Investment Beliefs of Endowments*

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Abstract

U.S. university and college endowments now hold close to one-third of their portfolios in private equity and hedge funds. We estimate the implied beliefs of endowments on these alternative assets' returns relative to equities and bonds. At the end of 2012, the typical endowment believes that its private equity investments will outperform a portfolio of conventional assets by 3.9% per year, and hedge funds will outperform by 0.7% per year. The perceived alpha for private equity is much higher than the historical performance of this asset class, but the opposite is true for the perceived hedge fund alpha. These out-performance beliefs have increased over time. Private universities tend to be less risk averse and have more optimistic beliefs. Universities with larger endowments, higher spending rates, and those that rely more on the endowment to meet operational budgets tend to believe that alternatives deliver higher alphas. Taking into account the implied equity exposures in alternative asset positions, the effective equity holding of endowments is approximately 60%.

1 Introduction

In recent years, important institutional investors such as university endowments, sovereign wealth funds and pension funds have shifted their asset allocation away from standard asset classes like stocks and bonds into alternative investments such as private equity and hedge funds. This suggests that their portfolio managers either believe that alternatives earn high net abnormal returns or they provide diversification benefits.

This paper focuses on investors' views on the level of net abnormal returns. We assume that asset returns follow a factor model and endowments solve a standard portfolio allocation problem. Using a Bayesian framework, we use information on asset returns and cross-sectional asset allocations to estimate the views on net abnormal returns—which we term alpha—that justify the observed weights on alternative investments. We find that the views on private equity's alpha are much higher than its time-series average abnormal return. The opposite is true for hedge funds: our estimated alpha is lower than the historical alpha of our proxy. Additionally, we find that endowments' views on expected abnormal returns have increased, reflecting the monotonic increase in allocations to alternatives during the period of our study.

Our analysis of investor beliefs is based on a comprehensive panel dataset of university endowment asset allocations and performance. Universities have been leaders in the recent trend towards alternative investments. David Swensen's *Pioneering Portfolio Management* articulated the value proposition of this investment style; first, accessing factor returns through non-marketable investments offers an additional liquidity premium to patient investors and second, inefficient asset markets offer astute investors the chance to capture positive alpha by identifying skilled managers. A handful of university endowment officers put these principles into practice in the 1990s and 2000s and were highly successful. Many other institutions followed suit (cf. Goetzmann and Oster, 2012).

With the widespread adoption of the alternative investment paradigm, a fundamental concern is whether the experience of the industry first-movers can be successfully imitated. By adopting a new style of investing, are investment managers also expecting to realize future risk-adjusted returns commensurate with the past performance of its most successful practitioners? Our evidence in this paper provides some support for this conjecture. We find a trend towards higher average expected excess returns to private equity and hedge fund investment since 2006, despite the broader increase in institutional interest in these asset classes. But there are significant cross-sectional differences around this trend. Private universities with larger endowments have relatively higher alpha expectations compared to smaller endowments. This may reflect an implicit assumption of increasing risk-adjusted returns to scale; Barber and Wang (2012) document a 3.15% to 3.82% positive alpha earned by Ivy League schools (the early adopters of alternative investing). This range is consistent with the expectations of the average endowment for the alpha generated by their private equity investment alone.

The Bayesian framework allows us to estimate implicit investor beliefs about their capacity to capture excess returns conditional upon both optimistic and pessimistic prior positions with respect to market efficiency, and their past historical performance. Our basic findings are robust to both of these specifications. Investors have high expectations for capturing excess returns through private equity investments—either via manager skill, or an illiquidity premium, or both. We also find that past positive experience influences expectations. This is consistent with the finding of Barber and Wang (2012) who document endowment performance persistence. It can also reflect endowments changing investment beliefs upwards after a period of high returns, which are effects Malmendier and Nagel (2011, 2013) document for individual investors.

Our analysis has implications for the future of university endowments and for other institutions strategically committed to large allocations in alternative asset classes. In the fiscal year ending in 2012, the typical endowment expects to earn an alpha of 3.9% in private equity and 0.7% in hedge funds after adjusting these alternatives for their equity and fixed income risk exposures. Elite institutions might continue to earn alpha on their alternative asset portfolios. However, if the expectations of the average institution are overly optimistic, the long-term consequences are at best a growing resource gap between the top and the middle, and at worst a long-term decline in university spending power, should illiquidity premiums and alpha in alternative asset classes disappear.

This paper is organized as follows. Section 2 reviews current evidence regarding excess returns to alternative investment. Section 3 presents an asset allocation model where alternative assets deliver out-performance compared to standard equity and bond factors. Section 4 describes the endowment data. Section 5 contains the main results of the paper and estimates investment beliefs of endowments with various priors. Section 6 concludes.

2 Background

Because the rationale for investing in hedge funds and private equity is in part the potential for delivering positive alpha over and above a passive benchmark it is useful to examine the current evidence on hedge fund and private equity manager skill. Expected returns to private equity comprise an illiquidity premium as well as an alpha component.

2.1 Hedge Funds

Evidence on abnormal returns earned by marketable alternative strategies is mixed. The early academic evidence reported positive abnormal returns for the industry. Fung and Hsieh (1997) documented considerable non-linearity in hedge fund returns with respect to standard asset pricing factors and then introduced additional controls. They found that hedge funds over the period of their study were a good investment. Ackermann et al. (1999) found that hedge funds outperformed mutual funds over the period 1988 through 1995, but do not on average, provide positive risk adjusted returns. In contrast, Brown et al. (1999a) found evidence of positive risk-adjusted performance in a database of off-shore hedge funds over the same time period. Presumably these and related studies that followed influenced institutional investor expectations about the potential for positive alpha.

Subsequent studies modified these early results to some extent. Bailey et al. (2004) documented the outperformance of hedge funds under the null of no arbitrage, even when non-linear factor payoffs are considered. Kosowski et al. (2007) examined the risk adjusted performance of hedge funds over the period 1990 to 2002 using fairly sophisticated measures. Their results concur that hedge funds over this extended period appear to have delivered positive performance persistent at the annual horizon. More recently, Ibbotson et al. (2011) have found that hedge funds delivered an average alpha of 3% per year over 1995 to 2009. A recent update of the study (unpublished) by the authors through 2012 lowers this to about 2.5% per year due to lower industry returns since 2009. Dichev and Yu (2011) report that hedge funds have under-performed on a risk-adjusted basis, with the dollar-weighted returns of hedge funds being reliably lower than equity market returns.

In addition to aggregate studies of the hedge fund industry, a number of researchers have examined conditional strategies for accessing manager outperformance. Avramov et al. (2011) for example, show that interacting macroeconomic conditions with manager selection yields positive results. Some studies of manager persistence support the potential for benefiting from "hot hands" in the hedge fund industry. Capocci and Hübner (2004), Kosowski et al. (2007), and Fung et al. (2008) show that even though there appears to be some short-run persistence, only a small group of hedge funds are able to generate alpha over longer horizons (one to three years). Brown et al. (1999b) likewise found little evidence of skill persistence. On the other hand, Jagannathan et al. (2010) report significance persistence in hedge fund returns.

Some papers questioned or re-interpreted the historical evidence of positive hedge fund alphas. Griffin and Xu (2009) find little evidence of differential or superior trading skill by hedge funds during the tech bubble. Malkiel and Saha (2005) argue that survivorship bias and backfill bias loom large in any reliance on historical hedge fund data and on this basis question whether prior empirical evidence is reliable enough for forming expectations of future performance. Aiken et al. (2013) point out that the voluntary nature of hedge fund reporting to commercial databases means that the worst performers are not represented and thus the severity of the lower till of hedge fund returns is biased upwards.[1]

In sum, hedge fund researchers have documented positive risk adjusted returns within the hedge fund universe, using imperfect but commercially available databases. These excess returns have declined in recent years and scholars have cautioned that biases in the databases may be strongly biasing the evidence. Research on the effect of selective reporting, survivorship and backfilling has yet to yield a comprehensive approach to proper adjustment of expectations. Nevertheless, for the purposes of our analysis, we presume that the academic studies cited above, in addition to related studies from academia and practice were inputs to the formation of investor expectations with respect to hedge funds, and that priors of positive alphas would have been consistent with the reported empirical evidence over the period 1997 through 2011.

[1] The issue of survival bias in the databases may cut two ways, however. Linnainmaa (2013) estimates that mutual fund performance is downward biased by certain measures due to fund closings due to negative exogenous shocks as opposed to poor skill. Presumably the same holds true for hedge funds as well.

2.2 Private Equity

Estimating abnormal returns to private equity investment is empirically challenging because of the lack of time-series market-based valuation. This problem was pointed out clearly by Gompers and Lerner (1997). Traditional measures used to evaluate private equity, such as the internal rate of return do not lend themselves to adjustment for systematic risk exposures. Techniques such as comparison to public market equivalent investments are used in lieu of time-series data but provide only rough estimates of the capacity for private equity investment to outperform an equivalent investment in marketable securities. Most academic research on private equity has found little empirical evidence of net abnormal performance compared to appropriate public equity benchmarks.

Some of the earliest academic studies relevant to the formation of expectations by managers in the dataset we study are Moskowitz and Vissing-Jørgensen (2002) who find that private equity investment underperforms, using entrepreneurial returns as a measure of private equity, and Kaplan and Schoar (2005) who find no evidence of outperformance over the S&P 500 net of fees. Most later work, using a variety of imperfect commercially available data generally confirms the lack of evidence for positive alpha—on average—in the private equity asset class. These studies include Edwards and Caglayan (2001), Phalippou and Gottschalg (2009), and Franzoni et al. (2012). Most recent evidence suggests that after correctly accounting for liquidity risk, investments on private equity do not generate positive alpha on average.

There has also been some evidence for conditional superior performance. Lerner et al. (2007) note that universities have been relatively successful at selecting private equity investments. Other studies have found evidence of performance persistence consistent with the presumption that access to top managers may deliver consistently higher returns. Harris et al. (2012) use a proprietary database and finds significant outperformance for U.S. private equity funds net of an S&P 500 benchmark. This recent work is not likely to have been informative over the period of our study.

Finally, it is taken as a stylized fact that there is a large, persistent dispersion of private equity returns—even if the average private equity fund does not outperform.

2.3 Endowment Performance

Endowment income constitutes a significant, and growing fraction, of universities' operating budgets. Brown et al. (2013) document that universities practice "endowment hoarding" where universities seek to preserve the value of their endowment following negative shocks. Consequently, performance of endowments impacts many universities' operations and aspirations.

