Portfolio Choice and Asset Pricing in a Status Conscious Economy *

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Abstract

I examine the impact of status-seeking considerations on investors' portfolio choices and asset prices in a general equilibrium setting. The economy I study consists of traditional ("Markowitz") investors as well as status seekers who are concerned about relative wealth. The model highlights the strategic and interdependent nature of portfolio selection in such a setting: low-status investors look for portfolio choices that maximize their chances of moving up the ladder while high-status investors look to maintain the status quo and hedge against these choices of the low-status investors. In equilibrium, asset returns obey a novel two-factor model in which one factor is the traditional market factor and the other is a particular "high volatility factor" that does not appear to have been identified so far in the theoretical or empirical literature. I test this two-factor model using stock market data and find significant economical and statistical support for it. Of particular interest, the model and the empirical results attribute the low returns on idiosyncratic volatility stocks documented by Ang, Hodrick, Xing and Zhang (2006) to their covariance with the portfolio of highly volatile stocks held by investors with relatively low status.

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1 Introduction

The empirical finance literature has provided many challenges to traditional portfolio choice and asset pricing theories. A key insight of modern portfolio theory is the merits of diversification. However, many households that hold individual stocks directly hold only a single stock, and the median number of stocks held is only about three [30]. In addition, less wealthy investors underdiversify more than wealthier investors. A fundamental principle of asset pricing theory is that idiosyncratic risk should not be priced, or in a market in which investors cannot properly diversify, one would expect idiosyncratic risk to be positively related to expected returns (Merton, 1987) [32]. However, one of the most puzzling recent empirical findings documented by Ang, Hodrick, Xing, and Zhang (2006) [3] is that stocks with high idiosyncratic volatility have abysmally low average returns. Another recent empirical study by Bali, Cakici, and Whitelaw (2009) [4] shows that stocks with lottery-like payoffs also have significantly low returns.

In this paper, I argue that taking status concerns into account sheds some light on these puzzles. Status concerns are prevalent in the decisions made by financial market participants. These concerns can take the form of a mutual fund tournament, in which fund managers compete against each other over past-returns. One reason for such competition is the findings that investors prefer to invest in funds that performed well relative to others. Status concerns can also be manifested by investors who care about their wealth or social status relative to other individuals, and compete over wealth-based rank. There is a growing literature in psychology, sociology and, more recently, economics documenting the importance of relative wealth or relative income concerns on self-reported well-being. For example, Luttmer (2005) [31] finds a negative effect of increases in neighbors' earnings on own wellbeing.

I devise a stylized model that abstracts from the exact nature of the underlying competition and concentrates on several salient characteristics of a competition between investors. First, the concern for status is modeled as a concern of investors for their ordinal rank in the competition. Modelling status as indicated by the ordinal rank in the distribution of wealth was pioneered by Frank (1985) [19] in a study of the demand for positional and non-positional goods. Robson (1992) [35] and Becker, Murphy and Werning (2005) [6] considered preferences over absolute wealth as well as ordinal rank in wealth. Second, investors are heterogeneous in their attitudes toward their current rank in the competition. Some competitors are ahead in the competition, satisfied with their position, and are interested in maintaining the statusquo. I call these investors "leaders". Other competitors are falling behind, dissatisfied with their position and are interested in "reshuffling" the ranks of the competitors. I call these investors "laggards". I will focus on the context of a two-player competition, where the leader is the richer investor, while the laggard is the poorer investor, and the players compete over wealth based rank.

The heterogeneity in attitude toward status risk is examined in the mutual fund tournament literature, spawned by Brown, Harlow and Starks (1996) [7] and Chevalier and Ellison (1997) [12]. The literature examines tournament behavior, that is, whether underperforming mutual funds increase risk in the latter part of the year. Taking the competition analogy into the social context, Kumar (2009) [27] finds that two of the characteristics of investors who hold lottery-like securities are being poor in absolute terms and being poor in relative terms, that is, earn less than their neighbors. Kumar concludes that "this evidence indicates that to some extent gambling-motivated investments are likely to be influenced by a desire to maintain or increase upward social mobility". In the context of this paper, these investors are the laggards in the competition over social status.

The third feature of the model is the strategic interaction between the players. If investors care about their performance relative to other investors, then their optimal investment strategy should take the choices of others into account. Anecdotal evidence of this sort is stated by Professor Harrison Hong (2008) during an interview to the WSJ regarding the High-Tech bubble: "My sister's getting rich. My friends are getting rich....I think this is all crazy, but I feel so horrible about missing out, about being left out of the party. I finally caved in, I put in some money just as a hedge against other people getting richer than me and feeling better than me".

The heart of the model is a game between two players, a laggard and a leader, who compete against each other over who will be the leader at the end of the period. Each player cares only about his or her position relative to the other player at the end of the game. The attitudes of investors toward the moments of the return on their portfolios are determined endogenously as a function of their initial status. While both investors seek to increase the expected return on their portfolio, the laggard investor pursues a volatile portfolio with low correlation with the portfolio of the leader as he has "nothing to lose" and he must obtain a portfolio that is different than that of the leader in order to overtake her. The leader tries to maintain her position and she is exposed to "status risk", the risk of losing her leadership. Status risk has two components. The first risk is that she will obtain a low return and fall behind the laggard, and the second is the risk that the laggard will obtain a high return and overtake her. To manage the first risk, the leader tries to minimize the variance of her portfolio, and so practically, this risk is the same as the risk bared by a Markowitz investor. To manage the second risk, the leader is interested in increasing the correlation of her portfolio with the portfolio of the laggard.

