and  $\alpha_{2,k} = \frac{1}{\zeta_k - 1}(1 - \delta_k - \Delta N_{ss})$ .

<sup>5</sup>The parameters  $\alpha_{1,n}$  and  $\alpha_{2,n}$  are set to values so that there are no adjustment costs in the deterministic

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The source of funds constraint is

$$\mathcal{D}_{i,t} = \mathcal{P}_t F(K_{i,t}, N_{i,t}, L_{i,t}; A_t, N_t, Y_t) - \mathcal{W}_{i,t} L_{i,t} - \mathcal{P}_t I_{i,t} - \mathcal{P}_t S_{i,t} - \mathcal{P}_t G(\mathcal{P}_{i,t}, \mathcal{P}_{i,t-1}; \mathcal{P}_t, Y_t), \quad (18)$$

where  $\mathcal{D}_{i,t}$  and  $\mathcal{W}_{i,t}$  are the nominal dividend and wage rate, respectively.

Firm i takes the pricing kernel  $M_t$  and the vector of aggregate states  $\Upsilon_t = [\mathcal{P}_t, K_t, N_t, Y_t, A_t]$  as given and solves the following recursive problem to maximize shareholder value,  $V_{i,t} \equiv V^{(i)}(\cdot)$ :

$$V^{(i)}(\mathcal{P}_{i,t-1}, K_{i,t}, N_{i,t}; \Upsilon_{\mathbf{t}}) = \max_{\mathcal{P}_{i,t}, I_{i,t}, S_{i,t}, K_{i,t+1}, N_{i,t+1}, L_{i,t}} \frac{\mathcal{D}_{i,t}}{\mathcal{P}_{t}} + E_{t} \left[ M_{t+1} V^{(i)}(\mathcal{P}_{i,t}, K_{i,t+1}, N_{i,t+1}; \Upsilon_{\mathbf{t}+1}) \right]$$

subject to

$$\begin{split} \frac{\mathcal{P}_{i,t}}{\mathcal{P}_{t}} &= J(K_{i,t}, N_{i,t}, L_{i,t}; A_{t}, N_{t}, Y_{t}), \\ K_{i,t+1} &= (1 - \delta_{k})K_{i,t} + \Phi_{k}\left(\frac{I_{i,t}}{K_{i,t}}\right)K_{i,t}, \\ N_{i,t+1} &= (1 - \delta_{n})N_{i,t} + \Phi_{n}\left(\frac{S_{i,t}}{N_{i,t}}\right)N_{i,t}, \\ \mathcal{D}_{i,t} &= \mathcal{P}_{t}F(K_{i,t}, N_{i,t}, L_{i,t}; A_{t}, N_{t}, Y_{t}) - \mathcal{W}_{i,t}L_{i,t} - \mathcal{P}_{t}I_{i,t} - \mathcal{P}_{t}G(\mathcal{P}_{i,t}, \mathcal{P}_{i,t-1}; \mathcal{P}_{t}, Y_{t}). \end{split}$$

The corresponding first-order conditions are derived in Appendix B.

### 2.3 Central Bank

The central bank follows a modified Taylor rule that depends on the lagged interest rate, and output and inflation deviations:

$$\ln\left(\frac{\mathcal{R}_{t+1}}{\mathcal{R}_{ss}}\right) = \rho_r \ln\left(\frac{\mathcal{R}_t}{\mathcal{R}_{ss}}\right) + (1 - \rho_r) \left(\rho_\pi \ln\left(\frac{\Pi_t}{\Pi_{ss}}\right) + \rho_y \ln\left(\frac{\widehat{Y}_t}{\widehat{Y}_{ss}}\right)\right) + \sigma_\xi \xi_t, \tag{19}$$

where  $\mathcal{R}_{t+1}$  is the gross nominal short rate,  $\hat{Y}_t \equiv \frac{Y_t}{N_t}$  is detrended output, and  $\xi_t \sim N(0,1)$  is a monetary policy shock. Variables with an ss-subscript denote steady-state values.

### 2.4 Symmetric Equilibrium

In the symmetric equilibrium, all intermediate firms make identical decisions:  $\mathcal{P}_{i,t} = \mathcal{P}_t$ ,  $X_{i,t} = X_t$ ,  $K_{i,t} = K_t$ ,  $L_{i,t} = L_t$ ,  $N_{i,t} = N_t$ ,  $I_{i,t} = I_t$ ,  $S_{i,t} = S_t$ ,  $D_{i,t} = D_t$ ,  $V_{i,t} = V_t$ . Also,  $\mathcal{B}_t = 0$ . The aggregate resource constraint is

$$Y_t = C_t + S_t + I_t + \frac{\phi_R}{2} \left( \frac{\Pi_t}{\Pi_{ss}} - 1 \right)^2 Y_t, \tag{20}$$

where  $\Pi_t \equiv \frac{\mathcal{P}_t}{\mathcal{P}_{t-1}}$  is the gross inflation rate.

# 2.5 Bond Pricing

The price of an n-period nominal bond  $\mathcal{P}_t^{(n)\$}$  can be written recursively as:

$$\mathcal{P}_{t}^{(n)\$} = E_{t} \left[ \mathcal{M}_{t+1}^{\$} \mathcal{P}_{t+1}^{(n-1)\$} \right], \tag{21}$$

where  $\mathcal{M}_{t+1}^{\$} \equiv \frac{M_{t+1}}{\Pi_{t+1}}$  is the nominal stochastic discount factor and  $\mathcal{P}_t^{(0)\$} = 1$  and  $\mathcal{P}_t^{(1)\$} = \frac{1}{\mathcal{R}_{t+1}}$ . Assuming that  $\mathcal{M}_t^{\$}$  is conditionally lognormally distributed, then Eq. (21) can be expressed in logs as

$$p_t^{(n)\$} = E_t \left[ p_{t+1}^{(n-1)\$} + m_{t+1}^{\$} \right] + \frac{1}{2} \text{var}_t \left[ p_{t+1}^{(n-1)\$} + m_{t+1}^{\$} \right],$$

and recursively substituting out prices,

$$p_t^{(n)\$} = E_t \left[ \sum_{j=1}^n m_{t+j}^\$ \right] + \frac{1}{2} \text{var}_t \left[ \sum_{j=1}^n m_{t+j}^\$ \right].$$
 (22)

The yield-to-maturity on the n-period nominal bond is defined as

$$y_t^{(n)\$} \equiv -\frac{1}{n} p_t^{(n)\$},$$

which after substituting in Eq. (22) can be expressed as

$$y_t^{(n)\$} = -\frac{1}{n} E_t \left[ \sum_{j=1}^n m_{t+j}^{\$} \right] - \frac{1}{2n} \text{var}_t \left[ \sum_{j=1}^n m_{t+j}^{\$} \right].$$
 (23)

As evident from Eq. (23), movements in nominal yields are driven by the conditional mean and variance of the nominal stochastic discount factor, which in turn depends on inflation and consumption growth.

Similarly, the price of a n-period real bond can be written as

$$P_t^{(n)} = E_t \left[ M_{t+1} P_{t+1}^{(n-1)} \right],$$

and the corresponding yield-to-maturity is defined as

$$y_t^{(n)} \equiv -\frac{1}{n} p_t^{(n)}$$

$$= -\frac{1}{n} E_t \left[ \sum_{j=1}^n m_{t+j} \right] - \frac{1}{2n} \operatorname{var}_t \left[ \sum_{j=1}^n m_{t+j} \right].$$

# 2.6 Equilibrium Growth and Inflation

The model endogenously generates (i) low-frequency movements in growth and inflation and (ii) a negative relationship between expected growth and inflation, which have important implications for the term structure. In particular, a negative link between growth and inflation implies that long-maturity nominal bonds have lower payoffs than short-maturity ones when long-term growth is expected to be low. With recursive preferences, these dynamics lead to a positive and sizeable average nominal term spread.