There is a small but growing empirical literature on university endowment performance. Lerner et al. (2008) studied the same database we use in this paper and found that the largest endowments and endowments for the most elite academic institutions out-performed—and these were also the group that relied most on alternative investments. Brown et al. (2010) use the same data to study whether endowments added value through allocation timing decisions. They found some evidence of skill which they believed to be under-utilized. Barber and Wang (2012) find no evidence that university endowments on average added alpha through timing, manager or security selection although they find no signs of negative risk-adjusted returns.

A review of the current evidence about hedge funds, private equity funds and university endowments suggests that, on average, universities as a group should have little expectation of beating standard benchmarks derived from publicly traded stocks and bonds. There is some evidence that larger endowments using alternative investments may have been able to access superior private equity managers and that, as a group, universities have a comparative advantage in this asset domain.

With respect to hedge funds, the literature dating to the early period in our sample is generally sanguine about the potential to earn positive risk adjusted returns although more recently, negative alphas have likely caused a revision in forward-looking expectations. Consequently, we should expect universities in our sample to have positive priors about hedge fund returns. With respect to private equity, the academic literature, particularly the papers published in the early 2000s, offered little hope of superior returns. On the other hand the success of leading universities who invested heavily in private equity in the 1990s and 2000s would likely have been relevant to the formation of positive expectations.

3 Model

Investors can cheaply invest in passively managed equity and fixed income funds. Alternative asset classes, which include private equity and hedge funds, are actively managed at much higher cost. Even if equities and fixed income are also actively managed, the fees on alternative investments are usually much higher than fees on traditional equity and bond products. Taking this as a starting point, we develop an asset allocation model based on Treynor and Black (1973) which takes equities and fixed income as factors. Alternative assets are exposed to factor risk, and they may exhibit alpha—out-performance that cannot be attributed to the equity and fixed income factor exposures, but which is generated with idiosyncratic risk. The methodology accommodates prior views, which may be held by investors or the econometrician, on the risk premiums of alternative assets.

3.1 Factors and Alternative Assets

We extend the Treynor and Black (1973) model to multiple factors and assets with non-zero risk-adjusted returns. We assume there are N_f tradable factors whose excess returns, f, can be written as

$$f = \mu_f + \varepsilon_f, \tag{1}$$

where μ_f is a size N_f column vector of expected excess returns and ε_f is a vector of independent and identically distributed (iid) normal shocks with covariance matrix Σ_f. The covariance matrix need not be diagonal, but must be full rank. We take U.S. equities, foreign equities, and U.S. bonds as factors in our empirical work.

There are N_a alternative assets whose excess returns, r_a, follow:

$$r_a = \alpha + \beta f + \varepsilon_a. \tag{2}$$

In our empirical work, we take private equity and hedge funds as alternative assets. We capture the co-movement of these alternative asset classes with equity and bond factors through the factor loadings β, which is an $N_a \times N_f$ matrix. The alternative assets have idiosyncratic shocks,

ε_a, which are assumed to be iid normal with covariance matrix Σ_a. We assume a factor model structure, so the idiosyncratic shocks are orthogonal to the factor shocks, $\varepsilon_f \perp \varepsilon_a$. However, the idiosyncratic shocks, ε_a, may have non-zero cross-correlations.

The alternative assets exhibit abnormal returns, α, which is an $N_a \times 1$ column vector. Alpha is the mean excess return that the alternative assets have in excess of their factor exposures. It can reflect mis-pricing or the fact that our set of N_f factors is incomplete. Either interpretation is consistent with endowments holding alternatives to seek returns which cannot be generated by holding plain-vanilla equities and bonds.[2]

3.2 Portfolio Allocation

The investor maximizes a mean-variance utility function with risk aversion γ:

$$\max_{\pi} E(r_p) - \frac{\gamma}{2}\gamma\sigma_p^2, \tag{3}$$

where $E(r_p)$ and σ_p^2 are the expected return and variance, respectively, of the investor's portfolio. The investor's portfolio weights in risky assets, π, can be partitioned into holdings on factor securities and alternative assets, $\pi = [\pi_f^\top, \pi_a^\top]^\top$, which has dimension $N_f + N_a$. The remaining weight in the risk-free asset, with return r_f, ensures the portfolio weights sum to one.

The portfolio's expected excess return, $\mu_p = E(r_p) - r_f$, is given by

$$\begin{aligned} \mu_p &= \pi_f^\top \mu_f + \pi_a^\top (\beta\mu_f + \alpha) \\ &= \tilde{\pi}_f^\top \mu_f + \pi_a^\top \alpha \end{aligned} \tag{4}$$

where

$$\tilde{\pi}_f = \pi_f + \beta^\top \pi_a. \tag{5}$$

We can interpret $\tilde{\pi}_f$ as the total implicit portfolio weight on factors since the alternative assets co-move with the factors. We examine this implicit factor exposure in our empirical work for different optimal portfolios.

[2] We assume that both μ_f and α are constant, and therefore, investment opportunities do not change over time. This is done for tractability, but time-varying expected returns can be easily incorporated in this framework, generating an additional hedging demand from investors.

Since shocks to the excess return of the portfolio can be written as

$$
\begin{aligned}
(r_p - r_f) - \mu_p &= \pi_f^\top \varepsilon_f + \pi_a^\top (\beta \varepsilon_f + \varepsilon_a) \\
&= \pi_f^\top \varepsilon_f + \pi_a^\top \varepsilon_a, \qquad (6)
\end{aligned}
$$

the variance of portfolio returns is equal to

$$
\sigma_p^2 = \tilde{\pi}_f^\top \Sigma_f \tilde{\pi}_f + \pi_a^\top \Sigma_a \pi_a. \qquad (7)
$$

The optimal portfolio allocations to maximizing mean-variance utility in equation (3) are given by

$$
\begin{aligned}
\pi_f^* &= \frac{1}{\gamma}\left(\Sigma_f^{-1}\mu_f - \beta \Sigma_a^{-1}\alpha\right) \qquad (8) \\
\pi_a^* &= \frac{1}{\gamma}\Sigma_a^{-1}\alpha. \qquad (9)
\end{aligned}
$$

The optimal factor holdings in equation (8) can be broken into two terms. The first is the standard mean-variance holding of factors, $\frac{1}{\gamma}\Sigma_f^{-1}\mu_f$, in the case where there are no alternative assets available. The second term, $\frac{1}{\gamma}\beta\Sigma_a^{-1}\alpha$, adjusts the benchmark factor allocations by taking into account the factor exposures of the alternative assets. Equation (9) shows that the investor holds alternative assets only if they have non-zero alpha.

Combining the previous expressions, we can express the risk aversion coefficient, γ, as

$$
\gamma = \frac{\sigma_p^2}{\mu_p} = \frac{\tilde{\pi}_f^{*\top}\Sigma_f \tilde{\pi}_f^* + \pi_a^{*\top}\Sigma_a \pi_a^*}{\tilde{\pi}_f^{*\top}\mu_f + \pi_a^{*\top}\alpha}. \qquad (10)
$$

Thus, portfolio holdings in the data can be used to estimate endowments' risk aversion.

3.3 Endowment Beliefs

It is natural to estimate the implied investment beliefs of endowments in a Bayesian framework. We use the model in Section 3.2 and infer endowments' beliefs, given the data of their holdings and asset returns, following Pástor and Stambaugh (1999, 2000), Avramov (2002), Avramov and Zhou (2010) and others. Our approach is similar in that we treat some assets as factors

(U.S. equity, international equity, and bonds), and the others (private equity and hedge funds) as active returns with alpha, but these studies only use the time-series of returns to conduct statistical inference about alphas. In our approach, we use both past returns and actual portfolio holdings of asset classes to infer investors' beliefs.

Denoting the return history and the portfolio holdings as $\mathbf{X}$, we wish to estimate the distribution of alternative assets' alphas given the observed data and a prior belief. To illustrate the approach, consider the case where we only estimate the parameter α. We construct the posterior distribution

$$p(\alpha|\mathbf{X}) \propto p(\mathbf{X}|\alpha)p(\alpha). \tag{11}$$

To construct the likelihood function, $p(\mathbf{X}|\alpha)$, we assume that the portfolio weights π_f and π_a in data are equal to the weights in equations (8) and (9), respectively, plus observation error:

$$\pi_f = \pi_f^* + u_1 \tag{12}$$

$$\pi_a = \pi_a^* + u_2, \tag{13}$$

where u_1 and u_2 are iid standard normal random variables with diagonal covariance matrices $\Sigma_{\pi f}$ and $\Sigma_{\pi a}$, respectively. The errors u_1 and u_2 are orthogonal to each other, and are orthogonal to the factor shocks, ε_f, and the shocks to the alternative assets, ε_a.

We assume several prior beliefs, $p(\alpha)$. In the uninformative, or flat, prior, alpha is estimated using only data on returns and portfolio holdings. We also use informative priors: a pessimistic prior which assumes that alternative assets have a negative return of -4% per year and an optimistic prior with a return of 4%, each with a standard deviation of 2%.

The posterior distribution, $p(\alpha|\mathbf{X})$, can be used in several ways. First, the posterior mean, $E(\alpha|\mathbf{X})$, can be interpreted as the implied investment belief that a typical endowment possesses in order to justify its portfolio holdings in alternative assets. The posterior distribution can also be used to compute other moments and confidence intervals. This gives a picture of the dispersion of endowments' beliefs and also can be used to judge statistical significance. Finally, by computing the posterior distribution of alpha for various prior beliefs, we can gauge how robust the investment views of endowments to be consistent with their allocations to alternative assets.

In our empirical work, we estimate the posterior distribution of all parameters, not just α. The full set of parameters is $\Theta = \{\mu_f, \Sigma_f, \alpha, \beta, \Sigma_a, \Sigma_{\pi,f}, \Sigma_{\pi,a}, \gamma\}$. We use flat priors for all parameters except α and μ_f. We motivate the informative priors for μ_f as follows. Our sample for factor returns is longer than the sample we use for alternatives. Since mean-variance portfolio weights are sensitive to the mean parameters, we parameterize the prior distribution of the factor excess returns, μ_f, in a way that allows us to change the weight given to the return data vs. the asset allocation data.[3] In particular, we assume a prior density centered on the time-series mean and scale proportional to time-series covariance matrix. The parameter ν controls the informativeness of the prior distribution, so that higher values of ν increase the weight given to factor returns. For example, if $\nu = T_f/(T_f + T_i)$, where T_f is the length of the factor sample and T_i is the length of the data on asset holdings, the prior distribution is flat and the posterior of μ_f is proportional to the likelihood. If $\nu = 1$ the posterior distribution is degenerate at the historical average excess return, so only holdings information is used to estimate μ_f. We consider the uninformative prior as our baseline specification, but estimate the model with other values for ν for robustness.