To study portfolio choices and the cross-section of asset returns in this status conscious economy, I introduce an economy with many groups of similar assets. There are many pairs of laggard and leader investors (status investors) and many Markowitz investors, who care only about maximizing expected return and minimizing volatility. I find a Nash-Equilibrium, in which the laggard player uses a mixed strategy to invest in a single stock from a specific group ("group V"), which is characterized by the high volatility of its assets. I obtain a closed form solution for the leader's response. The leader's portfolio reflects her two-folded concern for reducing her variance and increasing her covariance with the laggard as she invests in a linear combination of the tangency portfolio and group V. The tradeoff of the leader between the tangency portfolio and group V depends on her "hedging demands", which captures the extent to which the leader can hedge against the laggard. As the correlation within group V increases, the leader can better hedge against the laggard using group V, and accordingly she increases the portion she invests in group V relative to her investment in the tangency portfolio.

I obtain exact solutions for asset prices and show that they follow a twofactor beta pricing model where one factor is the market and the other is group V minus the market (VM). Assets with high exposure to VM obtain lower expected returns as they provide better hedge against the laggard. The negative premium for assets with high exposure to VM depends on the hedging demands, derived from the correlation within group V, and on the variance VM. When the variance of VM is low, the leader can use other assets in the economy to hedge against the laggard and exposure to VM is less appreciated. However, when the variance of VM is high, group V becomes more special in terms of hedging efficiency against the laggard and the price of exposure to VM increases. The model provides both portfolio choice and asset pricing implications. It can explain why some investors (the laggards in the model) take on undiversified portfolios, concentrated in stocks with lottery like payoffs, and why other investors (the leaders in the model) give high weight to these stocks in their portfolio. The model provides cross-sectional asset pricing implication in the form of a two-factor pricing equation. It predicts that assets with higher exposure to the most volatile group of stocks in the market should have negative premium, as they provide a hedge against "status risk". In addition, the model expresses the determinants of the price of the most volatile group of stocks.

I examine the cross-sectional implications by constructing 5x5 = 25 dynamic portfolios, sorting stocks first by their exposure to the market and then by their exposure to VM. Group V is proxied by selecting stocks with the highest total volatility. I examine the monthly returns on these portfolios and show that portfolios with higher exposure to VM achieve significantly lower returns. In addition, I conduct time series, Fama-MacBeth and SDF-GMM tests to show that the two factor pricing model resulted from my model is supported by the data. The model can explain the idiosyncratic volatility puzzle, as idiosyncratic risk has a prediciting power over exposure to VM and sorting by it creates a spread in exposure to VM.

2 Related Literature

The idea that individuals are often motivated in their behavior by a quest for social status has lately received growing attention in economics application (See Heffez and Frank (2008) [23] for an excellent review). There is a growing literature in Finance studying the effects of relative wealth concerns on portfolio choice and asset prices. Cole, Mailath, and Postlewaite (2001) [14] study the effects of relative wealth concerns on investment choice in a general framework and show that these concerns can have two opposite effects - investors can bias their portfolios either toward or away from the portfolios held by other investors.

Abel (1990) [1] and most of the papers studying interdependent preferences in Finance modeled relative wealth using utility functions that exhibit the first effect, which is commonly termed as "keeping up with the joneses". Since investors in these models tend to bias their portfolio toward the portfolio held by the reference group, such models yield herding behavior. For example, DeMarzo, Kaniel, and Kremer (2004) show that preference for a local good can give rise to relative wealth concerns, leading to undiversified portfolios, with households in each community tilting their portfolios toward community-specific assets. Others, such as Lauterbach and Reisman (2004) [28] and Cole, Mailath, and Postlewaite [14] have used such preferences to explain the home bias.

Roussanov (2009) [37] is probably the first paper in the Finance literature that specifies a utility function that takes the opposite approach and leads investors to "get ahead of the Joneses" and seek portfolios that are biased away from the aggregate portfolio. He motivates this approach referring to Friedman and Savage (1948) [21] who suggest that as people move to a higher social class their marginal utility of wealth rises. Consequently, they take great risks to distinguish themselves" (p. 299), potentially exhibiting risk-loving behavior.

However, Friedman and Savage (1948) provides a framework that supports both approaches to status as they interpret the convex segment in their utility function as a transition between two socio-economic levels. If this is the case, then individuals with high wealth relative to their socio-economic class will exhibit risk-loving behavior, while individuals with low wealth relative to their socio-economic class will exhibit risk-averse behavior. My model takes the view that merging these two effects into a single framework is essential to understand the effects of status on portfolio choice and asset pricing, since the investors who are satisfied with their position relative to others will purse the "keeping up with the Joneses" approach and will bias their portfolios towards the portfolio of others, while other investors who are dissatisfied with their position relative to others will pursue the "getting ahead of the Joneses" approach and will bias their portfolio away from the portfolios of others. Confronting these two approaches gives rise to a strategic game as the decision of both types of investors depends on each other. This paper is a first step in studying such models.