Low-frequency movements in growth rates (i.e., long-run risks) arise endogenously through the firms' R&D investments as in Kung and Schmid (2013). Imposing the symmetric equilibrium conditions implies that the aggregate production function is

$$Y_t = K_t^{\alpha} \left( Z_t L_t \right)^{1-\alpha},$$

where  $Z_t \equiv A_t N_t$  is measured aggregate productivity. Second, assuming that  $A_t$  is a persistent process in logs, expected log productivity growth can be approximated as

$$E_{t-1}[\Delta z_t] \approx \Delta n_t$$
.

Thus, low-frequency movements in growth are driven by the accumulation of R&D.

As standard in New Keynesian models, inflation dynamics depend on real marginal costs and expected inflation:

$$\widetilde{\pi}_t = \gamma_1 \widetilde{m} c_t + \gamma_2 E_t [\widetilde{\pi}_{t+1}],$$

where  $\gamma_1 = \frac{\nu - 1}{\phi_R} > 0$ ,  $\gamma_2 = \beta \Delta Y_{ss}^{1 - \frac{1}{\psi}} > 0$ , and lowercase tilde variables denote log deviations from the steady-state (see Appendix C for a derivation). Recursively substituting out future inflation terms implies that inflation is related to current and discounted expected future real marginal costs. Hence, persistence in marginal costs leads to low-frequency movements in inflation.

To understand the negative long-run relationship between growth and inflation, first suppose there is a positive productivity shock. In response to this positive shock, firms increase investment, which boosts expected productivity growth prospects persistently. Also, the prolonged increase in productivity lowers real marginal costs persistently so that inflation declines persistently as well. In short, the model endogenizes the consumption growth and inflation dynamics specified in Piazzesi and Schneider (2007) and Bansal and Shaliastovich (2012).

# 3 Quantitative Results

This section discusses the quantitative implications of the model. The model is solved in Dynare++ using a third-order approximation. The policies are centered around a fix-point that takes into account the effects of volatility on decision rules. A description of the data is in Appendix A.

#### 3.1 Calibration

Table 1 presents the quarterly calibration. I begin with the preference parameters. The elasticity of intertemporal substitution  $\psi$  is set to 2.0 and the coefficient of relative risk aversion  $\gamma$  is set to 10.0, which are standard values in the long-run risks literature.<sup>6</sup> The subjective discount factor  $\beta$  is set to a value of 0.997 to be consistent with the level of the real (risk-free) short-term rate.

Next, I move to the standard production parameters. The price elasticity of demand  $\nu$  is set at 6 (which corresponds to a markup of 0.2), the capital share  $\alpha$  is set to 0.33, and the depreciation rate of capital  $\delta_k$  is set to 0.02. The calibration of these three parameters is standard and set to match steady-state evidence. The price adjustment cost parameter  $\phi_R$  is set to 30, and is calibrated to match the impulse response of output to a monetary policy shock. This value of  $\phi_R$  implies that the average magnitude of the price adjustment costs are small (0.22% of output), consistent with empirical estimates.<sup>7</sup> The capital adjustment cost parameter  $\zeta_k$  is set at 4.7 to match the relative volatility of investment growth to consumption growth (reported in Table 2). This value of  $\zeta_k$  implies that the average magnitude of capital adjustment costs are quite small (0.08% of the capital stock), as in the data.<sup>8</sup>

The parameters related to R&D are calibrated to match R&D data. The depreciation rate of the R&D capital stock  $\delta_n$  is calibrated to a value of 0.0375 which corresponds to an annualized depreciation rate of 15%, and is the value used by the Bureau of Labor Statistics (BLS) in the R&D stock calculations. The R&D capital adjustment cost parameter  $\zeta_n$  is set at 3.3 to match the relative volatility of R&D investment growth to consumption growth (reported in Table 2). This value of  $\zeta_n$  implies that the average magnitude of R&D adjustment costs are small (0.05% of the R&D capital stock). The degree of technological appropriability  $\eta$  is set to match the steady-state value of the R&D investment rate.

The parameters governing the productivity process are calibrated to replicate salient features

<sup>&</sup>lt;sup>6</sup>This parametrization is also supported empirically by the GMM estimates from Bansal, Kiku, and Yaron (2007).

<sup>&</sup>lt;sup>7</sup>For example, in a log-linear approximation, the parameter  $\phi_R$  can be mapped directly to a parameter that governs the average price duration in a Calvo pricing framework. In this calibration,  $\phi_R = 30$  corresponds to an average price duration of 3.3 months, which accords with micro evidence from Bils and Klenow (2004). See Appendix C for details of this mapping.

<sup>&</sup>lt;sup>8</sup>For example, Cooper and Haltiwanger (2006) find that the average magnitude of capital adjustment costs is 0.91% using micro-data.

of measured productivity, which are reported in Table 3. The unconditional volatility parameter  $\bar{\sigma}$  is set at 1.20% to match the unconditional volatility of measured productivity growth. The persistence parameter  $\rho$  is set at 0.983 to match the first autocorrelation of expected productivity growth. Furthermore, the first autocorrelations of key macroeconomic aggregates are broadly consistent with the data (presented in Table 4). The parameters  $\lambda$  and  $\sigma_e$  of the volatility process are calibrated to match the first autocorrelation and standard deviation of realized consumption volatility, respectively (reported in Table 5).

The values for the interest rate rule parameters are consistent with estimates from the literature.<sup>9</sup> The parameter governing the sensitivity of the interest rate to inflation  $\rho_{\pi}$  is set to 1.5. The parameter determining the sensitivity of the interest rate to output  $\rho_{y}$  is set to 0.10. The persistence of the interest rate rule  $\rho_{R}$  is calibrated to 0.70. The volatility of interest shocks  $\sigma_{\xi}$ is set to 0.3%.<sup>10</sup> Steady-state inflation  $\Pi_{ss}$  is calibrated to match the average level of inflation. Overall, the nominal short rate dynamics implied by this calibration closely match the data, as shown in Table 6.

# 3.2 Bond Market Implications

The top half of Table 7 shows the means, volatilities, and first autocorrelations of nominal bond yields of different maturities. The model matches the slope of the nominal yield curve from the data very closely. In particular, the average five-year minus one-quarter nominal yield spread is around 1% in both the model and the data, which are reported in Table 6. Note that from the last two columns of Table 6, monetary policy shocks and uncertainty shocks play a small role in determining the average slope of the yield curve.

The positive nominal yield spread in the model is due to inflation risk premia increasing with maturity. As described in Section 2.6, firms' price-setting and investment decisions in the model lead to a negative long-run relationship between inflation and consumption growth. Table 8 shows that the negative short-run and long-run correlations between inflation and consumption growth from the model closely match the empirical counterparts.<sup>11</sup> This negative inflation-growth link

<sup>&</sup>lt;sup>9</sup>See, for example, Clarida, Galí, and Gertler (1999) and Rudebusch (2002).

<sup>&</sup>lt;sup>10</sup>See, for example, Smets and Wouters (2007).

<sup>&</sup>lt;sup>11</sup>The long-run correlation is computed by isolating the low-frequency components of inflation and con-

implies that long-maturity nominal bonds have lower payoffs than short-maturity ones when longterm growth is expected to be low. Since agents with recursive preferences are strongly averse to low expected growth states, these dynamics lead to a positive and sizeable term premium.