3.4 Endowment Heterogeneity

In the mean-variance model of Section 3.2, portfolio weights are determined by investor risk aversion and assumptions on the data-generating process of returns. Risk aversion, however, varies across endowments, and different endowments are also likely to have different beliefs on the alpha of alternative assets. To capture this heterogeneity, we assume that risk aversion, γ, and the alpha belief, α, depend on endowment size, past returns, spending rules, and other characteristics. Denoting these observable characteristics as Z, we assume that the risk aversion and alpha for endowment i are given by, respectively,

$$\gamma_i = \gamma_0 + \gamma_1 Z_i \tag{14}$$

$$\alpha_i = \alpha_0 + \alpha_1 Z_i, \tag{15}$$

[3] See Best and Grauer (1991) and Green and Hollifield (1992).

where Z_i is a vector of endowment i's characteristics, γ_0 and α_0 are constants, and γ_1 and α_1 are vectors which allow endowments' risk aversion to linearly depend on the characteristics.

We construct the set of endowment characteristics $Z = \{Z_i\}$ such that it is mean zero and unit variance at a point in time. Thus, the parameters γ_0 and α_0 represent the average level of risk aversion and the average view on the magnitude of alternative assets' abnormal returns, respectively. This also allows us to interpret the γ_1 and α_1 coefficients as representing the effect of a one-standard deviation change across the cross section of endowment characteristics. We also assume that endowments agree on parameters other than γ and α.

In addition, we allow α_0 to vary over time. We can plot a time series of α_0, allowing us to examine the evolution of endowments' beliefs. In fixing the other parameters for the full sample, we assume that time-series changes in average allocations to alternative assets are mainly driven by changing views on α_0. This is reasonable, since we have a relatively long time series of factor returns, and estimates of covariance parameters contain much less sampling error than estimates of means (see, for example, Merton, 1980). There is also some time-series variation in γ and alphas that come from changing endowment characteristics.

3.5 Estimation

We estimate the model using a Bayesian Markov Chain Monte Carlo (MCMC) approach. The estimation procedure generates the posterior distributions of the parameters by iteratively drawing from conditional densities which take into account all the information contained in assets' time-series returns, the cross-section of investors' allocation, and prior distributions. For a detailed exposition of the estimation algorithm, please see the appendix.

4 Data

4.1 Asset Allocation of Endowments

The portfolio allocation of most college and university endowments in the United States are voluntarily reported to the National Association of College and University Business Officers (NACUBO) and Commonfund. We use the results of the survey NACUBO/Commonfund study for the years 2006 to 2012. The database contains approximately 800 public and private university endowments which are surveyed every year. In addition to asset allocations each year, we also have general information about universities: their size, spending rates, and past endowment returns. We use all data contained in the database even if a given endowment is not surveyed every year. Universities report numbers to NACUBO and Commonfund for their fiscal year ends, which for most universities is June 30.

NACUBO uses ten asset categories, which are listed in the left-hand column of Table 1. To obtain a more parsimonious group of asset classes, we form five groups: U.S. stocks, fixed income, foreign stocks, private equity, and hedge funds. We group private equity, real estate, venture capital, and private equity into a "private equity" class, and the "hedge fund" category includes energy and natural resources, commodities, managed futures, marketable alternative strategies, and distressed debt. We treat cash as a risk-free asset. Endowments are not restricted from using leverage; Harvard University, for example, had a -5% cash holding in 2008 and a -3% holding in 2009.[4] The majority of endowments, however, do not use short positions.

Using just five asset classes has several advantages. First, it minimizes the effects of parameter sensitivity to data errors and mitigates well-known problems of extreme portfolio positions resulting from estimating large number of parameters. Second, the classification of assets differs from endowment to endowment, so a hedge fund investing in distressed commercial mortgage assets might be defined as a "marketable alternative strategy" for one endowment, a "distressed debt" fund for another endowment, or even as a "private equities real estate" fund. Grouping assets minimizes these reporting biases. Third, using fewer asset class groups is consistent with our aim to estimate broad investment views of endowments as a whole.

Table 2 reports the average allocations in the five asset groups. Over the sample period, there is a strong trend towards divesting from domestic stocks and increasing holdings in alternative investments. In 2006, the average allocation to U.S. stocks was 46% while at the end of 2012 that value was only 32%. At the same time, the average share of funds allocated to private equity increased from 5% in 2006 to 9% at the end of 2012. The corresponding average allocation to hedge funds increased from 12% to 19%. This is shown clearly in Figure 1, which plots the average allocation to U.S. equities and to alternatives defined as the sum of the average

[4] See "Liquidating Harvard," Columbia CaseWorks #100312.

allocation to private equity and hedge funds. Average alternative asset holdings rose from 17% in 2006 to 28% in 2012.

Table 2 shows that there is significant cross-sectional dispersion in the allocation to U.S. equities and alternatives. We address this in our model in two ways. First, some heterogeneity in portfolio weights is captured by the endowment-specific observation error we specify around the model-implied weights (see equations (12) and (13)). We also capture heterogeneity by allowing endowment risk aversion and beliefs about alternative asset class alphas to depend on university-specific characteristics (see equations (14) and (15)). Panel A also shows that in contrast to the decreasing holdings of U.S. equities and the increasing weights on alternatives, the weights on fixed income and foreign stocks have stayed relatively constant in our sample. In addition, these asset classes also have lower cross-sectional standard deviations.

4.2 Endowment Characteristics

In Table 3, we report summary statistics for various endowment characteristics: the type of institution (public or private), the size of the endowment in millions of U.S. dollars, the percentage of the fund that is spent each year, the percentage of the university's budget that is funded by the endowment, and the performance of the endowment over the past year. Table 3 lists the number of observations available every year, and the number of non-missing values. In the estimation, we do not restrict ourselves to using only endowments for which all variables are observable; our algorithm is able to use the full sample and infer values for missing observations (see the appendix).

In our sample, approximately 60% of the endowments fund private colleges or universities. The average fund size over the sample period is $463 million. There are large differences in size both across time and across endowments. The average size reaches its peak in 2008 before the financial crisis, shrinks by 35% during 2009, and recovers during 2012 to $518 million. The smallest 10% of endowments have less than $13 million over our sample period, and the largest 10% manage more than $847. The largest endowment in our sample, well-known from other sources to be Harvard University, has assets totaling approximately $31 billion as of 2012.

Endowment income plays a very important role in meeting operational budgets for universities. The average spending rate from endowment funds is 4.4%, and this is very persistent over time. There is modest variation in the spending rate across universities. The share of the university budget funded by the endowment exhibits more cross-sectional variation, with the typical university relying on the endowment to meet around 10% of their operations. Finally, endowment performance has significantly varied across universities. This may reflect the different experiences of endowments in alternative investments, or their different abilities to market time.[5]

4.3 Asset Class Returns

For each asset class we choose a well-known index with two key characteristics. First, we focus on indices with a long history of returns. Time-series information is relevant for identification since it pins down the moments of the distribution of returns. Second, we require indices to be marketable in order to avoid the problems associated with appraisal-based pricing, which induces artificial smoothing. All our return data are at the monthly frequency. Since our implied beliefs of endowments may be sensitive to the estimates from the shorter samples of the alternative asset returns, we examine robustness with various priors in our empirical work.

We use the S&P 500 index, the Ibbotson Associates SBBI Long-Term Government index and the MSCI World ex-U.S.A index as proxies for domestic stocks, fixed income, and foreign stocks, respectively. Our data samples are January 1926 to December 2012 for domestic stocks and bonds, while we use returns starting from January 1973 for international stocks. As a proxy for alternative investments, we use the HFRI Fund of Funds index and the S&P Listed Private Equity index. In these cases, monthly returns are available starting from January 1990 and January 1994, respectively. Finally, we use the Ibbotson Associates SBBI 30-day T-Bill returns as a risk-free rate in order to construct excess returns.

Table 4 reports summary statistics for excess returns on the asset classes. Domestic and international stocks have the highest average excess returns, at 5.91% and 4.20%, respectively. They also exhibit similar levels of volatility and have Sharpe ratios of 0.31 and 0.24, respectively. Fixed income has a lower Sharpe ratio of 0.25.

Private equity has the lowest Sharpe ratio of 0.14 among the asset classes. This Sharpe

[5] Lerner et al. (2008) document considerable heterogeneity in endowment returns, some of which is due to their different holdings in equities and alternative assets. Brown et al. (2010) show that a significant fraction of cross-sectional differences in endowment performance comes from the (lack of) ability to market time asset classes.

ratio is significantly lower than the performance of private equity typically reported in some academic studies, such as Robinson and Sensoy (2011) and Harris et al. (2012). This is because we use a listed equity index for private equity, rather than an index representing direct, illiquid private equity investment. Infrequent trading, the use of appraisals, and selection bias where we tend to observe market valuations only when the underlying valuations are high, all potentially cause illiquid, direct private equity indices to substantially under-state their true volatility (see, Ang and Sorensen (2012), for a summary). The volatility of private equity during the 1994-2012 period is 24.4%, which is above the stock market volatility of 19.0% over the 1926-2012 period—which is expected since private equity funds typically hold non-diversified portfolios with high idiosyncratic volatility (see Ewens et al., 2013).

Hedge funds have the highest Sharpe ratio of 0.65, which is driven by the unusually low volatility of aggregate returns, at only 5.8%, in the sample. Hedge fund abnormal performance has declined over time, as Dichev and Yu (2011) and others note. Hedge funds have a lower correlation with equities, at 0.54, than private equity, which has a correlation with equities of 0.67; since private equity is a form of equity, it is not surprising that unlisted equity is highly correlated with listed equity.

5 Results

In Section 5.1, we report estimates of the model and the implied beliefs about the risk and return of investments. We track the implied beliefs of endowments about the alphas of alternative assets over time. Section 5.2 addresses the heterogeneity across endowments. In Section 5.3, we present results with informative priors, and priors which put different weights on the time-series of returns vs. the cross-section of asset holdings. We conduct a series of robustness checks in Section 5.4.

5.1 Time-Varying Investment Beliefs

The estimated parameters are shown in Table 5. In solving the mean-variance model (equation (3)), we assume a risk-free rate of 3.5%. The amount of risk-free holdings by endowments is small, at less than 5% (see Table 2), and so the results are quite insensitive to the choice of the risk-free rate.

Table 5 shows that the average level of risk aversion, γ, for an endowment over the entire sample is 7.48. We find that private endowments are significantly more risk tolerant. There is weak evidence that larger endowments are more risk tolerant, however the coefficient is not significant. Additionally, the spending rate is positively correlated with risk aversion, while the percentage of the university budget financed by the endowment and its past return are negatively related to γ. These latter effects are likewise not statistically significant.