This paper is not the first to show that relative wealth might lead to preference for volatility or lottery like securities. Robson (1996) [36] uses a biologically motivated model to show agents who are fundamentally risk neutral are induced to take Fair bets involving small losses and large gains in an environment in which the rewards are a function of relative wealth. Other papers that yield preference for lottery-like securities are Barberis and Huang (2008) [5] who study the implications of prospect theory (Tversky and Kahneman (1992) [24]) on asset prices. Errors in the probability weighting of investors cause them to over-value stocks that have a small probability of a large positive return. The optimal beliefs framework of Brunnermeier, Gollier and Parker (2007) [8] also predicts preference for lottery like securities. In this model, agents optimally choose to distort their beliefs about future probabilities in order to maximize their current utility.

Other studies that use relative wealth concerns to provide explanations to under-diversification are Roussanov [37] and DeMarzo, Kaniel, and Kremer (2004). Unlike these studies, this paper identifies the holders of underdiversified portfolio as investors who fall behind in the competition over status and identifies the assets held in these portfolios as the most volatile assets. A model that associates assets held in under-diversified portfolio with high risk assets is provided by Van Nieuwerburgh and Veldkamp (2010) [33] who argue that Information acquisition can rationalize investing in a concentrated set of assets. In particular, they formalize the conditions under which the informed investor would hold an under-diversified portfolio of the riskiest assets. Liu (2008) [30] argues that portfolio insurance leads the poorest investors to hold under-diversified portfolios with assets that have the highest expected return and highest risk. Unlike these papers, in my model the underdiversified portfolios held by poorer investors have assets that are inefficient in the mean-variance sense and is close to the findings of Kumar who shows that investors who invest disproportionately more in lottery-type stocks experience greater underperformance.

Another strand of literature related to this paper is the mutual fund tournament literature, which examines whether underperforming mutual funds (the laggards in our model) increase risk in the latter part of the year. This phenomenon has been studied using return data, and different studies have reached different conclusions (see a review at Elton, Gruber, Blake, Krasny, and Ozelge [16]). There are at least two issues with this literature that might hinder reaching a decisive conclusion about mutual fund tournament behavior. First, while most studies examined the risk taken by the leaders versus the laggards measured by volatility or beta, Chen and Penachi (2009) [11] shows that laggard funds should increase fund's "tracking error" volatility and not necessarily volatility. My paper provides support to both approaches at the same time as the laggard is interested in higher variance and lower covariance with the leader. The second issue with this literature is the identity of the leaders and the laggards. While the common and intuitive approach is to view the under-performers funds as the laggards and the top-performers as the leaders, this might not necessarily be the case. Chevalier and Ellison (1997) study the relationship between new cash flows and returns, and find that it is nonlinear - an extreme payoff from winning the tournament as opposed to being one of the last knights killed in the jousting competition. Such a relationship might suggest that the competition structure might be more complex and localized, where the best performers compete locally with each other and the worst performers compete locally with each other. What is more, it could be the case that funds compete within families as suggested by Kempf and Ruenzi (2004) [25]. My model lends an additional test to this literature, as it predicts that both the leader and (particularly) the laggards should increase holdings of highly volatile assets as the tournament-based incentives intensify (i.e. towards the end of the calendar year). Finally, additional support for the existence of status concerns is provided by Koijen (2008) [26], who studies the joint cross-sectional distribution of managerial ability and risk preference using structural portfolio management models. He shows that plausible estimation of managers risk aversion requires introducing relative-size concerns into the managers objective function.

I use the terminology leaders and laggard following Cabral (2002) [9], who analyzes the choice of variance and risk in a race between two players. Cabral (2003) [10] and Anderson and Cabral (2007) [2] provide conditions under which the laggard chooses a risky strategy, while the leader chooses a safe strategy. Cabral (2002) shows that when the competitors choose covariance, the laggard is willing to trade off lower expected value for lower correlation with respect to the leader. Both effects are consistent with my model.

3 Theory Section

In this section, I first define and examine the status game between the leader and the laggard in a general setting where assets are multivariate normal. Next, I define an economy with additional structure imposed on the distribution of assets and derive the Nash-Equilibrium in the two-player game. Finally, I add many pairs of leaders and laggards and Marokowitz investors and examine asset prices in this economy.

3.1 The Two Player Status Game

The model has two players: the laggard (referred to as a male) with an initial wealth normalized to 1, and the leader (referred to as a female) with an initial wealth normalized to k > 1. There are finitely many risky assets with returns

from a multivariate normal distribution as well as a risk free asset available for investment. Time is discrete and runs for one period. At the beginning of the period, each of the players chooses a portfolio. The gross return of the leader's portfolio is given by r_d and the gross return of the laggard's portfolio is r_g . The wealth of the leader at the end of the period is kr_d , and the wealth of the laggard is r_g . Infinite shorts sales are not allowed. The players care only about their wealth based rank at the end of the period. We denote the difference in the end-of-period wealth between the leader and the laggard by $D(r_d, r_g)$:

$$D(r_d, r_g) = kr_d - r_g \tag{1}$$

This is a zero-sum game, in which the leader (laggard) tries to maximize (minimize) the expression:

$$Pr(D(r_d, r_g) > 0) \tag{2}$$

Since D is normal, we can write the objective function of the leader as:

$$Max \ \Phi\left(\frac{kE(r_d) - E(r_g)}{\sqrt{k^2 Var(r_d) - 2kCov(r_d, r_g) + Var(r_g)}}\right)$$
(3)

In the above, $\Phi(x)$ is the CDF of a standard normal distribution. If infinite short sales are not allowed, the expected return on any portfolio must be finite. Therefore, the leader can guarantee an expected return on her portfolio at least as good as that of the laggard, and so the following proposition holds:

Proposition 3.1. In a Nash Equilibrium, the choices of the players must satisfy:

$$kE(r_d) - E(r_q) > 0$$
 Almost Surely (4)

Where r_d is the gross return of the leader, r_g is the gross return of the laggard, and k > 1 is the wealth ratio between the players.

