Asset Pricing with Countercyclical Household Consumption Risk

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Abstract

We present evidence that shocks to household consumption growth are negatively skewed, persistent, and countercyclical and play a major role in driving asset prices. We construct a parsimonious model with one state variable that drives the conditional cross-sectional moments of household consumption growth. The estimated model provides a good fit for the moments of the cross-sectional distribution of household consumption growth and the unconditional moments of the risk free rate, equity premium, market price-dividend ratio, and aggregate dividend and consumption growth. The explanatory power of the model does not derive from possible predictability of aggregate dividend and consumption growth as these are intentionally modeled as *i.i.d*. processes. Consistent with empirical evidence, the model implies that the risk free rate and price-dividend ratio are pro-cyclical while the expected market return and the variance of the market return and risk free rate are countercyclical.

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Introduction

We present evidence that shocks to household consumption growth are negatively skewed, persistent, and countercyclical and play a major role in driving asset prices. We construct a parsimonious model with one state variable that drives the conditional cross-sectional moments of household consumption growth. The aggregate dividend and consumption growth are modeled as *i.i.d*. processes to emphasize that the explanatory power of the model does not derive from such predictability. The estimated model provides a good fit for the moments of the cross-sectional distribution of household consumption growth. The model matches well the unconditional mean, volatility, and autocorrelation of the risk free rate, thereby addressing the risk free rate puzzle. It provides a good fit for the unconditional mean, volatility, and auto-correlation of the market return, thereby addressing the equity premium and excess volatility puzzles. The model matches well the mean, volatility, and auto-correlation of the market price-dividend ratio and the aggregate dividend growth, targets that challenge a number of other models. Consistent with empirical evidence, the model implies that the risk free rate and price-dividend ratio are pro-cyclical while the expected market return and its variance and the equity premium are countercyclical. The model is also consistent with the salient features of aggregate dividend and consumption growth observed in the data: realistic mean and variance and lack of predictability.

Figure 1 displays the time series of the volatility and third central moment of the cross-sectional distribution of quarterly household consumption growth over the period 1982-2009. The third central moment is highly negative and countercyclical, with correlation -21.9% with NBER recessions. The counter-cyclical nature of the third central moment drives the observed low risk free rate and price-dividend ratio and the high equity premium in recessions. Hereafter, we refer to the third central moment as the *household consumption risk*. The cross-sectional volatility of the quarterly household consumption growth is countercyclical with correlation 10.1% with NBER recessions.

Shocks to household consumption growth are persistent and so are the estimated moments of the cross-sectional distribution of household consumption growth: the auto-correlation of the volatility is 77.1% and the auto-correlation of the third central moment is 11.2%. These long-run risks play a pivotal role in matching the data, given that the estimated model implies that households exhibit strong preference for early resolution of uncertainty, in the context of recursive preferences.

Finally, a methodological contribution of our paper is to demonstrate, under certain conditions, the existence of equilibrium in a heterogeneous agent economy with recursive preferences and obtain in closed form the risk free rate, expected market return, and price-dividend ratio as functions of the single state variable, the consumption risk.

The paper draws on several strands of the literature. It builds upon the empirical evidence by Attanasio and Davis (1996), Blundell, Pistaferri, and Preston (2008), Cochrane (1991), and Townsend (1994) that consumption insurance is incomplete. Constantinides (1982) highlighted the pivotal role of complete consumption insurance, showing that the equilibrium of such an economy with households with heterogeneous endowments and vonNeumann-Morgenstern preferences is isomorphic to the equilibrium of a homogeneous-household economy. Constantinides and Duffie (1996) further showed that, in the absence of complete consumption insurance, given the aggregate income and dividend processes, any given (arbitrage-free) price processes can be supported in the equilibrium of a heterogeneous household economy with judiciously chosen persistent idiosyncratic income shocks. Our paper provides empirical evidence that these shocks are persistent and drive asset prices and risk premia.

The paper draws also on Brav, Constantinides, and Geczy (2002) and Cogley (2002) who addressed the role of incomplete consumption insurance in determining risk premia in the context of economies in which households have power utility. Brav *et al.* presented empirical evidence that the equity and value premia are consistent in the 1982–1996 period with a stochastic discount factor (SDF) obtained as the average of individual households' marginal rates of substitution with low and economically plausible values of the relative risk aversion (RRA) coefficient. Since these premia are not explained with a stochastic discount factor obtained as the per capita marginal rate of substitution with low value of the RRA coefficient, the evidence supports the hypothesis of incomplete consumption insurance. Cogley (2002) calibrated a model with incomplete consumption insurance that recognizes the variance and skewness of the shocks to the households' consumption growth and obtained an annual equity premium of 4.5-5.75% with RRA coefficient of 15. Being couched in terms of economies with households endowed with power utility, neither of these papers allowed for the RRA coefficient and the elasticity of intertemporal substitution (EIS) to be disentangled or addressed the level and time-series properties of the risk free rate and price-dividend ratio. In contrast to these two papers, the present investigation disentangles the RRA coefficient and the EIS with recursive preferences and addresses the level and time series properties of the risk free rate; in addition, it addresses the level and time-series properties of the price-dividend ratio and the market return.

More to the point of the current investigation, Brav, Constantinides, and Geczy (2002) identified the pivotal role of the third central moment of the cross-sectional distribution of consumption growth in explaining the market and value premia. Specifically, they showed that a Taylor series expansion of the SDF up to cubic terms (thereby including the third central moment of the cross-sectional distribution) does a much better job in explaining the premia than an expansion up to quadratic terms which suppresses the third central moment. Guvenen, Ozkan, and Song (2012) also provided evidence regarding the importance of the (negative) third moment of the cross-section of individual income growth by analyzing the confidential earnings histories of millions of individuals over the period 1978-2010. They found that the skewness, but not the variance, of shocks is strongly countercyclical.

Finally, the paper relates to the literature on macroeconomic crises initiated by Rietz (1988) and revisited by Barro (2006) and others as an explanation of the equity premium and related puzzles.[1] This literature builds on domestic and international evidence that macroeconomic crises are associated with a large and sustained drop in aggregate consumption which increases the marginal rate of substitution of the representative consumer. Thus the basic mechanism of macroeconomic crises is similar in spirit to our paper in that the incidence of a large drop in the consumption of some or all households increases the marginal rate**s** of substitution of these households. The two classes of models part ways in their quantitative implications. As Constantinides (2008) pointed out, Barro (2006) finds it necessary to calibrate the model by treating the peak-to-trough drop in aggregate consumption during macroeconomic crises (which on average last four years) as if this drop occurred in one year, thereby magnifying by a factor of four the size of the observed annual disaster risks. Similar *ad hoc* magnification of the annual aggregate consumption drop during macroeconomic crises is relied upon in a number of papers that follow Barro (2006). In any case, Julliard and Ghosh (2012) empirically rejected the rare events explanation of the equity premium puzzle, showing that in order to explain the

[1] Related references include Backus, Chernov, and Martin (2011), Barro and Ursùa (2008), Constantinides (2008), Drechler and Yaron (2011), Gabaix (2012), Gourio (2008), Harvey and Siddique (2000), Julliard and Ghosh (2012), Nakamura, Steinsson, Barro, and Ursùa (2011), Veronesi (2004), and Wachter (2013).

puzzle with expected utility preferences of the representative agent and plausible RRA once the multi-year nature of disasters is correctly taken into account, one should be willing to believe that economic disasters should be happening every 6.6 years. Moreover, Backus, Chernov, and Martin (2011) demonstrated that options imply smaller probabilities of extreme outcomes than have been estimated from international macroeconomic data.

In contrast to these models, our model relies on shocks to *household* consumption growth, with frequency and annual size consistent with empirical observation. These shocks support the observed time-series properties of the risk free rate, market return, and market price-dividend ratio. Furthermore, the shocks to household consumption "average out" across households and do not imply unrealistically large annual shocks on aggregate consumption growth.

The paper is organized as follows. The model and its implications on consumption growth and prices are presented in Section 1. We discuss the data in Section 2. The empirical methodology and results are presented in Section 3. We conclude in Section 4. Derivations are relegated to the appendices.

1. The Model

We consider an exchange economy with a single nondurable consumption good serving as the numeraire. There is an arbitrary number of traded securities (for example, equities, corporate bonds, default free bonds, and derivatives) in positive or zero net supply. Conspicuously absent are markets for trading the households' wealth portfolios. A household's wealth portfolio is defined as a portfolio with dividend flow equal to the household's consumption flow. It is in this sense that the market is incomplete thereby preventing households from insuring their idiosyncratic income shocks. The sum total of traded securities in positive net supply is referred to as the "market". The market pays net dividend D_t at time *t*, has ex-dividend price P_t, and normalized supply of one unit. We assume that households are endowed with an equal number of market shares at time zero but can trade in these shares and all other securities (except the wealth portfolios) thereafter.

Aggregate consumption is denoted by C_t, log consumption by $c_t \equiv \log\left(C_t\right)$, and consumption growth by $\Delta c_{t+1} \equiv c_{t+1} - c_t$. We assume that aggregate consumption growth is *i.i.d* normal: $\Delta c_{t+1} = \mu + \sigma_a \varepsilon_{t+1}$, $\varepsilon_t \sim N\left(0,1\right)$.[2] By construction, aggregate consumption growth has zero auto-correlation, is unpredictable, and is uncorrelated with business cycles. We have also considered the case where the expected growth in aggregate consumption is a function of the state variable that tracks the business cycle and obtained similar results. We choose to present the case where the expected growth in aggregate consumption is uncorrelated with the business cycle in order to explore and highlight the role of the variability of household consumption risk along the business cycle. The aggregate labor income is defined as $I_t = C_t - D_t$.