Panel B reports the average level of alpha beliefs for private equity and hedge funds. For both classes of assets, alpha beliefs are positive—and this is statistically and economically significant each year. We also find evidence of heterogeneity due to endowment characteristics. Beliefs about both private equity and hedge fund alphas are higher for private endowments, funds with more asset under management, endowments with higher spending rates (only significantly so for hedge funds), and endowments that fund a higher proportion of universities' operating budgets. For hedge funds, positive past year returns are significantly associated with positive alpha beliefs, however this is not true for private equity. Given that past year returns on private equity investments are not reliably observable by investors, this difference is not surprising.

Figure 2 shows how the average view on the level of mispricing has evolved over time. For both alternative asset classes our estimated alpha increases over the sample, which is consistent with the observed trend in endowment allocations into alternative investments. The average view on private equity alpha increases from 1.39% per year in 2006 to 3.89% per year in 2012. Interestingly, our model generates an alpha that is larger than the OLS estimate from historical returns (0.54% per year) and larger than the alphas reported in the academic literature on the subject available prior to, and concurrent with, most of the period of the study. Although our proxy for private equity performance is imperfect as a measure of long-term performance, and empirical studies are limited by available private equity data, this suggests the prevalence of fairly aggressive positive beliefs about private equity. Thus, despite limited and imperfect available information, endowments believe private equity has significant alpha.[6] Beliefs about

[6] A survey of institutional investors by Dhar and Goetzmann (2006) taken just prior to the sample period found that that managers were relatively less comfortable in basing their expectations about future performance on past returns to hedge funds and private equity, compared to traditional asset classes.

hedge funds' abnormal returns also rise over time. In 2006 the average view is 0.29% per year while in 2012 it increases to 0.66% per year. In contrast to the private equity case, the historical alpha (1.53% per year) is higher than the average belief by university endowments. It is also interesting to note that we find no significant effects of the Great Recession on endowments' beliefs and allocations. This result complements the findings of Brown et al. (2013) who document that endowments reduce payouts by more than their formal rules after negative financial shocks.

The results seem to indicate that the average endowment believes that there are significant gains from holding private equity investments. The expected level of alpha is significantly higher than our historical proxy and empirical evidence found in the literature. There are two possible explanations for this. First, endowments may think that they have superior selection skills and are able to pick the managers that generate the highest alpha. This selection skill is not reflected on historical measures of performance. Second, they may expect higher conditional risk premia on liquidity or other risk factors in the future.

Panel C of Table 5 reports the estimates for the rest of the parameters of the model. Compared to historical time-series data, our model generates significantly larger expected excess returns for domestic stocks and international equities. Estimated excess returns for U.S. stocks is 15.6% per year and for foreign stocks is 12.9% per year. On the contrary, fixed income excess return, at 2.8% per year, is very similar to historical performance. This result is interesting, given that publicly traded securities remain a substantial proportion of the average endowment portfolio despite the move toward alternatives.

Finally, Figure 3 shows the cross-sectional distributions of the risk aversion coefficient and the views on private equity and hedge fund alpha. The two-humped shape of the distribution of risk aversion in the top panel is due to the large fixed effect of the type of institution (public or private). Since this is a discrete variable it generates a bimodal distribution. Although Panel A of Table 5 documents that private universities have significantly lower risk aversions, the economic difference is not large. A striking result is the large difference in belief dispersion between private equity and hedge funds in the middle and bottom panels. The model is able to capture a large degree of heterogeneity in alpha for private equity. This is reflected by the large α_1 coefficient associated with the size characteristic. An endowment that is one standard deviation larger than the average fund expects a larger out-performance of 2.56% per year. This may reflect the fact that endowments have truly less disagreement on the level of mispricing of hedge funds, or that our model is not able to capture the cross-sectional dispersion because it is associated with missing or unobservable characteristics.

5.2 Endowment Holdings and Factor Exposures

In this section we address the extent to which we can explain the observed cross-sectional differences in endowment allocations. To do so, we use the estimated parameters to compute model-implied optimal asset allocation for each endowment. The results are shown on Table 6. The model is able to fit accurately the observed average allocations to the different asset classes. Our results, however, do not account for the cross-sectional variation observed in the data. Cross-sectional standard deviations of portfolio weights implied by the model are much smaller than in the data, especially for fixed income and foreign stocks. Model-implied average allocations also do not change over time as much as observed weights. Both results can be explained, to some extent, by the assumptions made in the model. In particular, we assume that endowments differ only in their risk aversion coefficient and their view on alpha. Allowing for changes in the average view on factors' excess returns could improve the fit of the model. The literature has found other factors that matter, which we do not observe. Goetzmann and Oster (2012) find that endowment decisions to change asset allocation is conditional upon strategic considerations, including rivals' performance. Gilbert and Hrdlicka (2013) argue that a number of other unobserved university fundamentals may explain relative level of investment in risky assets, including the marginal productivity of internal projects, the influence of self-interested stakeholders, and binding constraints on asset allocations.

Given the factor structure of our model, we ask what are the implicit allocations to factor securities? We use the estimated factor loadings, β, and the observed allocations to compute $\tilde{\pi}_f$ for each endowment in the sample. Figure 4 shows non-parametric kernel densities of the cross-sectional distribution of allocations to the three factor securities. Endowments' exposure to U.S. stocks and fixed income are larger than the explicit weight on those asset classes. In the case of international equities, the distribution of both implicit and explicit weights are similar. The difference between the actual and the effective allocations to equity is very pronounced. The average fund has close to 62% of its wealth allocated to stock-like securities after controlling for the equity factor exposure in private equity and hedge funds. Endowments not viewing their total factor exposure may be significantly under-estimating their exposure to equity risk—as many universities found out in 2008 and 2009, including Harvard University, when endowment values declined drastically.

5.3 Informative Priors

The results in previous sections assumed uninformative priors for the average level of mispricing α_0 and factor excess returns μ_f. In this section, we impose informative priors and investigate how posterior distributions and optimal allocations change.

First, we assume that an investor, before observing the asset dynamics and endowments allocations, has optimistic or pessimistic prior beliefs on alternative investments. In the optimistic case, we assume that the investor's prior on the level of abnormal returns for both private equity and hedge funds are normally distributed with an annualized mean of 4% and a standard deviation of 2%. In the pessimistic case, we specify a normal prior with a mean of -4% and a standard deviation of 2%.

Figure 5 graphs the average of the posterior distribution of alpha for both optimistic and pessimistic priors. The effect of the prior appears to be asymmetric. Under the optimistic prior, the posterior distribution for private equity shifts up by almost 1% per year. The estimated α_0 of the baseline specification is lower than the 5th percentile of the new distribution. When the prior is pessimistic, the estimated α_0 shifts down by less than 0.5% per year. The same behavior is observed for hedge funds. In sum, observed returns and allocations provide the investor with a sufficient amount of information to significantly update his prior belief on the alternatives' alpha. Even with an informative negative prior, endowments exhibit beliefs with positive abnormal returns on private equity and hedge funds.

We consider different prior distributions for the factor securities' excess returns μ_f since, as we show above, estimated mean excess returns in the baseline specification are significantly higher than historical values. Therefore, we examine the effect when we increase the weight given to historical returns. Table 7 shows the estimated average view on alpha for private equity and hedge funds. When ν is equal to $T_f/(T_f + T_i)$ the prior is uninformative and we are in the baseline case. As ν increases, the scale parameter of the prior decreases and more weight is given to time-series information. In all cases the model is able to fit average allocations reasonably well. Table 7 shows that investment beliefs on private equity and hedge fund alpha decrease as ν increases. The intuition for this result is that a more informative prior pushes μ_f down to its historical average. In order to fit the observed allocations to factor securities, the level of risk aversion also decreases. But a lower risk aversion increases the weights on mispriced securities unless the average level of alpha also decreases. Nevertheless, our general conclusions hold. The view on the level of alternatives' alpha increases over the sample, reaching its peak in 2012. Furthermore, when less weight is given to historical returns, investors seem to think that private equity is an even more attractive investment than hedge funds since estimated alphas are significantly larger for private equity than for hedge funds.

5.4 Robustness

We check the robustness of our results by re-estimating the model under different assumptions and specifications. We consider the following cases:

1. We use return data starting only from 1970.

2. Instead of including all endowments, we estimate the model using only the ten most well-known university endowments: Brown, Columbia, Cornell, Darthmouth, Harvard, Princeton, University of Pennsylvania, Yale, Stanford, and MIT. This considers only bigger and more successful funds.

3. We Winsorize the funds' observed characteristics at the 5th and 95th percentiles.

4. We change our proxy for private equity to the Cambridge Associates Private Equity Index. This index is only available on a quarterly basis starting from 1990, so we convert all observed monthly returns to quarterly returns.

5. We change our proxy for hedge funds to the Dow Jones Credit Suisse Hedge Funds Index, which is available starting from 1994.

6. We remove endowments with no holdings in hedge funds or private equity.

7. We collapse private equity and hedge funds to a single asset class. As a proxy for this broad alternative asset class, we use a weighted average of the baseline proxies for private equity and hedge funds. Weights are the cross-sectional average of the shares invested in each subclass (30% private equity and 70% hedge funds).

Table 8 shows the estimated risk aversion, γ, and the average alternatives' alpha, α_0. In all specifications, the view on the level of mispricing for both private equity and hedge funds is significantly greater than zero and increases over time. Also, as in our main specification, the view on private equity's alpha is significantly larger than the one for hedge funds with one exception: when we use the Cambridge Associates Private Equity index, both mispricing levels are similar. Another interesting result is that large endowments appear to have more aggressive views since estimated α_0 are approximately two times larger. Winsorizing endowments' characteristics has almost zero effect. We obtain similar results to the baseline specification when both asset classes are grouped into a single alternatives class. The average view on alpha is significantly greater than zero and increases over time reaching 1.3% in 2012. In sum, endowments expect alternatives' alpha to be high, perhaps because they have demonstrated past capability in capturing alpha in the past and expect to continue to do so in the future.

6 Conclusion

Colleges and universities rely to varying extent upon endowment income to support their missions. The asset allocation policy figures heavily in endowment management, and this, in turn depends upon expectations about the risk, return, correlations, and liquidity of various asset classes. There has been a pronounced recent trend towards alternative investments, despite the fact that the risk-return trade-offs of these asset classes are the least well understood. Their short, observed history of returns and there are imperfect quality and quantity of information on private equity and hedge funds. Larger holdings of alternative assets suggests that endowments are accepting higher levels of uncertainty in exchange for high expected returns.