That is, in equilibrium, the laggard cannot choose a portfolio such that his end-of-period wealth, on average, will exceed or be equal to that of the leader. Otherwise, the leader will match his strategy. Note that the equilibrium allocation can be random, in case one of the players uses a mixed strategy to choose his or her portfolio, and therefore the term "almost surely". The leader (laggard) chooses her (his) portfolio in order to increase (decrease) $Pr(D(\theta_1, \theta_2) > 0)$. Using proposition (3.1) and equation (3), we can gain some insight into the preferences of the players in their portfolio choice. Both players try to maximize the expected returns of their portfolios. Since the numerator in equation (3) is positive, the laggard (leader) tries to maximize (minimize) the variance of his (her) portfolio. Finally, the laggard (leader) tries to minimize (maximize) the covariance between the wealth of both players.

3.2 The Economy

To further examine portfolio choices and asset prices, we impose additional structure on the distribution of assets, and introduce G groups of assets. To simplify notation, we will use capital letters to denote variables that relate to groups, and small letters to denote variables that relate to individual assets.

Each group $I \in \{1, ..., G\}$ has N_I similar assets with the same distribution. All assets are multivariate normal. The return of asset $k \in \{1, ..., N_I\}$ in group I is denoted by r_i^k . The return of the equally weighted portfolio over all assets in group I is denoted by r_I . I refer to the "equally weighted portfolio over all assets in group I" as "group I" for brevity.

The expected return of every individual asset in the same group I is identical and denoted by μ_i . Since the return of group I is the average of the expected returns of individual assets, the return of group I, μ_I , is equal to the expected return of individual securities:

$$E(r_i^k) = \mu_i = E(r_I) = \mu_I$$

$$\forall k \in \{1, ..., N_I\}, I \in \{1, ..., G\}$$

The variance of every individual security in group I is the same and denoted by σ_i^2 . The correlation of every pair of assets in group I is the same and denoted by ρ_I . Hence, the N_I by N_I covariance matrix of group I follows:

$$Cov(r_i^k, r_i^l) = \left\{ \begin{array}{c} \sigma_i^2 \text{ if } k = l\\ \rho_I \sigma_i^2 \text{ if } k \neq l \end{array} \right\}$$

The correlation between two assets of different groups $I, J \in \{1, ..., G\}$ is denoted by $\rho_{i,j}$. Hence, the G by G covariance matrix for individual assets across different groups follows

$$Cov(r_i^k, r_j^k) = \sigma_{ij} = \left\{ \begin{array}{c} \sigma_i^2 \text{ if } i = j \\ \rho_{i,j} \sigma_i \sigma_j \text{ if } i \neq j \end{array} \right\}$$

The G by G covariance matrix for groups is denoted by Σ and follows:

$$Cov(r_I, r_J) = \sigma_{I,J} = \left\{ \begin{array}{l} \sigma_I^2 = \sigma_i^2(\rho_I + \frac{1-\rho_I}{N_I}) \text{ if } I = J\\ \rho_{i,j}\sigma_i\sigma_j \text{ if } I \neq J \end{array} \right\}$$

Note that the covariance of two individual assets of different groups is equivalent to the covariance of these two groups.

3.3 Nash Equilibrium

For the strategy of the leader, we restrict our attention to strategies that guarantee that her expected wealth in the next period is higher than that of the laggard, regardless of the response of the laggard (following proposition 3.1).

First, we characterize the laggard's response to the leader's strategy. In the appendix, we prove the following:

Theorem 3.2. As long as condition (4) holds, the best response of the laggard to the leader's strategy is to invest his entire wealth in a single risky asset or in the risk free asset.

The laggard is interested in maximizing volatility and minimizing covariance with the leader. Investing in a single risky asset serves both objectives. The laggard might choose to invest in the risk-free asset if the leader is invested in risky assets and either the expected returns on these risky assets are too low, or the correlation between the risky assets is too high. In either of these cases, the laggard is better off waiting on the sideline, hoping that the leader obtains a low return using the risky assets.

The laggard is indifferent between using a pure strategy or using a mixed strategy where he invests in a single risky asset chosen uniformly over a certain group. If the laggard invests in a pure strategy, the leader can match his investment and thus guarantee her first place rank. On the other hand, with the mixed strategy, the leader finds it impossible to exactly match the investment of the laggard, and a Nash Equilibrium can exist.

We now turn to the leader's response to the laggard's strategy of choosing a single risky asset using a uniform mixing strategy over a specific group that we denote group V. We first show that if the number of assets in group V is large enough, the leader chooses a strategy where she invests in an equally-weighted portfolio over every group.

Proposition 3.3. For an N_V large enough, and given that the laggard invests his wealth in a single asset, chosen uniformly by a mixed strategy over group V, the leader invests the same amount in each of the assets in the same group.