The volatilities and first autocorrelations of nominal yields from the model match the data reasonably well (reported in Table 7). As illustrated in Section 2.5, nominal yield dynamics are dictated by fluctuations in the conditional mean and volatility of the nominal pricing kernel. In the model, the conditional mean of the nominal pricing kernel is driven by expected consumption and inflation dynamics while the conditional volatility of the pricing kernel is driven by conditional heteroskedasticity in the productivity shocks. The growth and inflation propagation mechanisms of the model generate significant persistence and variability in expected growth and inflation. Consequently, the volatility and persistence of nominal yields are primarily a reflection of the low-frequency movements in consumption growth and inflation.

The bottom half of Table 7 displays the means, volatilities, and first autocorrelations of real bond yields of different maturities from the model. Notably, the average slope of the real yield curve is negative, as in standard long-run risks models.<sup>12</sup> A downward sloping real yield curve is due to positive autocorrelation in consumption growth, which implies that long-maturity real bonds have higher payoffs than short-maturity ones when expected consumption growth is low. Empirical evidence for the slope of the real yield curve is varied. Evans (1998) and Bansal, Kiku, and Yaron (2012) show that the real yield curve in the UK is downward sloping for the 1984-1995 and 1996-2008 samples, respectively. On the other hand, Beeler and Campbell (2012) report that real yield curve data in the US is upward sloping in the 1997-2012 sample.

According to the expectations hypothesis, excess bond returns are not predictable. However, there is strong empirical evidence showing that excess bond returns are in fact forecastable by a single factor, such as the forward premium and a linear combination of forward rates. Table 9 reports the Fama and Bliss (1987) regressions of n-period excess bond returns on n-period forward premiums. The benchmark model is able to do a reasonable job in reproducing the empirical estimates. Specifically, the model produces slope coefficients and  $R^2$ s that are about half the

sumption growth using a bandpass filter.

<sup>&</sup>lt;sup>12</sup>For example, see Piazzesi and Schneider (2007) and Bansal and Shaliastovich (2012).

magnitude of the empirical estimates. As a production-based benchmark, the model estimates are comparable to Jermann (2013). As with the average level of bond yields, monetary policy shocks play a small role in the forecasting results. By setting the volatility of the policy shocks to zero, the slope coefficients and  $R^2$ s increase modestly.

Table 10 shows the Cochrane and Piazzesi (2005) regressions of n-period excess bond returns on a single linear combination of forward rates for the data and the benchmark model. The model is able to replicate the empirical slope coefficients and corresponding standard errors closely while the  $R^2$ s are sizeable. Worth noting, the slope coefficients are positive and increasing with horizon. In sum, the model is able to produce quantitatively significant bond return predictability.

Time-varying bond risk premia in the model is driven by the fluctuating economic uncertainty. Specifically, a positive uncertainty shock to productivity increases uncertainty in real marginal costs. Since equilibrium inflation is driven by real marginal costs, this implies an increase in inflation uncertainty. As shown in Bansal and Shaliastovich (2012), when agents prefer an early resolution of uncertainty (i.e.,  $\psi > 1/\gamma$ ), an increase in inflation uncertainty raises nominal bond risk premia, consistent with empirical evidence.<sup>13</sup>

# 3.3 Yield Curve and Macroeconomic Activity

The slope of the nominal yield curve is empirically a strong predictor of economic growth and inflation at business-cycle frequencies.  $^{14}$  Table 11 reports output growth forecasting regressions using the five-year minus one-quarter nominal yield spread for horizons of one, four, and eight quarters. The slope coefficients and  $R^2$ s from the benchmark model are of a similar magnitude as the empirical estimates. Monetary policy uncertainty and time-varying uncertainty in productivity play a small role in these forecasting results. Indeed, by setting the volatility parameters of the policy and volatility shocks to zero, the slope coefficients increase while the  $R^2$ s decrease slightly.

The positive relationship between the slope of the yield curve and expected growth is linked to the Taylor rule. In the model, a positive productivity shock increases expected consumption growth and decreases inflation. The monetary authority responds to the decline in inflation by lowering

<sup>&</sup>lt;sup>13</sup>Bansal and Shaliastovich (2012) find empirically that future bond returns load positively on inflation uncertainty.

<sup>&</sup>lt;sup>14</sup>For example, see Estrella (2005) and Ang, Piazzesi, and Wei (2006).

the nominal short rate aggressively. A temporary decrease in the short rate implies that future short rates are expected to rise. Consequently, the slope of the nominal yield curve increases.

Similarly, Table 12 and Table 13 show that the slope of the yield curve also forecasts consumption growth and inflation. The slope coefficients, standard errors, and  $R^2$ s from the benchmark model are similar to the empirical estimates.

Piazzesi and Schneider (2007) show that the high- and low-frequency components of the nominal yield spread and inflation are closely related. The first two rows of Table 14 show that there is a strong negative correlation between the yield spread and inflation at both high and low frequencies, and the model is able to match the empirical correlations quite well. The negative correlations are a reaffirmation of the inflation forecasting regressions. Fig. 1 provides a visual depiction of the negative relationship between the term spread (thin line) and inflation (thick line). Note that during periods of high inflation, such as the late 1970s/early 1980s, the term spread is negative. Similarly, from Fig. 1, periods of high inflation in model simulations are also associated with a negative term spread. In the model, when inflation rises sharply, the monetary authority aggressively increases the short rate, which decreases the slope of the yield curve. Thus, if the rise in inflation is high enough, the yield curve slopes downwards.

Interestingly, the model predicts a strong positive long-run relationship between R&D and the nominal yield spread. As shown in the bottom row of Table 14, both the model and the data exhibit a strong positive low-frequency correlation between the R&D rate and the term spread. A positive productivity shock increases R&D and decreases inflation persistently. Furthermore, a drop in inflation leads to a decline in the short rate, which implies an increase in the slope of the yield curve. Notably, the R&D boom of the 1990s was preceded by a persistent rise in the nominal term spread from the late 1980s through the early 1990s.

# 3.4 Additional Implications

This section explores additional results of the model. Table 15 reports the means and volatilities of the equity premium and the short-term real rate. As in Kung and Schmid (2013), the growth channel generates endogenous long-run risks, which allows the model to generate a sizeable equity premium and a low and stable real short rate. While return volatility falls short of the empirical es-

timate, incorporating real wage rigidities help generate substantially more volatility, as in Favilukis and Lin (2012). Following Blanchard and Galí (2007), assume the following real wage process:

$$\ln\left(\frac{\mathcal{W}_t}{\mathcal{P}_t}\right) = \kappa \ln\left(\frac{\mathcal{W}_{t-1}}{\mathcal{P}_{t-1}}\right) + (1 - \kappa) \ln\left(\frac{\tau C_t}{\overline{L} - L_t}\right),\tag{24}$$

where  $\kappa \in [0, 1]$  captures the degree of wage rigidity. In this specification, equity return volatility increases from 6.68% to 9.18%.

Fama and French (1989) empirically document that the term spread forecasts excess stock returns. Table 16 reports excess stock return forecasts using the five-year minus one-quarter nominal yield spread. The model regressions produce positive slope coefficients and sizeable  $R^2$ s, as in the data. While the slope coefficients are smaller than in the data, they are consistent with the model estimates from Jermann (2013), a production-based benchmark. Furthermore, Cochrane and Piazzesi (2005) show that a linear combination of forward rates can also forecast excess stock returns. Table 17 reports excess stock return forecasts using the Cochrane-Piazzesi factor for horizons of one to five years. The model forecasts produce positive slope coefficients that match the empirical estimates closely. Additionally, the slope coefficients are statistically significant and the  $R^2$ s are sizeable.