There are an infinite number of distinct households and their number is normalized to be one. Household *i* is endowed with labor income $I_{i,t} = \delta_{i,t} C_t - D_t$ at date *t*, where

$$\delta_{i,t} = \exp\left\{ \sum_{s=1}^{t} \left(j_{i,s}^{1/2} \sigma \theta_{i,s} - j_{i,s} \sigma^2 / 2 \right) + \left(\hat{j}_{i,s}^{1/2} \hat{\sigma} \hat{\theta}_{i,s} - \hat{j}_{i,s} \hat{\sigma}^2 / 2 \right) \right\}. \qquad (1)$$

The exponent consists of two terms. The first term captures shocks to household income that are related to the business cycle, for example, the event of job loss by the prime wage-earner in the household. The business cycle is tracked by the single state variable in the economy, $\omega_t > 0$, that follows a Markov process to be specified below. The state variable drives the household income shocks through the random variable $j_{i,s}$ which is exponentially distributed with $prob\left(j_{i,s} = n\right) = e^{-\omega_s} \omega_s^n / n!, \quad n = 0,1,...\infty, \quad E\left(j_{i,s}\right) = \omega_s$, and independent of all primitive random variables in the economy. The term $\theta_{i,s} \sim N\left(0,1\right)$ and *i.i.d.* is a random variable independent of all primitive random variables in the economy. Thus the first term is the sum of variables, $j_{i,s}^{1/2} \sigma \theta_{i,s} - j_{i,s} \sigma^2 / 2$, which are normal, conditional on the realization of $j_{i,s}$. The volatility of the

[2] In both the theoretical development of the model and its empirical implementation, we limit our attention to "stockholders", the subset of households that are marginal investors in the stock market according to some cut-off criterion based on stock market holdings. Therefore, the aggregate labor income and consumption should be understood as those of the stockholders. Empirical evidence for the importance of this distinction is presented in Brav, Constantinides, and Geczy (2002), Mankiw and Zeldes (1991), and Vissing-Jorgensen (2002).

conditional normal variable is $j_{i,s}^{1/2}\sigma$ and is driven by the variable $j_{i,s}$ with distribution driven by the state variable.[3] [4] The second term captures shocks to household income that are unrelated to the business cycle, for example, the death of the prime wage-earner in the household. It is defined in a similar manner as the first term with the major difference that $\widehat{\omega}$ is a parameter instead of being a state variable.[5]

This particular specification of household income captures several key features of household income and consumption. First, since the income of the i^{th} household at date t is determined by the sum of all past idiosyncratic shocks, household income shocks are permanent, generally consistent with the empirical evidence that household income shocks are persistent. Second, the joint assumptions that the number of households is infinite and their income shocks are symmetric across households allow us to apply the law of large numbers and show that the identity $I_t = C_t - D_t$ is respected.[6] Third, this particular specification of household income, combined with the symmetric and homogeneous household preferences to be defined below, is shown to imply that households choose not to trade and household consumption is simply given by $C_{it} = I_{it} + D_t = \delta_{it} C_t$. Finally, the cross-sectional distribution of the relative household consumption growth, $\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)$, has negative third central moment. Its moments depend on the parameters of the distribution of $j_{i,s}$ which, in turn, are driven by the state variable. Hereafter we refer to the state variable as "household risk".

We assume that households have identical recursive preferences:

$$U_{i,t} = \left\{ (1-\delta)\left(C_{i,t}\right)^{1-1/\psi} + \delta\left(E\left[\left(U_{i,t+1}\right)^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}} \right\}^{1/(1-1/\psi)} \tag{2}$$

[3] The probability distribution of the random variable $j_{i,s}^{1/2}\sigma\theta_{i,s}$ is known as a Poisson mixture of normals. This distribution is tractable because it is normal, conditional on $j_{i,s}$.

[4] We also considered a variation of the model where σ is a second state variable but chose to proceed with the parsimonious model with a single state variable because the second state variable does not lead to a better fit of the model to the data.

[5] We also considered a variation of the model where $\widehat{\sigma}$ is a second state variable but chose to proceed with the parsimonious model with a single state variable because the second state variable does not lead to a better fit of the model to the data.

[6] The argument is due to Green (1989) and is elaborated in Appendix A.

where δ is the subjective discount factor, γ is the RRA coefficient, ψ is the EIS, and $\theta \equiv \frac{1-\gamma}{1-1/\psi}$.[7] As shown in Epstein and Zin (1989), the SDF of household *i* is

$$SDF_{i,t+1} = \exp\left(\theta \log \delta - \frac{\theta}{\psi} \Delta c_{i,t+1} + \left(\theta - 1\right) r_{i,c,t+1}\right) \tag{3}$$

where $\Delta c_{i,t+1} \equiv \log\left(C_{i,t+1}\right) - \log\left(C_{i,t}\right)$ and $r_{i,c,t+1}$ is the log return on the i^{th} household's private valuation of its wealth portfolio. The assumption of recursive preferences appears to be necessary: in all subperiods and data frequencies, the estimated value of the EIS is substantially higher than the inverse of the RRA coefficient.

We conjecture and verify that autarchy is an equilibrium. Autarchy implies that the consumption of household *i* at date *t* is $C_{i,t} = I_{i,t} + D_t = \delta_{i,t} C_t$ and household consumption growth $C_{i,t+1} / C_{i,t} = \delta_{i,t+1} C_{t+1} / \delta_{i,t} C_t$ is independent of the household's consumption level.[8] This, combined with the property that the household's utility is homogeneous of degree $1-1/\psi$ in the household's consumption level, implies that the return on the household's private valuation of its wealth portfolio is independent of the household's consumption level. The SDF of household *i* is independent of the household's consumption level; it is specific to household *i* only through the term $\delta_{i,t+1} / \delta_{i,t}$. In pricing any security, other than the households' wealth portfolios, the term $\delta_{i,t+1} / \delta_{i,t}$ is integrated out of the pricing equation and the private valuation of any security is common across households. This verifies the conjecture that autarchy is an equilibrium.[9] We formalize this argument in Appendix B.

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[7] Recursive preferences were introduced by Kreps and Porteus (1978) and adapted in the form used here by Epstein and Zin (1989) and Weil (1990).

[8] Essentially, we build into the model the assumption that the consumption growth of all households in a given period is independent of each household's consumption level. A richer model would allow for the consumption growth of each household in a given period to depend on the household's consumption level, consistent with the empirical findings of Guvenen, Ozkan, and Song (2012). Guvenen *et al.* analyzed the confidential earnings histories of millions of individuals over the period 1978-2010 and found that the earning power of the lowest income workers and the top 1% income workers erodes the most in recessions, compared to other workers.

[9] The interpretation of the model that there is no trade in equilibrium may be easily modified by assuming outright that $C_{i,t} = \delta_{i,t} C_t$ is the post-trade consumption of the i^{th} household.

The logarithm of the cross-sectional relative household consumption growth is

$$\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)=\delta_{i,t+1}-\delta_{i,t}=j_{i,s}^{1/2}\sigma\theta_{i,s}-j_{i,s}\sigma^2/2+\hat{j}_{i,s}^{1/2}\hat{\sigma}\hat{\theta}_{i,s}-\hat{j}_{i,s}\hat{\sigma}^2/2$$

with conditional central moments calculated in Appendix C as follows:

$$\mu_1\left(\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{it}/C_t}\right)\right)=-\sigma^2\omega_{t+1}/2-\hat{\sigma}^2\hat{\omega}/2 \tag{4}$$

$$\mu_2\left(\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{it}/C_t}\right)\right)=\left(\sigma^2+\sigma^4/4\right)\omega_{t+1}+\left(\hat{\sigma}^2+\hat{\sigma}^4/4\right)\hat{\omega} \tag{5}$$

and

$$\mu_3\left(\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)\right)=-\left(3\sigma^4/2+\sigma^6/8\right)\omega_{t+1}-\left(3\hat{\sigma}^4/2+\hat{\sigma}^6/8\right)\hat{\omega} \tag{6}$$

The variance of the cross-sectional relative household consumption growth increases and the third central moment becomes more negative as household risk increases. Therefore, we associate a high level of household risk with recessions.

For computational convenience, we define the variable x_t in terms of the state variable ω_t as $x_t \equiv \left(e^{\gamma(\gamma-1)\sigma^2/2}-1\right)\omega_t$. In our estimation and calibration, we limit the range of the RRA coefficient as $\gamma>1$ which implies that $x_t>0$. Since the mapping from ω_t to x_t is unique, we sometimes refer to x_t as the household risk, in place of ω_t. We assume the following dynamics for the household risk:

$$x_{t+1}=x_t+\kappa\left(\bar{x}-x_t\right)+\sigma_x\sqrt{x_t}\varepsilon_{x,t+1} \tag{7}$$

where $\varepsilon_{x,t+1} \sim N(0,1)$, *i.i.d.*, and independent of all primitive random variables; $\bar{x} > 0$; and $2\kappa\bar{x} > \sigma_x^2$.[10] The auto-correlation of household risk is $1-\kappa$. As we show later on, the interest rate, price-dividend ratio, and expected market return are affine functions of household risk and, therefore, their auto-correlation is $1-\kappa$ also.

The heteroskedasticity of the innovation of household risk implies that the volatility of household risk increases in recessions, $\operatorname{var}(x_{t+1} \mid x_t) = \sigma_x^2 x_t$. This property drives key features of the economy. As we shall see shortly, the model implies that the variances of the risk free rate, price-dividend ratio of the stock market, and expected market return increase in recessions.

In Appendix D, equation (D.4), we calculate the households' common SDF as

$$(SDF)_{t+1} = e^{\theta \log\delta + \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1\right) - \gamma\Delta c_{t+1} + (\theta-1)\left\{h_0 + h_1 A_0 - (A_0 + A_1 x_t)\right\} + \lambda x_{t+1}} \tag{8}$$

where the parameters h_0, h_1, A_0, A_1, and λ are defined in Appendix D by equations (D.2), (D.3), and (D.5).

The log risk free rate is calculated in Appendix D, equation (D.6), as

$$\begin{aligned} r_t = &-\theta\log\delta - \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1\right) - (\theta-1)(h_0 + h_1 A_0 - A_0) + \gamma\mu - \gamma^2\sigma_a^2/2 - \lambda\kappa\bar{x} \\ &- \left\{\lambda(1-\kappa) + \lambda^2\sigma_x^2/2 - (\theta-1)A_1\right\}x_t \end{aligned} \tag{9}$$

In recessions, the conditional variance of household risk is high. Thus the model implies that, in recessions, the variance of the risk free rate is high. The model also implies that the risk free rate is low in recessions since in the estimated model the coefficient of x_t in equation (9) is negative. Both of these implications are consistent with observation. Finally, the unconditional mean of the risk free rate is

[10] The Feller condition $2\kappa\bar{x} > \sigma_x^2$ decreases the probability that the state variable takes negative values. In the continuous-time limit of equation (7), the square-root process $dx(t) = \kappa\left(\bar{x} - x(t)\right)dt + \sigma_x\sqrt{x(t)}dW(t)$, the Feller condition guarantees that the state variable is strictly positive.

$$\overline{r_t} = -\theta \log \delta - \widehat{\omega}\left(e^{\gamma(\gamma+1)\widehat{\sigma}^2/2} - 1\right) - (\theta-1)\left(h_0 + h_1 A_0 - A_0\right) + \gamma\mu - \gamma^2\sigma_a^2/2 - \lambda\kappa\overline{x}$$
$$-\left\{\lambda(1-\kappa) + \lambda^2\sigma_x^2/2 - (\theta-1)A_1\right\}\overline{x} \qquad (10)$$

and its unconditional variance is

$$\operatorname{var}(r_t) = \left\{\lambda(1-\kappa) + \lambda^2\sigma_x^2/2 - (\theta-1)A_1\right\}^2 \frac{\sigma_x^2\overline{x}}{2\kappa-\kappa^2} \qquad (11)$$

We assume that the log dividend growth of the *stock* market follows the process[11]

$$\Delta d_{t+1} = \mu_d + \sigma_d \varepsilon_{d.t+1} \qquad (12)$$

where $\varepsilon_{d,t+1} \sim N(0,1)$ is *i.i.d.* and independent of all primitive random variables. By construction, dividend growth has zero auto-correlation, is unpredictable, and is uncorrelated with the business cycle. We have also considered the case where the expected growth in aggregate dividend is a function of the state variable that tracks the business cycle and obtained similar results. We choose to present the case where the expected growth in aggregate dividend is uncorrelated with the business cycle in order to explore and highlight the role of the variability of household consumption risk along the business cycle. Note also that aggregate consumption and dividend are not co-integrated. We also considered a co-integrated version of the model and obtained similar results.