In this paper we address this tradeoff by modeling the allocation decision in a Bayesian framework. We use observed asset allocations of private equity and hedge funds, together with past data on these alternatives' past returns and standard equity and bond factors, to extract expectations on alternatives' alpha. We find that the typical endowment expects these asset classes to generate returns in excess of a benchmark adjusted for systematic risk exposure. There are significant differences in expectations across private equity and hedge funds, however. Despite prior empirical evidence on positive alphas in the hedge fund industry and neutral alphas in the private equity industry, the average endowment expects private equity to outperform hedge funds on a risk-adjusted basis. Our results may be due to the extrapolation of the success of the largest and most successful university endowments. It may be due to a preference for relative uncertainty. Further work on the expectation-formation mechanism is warranted.

Estimation Appendix

A Gibbs Sampler

Denote $X = (f, r_a, \pi_f, \pi_a)$ as the set of all time-series and cross-sectional information. The objective is to estimate the parameters' joint posterior distribution given observed factor returns, alternative strategies returns, endowments' asset allocations and their observable characteristics, Z:

$$p(\Theta|X, Z), \tag{A-1}$$

where $\Theta = \{\mu_f, \Sigma_f, \alpha_0, \alpha_1, \beta, \Sigma_a, \Sigma_{\pi,f}, \Sigma_{\pi,a}, \gamma_0, \gamma_1\}$ is the set of all parameters. We denote Θ_- as the set of parameters less the parameter of interest. We use a Gibbs sampler to sample from the joint posterior by specifying conditional distributions for each parameter: $p(\mu_f|\Theta_-, X, Z)$, ..., $p(\gamma|\Theta_-, X, Z)$. The posterior conditional distributions can be found using Bayes' theorem as the product of the likelihood function and the parameter's prior.

$$\begin{aligned} p(\mu_f|\Theta_-, X, Z) &\propto p(\pi_f, f, r_a|\Theta, Z)p(\mu_f^0) \\ &\vdots \\ p(\gamma|\Theta_-, X, Z) &\propto p(\pi_f, \pi_a, f, r_a|\Theta, Z)p(\gamma^0). \end{aligned} \tag{A-2}$$

For μ_f and the measurement error variances, Σ_π, $\Sigma_{\pi f}$, and $\Sigma_{\pi a}$, the posterior distributions are closed form. It is not possible to get analytical expressions for the remaining parameters, so we use the Random Walk Metropolis-Hasting algorithm. Given some initial parameters values θ_0, we assume candidate draws for the nth iteration follow a multivariate random walk,

$$\hat{\theta}_{n+1} = \theta_n + \sigma_\theta w_{n+1}, \tag{A-3}$$

where w_{n+1} is a standardized normal, and σ_θ is a scaling parameter. The new draw is accepted with probability

$$\min\left\{\frac{p(X|\hat{\theta}_{n+1}, \Theta, Z)q(\theta_n)}{p(X|\theta_n, \Theta)q(\hat{\theta}_{n+1}, Z)}, 1\right\}, \tag{A-4}$$

where $q(\cdot)$ is the prior distribution density. If the draw is accepted, then $\theta_{n+1} = \hat{\theta}_{n+1}$, otherwise $\theta_{n+1} = \theta_n$. Note that since $\hat{\theta}_{n+1}$ follows a random walk, $q(\hat{\theta}_{n+1}) = q(\theta_n)$ and the last term from the numerator and the denominator cancel.

We run the MCMC algorithm one million times. The first 800,000 draws are used to calibrate the diffusion coefficients σ_θ and are discarded at the end. Moreover, during this calibration stage if the Metropolis-Hastings' acceptance ratios are below 5%, diffusion parameters are reduced in half. Similarly, if acceptance ratios are above 50% the variance is doubled. The posterior distribution is then computed using the last 200,000 draws. Convergence is fast.

B Factors' Expected Excess Returns, μ_f.

Assume a multivariate normal prior $N(\mu_0, \Sigma_0)$, where μ_0 is an N_f column vector and Σ_0 is a $N_f \times N_f$ matrix. Since both the likelihood and the prior are conditionally normal, we obtain an analytical expression for the posterior by completing the square.

$$
\begin{aligned}
p(\mu_f|\Theta_-, X, Z) &\propto p(\pi_f, f|\Theta)p(\mu_f) \\
&\propto p(\pi_f|\Theta)p(f|\Theta)p(\mu_f) \\
&\propto N(\mu_f|\mu_n, \sigma_n)
\end{aligned}
\tag{A-5}
$$

where the second line follows from the independence assumption between factor dynamics and allocation measurement errors. The mean and variance of the posterior distribution are

$$
\begin{aligned}
\Sigma_n &= \left(\Sigma_0^{-1} + T_f \Sigma_f^{-1} + \sum_i \tilde{\Sigma}_{\pi,f}^{-1}\right)^{-1} \\
\mu_n &= \Sigma_n \left(\Sigma_0^{-1}\mu_0 + T_f \Sigma_f^{-1} \bar{f} + \sum_i \tilde{\Sigma}_{\pi,f}^{-1} \tilde{\pi}_f\right),
\end{aligned}
\tag{A-6}
$$

where

$$
\begin{aligned}
\tilde{\pi}_f &= \gamma \Sigma_f \pi_f + \Sigma_f \beta \Sigma_a^{-1} \alpha_a \\
\tilde{\Sigma}_{\pi,f}^{-1} &= \frac{1}{\gamma^2} \left(\Sigma_f \Sigma_{\pi,f} \Sigma_f^{\top}\right)^{-1}.
\end{aligned}
$$

Both expressions are intuitive. The posterior variance is the inverse of the sum of the likelihood and prior inverse variances, weighted by the number of observations. The posterior mean is the average of the likelihood and prior means weighted by the information matrix and the number of observations. The first term is a function of the prior distribution parameters, while the second and third terms are functions of the observed excess return mean and average allocations. The parameter Σ_0 characterizes the informativeness of the prior. If it is large, the prior is diffuse and the posterior mean does not depend much on the prior. If it is small, the posterior mean remains close to the prior. The more observations on factor returns and the lower the variance of those observations, the higher the weight of the factors' time series mean on the value of μ_n. The same intuition applies for the information provided by the data on asset allocations.

Assume that $\Sigma_0 = \Sigma_f / T_0$ and $\mu_0 = \bar{f}$. We can rewrite equation (A-6) as

$$
\begin{aligned}
\Sigma_n &= \left(\nu \Sigma_f^{-1} + (1-\nu)\Sigma^{-1}\right)^{-1} \frac{1}{\tilde{T}_f + T_i} \\
\mu_n &= \Sigma_n (\tilde{T}_f + T_i) \left(\nu \Sigma_f^{-1} \bar{f} + (1-\nu)\Pi_f\right),
\end{aligned}
\tag{A-7}
$$

where

$$\begin{aligned}\nu &= \frac{\tilde{T}_f}{\tilde{T}_f + T_i} \\ \tilde{T}_f &= T_0 + T_f\end{aligned}$$

and

$$\begin{aligned}\Sigma^{-1} &= \frac{1}{T_i}\sum_i \tilde{\Sigma}_{\pi,f}^{-1} \\ \Pi_f &= \frac{1}{T_i}\sum_i \tilde{\Sigma}_{\pi,f}^{-1}\tilde{\pi}_f.\end{aligned}$$

If $T_0 = 0$, the prior is uninformative and the posterior is unchanged. If $T_0 \to \infty$, then $\nu = 1$ and we have a degenerate posterior distribution at the time-series mean. As we increase T_0, the weight given to the time-series information increases.

C Error Variances

Assume an inverse Wishart prior $IW(\nu_0, \Psi_0)$. By Bayes' theorem,

$$\begin{aligned}p(\Sigma_u|\Theta_-, X, Z) &\propto p(\pi|\Theta)p(\Sigma_u) \\ &\propto IW(\nu_N, \Psi_N), \end{aligned} \tag{A-8}$$

where

$$\begin{aligned}\nu_N &= \nu_0 + T_\pi \\ \Psi_N &= \Psi_0 + \sum_{t=1}^{T_\pi}(\pi_t - \pi^*(\Theta))(\pi_t - \pi^*(\Theta))^\top.\end{aligned}$$

Since measurement errors for factor allocations and mispriced securities allocations are independent, the same procedure can be used for drawing $\Sigma_{\pi f}$ and $\Sigma_{\pi a}$.

D Factors' Covariance, Σ_f

By Bayes' theorem the posterior of Σ_f is given by

$$p(\Sigma_f|\Theta_-, X, Z) \propto p(\pi_f|\Theta, Z)p(f,|\Theta)p(\Sigma_f). \tag{A-9}$$

We use Metropolis-Hastings to draw from the posterior. We specify a candidate draw $\hat{\sigma}_f$, which is accepted with probability

$$\min\left\{\frac{p(\pi_f|\hat{\sigma}_f, \Theta_-, Z)p(f|\hat{\sigma}_f, \Theta_-)}{p(\pi_f|\Sigma_f, \Theta_-, Z)p(f|\Sigma_f, \Theta_-)}, 1\right\}. \tag{A-10}$$

E Alternatives' Alpha Parameters, α_0 and α_1

By Bayes' theorem we have

$$p(\alpha_0, \alpha_1|\Theta_-, X, Z) \propto p(\pi_f|\Theta, Z)p(\pi_a|\Theta, Z)p(f, r_a|\Theta)p(\alpha_0, \alpha_1). \tag{A-11}$$

We use Metropolis-Hastings with an acceptance probability of

$$\min\left\{\frac{p(\pi_f|\hat{\alpha}_a, \Theta_-, Z)p(\pi_a|\hat{\alpha}_a, \Theta_-, Z)p(f, r_a|\hat{\alpha}_A, \Theta_-)}{p(\pi_f|\alpha, \Theta_-, Z)p(\pi_a|\alpha, \Theta_-, Z)p(f, r_a|\alpha, \Theta_-)}, 1\right\}. \tag{A-12}$$

F Risk Aversion Parameters, γ_0 and γ_1

By Bayes' theorem we have

$$p(\gamma_0, \gamma_1|\Theta_-, X, Z) \propto p(\pi_f|\Theta, Z)p(\pi_a|\Theta, Z)p(\gamma_0, \gamma_1). \tag{A-13}$$

Metropolis-Hastings algorithm is used with an acceptance probability of

$$\min\left\{\frac{p(\pi_f|\hat{\alpha}_a, \Theta_-, Z)p(\pi_a|\hat{\alpha}_a, \Theta_-, Z)}{p(\pi_f|\alpha, \Theta_-, Z)p(\pi_a|\alpha, \Theta_-, Z)}, 1\right\}. \tag{A-14}$$

G Factor Loadings, β.