As in the pure production-based framework of Jermann (2013), <sup>15</sup> capital depreciation rates and adjustment costs play an important role for the nominal term premium. In Jermann (2013), depreciation rates and the curvature of the adjustment costs affect the term premium through the short rate. In contrast, in my model, these parameters impact the term premium through consumption and inflation. The top panel in Table 18 reports the comparative statics results from varying depreciation rates,  $\delta_k$  and  $\delta_n$ , from 1% to 3% separately. Higher depreciation rates make it more difficult for the households to smooth consumption, which is reflected in higher consumption volatility. Higher consumption volatility increases the quantity of risk and therefore results in a higher term premium. The bottom panel of Table 18 reports the comparative statics results from varying the curvature of adjustment costs,  $\zeta_k$  and  $\zeta_n$ . Increasing the curvature (i.e., moving from 8.0 to 2.0) dampens the response of capital and R&D investment to productivity shocks, which

 $<sup>^{15}</sup>$ Jermann (2013) introduces exogenous inflation dynamics to the two-sector production-based asset pricing framework of Jermann (2010) to analyze term structure implications.

weakens the negative correlation between growth and inflation. A weaker correlation reduces the riskiness of long nominal bonds.

In addition to the production parameters, the monetary policy parameters are also important for the nominal term premium. Fig. 2 illustrates the effects of varying inflation stabilization. More aggressive inflation smoothing (i.e., higher  $\rho_{\pi}$ ) decreases the quantity of nominal risks, which lowers the term premium. On the other hand, as inflation and growth are negatively related, higher inflation smoothing amplifies growth dynamics.<sup>16</sup> Larger real risks increase the equity premium. Analogously, Fig. 3 shows that increasing output stabilization (i.e., higher  $\rho_y$ ) decreases the equity premium but increases the term premium.<sup>17</sup>

# 4 Conclusion

This paper relates the term structure of interest rates to macroeconomic fundamentals using a stochastic endogenous growth model with imperfect price adjustment. The model matches the means and volatilities of nominal bond yields reasonably well and captures the failure of the expectations hypothesis. The production and price-setting decisions of firms generate a negative long-term relationship between expected growth and inflation. Consequently, the positive nominal term premium is attributed to inflation risks increasing with maturity. Monetary policy plays a crucial role in reconciling the empirical growth and inflation forecasts with the slope of the yield curve. In short, this paper highlights the importance of the growth channel in explaining the term structure of interest rates.

 $<sup>^{16}</sup>$ A larger value of  $\rho_{\pi}$  implies that the nominal short rate, and therefore the real rate (due to sticky prices), will rise more after an increase in inflation. Since inflation and R&D rates are negatively correlated, a larger rise in the real rate will further depress R&D and thus, amplify R&D rates. More volatile R&D amplifies growth.

<sup>&</sup>lt;sup>17</sup>Croce, Kung, Nguyen, and Schmid (2012) and Croce, Nyugen, and Schmid (2012) are related papers that explore how fiscal policy distorts expected growth rates.

# Appendix A. Data

Annual and quarterly data for consumption, capital investment, and GDP are from the Bureau of Economic Analysis (BEA). Annual data on private business R&D investment is from the survey conducted by the National Science Foundation (NSF). Annual data on the stock of private business R&D is from the Bureau of Labor Statistics (BLS). Annual productivity data is obtained from the BLS and is measured as multifactor productivity in the private nonfarm business sector. Quarterly total wages and salaries data are from the BEA. Quarterly hours worked data are from the BLS. The wage rate is defined as the total wages and salaries divided by hours worked. The sample period is for 1953-2008, since R&D data is only available during that time period. Consumption is measured as expenditures on nondurable goods and services. Capital investment is measured as private fixed investment. Output is measured as GDP. The variables are converted to real using the Consumer Price Index (CPI), which is obtained from the Center for Research in Security Prices (CRSP). The inflation rate is computed by taking the log return on the CPI index.

Monthly nominal return and yield data are from CRSP. The real market return is constructed by taking the nominal value-weighted return on the New York Stock Exchange (NYSE) and American Stock Exchange (AMEX) and deflating it using the CPI. The real risk-free rate is constructed by using the nominal average one-month yields on treasury bills and taking out expected inflation. Nominal yield data for maturities of 4, 8, 12, 16, and 20 quarters are from the CRSP Fama-Bliss discount bond file. The 1 quarter nominal yield is from the the CRSP Fama risk-free rate file.

# Appendix B. Intermediate Goods Firm Problem

The Lagrangian for intermediate firm i's problem is  $^{19}$ 

$$V^{(i)}(\mathcal{P}_{i,t-1}, K_{i,t}, N_{i,t}; \mathbf{\Upsilon_{t}}) = F(K_{i,t}, N_{i,t}, L_{i,t}; A_{t}, N_{t}, Y_{t}) - \frac{\mathcal{W}_{i,t}}{\mathcal{P}_{t}} L_{i,t} - I_{i,t} - S_{i,t} - G(\mathcal{P}_{i,t}, \mathcal{P}_{i,t-1}; \mathcal{P}_{t}, Y_{t})$$

$$+ E_{t} \left[ M_{t+1} V^{(i)}(\mathcal{P}_{i,t}, K_{i,t+1}, N_{i,t+1}; \mathbf{\Upsilon_{t+1}}) \right]$$

$$+ \Lambda_{i,t} \left\{ \frac{\mathcal{P}_{i,t}}{\mathcal{P}_{t}} - J(K_{i,t}, N_{i,t}, L_{i,t}; A_{t}, N_{t}, Y_{t}) \right\}$$

$$+ Q_{i,k,t} \left\{ (1 - \delta_{k}) K_{i,t} + \Phi_{k} \left( \frac{I_{i,t}}{K_{i,t}} \right) K_{i,t} - K_{i,t+1} \right\}$$

$$+ Q_{i,n,t} \left\{ (1 - \delta_{n}) N_{i,t} + \Phi_{n} \left( \frac{S_{i,t}}{N_{i,t}} \right) N_{i,t} - N_{i,t+1} \right\}$$

 $<sup>^{18}</sup>$ The monthly time series process for inflation is modeled using an AR(4).

<sup>&</sup>lt;sup>19</sup>Note that for the real revenue function  $F(\cdot)$  to exhibit diminishing returns to scale in the factors  $K_{i,t}$ ,  $L_{i,t}$ , and  $N_{i,t}$  requires the following parameter restriction  $\left[\alpha + (\eta + 1)(1 - \alpha)\right] \left(1 - \frac{1}{\nu}\right) < 1$  or  $\eta(1 - \alpha)(\nu - 1) < 1$ .