In Appendix D, equation (D.8), we calculate the price-dividend ratio as

$$z_{m,t} = B_0 + B_1 x_t \qquad (13)$$

the expected stock market return (equation (D.11)) as

[11] We draw a distinction between the stock market and the "market" which we defined earlier as the sum total of all assets in the economy. Δd_{t+1} is the log dividend growth of the stock market.

$$E\left[r_{m,t+1} \mid \omega_t \right] = k_0 + k_1 B_0 + k_1 B_1 \kappa \bar{x} - B_0 + \mu_d + \left\{ k_1 B_1 \left(1-\kappa\right) - B_1 \right\} x_t \tag{14}$$

and the unconditional variance of the stock market return (equation (D.12)) as

$$\mathrm{var}\left(r_{m,t+1} \right) = k_1^2 B_1^2 \frac{\sigma_x^2 \bar{x}}{2\kappa - \kappa^2} + \sigma_d^2 / 2 \tag{15}$$

where the parameters are determined in Appendix D.

The model implies that, in recessions, the variances of the price-dividend ratio of the stock market and its expected return are high. In the estimated model, the coefficient of x_t in equation (13) is negative, implying that the price-dividend ratio of the stock market is low in recessions. Finally, the coefficient of x_t in equation (14) is positive, implying that the expected return of the stock market is high in recessions. All these implications are consistent with observation.

2. Data Description

2.1 Prices and dividends

We use monthly data on prices and dividends from January 1929 through December 2012. The proxy for the market is the Centre for Research in Security Prices (CRSP) value-weighted index of all stocks on the NYSE, AMEX, and NASDAQ. The monthly portfolio return is the sum of the portfolio price and dividends at the end of the month, divided by the portfolio price at the beginning of the month. The annual portfolio return is the sum of the portfolio price at the end of the year and uncompounded dividends over the year, divided by the portfolio price at the beginning of the year. The real annual portfolio return is the above annual portfolio return deflated by the realized growth in the Consumer Price Index.

The proxy for the real annual risk free rate is obtained as in Beeler and Campbell (2012). Specifically, the quarterly nominal yield on 3-month Treasury Bills is deflated using the realized growth in the Consumer Price Index to obtain the ex post real 3-month T-Bill rate. The ex-ante quarterly risk free rate is then obtained as the fitted value from the regression of the ex post 3-month T-Bill rate on the 3-month nominal yield and the realized growth in the Consumer Price Index over the previous year. Finally, the ex-ante quarterly risk free rate at the beginning of the year is annualized to obtain the ex-ante annual risk free rate.

The annual price-dividend ratio of the market is the market price at the end of the year, divided by the sum of dividends over the previous twelve months. The dividend growth rate is the sum of dividends over the year, divided by the sum of dividends over the previous year and is deflated using the realized growth in the Consumer Price Index.

2.2 Household consumption data[12]

The household-level quarterly consumption data is obtained from the Consumer Expenditure Survey (CEX) produced by the Bureau of Labor Statistics (BLS). This series of cross-sections covers the period 1980:Q1-2011:Q4. Each quarter, roughly 5,000 U.S. households are surveyed, chosen randomly according to stratification criteria determined by the U.S. Census. Each household participates in the survey for five consecutive quarters, one training quarter and four regular ones, during which their recent consumption and other information is recorded. At the end of its fifth quarter, another household, chosen randomly according to stratification criteria determined by the U.S. Census, replaces the household. The cycle of the households is staggered uniformly across the quarters, such that new households replace approximately one-fifth of the participating households each quarter.[13,14] If a household moves away from the sample address, it is dropped from the survey. The new household that moves into this address is screened for eligibility and is included in the survey.

The number of households in the database varies from quarter to quarter. The survey attempts to account for an estimated 95% of all quarterly household expenditures in each

[12] Our description and filters of the household consumption data closely follows Brav, Constantinides, and Geczy (2002).
[13] If we were to exclude the training quarter in classifying a household as being in the panel, then each household would stay in the panel for *four* quarters and new households would replace *one-fourth* of the participating households each quarter.
[14] The constant rotation of the panel makes it impossible to test hypotheses regarding a specific household's behavior through time for more than four quarters. A longer time series of individual households' consumption is available from the PSID database, albeit only on *food* consumption.

consumption category from a highly disaggregated list of consumption goods and services. At the end of the fourth regular quarter, data is also collected on the demographics and financial profiles of the households, including the value of asset holdings as of the month preceding the interview. We use consumption data only from the regular quarters, as we consider the data from the training quarter unreliable. In a significant number of years, the BLS failed to survey households not located near an urban area. Therefore, we consider only urban households.

The CEX survey reports are categorized in three tranches that we term the *January*, *February*, and *March* tranches. For a given year, the first quarter consumption of the January tranche corresponds to consumption over January, February, and March; for the February tranche, first quarter consumption corresponds to consumption over February, March, and April; for the March tranche, first quarter consumption corresponds to consumption over, March, April, and May; and so on for the second, third, and fourth quarter consumption. Whereas the CEX consumption data are presented on a monthly frequency for some consumption categories, the numbers reported as monthly are simply quarterly estimates divided by three.[15] Thus, utilizing monthly consumption is not an option.

Following Attanasio and Weber (1995), we discard from our sample the consumption data for the years 1980 and 1981 because they are of questionable quality. Starting in interview period 1986:Q1, the BLS changed its household identification numbering system without providing the correspondence between the 1985:Q4 and 1986:Q1 identification numbers of households interviewed in both quarters. This change in the identification system makes it impossible to match households across the 1985:Q4 - 1986:Q1 gap and results in the loss of some observations. This problem recurs between 1996:Q1 and 1997:Q1.

2.3 Definition of the household consumption variables

For each tranche, we calculate each household's quarterly *nondurables and services* (NDS) consumption by aggregating the household's quarterly consumption across the consumption categories that comprise the definition of nondurables and services. We use consumption categories that adhere to the National Income and Product Accounts (NIPA) classification of

[15] See Attanasio and Weber (1995) and Souleles (1999) for further details regarding the database.

NDS consumption. Since the quantity of interest to us is the *relative* household consumption growth, $\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)$, it is unnecessary to either deflate or seasonally adjust consumption.

The *per capita* consumption of a set of households is calculated as follows. First, the *total consumption* in a given quarter is obtained by summing the nondurables and services consumption of all the households in that quarter. Second, the *per capita consumption* in a given quarter is obtained by dividing the total consumption in that quarter by the sum of the number of family members across all the households in that quarter. The *per capita* consumption *growth* between quarters *t* - 1 and *t* is defined as the *ratio* of the *per capita* consumption in quarters *t* and *t* - 1.

2.4 Household selection criteria

In any given quarter, we delete from the sample households that report in that quarter as zero either their total consumption, or their consumption of nondurables and services, or their food consumption. In any given quarter, we also delete from the sample households with missing information on the above items.

We define a household's beginning *total assets* as the sum of the household's market value of stocks, bonds, mutual funds, and other securities *at the beginning of the first regular quarter*.[16] We define as *asset holders* the households that report total assets exceeding a certain threshold. We present results for threshold values ranging from $0 to $20,000 in 1996:Q1 dollars. The number of households that are included as asset holders in our sample varies across quarters and across thresholds.

We mitigate observation error by subjecting the households to a *consumption growth filter*. The filter consists of the following selection criteria. First, we delete from the sample households with consumption growth reported in fewer than three consecutive quarters. Second, we delete the consumption growth rates $C_{i,t}/C_{i,t-1}$ and $C_{i,t+1}/C_{i,t}$, if $C_{i,t}/C_{i,t-1} < 1/2$ *and* $C_{i,t+1}/C_{i,t} > 2$, and vice versa. Third, we delete the consumption growth $C_{i,t}/C_{i,t-1}$, if it is

[16] During the fifth and last interview, the household is asked to report both the end-of-period asset holdings and the change of these asset holdings relative to a year earlier. From this, we calculate the household's asset holdings at the beginning of the first regular quarter.

greater than five. The surviving sub-sample of households is substantially smaller than the original one.

2.5 Household consumption statistics

In Table 1, we present summary statistics of the moments of the cross-sectional quarterly relative household consumption growth, $\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)$, over the period 1982:Q1-2009:Q4, in 1996:Q1 dollars. The results are largely similar across the January, February, and March tranches. The relatively mild filter on households with assets exceeding $2,000 eliminates about 80% of the households and stricter filters further eliminate households to the point that statistics with a small number of households become unreliable. The sample mean, μ_1, is practically zero across asset levels and tranches, as expected. The sample volatility, $\mu_2^{1/2}$, is fairly constant across asset levels and tranches and is highly auto-correlated. The sample third central moment, μ_3, is negative, as expected, across tranches when asset filters are not imposed, but becomes statistically insignificant when asset filters reduce the sample size. The sign of the auto-correlation of the third central moment varies across asset levels and tranches and we attribute this to the noisy estimate of the third central moment.

At each quarter, an indicator variable, I_{rec}, takes the value of one if there is an NBER-designated recession in any of the three months of the quarter. In Table 2, we present the correlation of the cross-sectional volatility and third central moment with NBER-designated recessions. In recessions, volatility increases and the third central moment becomes more negative, as expected.

3. Empirical Methodology and Results

3.1 Empirical methodology

The model has thirteen parameters: the mean, μ, and volatility, σ_a, of aggregate consumption growth; the three parameters of the household income shocks, $\sigma, \hat{\sigma},$ and $\hat{\omega}$; the three parameters of the dynamics of the state variable, $\bar{x}, \kappa,$ and σ_x; the mean, μ_d, and volatility, σ_d, of aggregate dividend growth; and the three preference parameters: the subjective discount factor, δ, the RRA coefficient, γ, and the elasticity of intertemporal substitution, ψ. We reduce the number of parameters to twelve by setting $\hat{\sigma} = \sigma$. We estimate the twelve model parameters using GMM to match the following twelve moments: the mean and variance of aggregate consumption growth, dividend growth, and market return; and the mean, variance, and autocorrelation of the risk free rate and market-wide price-dividend ratio. We use a diagonal weighting matrix with a weight of one on all the moments except for the unconditional means of the market return and risk free rate that have weights of 100.[17]

3.2 Results with annual data 1929-2009

We first present results at the annual frequency for the entire available sample period 1929-2009. The parsimonious model with just one state variable fits the sample moments of the risk free rate, market return, and price-dividend ratio very well. The model fit and parameter estimates are presented in Table 3. The *J*-stat is 8.82 and the model is not rejected at the 10% level of significance. The asymptotic 90% critical value is 10.1.