By Bayes' theorem,

$$p(\beta|\Theta_-, X, Z) \propto p(\pi_f|\Theta, Z)p(f, r_a|\Theta)p(\beta). \tag{A-15}$$

The acceptance probability of the Metropolis-Hastings step is equal to

$$\min\left\{\frac{p(\pi_f|\hat{\beta}, \Theta_-, Z)p(f, r_a|\hat{\beta}, \Theta_-)}{p(\pi_f|\beta, \Theta_-, Z)p(f, r_a|\beta, \Theta_-)}, 1\right\}. \tag{A-16}$$

H Alternatives Covariance, Σ_a.

By Bayes' theorem, the posterior of Σ_a is given by

$$p(\Sigma_a|\Theta_-, X, Z) \propto p(\pi_f|\Theta, Z)p(\pi_a|\Theta, Z)p(f, r_a, |\Theta)p(\Sigma_a). \tag{A-17}$$

We use Metropolis-Hastings with acceptance probability

$$\min\left\{\frac{p(\pi_f|\hat{\sigma}_a, \Theta_-, Z)p(\pi_a|\hat{\sigma}_a, \Theta_-, Z)p(f, r_a|\hat{\sigma}_a, \Theta_-)}{p(\pi_f|\Sigma_a, \Theta_-, Z)p(\pi_a|\Sigma_a, \Theta_-, Z)p(f, r_a|\Sigma_a, \Theta_-)}, 1\right\}. \tag{A-18}$$

I Inferring Missing Endowment Characteristics

We assume that endowments' observable characteristics, Z, can be modeled as a multivariate normal random variables with mean μ_Z and covariance matrix Σ_Z. When some fund-year pair characteristics are missing, we group the known variables into Z_1 and the unobserved ones in the vector Z_2:

$$\left(\begin{array}{c} Z_1 \\ Z_2 \end{array} \right) = N \left(\left[\begin{array}{c} \mu_{Z1} \\ \mu_{Z2} \end{array} \right], \left[\begin{array}{cc} \Sigma_{Z,11} & \Sigma_{Z,12} \\ \Sigma_{Z,21} & \Sigma_{Z,22} \end{array} \right] \right). \tag{A-19}$$

We can compute then the conditional mean and the conditional covariance matrix of Z_2 given the observed information Z_1:

$$\begin{aligned} \hat{\mu}_{Z2} &= \mu_{Z2} + \Sigma_{Z,21}\Sigma_{Z,11}^{-1}\left(Z_1 - \mu_{Z1}\right) \\ \hat{\Sigma}_{Z,22} &= \Sigma_{Z,22} - \Sigma_{Z,21}\Sigma_{Z,11}^{-1}\Sigma_{Z,12} \end{aligned} \tag{A-20}$$

Given μ_{Z2} and $\Sigma_{Z,22}$, we draw new values for the unobserved characteristics in each iteration of the Gibbs sampler for the missing Z_2:

$$\hat{Z}_2 \sim N\left(\hat{\mu}_{Z2}, \hat{\Sigma}_{Z22}\right). \tag{A-21}$$

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Table 1: Asset Allocations of Endowments

The table lists the asset allocation categories in the NACUBO-Commonfund study in the left-hand column, and our classification in the right-hand column.

NACUBO Category	Group
Cash	Cash
U.S. stocks	U.S. stocks
Fixed income	Fixed income
Foreign stocks	Foreign stocks
Private equities real estate Venture capital Private equity	Private equity
Energy and natural resources Commodities managed futures Marketable alternative strategies Distressed debt	Hedge funds

Table 2: Endowments' Asset Allocation

The sample consists of university endowments in the NACUBO-Commonfund Study of Endowments from 2006 through 2012. We list cross-sectional means and standard deviations (in parentheses) of endowments' allocations to domestic stocks, fixed income, international stocks, private equity, and hedge funds using the groupings of the original NACUBO assets in Table 1. Allocations are in percent.

Year	2006	2007	2008	2009	2010	2011	2012	Average
Cash	4.34	5.16	4.14	7.46	6.02	5.70	5.37	5.63
	(8.49)	(11.49)	(7.93)	(11.31)	(9.01)	(9.17)	(8.26)	(9.56)
U.S. stocks	45.55	42.37	38.07	33.58	32.44	32.51	31.95	35.74
	(16.94)	(16.84)	(17.72)	(16.25)	(16.46)	(16.39)	(15.83)	(17.22)
Fixed income	20.00	17.79	18.98	21.55	21.78	18.99	19.84	20.03
	(11.43)	(9.26)	(10.4)	(11.51)	(11.71)	(10.56)	(11.35)	(11.09)
Foreign stocks	13.49	15.76	15.03	14.29	14.75	16.42	15.25	15.03
	(9.19)	(9.66)	(9.13)	(8.44)	(8.05)	(8.26)	(8.03)	(8.62)
Private equity	5.06	5.80	8.03	7.28	7.25	8.06	8.79	7.31
	(6.08)	(6.67)	(8.38)	(8.78)	(8.89)	(9.29)	(9.95)	(8.66)
Hedge funds	12.08	13.12	15.73	15.85	17.75	18.29	18.74	16.31
	(12.96)	(12.93)	(14.47)	(14.84)	(15.52)	(14.92)	(14.78)	(14.69)

Table 3: Endowments' Characteristics

The sample consists of university endowments in the NACUBO-Commonfund Study of Endowments from 2006 through 2012. The table reports summary statistics for the following characteristics: the number of private vs. public universities, assets under management (in million of dollars), the spending rate, which is the percentage of the fund that is spent each year, the percentage of the University's budget funded by the endowment, and the funds' performance during the previous year. We report the cross-sectional mean and standard deviation each year, along with various percentiles of the cross-sectional distribution. We also report the number of non-missing observations, N.

		2006	2007	2008	2009	2010	2011	2012	Average
Private	Count	298	351	295	502	507	489	483	2,925
	N	448	534	453	791	799	768	766	4,559
Size	Mean	437	518	529	343	422	511	518	463
	10%	9	12	13	12	13	17	14	13
	50%	75	104	117	68	73	94	92	84
	90%	683	1,003	1,065	665	779	914	933	847
	Stdev.	2047	1762	1814	1166	1615	1933	1934	1748
	N	448	534	453	791	799	768	766	4,559
Spending Rate	Mean	4.62	4.47	4.40	4.37	4.53	4.57	4.15	4.43
	10%	3.10	3.00	3.20	2.27	1.85	2.56	2.47	2.58
	50%	4.54	4.50	4.40	4.50	4.90	4.70	4.25	4.50
	90%	6.00	5.80	5.75	5.75	6.38	5.99	5.40	5.98
	Stdev.	1.77	1.63	1.47	1.88	1.89	3.32	1.52	2.09
	N	437	517	441	758	771	753	747	4,424
% Budget	Mean	.	8.85	9.94	13.41	10.59	9.30	8.63	10.31
	10%	.	0.20	0.00	0.00	0.00	0.00	0.00	0.00
	50%	.	4.31	4.35	4.70	3.25	3.15	3.01	3.80
	90%	.	22.00	26.30	41.84	30.00	24.12	23.00	29.00
	Stdev.	.	13.00	16.71	21.08	18.60	16.39	15.40	17.57
	N	0	344	358	721	718	676	672	3,489
Past Return	Mean	10.54	17.33	-2.69	-18.76	11.95	19.26	-0.33	4.86
	10%	6.87	13.40	-7.13	-24.00	8.40	14.42	-3.20	-18.30
	50%	10.22	17.50	-2.85	-19.10	12.20	19.81	-0.50	8.80
	90%	14.70	20.87	2.14	-12.90	15.40	23.50	2.39	20.10
	Stdev.	3.57	3.38	3.78	5.26	3.23	4.31	2.67	13.63
	N	418	512	437	748	769	740	750	4,374

Table 4: Asset Class Excess Returns

The table shows annualized averages, standard deviations, and correlations for excess returns on the following asset classes: domestic stocks, fixed income, international equities, private equity, and hedge funds. The statistics are computed from monthly returns. U.S. equities are proxied by Standard & Poors' 500 from 1926 through 2012. Fixed income is represented by the Ibbotson U.S. Long-Term Government Bond Index for the same period. For international stocks, we use the MSCI International World ex-U.S. Index for the period 1970 through 2012. For private equity and hedge funds we use Standard & Poors' Listed Private Equity Index, starting in 1994, and the HFRI Fund of Funds Composite Index, starting in 1990, respectively. Available returns start in 1994 for the former and in 1990 for the latter. Returns are in excess of Ibbotson U.S. 30-day Treasury Bill returns. Correlations are computed using the longest available common data sample between each variable.

Asset Class	U.S. Stocks	Fixed Income	Foreign Stocks	Private Equity	Hedge Funds
Period	1926 - 2012	1926 - 2012	1970 - 2012	1994 - 2012	1990 - 2012
Mean	0.0591	0.0206	0.0420	0.0338	0.0375
Volatility	0.1903	0.0823	0.1750	0.2442	0.0575
Sharpe Ratio	0.31	0.25	0.24	0.14	0.65
Correlations					
U.S. Stocks	1.00	0.09	0.66	0.73	0.54
Fixed Income		1.00	0.05	-0.27	-0.11
Foreign Stocks			1.00	0.72	0.56
Private Equity				1.00	0.67
Hedge Funds					1.00

Table 5: Parameter Estimates

The table lists parameters of the model estimated using asset returns from 1926 to 2012 and endowment allocations from 2006 to 2012. We use uninformative priors for all parameters. Both the view on the alpha of alternative investments and the risk aversion coefficients are assumed to be linear functions of funds' observable characteristics (see equation (14)): whether the college is private, endowment size, spending rate, the proportion of the budget met by endowment revenue, and the return over the past year. The characteristics are cross-sectionally normalized at each point in time. Parameter estimates are annualized. We report posterior means and standard deviations (in parentheses).