The reason that we need a large enough number of assets in group V can be illustrated by the following example. Suppose there is only one group in the economy, and there are only two assets in that group. In addition, the wealth ratio, k, is very close to 1. In this case, if the leader takes the equally weighted portfolio, the probability she remains the leader is a little more than 0.5, since the result of the game depends on whether the laggard's chosen asset performs better than the other asset. However, in the case she invests all in one asset, she will obtain a probability of a little more than 0.75, because if she "catches" the laggard, and invests in the same asset, she will remain the leader for sure. Having a large enough number of assets is important to discourage the leader from pursuing such strategies.

Since the leader invests in equally weighted portfolios over groups, and the laggard invests in an asset v, we can treat the leader's problem as choosing a size N vector θ over groups. We can write the problem of the leader:

$$Max_{\theta} \frac{k\left(\theta'\tilde{\mu}+1\right)-\left(\tilde{\mu}_{v}+1\right)}{\sqrt{k^{2}\theta'\Sigma\theta-2k\theta'\Sigma E_{v}+\sigma_{v}^{2}}}$$
(5)

Where $\tilde{\mu}$ is the vector of expected excess returns over the N groups, $\tilde{\mu}_v$ is the expected excess return of a group V asset, Σ is the covariance-variance matrix over groups, and E_v is a vector of zeros except for entry v, which is 1. The leader is interested in maximizing expected return, minimizing variance and maximizing covariance with group V. The leader's problem is a special case of the problem of an ICAPM investor who cares about her covariance with state variables. In our model, the only "state" variable is the return on group V and naturally the mimicking portfolio for group V is the return on group V. Following Fama (1996) [17], we conclude that the leader chooses an MMV portfolio - a combination of the risk free asset, the tangency portfolio and group V. Hence, we can write the leader's risky portfolio in the following way:

$$\theta = x \Sigma^{-1} \tilde{\mu} + y E_v \tag{6}$$

Using this insight, and restricting the players to choose strategies that are symmetric within groups if they are indifferent among several strategies, we can now solve for a unique Nash Equilibrium:

Theorem 3.4. There exists a unique Nash-Equilibrium in which the laggard chooses a single asset using a uniform mixed strategy over group V assets, and the leader invests in a risky portfolio θ over groups, where

$$\theta = \frac{\sigma_v^2 - \sigma_V^2}{k(k-1)} \Sigma^{-1} \tilde{\mu} + \frac{1}{k} E_v \tag{7}$$

Given that N_V is large enough and the following conditions hold: (1) $\sigma_v \ge \sqrt{2}\sigma_j \ \forall j \in \{1, ..., g\}, \ (2) \ \sigma_{v,j} \ge 0 \ \forall j, \ (3) \ \sigma_v \ge 2\sigma_V,$ (4) $0 < \psi^{-1}\iota'\Sigma^{-1}\tilde{\mu} < k-1, \ (5) \ E_j\Sigma^{-1}\tilde{\mu} > 0 \ \forall j, \ (6) \ E_v\Sigma^{-1}\tilde{\mu} > -\psi$

Where $\sigma_{v,j}$ is the covariance between an asset in group V and an asset in group J, σ_j is the volatility of an individual asset of group J, σ_V is the volatility of group V, Σ is the covariance-variance matrix for groups, $\tilde{\mu}$ is the expected excess return over the groups, E_v is a vector of zeros except entry v, which is 1, N_V is the number of assets in group V, and $\psi = \frac{k-1}{\sigma_v^2 - \sigma_V^2}$.

The leader uses the risky assets in order to optimally compete against the laggard. Her two-folded concern for low variance and high covariance with the laggard is illustrated by the two terms in (7). To obtain high covariance with the laggard she matches his investment in group V in the second term $\frac{1}{k}E_v$. To manage her implicit risk-aversion, she finds an efficient mean-variance portfolio and invests in the tangency portfolio. The balance between the two terms is the function of the following term:

$$\psi = \frac{k-1}{\sigma_v^2 - \sigma_V^2} \tag{8}$$

 ψ reflects the hedging demands of the leader. When the volatility of an individual asset of group V (asset v) relative to the volatility of group V increases, the correlation within group V decreases, it is harder for the leader to obtain a high covariance with the laggard and so her hedging demands, reflected in ψ , decrease. When hedging demands are high, the leader concentrates her efforts in the covariance with the laggard and decreases her investment in the tangency portfolio. When hedging demands are low, she cannot obtain a high covariance with the laggard, and she channels her concerns to the variance term by investing more in the tangency portfolio, obtaining a more efficient risk-return tradeoff.

The three first conditions stated in theorem (3.4) are sufficient to have the laggard not deviating form investing in asset v. Condition (1) says that the variance of asset v should be high enough relative to assets of other groups in order to encourage the laggard to invest in it. This condition identifies the attribute that makes group V the laggard's choice. It is the high volatility of group V that distinguishes it from other groups in the economy. Condition (2) tells us that there is no group with negative covariance with group V. If there were one, the laggard might have been enticed to invest in it in order to obtain a negative correlation with the leader who tilts her portfolio towards group V. Condition (3) says that the volatility of asset v should be high enough relative to the volatility of group V. In other words, the correlation within group V should be low enough, otherwise, the leader can easily hedge against the laggard. To have the laggard not deviating to the risk free asset, a sufficient condition is $\sigma_v \geq \sqrt{2}\sigma_V$, which is included in the above conditions. Conditions 4 to 6 are necessary to ensure that the leader refrains from taking a short position in the risk free asset or in any of the risky assets.