The model generates mean risk free rate close to zero and stock market return 5.5%, both very close to their sample values of 0.6% and 6.2%, respectively. Therefore, the model provides an explanation of the equity premium and risk free rate puzzles. The model generates volatility

[17] The pre-specified weighting matric has two advantages over the efficient weighting matrix. First, it has superior small-sample properties (see e.g., Ahn and Gadarowski (1999), Ferson and Foerster (1994), and Hansen, Heaton, and Yaron (1996)). Second, the moment restrictions included in the GMM have different orders of magnitude, with the mean of the price-dividend ratio being a couple of orders of magnitude larger than the means of the market return and risk free rate. Therefore, placing larger weights on the latter two moments enables the GMM procedure to put equal emphasis in matching all these moments. We repeated our estimation using the efficient weighting matrix and obtained similar results that are available upon request.

2.5% and first-order autocorrelation 0.904 of the risk free rate, close to the sample values of 3% and 0.672, respectively. The model also generates volatility 21.2% of the market return, close to its sample value of 19.8%. The model-implied mean of the market-wide price-dividend ratio is 3.326, very close to its sample value of 3.377. More importantly, the model generates the high volatility of the price-dividend ratio observed in the data (34.6% versus 45%), thereby explaining the excess volatility puzzle. Note that most asset pricing models, including those with long run risks and rare disasters, have difficulty in matching the latter moment and, therefore, at explaining the high volatility of stock prices (see e.g., Beeler and Campbell (2012) and Constantinides and Ghosh (2011)). The model-implied first-order autocorrelation of the market-wide price-dividend ratio is 0.904, very close to its sample value of 0.877.

The model is calibrated to match exactly the unconditional mean and volatility of the aggregate consumption growth rate. Note that models that rely on the incidence of shocks to aggregate, as opposed to household, consumption growth in order to explain the equity premium and excess volatility puzzles require unrealistically high variance of the aggregate consumption growth: the Barro (2006) rare disasters model implies an aggregate consumption growth volatility of 4.6%). By contrast, the incidence of shocks to household consumption growth, as modeled in our paper, does not affect the volatility of the aggregate consumption growth.

The model generates 2% mean and 15% volatility of the aggregate dividend growth rate, compared to their sample counterparts of 1% and 11.7%, respectively. The sample autocorrelation of the aggregate dividend growth rate is 16.3%. By construction, the autocorrelation in our model is zero, consistent with the broader evidence that dividend growth is unpredictable. This contrasts with long run risks models that rely on implausibly high levels of persistence in the dividend growth process.

The model also generates the empirically observed dynamics of the risk free rate, price-dividend ratio, and stock market return. Recall that high values of the household consumption risk imply that the variance of the cross-sectional distribution of household-level consumption growth relative to per capita aggregate consumption growth is high and the third central moment is very negative. Therefore, high values of the household consumption risk are associated with recessions. Since the volatility of the household consumption risk is high when the household consumption risk is high and since the risk free rate, price-dividend ratio, and the conditional expected market return are affine functions of the household consumption risk, the model implies that the volatilities of the risk free rate, price-dividend ratio, and the conditional expected market return are countercyclical, consistent with observation.

We use the point estimates of the model parameters in Table 3 to calculate the sign of the coefficients of the household consumption risk in the equations that determine the risk free rate, price-dividend ratio and the conditional expected market return: $r_{f,t} = .024 - .465\, x_t$, $z_{m,t} = 3.66 - 6.46\, x_t$, and $E\left[r_{m,t+1} \mid x_t \right] = .012 + .822\, x_t$. Consistent with empirical evidence, the model implies that the risk free rate and price-dividend ratio are pro-cyclical while the expected market return is countercyclical.

The estimated preference parameters are reasonable: the risk aversion coefficient is 6.47 and the EIS is close to one. The EIS is much higher than the inverse of the risk aversion coefficient, thereby highlighting the importance of recursive preferences and pointing towards strong preference for early resolution of uncertainty.

The parameters $\kappa, \overline{x}$, and σ_x govern household risk. The auto-correlation of household risk is $1-\kappa = 0.904$ and this renders the auto-correlation of the interest rate and price-dividend ratio to be 0.904 also, close to their sample values. The parameters $\overline{x}$ and σ_x govern the variance of household risk and render the variance of the interest rate, expected market return, and price-dividend ratio close to their sample values.

3.3 Results with quarterly data 1947:Q1-2009:Q4

We re-estimate the model using quarterly data over the sub-period 1947:Q1-2009:Q4, the period over which quarterly consumption data is available. The model fit and parameter estimates are presented in Table 4. The reported returns and growth rates are quarterly. The *J*-stat is 16.27 and the model is not rejected at the 10% level of significance. The asymptotic 90% critical value is 37.97. The model matches well the moments of the risk free rate, stock market return, and price-dividend ratio, except that it generates a slightly higher value of the mean market return (2.5%) than its sample value (1.7%).

The model generates the empirically observed dynamics of the risk free rate, price-dividend ratio, and stock market return. The model implies that the volatilities of the risk free rate, price-dividend ratio, and the conditional expected market return are countercyclical, consistent with observation. We use the point estimates of the model parameters in Table 4 to calculate the sign of the coefficients of the household consumption risk in the equations that determine the risk free rate, price-dividend ratio and the conditional expected market return: $r_{f,t} = .011 - .163\, x_t$, $z_{m,t} = 4.34 - 9.88\, x_t$, and $E\left[r_{m,t+1} \mid x_t \right] = .009 + .256 x_t$. Consistent with empirical evidence, the model implies that the risk free rate and price-dividend ratio are pro-cyclical while the expected market return is countercyclical. The estimated preference parameters are reasonable: the risk aversion coefficient is 14.47 and the EIS is close to one.

Overall, we conclude that the model is more naturally interpreted at the annual rather than quarterly frequency. However, we presented the estimation and fit of the model interpreted at the quarterly frequency because data on relative household consumption growth is available only at the quarterly frequency. In the next section, we discuss the implications of the quarterly model regarding the unconditional moments of the relative household consumption growth.

3.4 Results with quarterly data 1982:Q1-2009:Q4

Data on relative household consumption growth is available only at the quarterly frequency over the period 1982:Q1-2012:Q4. We re-estimate the model at the quarterly frequency over this sub-period 1982:Q1-2009:Q4 in order to test the fit of the model-generated unconditional moments of the cross-sectional distribution of relative quarterly household consumption growth to their empirical counterparts. The model fit and parameter estimates are presented in Table 5.

The *J*-stat is 10.11 and the model is not rejected at the 5% level of significance. The asymptotic 95% critical value is 13.20. The model matches well the moments of the risk free rate, stock market return, and price-dividend ratio, except that it generates a slightly higher value of the mean market return (2.4%) than its sample value (1.9%) and a lower value of the mean risk free rate (-1.8%) than its sample value (.005%). The model generates the empirically observed dynamics of the risk free rate, price-dividend ratio, and stock market return. The model implies that the volatilities of the risk free rate, price-dividend ratio, and the conditional expected market return are countercyclical; the risk free rate and price-dividend ratio are pro-cyclical; and the expected market return is countercyclical. The estimated preference parameters are reasonable: the risk aversion coefficient is 1.20 and the EIS is close to one.

More to the point, the model matches very well the third central moment (but lesser so the volatility) of the cross-sectional distribution of household consumption growth, thereby providing the punch line: *we explain the time-series properties of the targeted financial variables with the third central moment of the cross-sectional distribution of household consumption growth comparable to its sample counterpart*.

Recall that the cross-sectional variance of the relative household consumption growth is given in equation (5) as $\mu_2\left(\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{it}/C_t}\right)\right)=\left(\sigma^2+\sigma^4/4\right)\omega_{t+1}+\left(\hat{\sigma}^2+\hat{\sigma}^4/4\right)\hat{\omega}$. The first component is driven by the state variable and, therefore, by the business cycle. The second component is driven by shocks to household income unrelated to the business cycle, for example, the death of the primary wage earner in the household. Given the parameter estimates in Table 5, we calculate the relative importance of the first component as $\left(\sigma^2+\sigma^4/4\right)E\left[\omega_t\right]/\left\{\left(\sigma^2+\sigma^4/4\right)E\left[\omega_t\right]+\left(\hat{\sigma}^2+\hat{\sigma}^4/4\right)\hat{\omega}\right\}=E\left[\omega_t\right]/\left\{E\left[\omega_t\right]+\hat{\omega}\right\}=0.094$, where $E\left[\omega_t\right]=\left(e^{\gamma(\gamma-1)\sigma^2/2}-1\right)^{-1}\bar{x}$ since we set $\hat{\sigma}=\sigma$. Likewise, the third central moment is given in equation (6) as $-\left(3\sigma^4/2+\sigma^6/8\right)\omega_{t+1}-\left(3\hat{\sigma}^4/2+\hat{\sigma}^6/8\right)\hat{\omega}$. The relative importance of the first component is $\left(3\sigma^4/2+\sigma^6/8\right)E\left[\omega_t\right]/\left\{\left(3\sigma^4/2+\sigma^6/8\right)E\left[\omega_t\right]+\left(3\hat{\sigma}^4/2+\hat{\sigma}^6/8\right)\hat{\omega}\right\}=0.094$. Only about one-tenth of the shocks to household income are related to the business cycle.[18]

5. Concluding Remarks

We explore the cross-sectional variation of household income shocks as a channel that drives the time series properties of the risk free rate, market return, and market price-dividend ratio. We focus on this channel by suppressing potential predictability of the aggregate consumption and

[18] Using the parameter estimates of the model interpreted at the annual frequency in Table 3, the relative importance of the first component is $E\left[\omega_t\right]/\left\{E\left[\omega_t\right]+\hat{\omega}\right\}=0.074$, very similar to the relative importance of the first component above. This is remarkable because the estimation of the model interpreted at the annual frequency does not even target the moments of the cross-sectional relative household consumption growth.

dividend growth rates and modeling them as *i.i.d*. processes. The model is parsimonious with only one state variable that is counter-cyclical and drives the cross-sectional distribution of household consumption growth. Despite this enforced parsimony, the model fits reasonably well both the unconditional and conditional price moments, particularly the moments of the market price-dividend ratio, a target that has eluded a number of other models. More to the point, the model-generated volatility and third central moment of the cross-sectional distribution of household consumption match very well their sample counterparts.