Panel A: Risk Aversion

	γ_1				
γ_0	Private	Size	Spending	% Budget	Past Ret
7.48	-0.2594	-0.0700	0.0118	-0.0088	-0.0527
(0.77)	(0.1028)	(0.0506)	(0.0463)	(0.0483)	(0.0463)

Panel B: Beliefs

Year		2006	2007	2008	2009	2010	2011	2012
α_0	Private Equity	0.0139	0.0230	0.0311	0.0311	0.0325	0.0370	0.0389
		(0.0024)	(0.0023)	(0.0029)	(0.0024)	(0.0026)	(0.0028)	(0.0030)
	Hedge Funds	0.0029	0.0041	0.0053	0.0054	0.0058	0.0064	0.0066
		(0.0004)	(0.0004)	(0.0006)	(0.0005)	(0.0006)	(0.0006)	(0.0006)
		Private	Size	Spending	% Budget	Past Ret		
α_1	Private Equity	0.0085	0.0256	0.0012	0.0015	-0.0002		
		(0.0012)	(0.0020)	(0.0007)	(0.0007)	(0.0007)		
	Hedge Funds	0.0010	0.0038	0.0002	0.0002	0.0010		
		(0.0002)	(0.0003)	(0.0001)	(0.0001)	(0.0002)		

Table 5 Continued

Panel C: Other Parameters

		U.S. Stocks	Fixed Income	Foreign Stocks	Private Equity	Hedge Funds
Factors						
μ_f		0.1558	0.0283	0.1287		
		(0.0163)	(0.0030)	(0.0135)		
Σ_f	U.S. Stocks	0.0347	0.0034	0.0254		
		(0.0016)	(0.0002)	(0.0011)		
	Fixed Income		0.0072	0.0020		
			(0.0003)	(0.0001)		
	Foreign Stocks			0.0360		
				(0.0013)		
Alternative Assets						
β_a	Private Equity	1.5642	0.6873	-0.4657		
		(0.0787)	(0.0635)	(0.0542)		
	Hedge Funds	0.1802	0.1127	0.0361		
		(0.0269)	(0.0251)	(0.0227)		
Σ_a	Private Equity				0.0565	0.0057
					(0.0042)	(0.0004)
	Hedge Funds				0.0026	
					(0.0002)	
Observation Errors						
$\Sigma_{\pi f}$		0.0809	0.0424	0.0289		
		(0.0036)	(0.0019)	(0.0013)		
$\Sigma_{\pi a}$					0.0208	0.0590
					(0.0009)	(0.0026)

Table 6: Model-Implied Asset Allocations

The table reports the posterior distribution of the optimal allocations implied by the model. We report the cross-sectional average weight and the cross-sectional standard deviation in parentheses.

Year	2006	2007	2008	2009	2010	2011	2012	Average
Cash	6.82	6.82	6.39	6.05	4.99	5.42	5.46	5.99
	(2.83)	(2.8)	(2.74)	(2.52)	(2.63)	(2.66)	(2.65)	(2.69)
U.S. stocks	42.18	38.75	35.63	35.68	35.11	33.37	32.67	36.20
	(9.61)	(9.71)	(9.78)	(9.94)	(9.99)	(9.86)	(10.01)	(9.84)
Fixed income	23.19	21.61	20.15	20.15	19.84	19.06	18.73	20.39
	(4.42)	(4.47)	(4.51)	(4.61)	(4.63)	(4.56)	(4.63)	(4.54)
Foreign stocks	13.51	14.31	14.99	14.91	14.93	15.40	15.56	14.80
	(2.62)	(2.62)	(2.6)	(2.51)	(2.56)	(2.56)	(2.57)	(2.58)
Private equity	3.85	5.76	7.44	7.33	7.50	8.55	8.93	7.05
	(5.76)	(5.78)	(5.79)	(5.72)	(5.79)	(5.75)	(5.81)	(5.77)
Hedge funds	10.47	12.75	15.39	15.89	17.63	18.21	18.65	15.57
	(8.39)	(8.49)	(8.55)	(8.75)	(8.8)	(8.66)	(8.8)	(8.63)

Table 7: Trading Off Time-Series Returns and Cross-Sectional Asset Holdings

We report the posterior mean and standard deviation of private equity and hedge fund alphas for different priors, indexed by ν. The case of $\nu = T_f/(T_f + T_i) = 0.19$ corresponds to an uninformative prior, where T_f is the length of the returns data and T_i is the number of cross sections of endowment allocations. The case of $\nu = 0$ corresponds to placing all weight on return time series, and the case of $\nu = 1$ corresponds to using only asset allocations.

		2006	2007	2008	2009	2010	2011	2012
OLS	Private Equity	0.0054						
		(0.0104)						
	Hedge Fund	0.0153						
		(0.0029)						
$\nu = 0.19$	Private Equity	0.0139	0.0230	0.0311	0.0311	0.0325	0.0370	0.0389
		(0.0024)	(0.0023)	(0.0029)	(0.0024)	(0.0026)	(0.0028)	(0.0030)
	Hedge Fund	0.0029	0.0041	0.0053	0.0054	0.0058	0.0064	0.0066
		(0.0004)	(0.0004)	(0.0006)	(0.0005)	(0.0006)	(0.0006)	(0.0006)
$\nu = 0.50$	Private Equity	0.0002	0.0077	0.0152	0.0147	0.0164	0.0198	0.0212
		(0.0028)	(0.0023)	(0.0025)	(0.0023)	(0.0026)	(0.0032)	(0.0033)
	Hedge Fund	0.0010	0.0020	0.0031	0.0030	0.0034	0.0038	0.0040
		(0.0028)	(0.0023)	(0.0025)	(0.0023)	(0.0026)	(0.0032)	(0.0033)
$\nu = 1.00$	Private Equity	0.0093	0.0139	0.0175	0.0173	0.0179	0.0202	0.0206
		(0.0013)	(0.0015)	(0.0016)	(0.0018)	(0.0017)	(0.0017)	(0.0017)
	Hedge Fund	0.0017	0.0023	0.0028	0.0028	0.0030	0.0033	0.0033
		(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)

Table 8: Robustness

Different robustness checks are considered. Instead of using the full sample of returns, we consider the subsample starting from 1970. We use only the allocations of endowments belonging to Ivy league universities plus MIT and Stanford University. We Winsorize funds' characteristics at the 5th and 95th percentiles. We change the indices representing private equities and hedge funds returns to the Cambridge Associates Private Equity Index and the Dow Jones Credit Suisse Hedge Fund Index, respectively. We exclude all endowments with zero weights in alternative assets. We collapse both asset classes into one alternative investment class. We report the posterior mean and standard deviation of average risk aversion γ_0, and the private equity and hedge fund alphas, α_0. All alphas are annualized.

		Alphas α_0					
		2006		2009		2012	
	γ_0	Priv. Eq.	HF	Priv. Eq.	HF	Priv. Eq.	HF
Subsample 1970-2012	11.12	0.0204	0.0043	0.0451	0.0081	0.0551	0.0097
	(0.97)	(0.0046)	(0.0006)	(0.0068)	(0.0009)	(0.0079)	(0.001)
Ivy League Plus	5.47	0.0357	0.0075	0.0512	0.0100	0.0485	0.0090
	(0.64)	(0.0046)	(0.0009)	(0.0058)	(0.0011)	(0.0055)	(0.001)
Winsorized Characteristics	7.04	0.0120	0.0026	0.0276	0.0050	0.0339	0.0060
	(0.63)	(0.0017)	(0.0003)	(0.0026)	(0.0004)	(0.0033)	(0.0005)
Cambridge Private Equity	6.65	0.0061	0.0051	0.0113	0.0092	0.0138	0.0110
	(0.72)	(0.0013)	(0.0008)	(0.0023)	(0.0012)	(0.0027)	(0.0014)
DJ Credit Suisse HF Index	7.14	0.0113	0.0038	0.0244	0.0066	0.0301	0.0078
	(0.65)	(0.0024)	(0.0006)	(0.003)	(0.0009)	(0.0035)	(0.001)
No Alternative Holdings	7.17	0.0082	0.0025	0.0235	0.0049	0.0290	0.0057
	(0.63)	(0.0023)	(0.0004)	(0.0031)	(0.0006)	(0.0036)	(0.0006)
One Alternative Asset Class	5.83	0.0060		0.0111		0.0133	
	(0.77)	(0.0012)		(0.0018)		(0.0022)	

Figure 1: Endowment Asset Allocations

Endowments average asset allocations to U.S. Stocks and Alternative investments from 2006 through 2012.

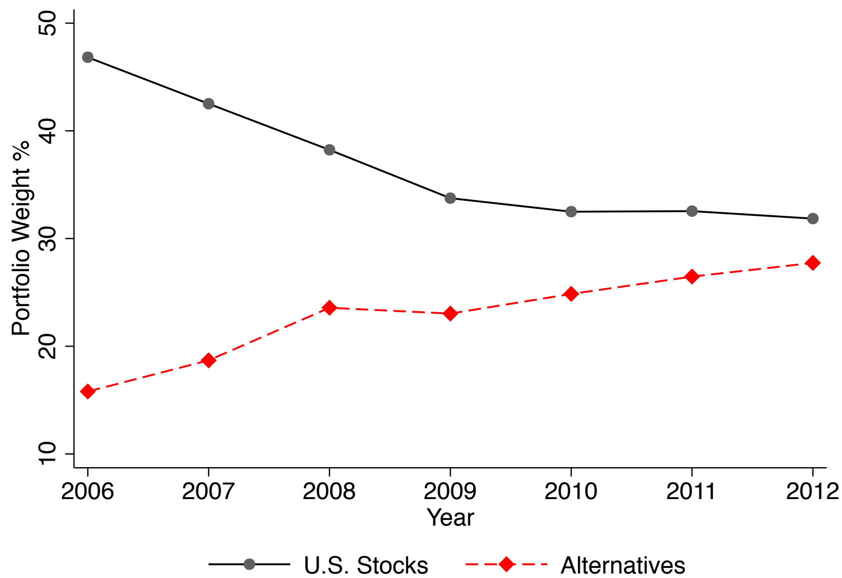


Figure 2: Endowments' Alpha Beliefs for Alternatives

The figure shows the average view on the level of mispricing for private equity (Panel A) and hedge funds (Panel B). We plot the posterior mean of α over time in the solid line, along with 5% and 95% percentiles of the posterior distribution in dotted lines. The dashed line with horizontal triangles represents the estimated alpha from time-series regressions. All numbers are annualized and are in percent.