The first-order conditions are

$$0 = -G_{i,1,t} + E_t[M_{t+1}V_{p,t+1}^{(i)}] + \frac{\Lambda_{i,t}}{\mathcal{P}_t}$$

$$0 = -1 + Q_{i,k,t}\Phi'_{i,k,t}$$

$$0 = -1 + Q_{i,n,t}\Phi'_{i,n,t}$$

$$0 = E_t[M_{t+1}V_{k,t+1}^{(i)}] - Q_{i,k,t}$$

$$0 = E_t[M_{t+1}V_{n,t+1}^{(i)}] - Q_{i,n,t}$$

$$0 = F_{i,l,t} - \frac{\mathcal{W}_{i,t}}{\mathcal{P}_t} - \Lambda_{i,t}J_{i,l,t}$$

The envelope conditions are

$$\begin{array}{lcl} V_{p,t}^{(i)} & = & -G_{i,2,t} \\ V_{k,t}^{(i)} & = & F_{i,k,t} - \Lambda_{i,t} J_{i,k,t} + Q_{i,k,t} \left( 1 - \delta_k - \frac{\Phi'_{i,k,t} I_{i,t}}{K_{i,t}} + \Phi_{i,k,t} \right) \\ V_{n,t}^{(i)} & = & F_{i,n,t} - \Lambda_{i,t} J_{i,n,t} + Q_{i,n,t} \left( 1 - \delta_n - \frac{\Phi'_{i,n,t} S_{i,t}}{N_{i,t}} + \Phi_{i,n,t} \right) \end{array}$$

where  $Q_{i,k,t}$ ,  $Q_{i,n,t}$ , and  $\Lambda_{i,t}$  are the shadow values of physical capital, R&D capital and price of intermediate goods, respectively.<sup>20</sup> Define the following terms from the equations above:

$$G_{i,1,t} = \phi_R \left( \frac{\mathcal{P}_{i,t}}{\Pi_{ss}\mathcal{P}_{i,t-1}} - 1 \right) \frac{Y_t}{\Pi_{ss}\mathcal{P}_{i,t-1}}$$

$$G_{i,2,t} = -\phi_R \left( \frac{\mathcal{P}_{i,t}}{\Pi_{ss}\mathcal{P}_{i,t-1}} - 1 \right) \frac{Y_t\mathcal{P}_{i,t}}{\Pi_{ss}\mathcal{P}_{i,t-1}}$$

$$\Phi_{i,k,t} = \frac{\alpha_{1,k}}{1 - \frac{1}{\zeta_k}} \left( \frac{I_{i,t}}{K_{i,t}} \right)^{1 - \frac{1}{\zeta_k}} + \alpha_{2,k}$$

$$\Phi_{i,n,t} = \frac{\alpha_{1,n}}{1 - \frac{1}{\zeta_n}} \left( \frac{S_{i,t}}{N_{i,t}} \right)^{1 - \frac{1}{\zeta_n}} + \alpha_{2,n}$$

$$\Phi'_{i,k,t} = \alpha_{1,k} \left( \frac{I_{i,t}}{K_{i,t}} \right)^{-\frac{1}{\zeta_n}}$$

$$\Phi'_{i,n,t} = \alpha_{1,n} \left( \frac{S_{i,t}}{N_{i,t}} \right)^{-\frac{1}{\zeta_n}}$$

 $<sup>\</sup>overline{\frac{20}{\Phi_{i,k,t}} \equiv \Phi_k\left(\frac{I_{i,t}}{K_{i,t}}\right), \Phi_{i,n,t} \equiv \Phi_n\left(\frac{S_{i,t}}{N_{i,t}}\right)}, \Phi'_{i,k,t} \equiv \alpha_{1,k}\left(\frac{I_{i,t}}{K_{i,t}}\right)^{-\frac{1}{\zeta_k}}, \Phi'_{i,n,t} \equiv \alpha_{1,n}\left(\frac{S_{i,t}}{N_{i,t}}\right)^{-\frac{1}{\zeta_n}} \text{ are defined for notational convenience.}$ 

$$F_{i,k,t} = \frac{\alpha \left(1 - \frac{1}{\nu}\right) Y_{t}^{\frac{1}{\nu}} X_{i,t}^{1 - \frac{1}{\nu}}}{K_{i,t}}$$

$$F_{i,n,t} = \frac{\eta (1 - \alpha) \left(1 - \frac{1}{\nu}\right) Y_{t}^{\frac{1}{\nu}} X_{i,t}^{1 - \frac{1}{\nu}}}{N_{i,t}}$$

$$F_{i,l,t} = \frac{\left(1 - \alpha\right) \left(1 - \frac{1}{\nu}\right) Y_{t}^{\frac{1}{\nu}} X_{i,t}^{1 - \frac{1}{\nu}}}{L_{i,t}}$$

$$J_{i,k,t} = \frac{-\left(\frac{\alpha}{\nu}\right) Y_{t}^{\frac{1}{\nu}} X_{i,t}^{-\frac{1}{\nu}}}{K_{i,t}}$$

$$J_{i,n,t} = \frac{-\left(\frac{\eta (1 - \alpha)}{\nu}\right) Y_{t}^{\frac{1}{\nu}} X_{i,t}^{-\frac{1}{\nu}}}{N_{i,t}}$$

$$J_{i,l,t} = \frac{-\left(\frac{1 - \alpha}{\nu}\right) Y_{t}^{\frac{1}{\nu}} X_{i,t}^{-\frac{1}{\nu}}}{L_{i,t}}$$

Substituting the envelope conditions and definitions above, the first-order conditions can be expressed as:

$$\begin{split} \frac{\Lambda_{i,t}}{\mathcal{P}_{t}} &= \phi_{R} \left( \frac{\mathcal{P}_{i,t}}{\Pi_{ss}\mathcal{P}_{i,t-1}} - 1 \right) \frac{Y_{t}}{\Pi_{ss}\mathcal{P}_{i,t-1}} - E_{t} \left[ M_{t+1}\phi_{R} \left( \frac{\mathcal{P}_{i,t+1}}{\Pi_{ss}\mathcal{P}_{i,t}} - 1 \right) \frac{Y_{t+1}\mathcal{P}_{i,t+1}}{\Pi_{ss}\mathcal{P}_{i,t}^{2}} \right] \\ Q_{i,k,t} &= \frac{1}{\Phi'_{i,k,t}} \\ Q_{i,n,t} &= E_{t} \left[ M_{t+1} \left\{ \frac{\alpha(1 - \frac{1}{\nu})Y_{t+1}^{\frac{1}{\nu}}X_{i,t+1}^{1 - \frac{1}{\nu}}}{K_{i,t+1}} + \frac{\Lambda_{i,t+1} \left( \frac{\alpha}{\nu} \right) Y_{t+1}^{\frac{1}{\nu}}X_{i,t+1}^{-\frac{1}{\nu}}}{K_{i,t+1}} \right\} \right] \\ &+ E_{t} \left[ M_{t+1}Q_{i,k,t+1} \left( 1 - \delta_{k} - \frac{\Phi'_{i,k,t+1}I_{i,t+1}}{K_{i,t+1}} + \Phi_{i,k,t+1} \right) \right] \\ Q_{i,n,t} &= E_{t} \left[ M_{t+1} \left\{ \frac{\eta(1 - \alpha)(1 - \frac{1}{\nu})Y_{t+1}^{\frac{1}{\nu}}X_{i,t+1}^{1 - \frac{1}{\nu}}}{K_{i,t+1}} + \frac{\Lambda_{i,t+1} \left( \frac{\eta(1 - \alpha)}{\nu} \right) Y_{t+1}^{\frac{1}{\nu}}X_{i,t+1}^{-\frac{1}{\nu}}}{N_{i,t+1}} \right\} \right] \\ &+ E_{t} \left[ M_{t+1}Q_{i,n,t+1} \left( 1 - \delta_{n} - \frac{\Phi'_{i,n,t+1}S_{i,t+1}}{N_{i,t+1}} + \Phi_{i,n,t+1} \right) \right] \\ \frac{\mathcal{W}_{i,t}}{\mathcal{P}_{t}} &= \frac{(1 - \alpha)(1 - \frac{1}{\nu})Y_{t}^{\frac{1}{\nu}}X_{i,t}^{1 - \frac{1}{\nu}}}{L_{i,t}} + \Lambda_{i,t} \frac{\left( \frac{1 - \alpha}{\nu} \right) Y_{t}^{\frac{1}{\nu}}X_{i,t}^{-\frac{1}{\nu}}}{L_{i,t}} \right] \end{aligned}$$