Appendix A: Proof that the identity $I_t = C_t - D_t$ is respected

Since the households are symmetric and their number is normalized to equal one, we apply the law of large numbers as in Green (1989) and claim that $I_t = E\left[I_{i,t} \mid C_t, D_t \right]$. Furthermore, since the household shocks are assumed to be conditionally normally distributed and independent of anything else in the economy, we obtain the following:

$$
\begin{aligned}
I_t &= E\left[I_{i,t} \mid C_t, D_t \right] \\
&= E\left[\exp\left(\sum_{s=1}^{t} j_{i,s}^{1/2} \sigma \theta_{i,s} - j_{i,s} \sigma^2 / 2 + \hat{j}_{i,s}^{1/2} \hat{\sigma} \hat{\theta}_{i,s} - \hat{j}_{i,s} \hat{\sigma}^2 / 2 \right) \right] C_t - D_t \\
&= E\left[E\left[\exp\left(\sum_{s=1}^{t} j_{i,s}^{1/2} \sigma \theta_{i,s} - j_{i,s} \sigma^2 / 2 + \hat{j}_{i,s}^{1/2} \hat{\sigma} \hat{\theta}_{i,s} - \hat{j}_{i,s} \hat{\sigma}^2 / 2 \right) \mid \left\{ j_{i,\tau}, \hat{j}_{i,\tau} \right\}_{\tau=1,\ldots,t} \right] \right] C_t - D_t \\
&= C_t - D_t
\end{aligned} \qquad \text{(A.1)}
$$

proving the claim.

Appendix B: Proof that autarchy is an equilibrium

We conjecture and verify that autarchy is an equilibrium. The proof follows several steps. First, we calculate the i^{th} household's private valuation of its wealth portfolio. Next we calculate the log return, $r_{i,c,t+1}$, on the i^{th} household's wealth portfolio and substitute this return in the household's SDF, as stated in equation (3). We integrate out of this SDF the household's idiosyncratic income shocks and show that households have common SDF. This implies that the private valuation of any security with given payoffs independent of the idiosyncratic income shocks is the same across households, thereby verifying the conjecture that autarchy is an equilibrium.

Let $P_{i,c,t}$ be the price of the i^{th} household's private valuation of its wealth portfolio, $Z_{i,c,t} \equiv P_{i,c,t} / C_{i,t}$, and $z_{i,c,t} \equiv \log\left(Z_{i,c,t}\right)$. We prove by induction that the price-to-consumption ratio is a function of only the state variable ω_t. We conjecture that $z_{i,c,t+1} = z_{c,t+1}\left(\omega_{t+1}\right)$. The Euler equation for $r_{i,c,t+1}$ is

$$E\left[e^{\theta \log \delta - \frac{\theta}{\psi}\Delta c_{i,t+1} + (\theta-1) r_{i,c,t+1} + r_{i,c,t+1}} \mid \Delta c_t, \omega_t, j_{i,t}, \theta_{i,t}, \hat{j}_{i,t}, \hat{\theta}_{i,t} \right] = 1 \tag{B.1}$$

We write

$$\begin{aligned} r_{i,c,t+1} &= \log\left(P_{i,c,t+1} + C_{i,t+1}\right) - \log P_{i,c,t+1} \\ &= \log\left(Z_{i,c,t+1} + 1\right) - \log\left(Z_{i,c,t}\right) + \log C_{i,t+1} - \log C_{i,t} \\ &= \log\left(e^{z_{c,t+1}} + 1\right) - z_{i,c,t} + \Delta c_{i,t+1} \end{aligned} \tag{B.2}$$

and substitute (B.2) in the Euler equation (B.1):

$$E\left[e^{\theta \log \delta - \frac{\theta}{\psi}\Delta c_{i,t+1} + \theta\left(\log\left(e^{z_{c,t+1}} + 1\right) - z_{i,c,t} + \Delta c_{i,t+1}\right)} \mid \Delta c_t, \omega_t, j_{i,t}, \theta_{i,t}, \hat{j}_{i,t}, \hat{\theta}_{i,t} \right] = 1$$

or

$$e^{\theta z_{i,c,t}} = E\left[e^{\theta \log \delta + (1-\gamma)\left(\mu + \sigma_a \varepsilon_{t+1} + j_{i,t+1}^{1/2} \sigma \theta_{i,t+1} - j_{i,t+1} \sigma^2 /2 + \hat{j}_{i,t+1}^{1/2} \hat{\sigma} \hat{\theta}_{i,t+1} - \hat{j}_{i,t+1} \hat{\sigma}^2 /2\right) + \theta \log\left(e^{z_{c.t+1}} + 1\right)} \mid \Delta c_t, \omega_t, j_{i,t}, \theta_{i,t}, \hat{j}_{i,t}, \hat{\theta}_{i,t} \right] \tag{B.3}$$

We integrate out of equation (B.3) the random variables $\varepsilon_{t+1}, \theta_{i,t+1}, j_{i,t+1}, \hat{\theta}_{i,t+1}$, and $\hat{j}_{i,t+1}$, leaving $z_{i,c,t}$ as a function of only ω_t, thereby proving the claim that $z_{i,c,t} = z_{c,t}(\omega_t)$.

The $(SDF)_{i,t+1}$ of the i^{th} household is

$$\begin{aligned}
(SDF)_{i,t+1} &= \exp\left(\theta \log \delta - \frac{\theta}{\psi} \Delta c_{i,t+1} + (\theta - 1) r_{i,c,t+1} \right) \\
&= \exp\left(\theta \log \delta - \gamma \left(\Delta c_{t+1} + j_{i,t+1}^{1/2} \sigma \theta_{i,t+1} - j_{i,t+1} \sigma^2 / 2 + \hat{j}_{i,t+1}^{1/2} \hat{\sigma} \hat{\theta}_{i,t+1} - \hat{j}_{i,t+1} \hat{\sigma}^2 / 2 \right) + (\theta - 1)\left(\log\left(e^{z_{c,t+1}} + 1\right) - z_{c,t} \right) \right)
\end{aligned} \tag{B.4}$$

In pricing any security, other than the households' wealth portfolios, we integrate out of the $(SDF)_{i,t+1}$ the household-specific random variables $\theta_{i,t+1}, j_{i,t+1}, \hat{\theta}_{i,t+1}$, and $\hat{j}_{i,t+1}$ and obtain a SDF common across households. Therefore, each household's private valuation of any security, *other than the households' wealth portfolios*, is common. This completes the proof that no-trade is an equilibrium.

Appendix C: Derivation of the cross-sectional moments of consumption growth

We use the following result:

$$e^{-\omega}\sum_{n=0}^{\infty} e^{kn}\omega^n / n! = e^{-\omega}\sum_{n=0}^{\infty}\left(e^k\omega\right)^n / n! = e^{-\omega}e^{e^k\omega} \tag{C.1}$$

Differentiating once, twice, and thrice with respect to *k* and setting $k = 0$ we obtain

$$\begin{aligned}
&e^{-\omega}\sum_{n=0}^{\infty} n\omega^n / n! = \omega \\
&e^{-\omega}\sum_{n=0}^{\infty} n^2\omega^n / n! = \omega^2 + \omega \\
&e^{-\omega}\sum_{n=0}^{\infty} n^3\omega^n / n! = \omega^3 + 3\omega^2 + \omega
\end{aligned} \tag{C.2}$$

We calculate the mean as follows:

$$\begin{aligned}
\mu_1 &= E\left[\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right) | \, \omega_{t+1}\right] \\
&= E\left[E\left[\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right) | \, j_{i,t+1}, \hat{j}_{i,t+1}\right] | \, \omega_{t+1}\right] \\
&= E\left[E\left[j_{i,t+1}^{1/2}\sigma\theta_{i,t+1} - j_{i,t+1}\sigma^2/2 + \hat{j}_{i,t+1}^{1/2}\hat{\sigma}\hat{\theta}_{i,t+1} - \hat{j}_{i,t+1}\hat{\sigma}^2/2 \, | \, j_{i,t+1}, \hat{j}_{i,t+1}\right] | \, \omega_{t+1}\right] \\
&= E\left[-j_{i,t+1}\sigma^2/2 - \hat{j}_{i,t+1}\hat{\sigma}^2/2 \, | \, \omega_{t+1}\right] \\
&= -\left(\sigma^2/2\right)\omega_{t+1} - \left(\hat{\sigma}^2/2\right)\hat{\omega}
\end{aligned} \tag{C.3}$$

We calculate the variance as follows:

$$
\begin{aligned}
\mu_2 &= \operatorname{var}\left(\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)\right) \\
&= \operatorname{var}\left(j_{i,t+1}^{1/2}\sigma\theta_{i,t+1} - j_{i,t+1}\sigma^2/2 + \hat{j}_{i,t+1}^{1/2}\hat{\sigma}\hat{\theta}_{i,t+1} - \hat{j}_{i,t+1}\hat{\sigma}^2/2\right) \\
&= \operatorname{var}\left(j_{i,t+1}^{1/2}\sigma\theta_{i,t+1} - j_{i,t+1}\sigma^2/2\right) + \operatorname{var}\left(\hat{j}_{i,t+1}^{1/2}\hat{\sigma}\hat{\theta}_{i,t+1} - \hat{j}_{i,t+1}\hat{\sigma}^2/2\right) \\
&= E\left[E\left[\left(j_{i,t+1}^{1/2}\sigma\theta_{i,t+1} - j_{i,t+1}\sigma^2/2\right)^2 \mid j_{i,t+1}\right] \mid \omega_{t+1}\right] - \left(\sigma^2\omega_{t+1}/2\right)^2 \\
&+ E\left[E\left[\left(\hat{j}_{i,t+1}^{1/2}\hat{\sigma}\hat{\theta}_{i,t+1} - \hat{j}_{i,t+1}\hat{\sigma}^2/2\right)^2 \mid \hat{j}_{i,t+1}\right]\right] - \left(\hat{\sigma}^2\hat{\omega}/2\right)^2 \\
&= E\left[j_{i,t+1}\sigma^2 + j_{i,t+1}^2\sigma^4/4 \mid \omega_{t+1}\right] - \left(\sigma^2\omega_{t+1}/2\right)^2 + E\left[\hat{j}_{i,t+1}\hat{\sigma}^2 + \hat{j}_{i,t+1}^2\hat{\sigma}^4/4\right] - \left(\hat{\sigma}^2\hat{\omega}/2\right)^2 \\
&= \sigma^2\omega_{t+1} + \left(\sigma^4/4\right)\omega_{t+1}\left(1+\omega_{t+1}\right) - \left(\sigma^2\omega_{t+1}/2\right)^2 + \hat{\sigma}^2\hat{\omega} + \left(\hat{\sigma}^4/4\right)\hat{\omega}\left(1+\hat{\omega}\right) - \left(\hat{\sigma}^2\hat{\omega}/2\right)^2 \\
&= \left(\sigma^2 + \sigma^4/4\right)\omega_{t+1} + \left(\hat{\sigma}^2 + \hat{\sigma}^4/4\right)\hat{\omega}
\end{aligned}
\tag{C.4}
$$