3.4 Asset Prices

To examine asset prices in this economy, we add many leader-laggard pairs (status investors) and many Markowitz investors whose risky portfolio is just the tangency portfolio. Since there are many laggards in the economy, and using the law of large numbers, the aggregation of the portfolios of the laggards is group V. As we have seen, the portfolio of the leader is a linear combination of the tangency portfolio and group V. Hence, the market portfolio, which is a linear combination of all investors, is a also a linear combination of the tangency portfolio and group V. This guarantees the existence of a two factor model as we will shortly see. To obtain a closed form solution for prices, we start by taking the first order condition of the leader's problem, equation (5), by θ and obtain:

$$\tilde{\mu} = \frac{k\left(\theta'\tilde{\mu}+1\right) - \left(\tilde{\mu}_v+1\right)}{k^2\theta'\Sigma\theta - 2k\theta'\Sigma E_v + \sigma_v^2} \left(k\Sigma\theta - \Sigma E_v\right) \tag{9}$$

Using the solution of equation (7) for the portfolio of the leader we further simplify:

$$\tilde{\mu} = \frac{k-1}{\sigma_v^2 - \sigma_V^2} \left(k \Sigma \theta - \Sigma E_v \right) = \psi \left(k \Sigma \theta - \Sigma E_v \right)$$
(10)

We can see that the expected return of an asset depends on its covariance with group V and on its covariance with the portfolio of the leader. Since the leader's portfolio is a combination of group V and the tangency portfolio, it is also a combination of the market and group V. Hence, we obtain a two factor beta pricing model, where one of the factors is the market and the other is group V.

3.4.1 Only Status Investors

Suppose that there are only status investors in the market. In this case, we can write the market portfolio as:

$$\theta_M = \frac{k\theta + E_v}{k\theta'\iota + 1} \tag{11}$$

Using the expression for the leader's portfolio we obtained in (7) and the pricing equation (10), we can write the expected excess return for an individual asset as:

$$\tilde{\mu}_i = \left[\iota' \Sigma^{-1} \tilde{\mu} + 2\psi\right] Cov(r_i, r_M) - 2\psi Cov(r_i, r_V)$$
(12)

An exposure of an asset to the market positively contributes to its expected return. The term $\iota'\Sigma^{-1}\tilde{\mu}$ is the expected excess return divided by the variance of the tangency portfolio. It reflects the implicit risk aversion of the leader. The term ψ reflects the hedging demands of the leader, as discussed above:

$$\psi = \frac{k-1}{\sigma_v^2 - \sigma_V^2}$$

If the hedging demands are high, the leader is inclined to invest more in group V at the expense of other groups, leading to a higher price for group V and a lower price for the market. A compact way to express the asset pricing relation as a two factor beta pricing model, is to have the first factor as the excess return of the market portfolio, and the second factor as the return on the most volatile group minus the return on the market. This form is not only algebraically simpler but also leads to sharp empirical predictions, as we will shortly see.

Theorem 3.5. In an economy with many pairs of leaders and laggards, where the Nash-Equilibrium described in theorem (7) holds, the expected excess return of asset i ($\tilde{\mu}_i$) is:

$$\tilde{\mu}_{i} = \beta_{i,MF}^{I} \left[\iota' \Sigma^{-1} \tilde{\mu} \right] Var\left(r_{MF} \right) + \beta_{i,VM}^{I} \left[-2\psi \right] Var\left(r_{VM} \right)$$
(13)

Where $\beta_{i,MF}^{I}$ is the slope from a univariate regression of the asset return on the excess return of the market return (r_M) , $\beta_{i,VM}^{I}$ is the slope from a univariate regression of the asset return on the return of the most volatile group minus the return on the market $(r_V - r_M)$, ι is a vector of ones, Σ is the covariance matrix of groups, and $\tilde{\mu}$ is a the expected excess returns of groups.

Equation (13) illustrates how exposure to the most volatile group of assets (group 1) translates into prices. Fixing the univariate beta of an asset on the market $(\beta_{i,MF}^{I})$, a higher univariate beta on the most volatile group of stocks minus the market $(\beta_{i,VM}^{I})$ leads to lower expected return. The negative premium on exposure to the most volatile group has two determinants. First, the hedging demands ψ - when there are higher hedging demands, the premium becomes more negative as the demand for exposure to group 1 increases. The second determinant is the variance of the return on group 1 minus the market. When $Var(r_V - r_M)$ increases it means that it is harder to hedge against the most volatile group using the market and so group 1 becomes more special in its effectiveness as a hedge.

To have the model in a multifactor beta form and to obtain the expected return of group 1 minus the market $(r_V - r_M)$, we need to express the model where the slopes of an asset on the market and on group 1 are obtained from a bivariate regression. The model in a **bivariate** beta form:

$$\tilde{\mu}_i = \beta_{i,MF}^{II} \left[\iota' \Sigma^{-1} \tilde{\mu} Var(r_M) - 2\psi Cov(r_V - r_M, r_M) \right] +$$
(14)

$$\beta_{i,VM}^{II} \left[(-2\psi) Var(r_V - r_M) + \iota' \Sigma^{-1} \tilde{\mu} Cov(r_V - r_M, r_M) \right]$$

Where $\beta_{i,MF}^{II}$ and $\beta_{i,VM}^{II}$ are the slopes from a bivariate regression of the asset return on both the excess market return (r_{MF}) and the return of the most volatile group minus the return on the market (r_{VM}) . Hence the expected return on the most volatile group minus the market is:

$$E(r_V - r_M) = (-2\psi)Var(r_V - r_M) + \iota' \Sigma^{-1} \tilde{\mu} Cov(r_V - r_M, r_M)$$
(15)

We can see that the two ingredients discussed above, the hedging demands (ψ) , and the variance of the most volatile group minus the market $(Var(r_V - r_M))$ drive the expected return of the most volatile group minus the market.