# Appendix C. Derivation of the New Keynesian Phillips

### Curve

Define  $MC_t \equiv \frac{W_t}{MPL_t}$  and  $MPL_t \equiv (1 - \alpha)\frac{Y_t}{L_t}$  for real marginal costs and the marginal product of labor, respectively. Rewrite the price-setting equation of the firm in terms of real marginal costs

$$\nu MC_t - (\nu - 1) = \phi_R \left(\frac{\Pi_t}{\Pi_{ss}} - 1\right) \frac{\Pi_t}{\Pi_{ss}} - E_t \left[ M_{t+1} \phi_R \left(\frac{\Pi_{t+1}}{\Pi_{ss}} - 1\right) \frac{\Delta Y_{t+1} \Pi_{t+1}}{\Pi_{ss}} \right]$$

Log-linearizing the equation above around the nonstochastic steady-state gives

$$\widetilde{\pi}_t = \gamma_1 \widetilde{m} c_t + \gamma_2 E_t [\widetilde{\pi}_{t+1}]$$

where  $\gamma_1 = \frac{\nu - 1}{\phi_R}$ ,  $\gamma_2 = \beta \Delta Y_{ss}^{1 - \frac{1}{\psi}}$ , and lowercase variables with a tilde denote log deviations from the steady-state.<sup>21</sup>

Substituting in the expression for the marginal product of labor and imposing the symmetric equilibrium conditions, real marginal costs can be expressed as

$$MC_t = \frac{W_t L_t}{(1 - \alpha) K_t^{\alpha} (A_t N_t L_t)^{1 - \alpha)}}$$

Define the following stationary variables:  $\overline{W}_t \equiv \frac{W_t}{K_t}$  and  $\overline{N}_t \equiv \frac{N_t}{K_t}$ . Thus, we can rewrite the expression above as

$$MC_t = \frac{\overline{W}_t L_t^{\alpha}}{(1 - \alpha)(A_t \overline{N}_t)^{1 - \alpha}}$$

Log-linearizing this expression yields

$$\widetilde{mc}_t = \widetilde{w}_t + \alpha \widetilde{l}_t - (1 - \alpha)\widetilde{a}_t - (1 - \alpha)\widetilde{n}_t$$

where lowercase variables with a tilde denote log deviations from the steady-state.

<sup>&</sup>lt;sup>21</sup>In a log-linear approximation, the relationship between the price adjustment cost parameter  $\phi_R$  and the fraction of firms resetting prices  $(1 - \theta_c)$  from a Calvo pricing framework is given by:  $\phi_R = \frac{(\nu - 1)\theta_c}{(1 - \theta_c)(1 - \beta\theta_c)}$ . Further, the average price duration implied by the Calvo pricing framework is  $\frac{1}{1 - \theta_c}$  quarters.

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Table 1: Quarterly Calibration

	D	3.6.1.1
Parameter	Description	Model
Preference Parameters		
eta	Subjective Discount Factor	0.997
$\psi$	Elasticity of Intertemporal Substitution	2.0
$\gamma$	Risk Aversion	10.0
Technology Parameters		
$\alpha$	Capital Share	0.33
$\phi_R$	Magnitude of Price Adjustment Costs	30.0
u	Price Elasticity for Intermediate Good	6.0
$\eta$	Degree of Technological Appropriability	0.1
$\delta_k$	Depreciation Rate of Capital Stock	0.02
$\delta_n$	Depreciation Rate of R&D Stock	0.0375
$\zeta_k$	Capital Adjustment Cost Parameter	4.8
$\zeta_n$	R&D Capital Adjustment Cost Parameter	3.3
Productivity Parameters		
ρ	Persistence of $a_t$	0.983
$\overline{\sigma}$	Volatility of Productivity Shock $\epsilon$	1.20%
$\lambda$	Persistence of Squared Volatility Process $\sigma_t^2$	0.997
$\sigma_e$	Volatility of Volatility Shock $e_t$	0.00008
Policy Parameters		
$\overline{ ho_r}$	Degree of Monetary Policy Inertia	0.7
$ ho_{\pi}$	Sensitivity of Interest Rate to Inflation	1.5
$ ho_y$	Sensitivity of Interest Rate to Output	0.10
$\sigma_{\xi}$	Volatility of $\xi_t$	0.3%

This table reports the parameter used for the quarterly calibration of the model. The table is divided into four categories: preference, technological, productivity, and policy parameters.

Table 2: Macroeconomic Moments

	Data	Model
First Moments		
$E(\Delta y)$	2.20%	2.20%
$E(\pi)$	3.74%	3.74%
Second Moments		
$\overline{\sigma_{\Delta c}/\sigma_{\Delta y}}$	0.64	0.60
$\sigma_{\Delta l}/\sigma_{\Delta y}$	0.92	0.95
$\sigma_{\Delta i}/\sigma_{\Delta c}$	4.38	4.31
$\sigma_{\Delta s}/\sigma_{\Delta c}$	3.44	3.30
$\sigma_{\Delta c}$	1.42%	1.60%
$\sigma_{\pi}$	1.64%	1.92%
$\sigma_w$	2.04%	2.72%

This table presents first and second macroeconomic moments from the model. The model is calibrated at a quarterly frequency and the reported means and volatilities are annualized.

Table 3: Measured Productivity Growth Dynamics

	Data	Model
$\sigma(\Delta z)$	2.59%	2.59%
$AC1(\Delta z)$	0.04	0.02
$\sigma(E(\Delta z))$	1.10%	1.05%
$AC1(E(\Delta z))$	0.93	0.93

This table presents the standard deviation and first autocorrelation of measured productivity growth and expected productivity growth for the data and the model. The model is calibrated at a quarterly frequency and the moments are annualized. The data MLE estimates of expected productivity growth are taken from Croce (2012), where the expected growth rate component of productivity is a latent variable that is assumed to follow an AR(1).

Table 4: Annual Autocorrelations

	Data	Model
$\Delta c$	0.37	0.43
$\Delta y$	0.32	0.17
s-n	0.93	0.93
i - k	0.86	0.92
$\pi$	0.73	0.75

This table presents annual autocorrelations of consumption growth, output growth, R&D intensity, the investment rate, and inflation from the data and the model. Model estimates are obtained from 200 simulations of 56 years of data at quarterly frequency, time-aggregated to annual frequency.

Table 5: Realized Consumption Growth Volatility Dynamics

	Data	Model
$\overline{\sigma(Vol_{t,t+K})}$	1.03%	1.00%
$AC1(Vol_{t,t+K})$	0.18	0.18

This table reports the standard deviation and first autocorrelation of annual realized consumption growth volatility from the data and the model. The series for realized consumption growth volatility is computed following Beeler and Campbell (2012) and Bansal, Kiku, and Yaron (2012). First, the consumption growth series is fitted to an AR(1):  $\Delta c_t = \beta_0 + \beta_1 \Delta c_{t-1} + u_t$ . Then, annual (four-quarter) realized volatility is computed as  $Vol_{t,t+4} = \sum_{j=0}^{4-1} |u_{t+j}|$ .