We calculate the third central moment as follows:

$$
\begin{aligned}
&\mu_3\left(\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)\right) \\
&= \mu_3\left(j_{i,t+1}^{1/2}\sigma\theta_{i,t+1} - j_{i,t+1}\sigma^2/2 + \hat{j}_{i,t+1}^{1/2}\hat{\sigma}\hat{\theta}_{i,t+1} - \hat{j}_{i,t+1}\hat{\sigma}^2/2\right) \\
&= \mu_3\left(j_{i,t+1}^{1/2}\sigma\theta_{i,t+1} - j_{i,t+1}\sigma^2/2\right) + \mu_3\left(\hat{j}_{i,t+1}^{1/2}\hat{\sigma}\hat{\theta}_{i,t+1} - \hat{j}_{i,t+1}\hat{\sigma}^2/2\right)
\end{aligned}
$$

But

$$
\begin{aligned}
&\mu_3\left(j_{i,t+1}^{1/2}\sigma\theta_{i,t+1} - j_{i,t+1}\sigma^2/2\right)\\
&= E\left[E\left[\left(j_{i,t+1}^{1/2}\sigma\theta_{i,t+1} + \left(\omega_{t+1} - j_{i,t+1}\right)\sigma^2/2\right)^3 \mid j_{i,t+1}\right] \mid \omega_{t+1}\right]\\
&= \sigma^3 E\left[E\left[\left(j_{i,t+1}^{1/2}\theta_{i,t+1} + \left(\omega_{t+1} - j_{i,t+1}\right)\sigma/2\right)^3 \mid j_{i,t+1}\right] \mid \omega_{t+1}\right]\\
&= \sigma^3 E\left[E\left[3 j_{i,t+1}\left(\omega_{t+1} - j_{i,t+1}\right)\sigma/2 + \left(\omega_{t+1} - j_{i,t+1}\right)^3\sigma^3/8 \mid j_{i,t+1}\right] \mid \omega_{t+1}\right]\\
&= \left(\sigma^4/2\right) E\left[E\left[3 j_{i,t+1}\omega_{t+1} - 3 j_{i,t+1}^2 + \left(\omega_{t+1}^3 - 3\omega_{t+1}^2 j_{i,t+1} + 3\omega_{t+1} j_{i,t+1}^2 - j_{i,t+1}^3\right)\sigma^2/4 \mid j_{i,t+1}\right] \mid \omega_{t+1}\right]\\
&= \left(\sigma^4/2\right)\left\{ 3\omega_{t+1}^2 - 3\left(\omega_{t+1}^2 + \omega_{t+1}\right) + \left(\omega_{t+1}^3 - 3\omega_{t+1}^3 + 3\omega_{t+1}\left(\omega_{t+1}^2 + \omega_{t+1}\right) - \left(\omega_{t+1}^3 + 3\omega_{t+1}^2 + \omega_{t+1}\right)\right)\sigma^2/4\right\}\\
&= -\left(3\sigma^4/2 + \sigma^6/8\right)\omega_{t+1}
\end{aligned}
$$

Likewise, we prove that $\mu_3\left(\hat{j}_{i,t+1}^{1/2}\hat{\sigma}\hat{\theta}_{i,t+1} - \hat{j}_{i,t+1}\hat{\sigma}^2/2\right) = -\left(3\hat{\sigma}^4/2 + \hat{\sigma}^6/8\right)\hat{\omega}$. Therefore

$$
\mu_3\left(\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t} \right)\right) = -\left(3\sigma^4/2 + \sigma^6/8\right)\omega_{t+1} - \left(3\hat{\sigma}^4/2 + \hat{\sigma}^6/8\right)\hat{\omega} \tag{C.5}
$$

Appendix D: Derivation of the common SDF, risk free rate, market price-dividend ratio, and expected market return

In Appendix B we proved that any household's consumption-wealth ratio is a function of only the state variable, that is, $z_{i,c,t} = z_{c,t}(\omega_t)$. We conjecture and verify that $z_{c,t} = A_0 + A_1 x_t$. We plug $z_{c,t} = A_0 + A_1 x_t$ in the Euler equation (B.3). We also log-linearize the term $\log\left(e^{z_{c,t+1}} + 1\right)$ as in Campbell and Shiller (1988) and obtain $\log\left(e^{z_{c,t+1}} + 1\right) \approx h_0 + h_1 z_{c,t+1}$, where $h_0 \equiv \log\left(e^{\overline{z_c}} + 1\right) - \frac{\overline{z_c} e^{\overline{z_c}}}{e^{\overline{z_c}} + 1}$, and $h_1 \equiv \frac{e^{\overline{z_c}}}{e^{\overline{z_c}} + 1}$:

$$e^{\theta z_{i,c,t}} = E\left[e^{\theta \log\delta + (1-\gamma)\left(\mu + \sigma_a \varepsilon_{t+1} + j_{i,t+1}^{1/2} \sigma \theta_{i,t+1} - j_{i,t+1}\sigma^2/2 + \hat{j}_{i,t+1}^{1/2} \hat{\sigma} \hat{\theta}_{i,t+1} - \hat{j}_{i,t+1} \hat{\sigma}^2/2 \right) + \theta \log\left(e^{z_{c.t+1}} + 1 \right)} \mid \Delta c_t, \omega_t, j_{i,t}, \theta_{i,t}, \hat{j}_{i,t}, \hat{\theta}_{i,t} \right]$$

or

$$e^{\theta(A_0 + A_1 x_t)} = E\left[e^{\theta \log\delta + (1-\gamma)\left(\mu + \sigma_a \varepsilon_{t+1} + j_{i,t+1}^{1/2} \sigma \theta_{i,t+1} - j_{i,t+1}\sigma^2/2 + \hat{j}_{i,t+1}^{1/2} \hat{\sigma} \hat{\theta}_{i,t+1} - \hat{j}_{i,t+1} \hat{\sigma}^2/2 \right) + \theta\left\{ h_0 + h_1 (A_0 + A_1 x_{t+1}) \right\}} \mid \Delta c_t, \omega_t, j_{i,t}, \theta_{i,t}, \hat{j}_{i,t}, \hat{\theta}_{i,t} \right]$$

or

$$E\left[e^{\theta \log\delta + (1-\gamma)\mu + (1-\gamma)^2 \sigma_a^2/2 + \gamma(\gamma-1)\left(j_{i,t+1} + \hat{j}_{i,t+1} \right) + \theta\left\{ h_0 + h_1 (A_0 + A_1 x_{t+1}) - A_0 - A_1 x_t \right\}} \mid \omega_t, j_{i,t}, \hat{j}_{i,t} \right] = 1$$

or

$$E\left[e^{\theta \log\delta + (1-\gamma)\mu + (1-\gamma)^2 \sigma_a^2/2 + x_{t+1} + \left(e^{\gamma(\gamma-1)\hat{\sigma}^2/2} - 1 \right)\hat{\omega} + \theta\left\{ h_0 + h_1 (A_0 + A_1 x_{t+1}) - A_0 - A_1 x_t \right\}} \mid \omega_t \right] = 1$$

since $e^{-\omega} \sum_{n=0}^{\infty} e^{kn} \omega^n / n! = e^{-\omega} \sum_{n=0}^{\infty} \left(e^k \omega \right)^n / n! = e^{-\omega} e^{e^k \omega}$ and $\left(e^{(\gamma-1)\gamma\sigma^2/2} - 1 \right)\omega_t = x_t$. Therefore,

$$e^{\theta \log\delta + (1-\gamma)\mu + (1-\gamma)^2 \sigma_a^2/2 + \left(e^{\gamma(\gamma-1)\hat{\sigma}^2/2} - 1 \right)\hat{\omega} + \theta\left(h_0 + h_1 A_0 - A_0 - A_1 x_t \right)} E\left[e^{(1+\theta h_1 A_1) x_{t+1}} \mid \omega_t \right] = 1$$

or

$$e^{\theta \log \delta + (1-\gamma)\mu + (1-\gamma)^2 \sigma_a^2/2 + \left(e^{\gamma(\gamma-1)\hat{\sigma}^2/2} - 1\right)\hat{\omega} + \theta\left(h_0 + h_1 A_0 - A_0 - A_1 x_t\right) + (1+\theta h_1 A_1)\left(x_t + \kappa\left(\bar{x} - x_t\right)\right) + (1+\theta \kappa_1 A_1)^2 \sigma_x^2 x_t/2} = 1 \tag{D.1}$$

Matching the constant, we obtain:

$$\theta \log \delta + (1-\gamma)\mu + (1-\gamma)^2 \sigma_a^2 / 2 + \left(e^{\gamma(\gamma-1)\hat{\sigma}^2/2} - 1\right)\hat{\omega} + \theta\left(h_0 + h_1 A_0 - A_0\right) + \left(1 + \theta h_1 A_1\right)\kappa \bar{x} = 0 \tag{D.2}$$

and matching the coefficient of x_t, we obtain:

$$-A_1\theta + \left(1 + \theta h_1 A_1\right)\left(1 - \kappa\right) + \left(1 + \theta h_1 A_1\right)^2 \sigma_x^2 / 2 = 0 \tag{D.3}$$

The solution of equations (D.2) and (D.3) produces values for the parameters A_0 and A_1 that verify the conjecture that $z_{c,t} = A_0 + A_1 x_t$. Since $\overline{z_c} = A_0 + A_1 \bar{x}$, $h_0 \equiv \log\left(e^{\overline{z_c}} + 1\right) - \frac{\overline{z_c} e^{\overline{z_c}}}{e^{\overline{z_c}} + 1}$, and $h_1 \equiv \frac{e^{\overline{z_c}}}{e^{\overline{z_c}} + 1}$, the parameters h_0 and h_1 are determined in terms of the parameters A_0, A_1, and $\bar{x}$.