To obtain the Stochastic Discount Factor in this economy, we can manipulate equation (12) to obtain:

$$M = 1 + \tilde{\mu}' \Sigma^{-1} \tilde{\mu} - \iota' \Sigma^{-1} \tilde{\mu} \left(r_{MF} \right) + 2\psi \left(r_{VM} \right)$$
(16)

The SDF expression is useful if we want to think about the model in conditional terms. So far our analysis focused on a one-period game. To extend the asset pricing implications to a multi period settings, we have to assume that our investors are myopic ones. In this case, every time period t we can write our SDF as:

$$M_{t+1} = 1 + \left(\tilde{\mu}' \Sigma^{-1} \tilde{\mu}\right)_t - \left(\iota' \Sigma^{-1} \tilde{\mu}\right)_t r_{M,t+1} + 2\psi_t (r_{V,t+1} - r_{M,t+1})$$
(17)

This form of the SDF implies that deriving an unconditional version of the beta pricing model will have to include not only the returns on the market and group V, but also the time varying hedging demands. In particular, in an unconditional model, an asset will obtain high price if it covarias with group 1 conditional on times when the hedging demands are especially high.

3.4.2 Status Investors and Markowitz Investors

Suppose that along with the status investors, there are Markowitz investors who invest solely in the tangency portfolio. In particular, suppose that for every pair of leader/laggard investors with wealth of (k + 1), there is one Markowitz investor that invests λ in the tangency portfolio. λ captures both the proportion of Markowitz investors relative to status investors and also their risk aversion. As both increase, λ increases. We can use the same derivation we used above. Now the market portfolio is:

$$\theta_M = \frac{k\theta + E_1 + \frac{\lambda \Sigma^{-1} \tilde{\mu}}{\iota' \Sigma^{-1} \tilde{\mu}}}{k\theta' \iota + 1 + \lambda}$$
(18)

And the expected return is:

$$\tilde{\mu}(\lambda) = \frac{\iota' \Sigma^{-1} \tilde{\mu} + (2+\lambda) \psi}{1 + \frac{\psi \lambda}{\iota' \Sigma^{-1} \tilde{\mu}}} \Sigma \theta_M - \frac{2\psi}{1 + \frac{\psi \lambda}{\iota' \Sigma^{-1} \tilde{\mu}}} \Sigma E_1$$
(19)

We see that as λ increases, the negative coefficient on the covariance with group V decreases. We can follow the derivation in the previous section (only status investors) to obtain an beta pricing expression of the expected return of an asset:

$$\tilde{\mu}_{i} = \beta_{i,MF}^{I} \left[\frac{\iota' \Sigma^{-1} \tilde{\mu} + \lambda \psi}{1 + \frac{\psi \lambda}{\iota' \Sigma^{-1} \tilde{\mu}}} \right] Var\left(r_{MF}\right) + \beta_{i,VM}^{I} \left[\frac{-2\psi}{1 + \frac{\psi \lambda}{\iota' \Sigma^{-1} \tilde{\mu}}} \right] Var\left(r_{VM}\right)$$
(20)

And an expression for the SDF:

$$M = 1 + \tilde{\mu}' \Sigma^{-1} \tilde{\mu} - \left[\frac{\iota' \Sigma^{-1} \tilde{\mu} + \lambda \psi}{1 + \frac{\psi \lambda}{\iota' \Sigma^{-1} \tilde{\mu}}} \right] r_{MF} - \left[\frac{-2\psi}{1 + \frac{\psi \lambda}{\iota' \Sigma^{-1} \tilde{\mu}}} \right] (r_{VM})$$
(21)

Not surprisingly, as λ goes to infinity, we converge to the CAPM world:

$$\lim_{\lambda \to \infty} \tilde{\mu}(\lambda) = \iota' \Sigma^{-1} \tilde{\mu} \Sigma \theta_M \tag{22}$$

4 Empirical Section

The model provides a linear two factor pricing model, where the factors are excess returns. The first factor is the return on the market minus the risk free rate (henceforth MF), and the second factor is the return over the most volatile group of stocks minus the market (henceforth VM). Since the model is a member in the family of linear factor models, we can use an array of statistical tests provided by the empirical asset pricing literature to evaluate it.

The model suggests that assets with higher exposure to the group of the most volatile stocks (henceforth V) should obtain lower returns. If the model is true, then such assets are over-priced relative to asset pricing models that do not take this negative premium into account. For example, our model suggests that the CAPM alpha follows:

$$\alpha_{capm} = \beta_{i,mf}^{I} \left(\lambda_{mf}^{I} - E(r_{MF}) \right) + \beta_{i,vm}^{I} \lambda_{vm}^{I}$$
(23)

Where $\beta_{i,mf}^{I}$ is the slope from a univariate regression of r_i , the asset return, on MF. The coefficient $\beta_{i,vm}^{I}$ is the slope from a univariate regression of r_i on VM. For a given $\beta_{i,mf}^{I}$, a higher $\beta_{i,vm}^{I}$ leads to lower CAPM alpha, since λ_{vm}^{I} is a negative number.