Table 6: Nominal Yield Spread and Short Rate

	Data	Model	Model	Model
			$(\sigma_e = 0)$	$(\sigma_e, \sigma_{\xi} = 0)$
$E(y^{(5)} - y^{(1Q)})$	1.02%	0.96%	0.94%	0.92%
$\sigma(y^{(5)} - y^{(1Q)})$	1.05%	1.08%	0.72%	0.33%
$E(y^{(1Q)})$	5.03%	5.05%	5.01%	5.00%
$\sigma(y^{(1Q)})$	2.96%	3.09%	2.33%	2.19%

This table presents annual first and second moments of the five-year nominal yield spread and the nominal short rate from the data, the benchmark model, the model with constant volatility ( $\sigma_e = 0$ ), and the model with constant volatility ( $\sigma_e = 0$ ) and with no policy uncertainty ( $\sigma_{\xi} = 0$ ). The model is calibrated at a quarterly frequency and the moments are annualized.

Table 7: Term Structure

		Maturity (Years)						
	1	2	3	4	5			
Nominal Yields								
Mean (Model)	5.26%	5.45%	5.64%	5.82%	5.99%			
Mean (Data)	5.29%	5.48%	5.66%	5.80%	5.89%			
Std (Model)	2.87%	2.68%	2.51%	2.35%	2.20%			
Std (Data)	2.99%	2.96%	2.88%	2.83%	2.77%			
AC1 (Model)	0.97	0.98	0.98	0.98	0.98			
AC1 (Data)	0.94	0.95	0.95	0.96	0.96			
Real Yields								
Mean (Model)	0.98%	0.91%	0.85%	0.80%	0.76%			
Std (Model)	0.78%	0.66%	0.63%	0.62%	0.62%			
AC1 (Model)	0.77	0.90	0.94	0.96	0.97			

This table presents summary statistics for the term structure of interest rates. The top half of the table presents the annual mean and standard deviation of the one-, two-, three-, four-, and five-year nominal yields from the model and the data. The bottom half of the table presents the annual mean and standard deviation of the real yields from the model. The model is calibrated at a quarterly frequency and the moments are annualized.

Table 8: Inflation-Growth Link

	Data	Model
$\overline{corr(\pi, \Delta c)}$	-0.56	-0.64
$corr(\pi, \Delta c)$ (low-freq)	-0.85	-0.86

The top row of this table reports the annual correlation between inflation and consumption growth. The bottom row reports the correlation between the low-frequency components of inflation and consumption growth. The low-frequency component is obtained using a bandpass filter and isolating frequencies between 20 and 50 years.

Table 9: Fama-Bliss Excess Return Regressions

	Horizon (Years)					
	2	3	4	5		
$\beta$ (Data) $R^2$ (Data)	1.076 0.175	1.476 0.190	1.689 0.185	1.150 0.068		
$\beta \text{ (Model)}$ $R^2 \text{ (Model)}$	$0.529 \\ 0.069$	$0.685 \\ 0.095$	$0.726 \\ 0.103$	$0.743 \\ 0.106$		
$\beta \text{ (Model) } (\sigma_{\xi} = 0)$ $R^2 \text{ (Model) } (\sigma_{\xi} = 0)$	0.743 0.091	0.765 0.106	0.775 0.112	$0.781 \\ 0.115$		

This table presents forecasts of one-year excess returns on bonds of maturities of two- to five-years using the forward spread for the data, the benchmark model, and the model with no policy uncertainty ( $\sigma_{\xi} = 0$ ). The regression,  $rx_{t+1}^{(n)} = \alpha + \beta(f_t^{(n)} - y_t^{(1)}) + \epsilon_{t+1}^{(n)}$ , is estimated using overlapping quarterly data.

Table 10: Cochrane-Piazzesi Excess Return Predictability

Maturity (Years)	Data				Model	
	$b_n$	S.E.	$R^2$	$b_n$	S.E.	$R^2$
2	0.455	0.027	0.379	0.423	0.019	0.109
3	0.862	0.014	0.415	0.833	0.008	0.119
4	1.229	0.011	0.446	1.204	0.006	0.123
5	1.449	0.030	0.421	1.540	0.020	0.125

This table presents forecasts of one-year excess returns on bonds of maturities of two- to five-years using the single factor model from Cochrane and Piazzesi (2005) for the data and the model. First, the factor is obtained by running the regression:  $\frac{1}{4}\sum_{n=2}^{5} rx_{t+1}^{(n)} = \gamma' \mathbf{f}_t + \overline{\epsilon}_{t+1}$ , where  $\gamma' \mathbf{f}_t \equiv \gamma_0 + \gamma_1 y_t^{(1)} + \gamma_2 f_t^{(2)} + \cdots + \gamma_5 f_t^{(5)}$ . Second, use the factor  $\gamma' \mathbf{f}_t$  obtained in the previous regression to forecast bond excess returns of maturity n:  $rx_{t+1}^{(n)} = b_n(\gamma' \mathbf{f}_t) + \epsilon_{t+1}^{(n)}$ . The forecasting regression use overlapping quarterly data. Newey-West standard errors are used to correct for heteroscedasticity.

Table 11: Output Forecasts with the Yield Spread

	Horizon (Quarters)			
	1	4	8	
$\beta$ (Data) $R^2$ (Data)	1.023 0.067	0.987 0.148	0.750 0.147	
$\beta \text{ (Model)}$ $R^2 \text{ (Model)}$	$0.709 \\ 0.051$	1.202 0.155	1.337 $0.258$	
$\beta \text{ (Model) } (\sigma_e, \sigma_{\xi} = 0)$ $R^2 \text{ (Model) } (\sigma_e, \sigma_{\xi} = 0)$	2.324 0.048	2.361 0.123	$2.057 \\ 0.176$	

This table presents output growth forecasts for horizons of one, four, and eight quarters using the five-year nominal yield spread for the data, the benchmark model, and the model with constant volatility ( $\sigma_e = 0$ ) and no policy uncertainty ( $\sigma_{\xi} = 0$ ). The *n*-quarter regression,  $\frac{1}{n}(\Delta y_{t,t+1} + \cdots + \Delta y_{t+n-1,t+n}) = \alpha + \beta(y_t^{(5)} - y^{(1Q)}) + \epsilon_{t+1}$ , is estimated using overlapping quarterly data.

Table 12: Consumption Forecasts with the Yield Spread

Horizon (Quarters)	Data				Model	
	$\beta$	S.E.	$\mathbb{R}^2$	$\beta$	S.E.	$R^2$
1	0.731	0.187	0.092	0.904	0.204	0.103
4	0.567	0.163	0.136	0.773	0.175	0.155
8	0.373	0.153	0.088	0.684	0.182	0.161

This table presents consumption growth forecasts for horizons of one, four, and eight quarters using the five-year nominal yield spread for the data and the model. The *n*-quarter regression,  $\frac{1}{n}(\Delta c_{t,t+1} + \cdots + \Delta c_{t+n-1,t+n}) = \alpha + \beta(y_t^{(5)} - y^{(1Q)}) + \epsilon_{t+1}$ , is estimated using overlapping quarterly data. Newey-West standard errors are used to correct for heteroscedasticity.

Table 13: Inflation Forecasts with the Yield Spread

Horizon (Quarters)	Data				Model		
	$\beta$	S.E.	$\mathbb{R}^2$	$\beta$	S.E.	$R^2$	
1	-1.328	0.227	0.180	-0.770	0.208	0.081	
4	-1.030	0.315	0.157	-0.955	0.273	0.118	
8	-0.649	0.330	0.071	-0.984	0.303	0.136	

This table presents log inflation forecasts for horizons of one, four, and eight quarters using the five-year nominal yield spread for the data and the model. The *n*-quarter regression,  $\frac{1}{n}(\pi_{t,t+1} + \cdots + \pi_{t+n-1,t+n}) = \alpha + \beta(y_t^{(5)} - y^{(1Q)}) + \epsilon_{t+1}$ , is estimated using overlapping quarterly data. Newey-West standard errors are used to correct for heteroscedasticity.