In pricing any security, other than the households' wealth portfolios, we integrate out of the *SDF* in equation (B.4) the household-specific random variables $\theta_{i,t+1}, j_{i,t+1}, \hat{\theta}_{i,t+1}$, and $\hat{j}_{i,t+1}$ and obtain a SDF common across households:

$$\begin{aligned}
(SDF)_{t+1} &= E\left[e^{\theta \log \delta - \gamma\left(\Delta c_{t+1} + j_{i,t+1}^{1/2}\sigma\theta_{i,t+1} - j_{i,t+1}\sigma^2/2 + \hat{j}_{i,t+1}^{1/2}\hat{\sigma}\hat{\theta}_{i,t+1} - \hat{j}_{i,t+1}\hat{\sigma}^2/2\right) + (\theta-1)\left(h_0 + h_1 z_{c,t+1} - z_{c,t}\right)} \mid c_t, c_{t+1}, \omega_t, \omega_{t+1} \right] \\
&= e^{\theta \log \delta - \gamma \Delta c_{t+1} + \omega_{t+1}\left(e^{\gamma(\gamma+1)\sigma^2/2} - 1\right) + \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1\right) + (\theta-1)\left(h_0 + h_1 z_{c,t+1} - z_{c,t}\right)} \\
&= e^{\theta \log \delta + \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1\right) - \gamma \Delta c_{t+1} + (\theta-1)\left\{h_0 + h_1 A_0 - \left(A_0 + A_1 x_t\right)\right\} + \lambda x_{t+1}}
\end{aligned} \tag{D.4}$$

where

$$\lambda \equiv \frac{e^{\gamma(\gamma+1)\sigma^2/2} - 1}{e^{\gamma(\gamma-1)\sigma^2/2} - 1} + (\theta - 1) h_1 A_1 \tag{D.5}$$

The Euler equation for the log risk free rate is

$$E\left[e^{\theta \log \delta + \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1 \right) - \gamma \Delta c_{t+1} + (\theta-1)\{h_0 + h_1 A_0 - (A_0 + A_1 x_t)\} + \lambda x_{t+1} + r_t} \mid \omega_t \right] = 1$$

or

$$e^{\theta \log \delta + \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1 \right) + (\theta-1)\{h_0 + h_1 A_0 - (A_0 + A_1 x_t)\} - \gamma\mu + \gamma^2 \sigma_a^2/2 + \lambda\left(x_t + \kappa\left(\bar{x} - x_t\right)\right) + \lambda^2 \sigma_x^2 x_t/2 + r_t} = 1$$

or

$$\begin{aligned} r_t = & -\theta \log \delta - \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1 \right) - (\theta - 1)(h_0 + h_1 A_0 - A_0) + \gamma\mu - \gamma^2 \sigma_a^2 / 2 - \lambda\kappa\bar{x} \\ & - \left\{ \lambda(1-\kappa) + \lambda^2 \sigma_x^2 / 2 - (\theta - 1) A_1 \right\} x_t \end{aligned} \tag{D.6}$$

We denote the log stock market return as $r_{m,t}$ and the stock market price-dividend ratio as $z_{m,t}$. As in Campbell-Shiller (1988), we write

$$r_{m,t+1} = k_0 + k_1 z_{m,t+1} - z_{m,t} + \Delta d_{t+1} \tag{D.7}$$

where $k_0 \equiv \log\left(e^{\bar{z}_m} + 1\right) - \frac{\bar{z}_m e^{\bar{z}_m}}{e^{\bar{z}_m} + 1}$ and $k_1 = \frac{e^{\bar{z}_m}}{e^{\bar{z}_m} + 1}$. We conjecture and verify that the price-dividend ratio of the stock market is

$$z_{m,t} = B_0 + B_1 x_t \tag{D.8}$$

and write

$$r_{m,t+1} = k_0 + k_1 \left(B_0 + B_1 x_{t+1} \right) - \left(B_0 + B_1 x_t \right) + \mu_d + \sigma_d \varepsilon_{d.t+1}$$

The Euler equation is

$$E\left[e^{\theta \log \delta + \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1 \right) + (\theta-1)\{h_0 + h_1 A_0 - (A_0 + A_1 x_t)\} - \gamma \Delta c_{t+1} + \lambda x_{t+1} + r_{m,t+1}} \mid \omega_t \right] = 1$$

or

$$E\left[e^{\theta \log \delta + \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1 \right) + (\theta-1)\{h_0 + h_1 A_0 - (A_0 + A_1 x_t)\} - \gamma \Delta c_{t+1} + \lambda x_{t+1} + k_0 + k_1(B_0 + B_1 x_{t+1}) - (B_0 + B_1 x_t) + \mu_d + \sigma_d \varepsilon_{d.t+1}} \mid \omega_t \right] = 1$$

or

$$e^{\theta \log \delta + \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1 \right) + (\theta-1)\{h_0 + h_1 A_0 - (A_0 + A_1 x_t)\} - \gamma\mu + \gamma^2 \sigma_a^2/2 + k_0 + k_1 B_0 - (B_0 + B_1 x_t) + \mu_d + \sigma_d^2/2} E\left[e^{(\lambda + k_1 B_1) x_{t+1}} \mid \omega_t \right] = 1$$

or

$$\begin{aligned} &\theta \log \delta + \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1 \right) + (\theta - 1)\left\{ h_0 + h_1 A_0 - (A_0 + A_1 x_t) \right\} - \gamma\mu + \gamma^2 \sigma_a^2 / 2 + k_0 + k_1 B_0 \\ &- (B_0 + B_1 x_t) + \mu_d + \sigma_d^2 / 2 + (\lambda + k_1 B_1)\left\{ x_t + \kappa\left(\bar{x} - x_t \right) \right\} + (\lambda + k_1 B_1)^2 \sigma_x^2 x_t / 2 \\ &= 0 \end{aligned}$$

We set the coefficients of B_0 and B_1 equal to zero and obtain two equations that determine these parameters:

$$\begin{aligned} &\theta \log \delta + \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1 \right) + (\theta - 1)(h_0 + h_1 A_0 - A_0) - \gamma\mu \\ &+ \gamma^2 \sigma_a^2 / 2 + k_0 + k_1 B_0 - B_0 + \mu_d + \sigma_d^2 / 2 + (\lambda + k_1 B_1)\kappa \bar{x} \\ &= 0 \end{aligned} \tag{D.9}$$

and

$$-(\theta - 1) A_1 - B_1 + (\lambda + k_1 B_1)(1 - \kappa) + (\lambda + k_1 B_1)^2 \sigma_x^2 / 2 = 0 \tag{D.10}$$

Note that the parameters k_0 and k_1 are determined in terms of the parameters B_0, B_1, and $\bar{x}$. The expected stock market return is

$$
\begin{aligned}
E\left[r_{m,t+1} \mid \omega_t \right] &= k_0 + k_1 B_0 + k_1 B_1 \left\{ x_t + \kappa \left(\bar{x} - x_t \right) \right\} - \left(B_0 + B_1 x_t \right) + \mu_d \\
&= k_0 + k_1 B_0 + k_1 B_1 \kappa \bar{x} - B_0 + \mu_d + \left\{ k_1 B_1 \left(1 - \kappa \right) - B_1 \right\} x_t
\end{aligned} \tag{D.11}
$$

and its unconditional variance is

$$
\operatorname{var}\left(r_{m,t+1} \right) = k_1^2 B_1^2 \frac{\sigma_x^2 \bar{x}}{2\kappa - \kappa^2} + \sigma_d^2 / 2 \tag{D.12}
$$

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Table 1: Summary Statistics of Household Consumption Growth,
Quarterly Data 1982:Q1-2009:Q4

January Tranche

						Number of Households		
	μ_1	$\mu_2^{1/2}$	μ_3	$AC1\left(\mu_2^{1/2}\right)$	$AC1(\mu_3)$	Minimum	Maximum	Mean
A>0	.010 (.006)	.382 (.016)	-.026 (.008)	.771	.112	19	1310	685
A>2,000	-.009 (.014)	.383 (.019)	-.016 (.007)	.190	.093	0	245	115
A>10,000	-.004 (.015)	.376 (.019)	-.012 (.007)	.151	.120	0	224	102
A>20,000	-.0003 (.006)	.384 (.020)	-.009 (.006)	.169	.118	0	201	92

February Tranche

						Number of Households		
	μ_1	$\mu_2^{1/2}$	μ_3	$AC1\left(\mu_2^{1/2}\right)$	$AC1(\mu_3)$	Minimum	Maximum	Mean
A>0	.004 (.005)	.383 (.017)	-.027 (.009)	.802	-.036	19	1313	713
A>2,000	.021 (.011)	.370 (.018)	-.040 (.023)	.122	-.022	1	233	113
A>10,000	.017 (.011)	.363 (.021)	-.044 (.025)	.201	-.012	0	202	99
A>20,000	.020 (.014)	.368 (.024)	-.024 (.013)	-.083	.037	0	179	89

March Tranche

						Number of Households		
	μ_1	$\mu_2^{1/2}$	μ_3	$AC1\left(\mu_2^{1/2}\right)$	$AC1(\mu_3)$	Minimum	Maximum	Mean
A>0	.003 (.004)	.385 (.015)	-.017 (.006)	.836	-.101	17	1319	709
A>2,000	-.006 (.015)	.375 (.027)	.001 (.004)	.509	-.119	0	240	113
A>10,000	-.001 (.011)	.333 (.021)	-.001 (.004)	.359	.026	0	213	101
A>20,000	.005 (.010)	.327 (.023)	-.003 (.005)	.501	.113	0	190	91

The January tranche is the sample of households with first quarter consumption in January, February, and March; the February tranche is the sample of households with first quarter consumption in February, March, and April; and the March tranche is the sample of households with first quarter consumption in March, April, and May. "A" is the minimum total assets of each household that passes the filter for inclusion in the sample. μ_1 is the mean, $\mu_2^{1/2}$ is the standard deviation, and μ_3 is the third central moment of the quarterly household consumption growth. $AC1$ stands for first-order auto-correlation.

Table 2: Correlation of Household Consumption Growth with Recessions, Quarterly Data 1982:Q1-2009:Q4

January Tranche			
	$corr(\mu_1, I_{rec})$	$corr(\mu_2, I_{rec})$	$corr(\mu_3, I_{rec})$
A>0	-.032	.101	-.219
A>2,000	.100	.019	.010
A>10,000	.130	.020	-.051
A>20,000	.150	.050	.030
February Tranche			
	$corr(\mu_1, I_{rec})$	$corr(\mu_2, I_{rec})$	$corr(\mu_3, I_{rec})$
A>0	-.085	.079	-.123
A>2,000	.066	.109	-.176
A>10,000	.105	.116	-.164
A>20,000	.224	.139	.020
March Tranche			
	$corr(\mu_1, I_{rec})$	$corr(\mu_2, I_{rec})$	$corr(\mu_3, I_{rec})$
A>0	-.059	.051	-.048
A>2,000	.044	.197	.037
A>10,000	.064	.056	.136
A>20,000	.040	.077	.135

The January tranche is the sample of households with first quarter consumption in January, February, and March; the February tranche is the sample of households with first quarter consumption in February, March, and April; and the March tranche is the sample of households with first quarter consumption in March, April, and May. “A” is the minimum total assets of each household that passes the filter for inclusion in the sample. I_{rec} is an indicator variable that takes the value of one if there is a NBER-designated recession in any of the three months of the quarter.