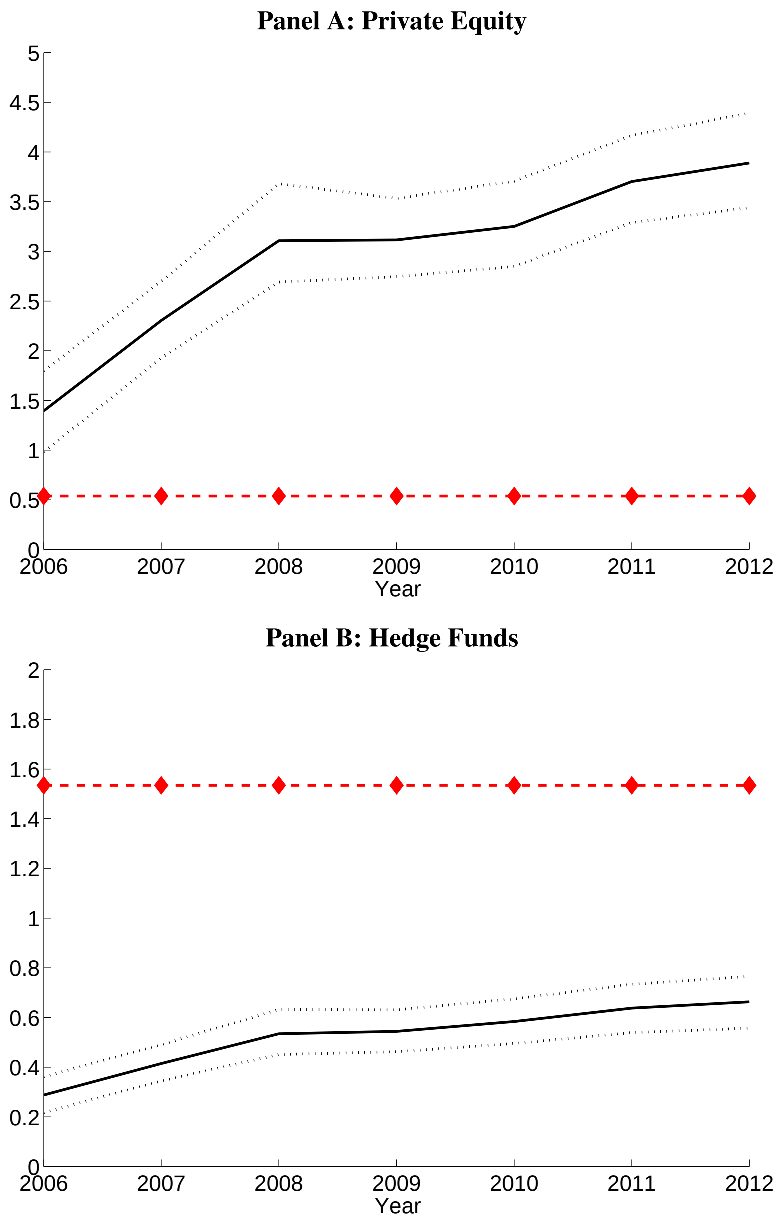


Figure 3: Endowments' Dispersion of Risk Aversion and Alternative Alphas

The figure plots the model-implied cross-sectional distribution of the risk aversion coefficient (top graph), the view private equity's alpha (middle graph), and the view on hedge funds' alpha (bottom graph).

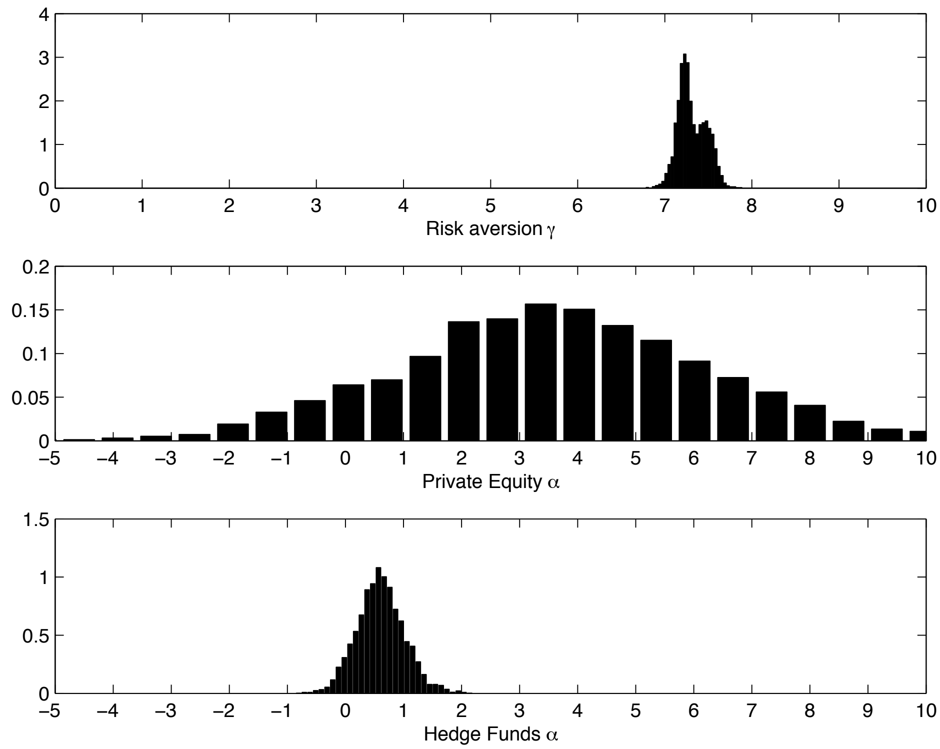


Figure 4: Endowments' Explicit and Implicit Weights on Factor Securities.

Using the estimated factor loadings of alternative investments, β_a, and the observed asset allocations, we compute the implicit weight on factor securities, $\tilde{\pi}_f = \pi_f + \beta_a^\top \pi_a$ for each endowment. Figures show kernel estimations of the (pooled) cross-sectional distribution of the explicit (observed) factor weights (solid black) and the implicit factor weight (dashed red).

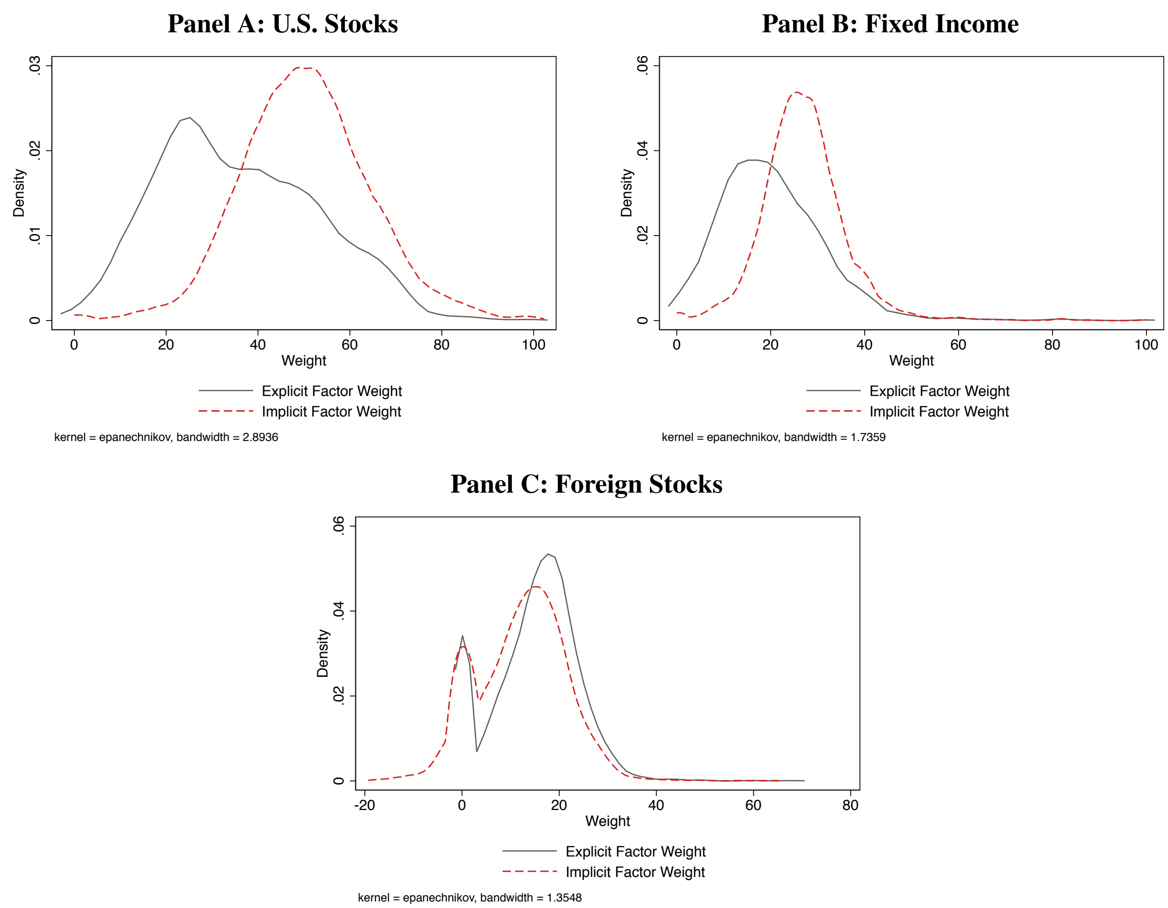
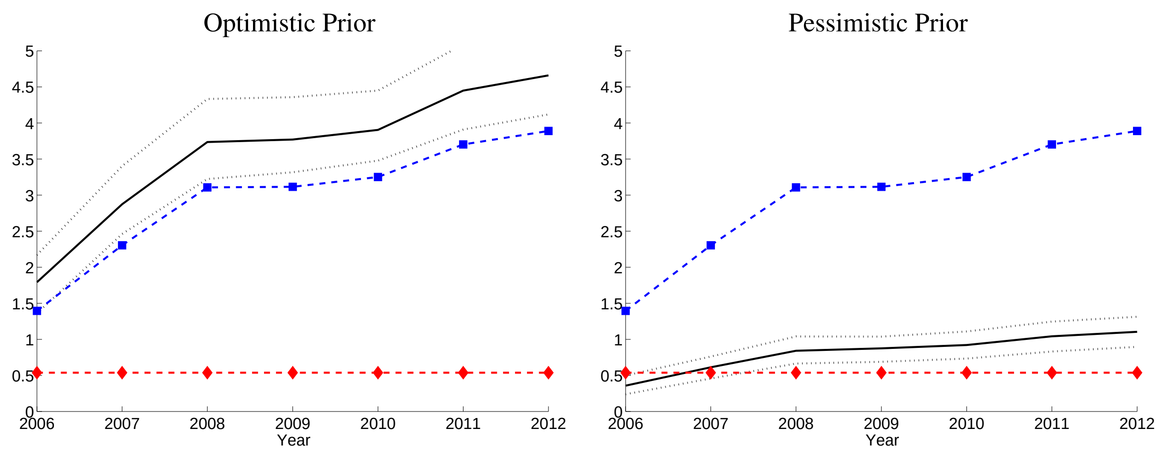


Figure 5: Alpha Beliefs Under Optimistic and Pessimistic Priors

We compute α_0 posterior distribution under two informative prior distributions. For the optimistic case, we assume a normal prior with an annualized mean of 4% and a standard deviation of 2%. For the pessimistic case, we assume a normal prior with a mean of -4% and a standard deviation of 2%. The solid line corresponds to the posterior distribution's mean, while the dotted lines are the distributions' 5th and 95th percentiles. The dashed lines linked by squares represent the posterior average from using a non-informative prior, which is the same as Figure 3.

Panel A: Private Equity



Panel B: Hedge Funds