Table 14: Yield Spread Dynamics

	Data	Model
$corr(\pi, y^{(5)} - y^{(1Q)})$	-0.40	-0.46
$\operatorname{corr}(\pi, y^{(5)} - y^{(1Q)}) \text{ (low-freq)}$	-0.69	-0.54
$corr(s - n, y^{(5)} - y^{(1Q)})$ (low-freq)	0.72	0.77

The top row of this table reports the annual correlation between log inflation and the five-year minus one-quarter nominal yield spread. The middle row reports the correlation between the low-frequency components of log inflation and the five-year minus one-quarter nominal yield spread. The bottom row reports the correlation between the low-frequency components of the R&D rate and the five-year minus one-quarter nominal yield spread. The low-frequency components are obtained using a bandpass filter and isolating frequencies between 20 and 50 years.

Table 15: Equity Premium and Real Rate

	Data	Model	Model-WR
$E(r_d - r_f)$	5.84%	3.17%	4.10%
$\sigma(r_d - r_f)$	17.87%	6.68%	9.18%
$E(r_f)$	1.62%	1.07%	0.72%
$\sigma(r_f)$	0.67%	0.68%	0.81%

This table presents annual first and second asset moments of the equity premium and the real risk-free rate from the data, the benchmark model (Model), and the model with wage rigidities (Model-WR). The model is calibrated at a quarterly frequency and the moments are annualized. Since the equity risk premium from the models is unlevered, I follow Boldrin, Christiano, and Fisher (2001) and compute the levered risk premium from the model as:  $r_{d,t+1}^* - r_{f,t} = (1+\kappa)(r_{d,t+1} - r_{f,t})$ , where  $r_d$  is the unlevered return and  $\kappa$  is the average aggregate debt-to-equity ratio, which is set to  $\frac{2}{3}$ .

Table 16: Excess Stock Return Forecasts with the Yield Spread

	Horizon (Years)					
	1	2	3	4	5	
$\beta$ (Data) $R^2$ (Data)	2.958 0.040	4.606 0.048		9.889 0.080	14.656 0.106	
$\beta \text{ (Model)}$ $R^2 \text{ (Model)}$	$1.017 \\ 0.074$	2.033 0.122	3.057 0.156		5.100 0.197	

This table presents forecasts of excess market stock returns using the five-year minus one-quarter yield spread for investment horizons of one- to five-years for both the data and the model. The *n*-year horizon regression,  $r_{t,t+n}^{ex} - y_t^{(n)} = \beta(y_t^{(5)} - y^{(1Q)}) + \epsilon_{t+1}$ , is estimated using overlapping quarterly data.

Table 17: Excess Stock Return Forecasts with the CP Factor

Horizon (Years)	Data				Model	Model		
	$b_n$	S.E.	$R^2$	$b_n$	S.E.	$R^2$		
1	1.718	0.815	0.078	1.847	0.606	0.113		
2	3.177	0.868	0.131	3.324	1.093	0.176		
3	3.220	1.074	0.081	4.657	1.537	0.216		
4	4.433	1.279	0.094	5.748	1.950	0.237		
5	6.975	1.757	0.140	6.682	2.333	0.248		

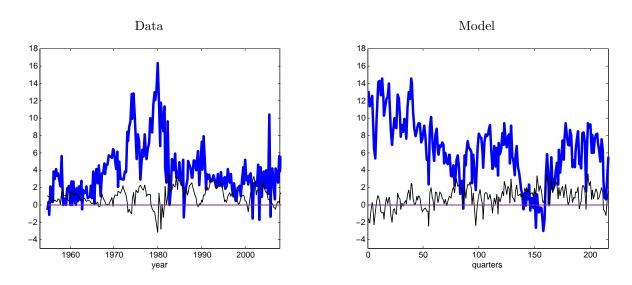
This table presents forecasts of excess market stock returns using the Cochrane and Piazzesi (2005) factor for investment horizons of one- to five-years for both the data and the model. First, the factor is obtained by running the regression:  $\frac{1}{4} \sum_{n=2}^{5} r x_{t+1}^{(n)} = \gamma' \mathbf{f}_t + \bar{\epsilon}_{t+1}$ , where  $\gamma' \mathbf{f}_t \equiv \gamma_0 + \gamma_1 y_t^{(1)} + \gamma_2 f_t^{(2)} + \cdots + \gamma_5 f_t^{(5)}$ . Second, use the factor  $\gamma' \mathbf{f}_t$  obtained in the previous regression to forecast excess stock returns of horizon n:  $r_{t,t+n}^{ex} - y_t^{(n)} = b_n \gamma' \mathbf{f}_t$ ) +  $\epsilon_{t+1}^{(n)}$ . The forecasting regressions use overlapping quarterly data. Newey-West standard errors are used to correct for heteroscedasticity.

Table 18: Comparative Statics

Depreciation Rates						
<b>-</b>						
	$\delta_k = 0.01$	$\delta_k = 0.02$	$\delta_k = 0.03$			
$E[y^{(5)} - y^{(1Q)}]$	0.78%	0.96%	1.01%			
$\sigma(\Delta c)$	1.47%	1.60%	1.69%			
	$\delta_n = 0.01$	$\delta_n = 0.02$	$\delta_n = 0.03$			
$E[y^{(5)} - y^{(1Q)}]$	0.70%	0.89%	0.94%			
$\sigma(\Delta c)$	1.17%	1.30%	1.51%			
Adjustment Costs						
	$\zeta_k, \zeta_n = 8.0$	$\zeta_k, \zeta_n = 5.0$	$\zeta_k, \zeta_n = 2.0$			
$E[y^{(5)} - y^{(1Q)}]$	0.93%	0.91%	0.83%			
$corr(\Delta c, \pi)$	-0.67	-0.61	-0.38			

The top half of this table reports the impact of varying the depreciation rates for physical capital and R&D capital on the average nominal yield spread. The bottom half of this table reports the impact of varying the curvature of the adjustment cost function for physical capital and R&D capital on the average 5-year minus 1-quarter nominal yield spread. The model is calibrated at a quarterly frequency and the moments are annualized.

Figure 1: Inflation and Yield Spread Dynamics



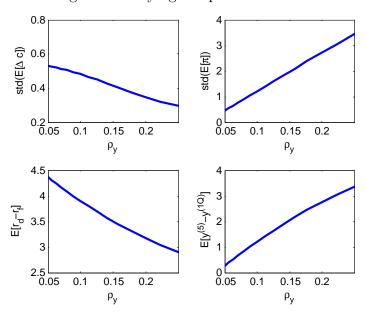
This figure plots inflation (thick line) and the five-year nominal yield spread (thin line) for the data (left panel) and the model (right panel). Data are quarterly and the values of the series are in annualized percentage units.

0.55 0.5 std(E[∆ c])  $std(E[\pi])$ 0.45 0.4 0.35 1.5 2 2.5 3 1.5 2 2.5 3  $\rho_{\pi}$  $\rho_{\pi}$ 4.5 2 1.5 2 2.5 3 1.5 2 2.5 3  $\rho_{\pi}$  $\rho_{\pi}$ 

Figure 2: Varying Inflation Stabilization

This figure plots the impact of varying the policy parameter  $\rho_{\pi}$  on the volatility of expected consumption growth, volatility of expected inflation, equity premium, and average nominal yield spread in the model. Values on y-axis are in annualized percentage units.

Figure 3: Varying Output Stabilization



This figure plots the impact of varying the policy parameter  $\rho_y$  on the volatility of expected consumption growth, volatility of expected inflation, equity premium, and average nominal yield spread in the model. Values on y-axis are in annualized percentage units.