Table 3: Model Fit and Parameter Estimates, Annual Data 1929-2009

Fit in Financial Data

	$E[r_f]$	$\sigma(r_f)$	$AC1(r_f)$	$E[r_m]$	$\sigma(r_m)$	$E[p/d]$	$\sigma(p/d)$	$AC1(p/d)$
Data	0.006	0.030	0.672	0.062	0.198	3.377	0.450	0.877
Model	-.001	0.025	0.904	0.055	0.212	3.326	0.346	0.904

Fit in Consumption and Dividend Data

	$E[\Delta c]$	$\sigma(\Delta c)$	$E[\Delta d]$	$\sigma(\Delta d)$	$AC1(\Delta d)$
Data	0.020	0.021	0.010	0.117	0.163
Model	0.020	0.020	0.020	0.150	0.0

Estimates of Preference Parameters

γ	ψ	δ
6.47	1.17	.953
(.0005)	(.002)	(.020)

Other Parameter Estimates

μ	σ_a	κ	$\bar{x}$	σ_x	σ	$\hat{\omega}$	μ_d	σ_d
.020	.020	.096	.052	.100	.095	.578	.020	.150
(.003)	(.004)	(.171)	(.069)	(.105)	(.039)	(.002)	(.019)	(.060)

$E[r_f]$, $\sigma(r_f)$, and $AC1(r_f)$ are the mean, standard deviation, and first-order auto-correlation of the risk free rate; $E[r_m]$ and $\sigma(r_m)$ are the mean and standard deviation of the market return; and $E[p/d]$, $\sigma(p/d)$, and $AC1(p/d)$ are the mean, standard deviation, and first-order auto-correlation of the price-dividend ratio; $E[\Delta c]$ is aggregate consumption growth and Δd is dividend growth. The preference parameters are the RRA coefficient, γ, the elasticity of intertemporal substitution, ψ, and the subjective discount factor, δ. The other parameters are: the mean, μ, and volatility, σ_a, of aggregate consumption growth; the parameters of the dynamics of the state variable, $\kappa, \bar{x}$, and σ_x; the parameters of the household income shocks, σ and $\hat{\omega}$; and the mean, μ_d, and volatility, σ_d, of aggregate dividend growth. Asymptotic standard errors are in parentheses. The *J*-stat is 8.82 and the model is not rejected at the 10% level of significance. The asymptotic 90% critical value is 10.1.

Table 4: Model Fit and Parameter Estimates, Quarterly Data 1947:Q1-2009:Q4

Fit in Price Data

	$E[r_f]$	$\sigma(r_f)$	$AC1(r_f)$	$E[r_m]$	$\sigma(r_m)$	$E[p/d]$	$\sigma(p/d)$	$AC1(p/d)$
Data	0.003	0.006	0.854	0.017	0.084	3.470	0.423	0.980
Model	0.001	0.007	0.999	0.025	0.073	3.686	0.420	0.999

Fit in Consumption and Dividend Data

	$E[\Delta c]$	$\sigma(\Delta c)$	$E[\Delta d]$	$\sigma(\Delta d)$	$AC1(\Delta d)$
Data	0.005	0.004	0.005	0.105	-0.70
Model	0.002	0.002	0.001	0.070	0.0

Estimates of Preference Parameter

γ	ψ	δ
14.47	1.01	.986
(10^{-9})	(10^{-6})	(10^{-6})

Other Parameter Estimates

μ	σ_a	κ	$\bar{x}$	σ_x	σ	$\hat{\omega}$	μ_d	σ_d
.002	.002	.001	.066	.008	.028	.426	.001	.070
(10^{-7})	(10^{-7})	(10^{-5})	(10^{-6})	(10^{-6})	(10^{-6})	(10^{-8})	(10^{-6})	(10^{-7})

$E[r_f]$, $\sigma(r_f)$, and $AC1(r_f)$ are the mean, standard deviation, and first-order auto-correlation of the risk free rate; $E[r_m]$ and $\sigma(r_m)$ are the mean and standard deviation of the market return; and $E[p/d]$, $\sigma(p/d)$, and $AC1(p/d)$ are the mean, standard deviation, and first-order auto-correlation of the price-dividend ratio; $E[\Delta c]$ is aggregate consumption growth and Δd is dividend growth. The preference parameters are the RRA coefficient, γ, the elasticity of intertemporal substitution, ψ, and the subjective discount factor, δ. The other parameters are: the mean, μ, and volatility, σ_a, of aggregate consumption growth; the parameters of the dynamics of the state variable, $\kappa, \bar{x}$, and σ_x; the parameters of the household income shocks, σ and $\hat{\omega}$; and the mean, μ_d, and volatility, σ_d, of aggregate dividend growth. Asymptotic standard errors are in parentheses. The *J*-stat is 16.27 and the model is not rejected at the 10% level of significance. The asymptotic 90% critical value is 37.97.

Table 5: Model Fit and Parameter Estimates, Quarterly Data 1982:Q1-2009:Q4

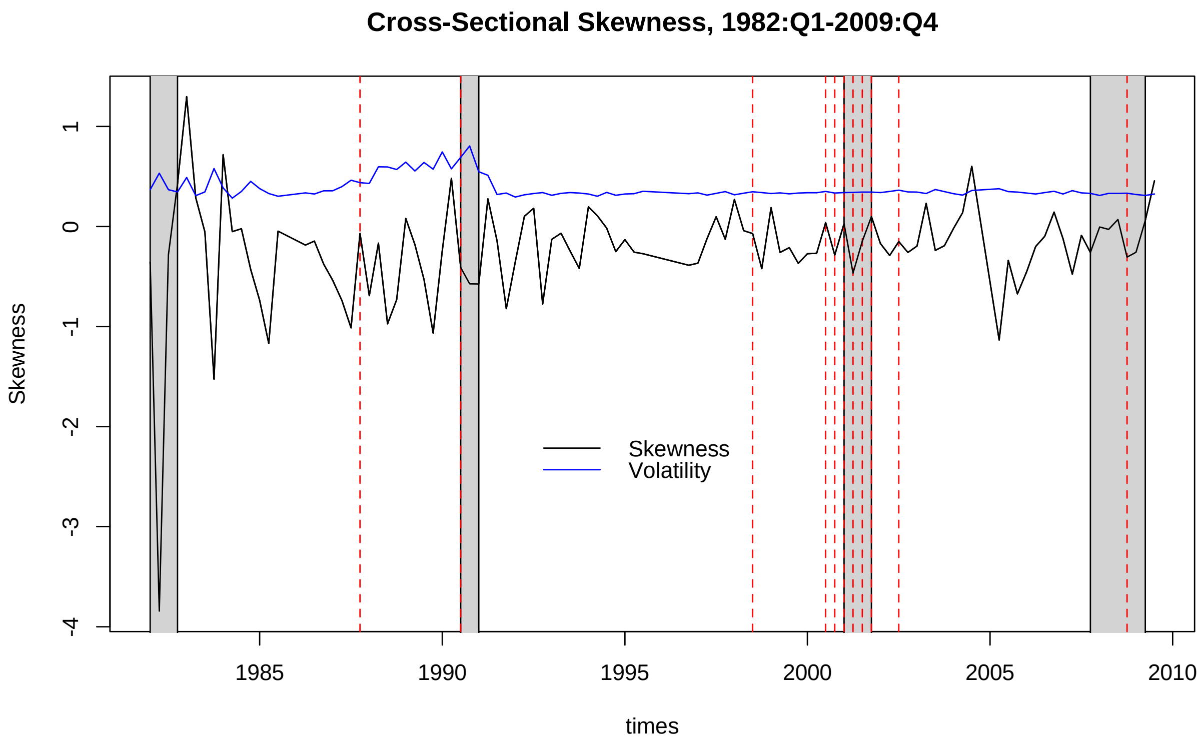
Fit in Price Data								
	$E[r_f]$	$\sigma(r_f)$	$AC1(r_f)$	$E[r_m]$	$\sigma(r_m)$	$E[p/d]$	$\sigma(p/d)$	$AC1(p/d)$
Data	.005	.005	.899	.019	.084	3.759	.414	.986
	(.001)	(.001)	(.188)	(.008)	(.007)	(.068)	(.032)	(.152)
Model	-.018	.008	.983	.024	.098	3.742	.390	.983

Fit in Consumption and Dividend Data								
	$E[\Delta c]$	$\sigma(\Delta c)$	$E[\Delta d]$	$\sigma(\Delta d)$	$AC1(\Delta d)$	$\mu_1(\Delta c_{CEX})$	$\mu_2^{1/2}(\Delta c_{CEX})$	$\mu_3(\Delta c_{CEX})$
Data	.005	.004	.005	.104	-.69	.010	.382	-.026
	(.0005)	(.0004)	(.006)	(.019)	(.264)	(.006)	(.016)	(.008)
Model	.008	.002	.001	.065	0.0	-.008	.142	-.022

Estimates of Preference Parameter		
γ	ψ	δ
1.20	1.01	.984
(.005)	(.137)	(.012)

Other Parameter Estimates								
μ	σ_a	κ	$\bar{x}$	σ_x	σ	$\hat{\omega}$	μ_d	σ_d
.008	.002	.017	.0002	.003	.898	.019	.001	.065
(.070)	(.0001)	(.038)	(.038)	(1.7×10^{-7})	(.001)	(.008)	(.012)	(.004)

$E[r_f]$, $\sigma(r_f)$, and $AC1(r_f)$ are the mean, standard deviation, and first-order auto-correlation of the risk free rate; $E[r_m]$ and $\sigma(r_m)$ are the mean and standard deviation of the market return; and $E[p/d]$, $\sigma(p/d)$, and $AC1(p/d)$ are the mean, standard deviation, and first-order auto-correlation of the price-dividend ratio; $E[\Delta c]$ is aggregate consumption growth and Δd is dividend growth. $\mu_1(\Delta c_{CEX})$, $\mu_2^{1/2}(\Delta c_{CEX})$, and $\mu_3(\Delta c_{CEX})$ are the first three unconditional central moments of the cross-sectional distribution of relative household consumption. The preference parameters are the RRA coefficient, γ, the elasticity of intertemporal substitution, ψ, and the subjective discount factor, δ. The other parameters are: the mean, μ, and volatility, σ_a, of aggregate consumption growth; the parameters of the dynamics of the state variable, $\kappa, \bar{x}$, and σ_x; the parameters of the household income shocks, σ and $\hat{\omega}$; and the mean, μ_d, and volatility, σ_d, of aggregate dividend growth. Asymptotic standard errors are in parentheses. The *J*-stat is 10.11 and the model is not rejected at the 5% level of significance. The asymptotic 95% critical value is 13.20.


Cross-Sectional Skewness, 1982:Q1-2009:Q4
Skewness
1
0
-1
-2
-3
-4
1985
1990
1995
2000
2005
2010
times
Skewness
Volatility