My empirical agenda is to first examine whether assets with high exposure to the portfolio of the high volatility stocks obtain low returns relatively to other asset pricing models. Then, I examine the two factor model directly using Time Series Regressions, Fama-MacBeth, and GMM-SDF tests. Since beta VM is correlated with idiosyncratic risk across our test asset, I examine the explanatory power of beta VM versus idiosyncratic risk. Finally, I examine the conditional version of the model using GMM.

In testing our asset pricing model, we first need to create test assets that have dispersion in their exposure to MF and VM. To successfully create such assets, we need to take into account time variation not only in the volatility of stock returns, but also in the cross-section of stocks volatility. In particular, the composition of the most volatile portfolio of stock may frequently change and therefore the sensitivity of an individual stock to V can dramatically change in a short period of time. To have an up-to-date estimators, estimating parameters over short windows with daily data is preferable. However, to obtain more accurate estimators longer windows are better. Most studies that estimated betas use a formation period of more than a year, on the other hand, Ang, Hodrick, Xing and Zhang (2006), use a formation period of one month to estimate idiosyncratic volatility. I choose a formation period of six month.

4.1 The Test Assets

Our model has a sharp prediction regarding the relationship between univariate MF and VM betas and stock expected returns. Higher univariate beta VM leads to lower expected return, while higher univariate MF beta leads to higher expected return. I follow the spirit of the model and construct strategies that select stocks based on their univariate slopes to MF and to VM. I first form portfolio V as the value-weighted top decile of stocks sorted by total volatility. I estimate total volatility using daily returns of the past six months. I then sort stocks into 5x5 = 25 portfolios. First, I sort stocks into quintiles according to their univariate market beta, estimated using daily returns of past six months. Next, for every quintile, I sort stocks into sub-quintiles based on their univariate VM beta.

Since a sound estimation of β_{VM}^{I} is crucial for our tests, we estimate β_{VM}^{I} using a cross-section predictive model that relates the expected β_{VM}^{I} of the following month to the lagged β_{VM}^{I} and other predictive variables detailed below.

4.1.1 Choosing Portfolio V

I use the following procedure to form portfolio V every month:

- 1. I include only stocks that have daily returns for all trading days in the past six months.
- 2. I exclude the lowest decile of stocks in terms of dollar volume.
- 3. I rank stocks according to their total volatility estimated in the previous six months, according to equation (24) below, and pick the valueweighted top decile as portfolio V. If the market share of portfolio V is less than 1%, I keep adding stocks until the 1% cap is reached.

We use the liquidity based filtration, because we are interested to capture the co-movements across volatile stocks and to measure sensitivities to VM. Hence, we refrain from using noisy stocks with low-quality daily return data that suffer from micro-structure issues and from the problem of zero returns that might obscure our estimations.

We estimate $Var(r_i)$ using daily returns of the past six months. Since non-synchronous trading of securities causes daily portfolio returns to be autocorrelated, I follow French, Schwert, and Stambaugh (1987) [20], and estimate $Var(r_i)$ as the sum of the squared daily return plus twice the sum of the products of adjacent return:

$$\hat{\sigma}_{i,t}^2 = \sum_{j=1}^{N_t} (r_{i,j,t})^2 + 2 \sum_{j=1}^{N_t-1} (r_{i,j,t})(r_{i,j+1,t})$$
(24)

Where there are N_t daily stock returns, $r_{i,j,t}$, in formation period t. After obtaining the variance for the entire six months, we divide by six to obtain an estimator for the monthly volatility.

Table (1) provides some statistics about VM and its relation with other well known factors. VM has an average monthly return of -0.86% and its monthly standard deviation 8.22% is, not surprisingly, the highest among the factors. VM has a correlation of 0.69 with SMB, and a correlation of -0.53% with HML, suggesting that the small growth stocks set the tone within portfolo V. Finally, VM has a low correlation with UMD, suggesting that its return is not driven by momentum effects. Finally, we see that the correlation of VM with MF is 0.52. Although the VM portfolio has a short position in the market portfolio, the market beta of VM is high enough to make the correlation between VM and MF positive.

4.1.2 Estimating β_{VM}^{I}

To estimate the next period β_{VM}^{I} for a specific stock, we start by regressing the daily return of the stock on the daily returns of VM. To account for nonsynchronous price movements in returns, we follow Lewellen and Nagel (2006) [29], who include four lags of factor returns, imposing the constraint that lags 2,3, and 4 have the same slope to reduce the number of parameters. The Lewellen and Nagel method is an extension of Dimson (1979) [15], who included current and lagged factor returns in the regression, and addressed the findings that small stocks tend to react with a week or more delay to common news (Lo and MacKinlay (1990), so a daily beta will miss much of the small-stock covariance with market returns. Specifically, we estimate β_X^I , where X is either MF or VM, using the following regression:

$$r_{i,t} = \alpha_i + \beta_{i,0} r_{X,t} + \beta_{i,1} r_{X,t-1} + \beta_{i,2} \left[r_{X,t-2} + r_{X,t-3} + r_{X,t-4} \right] + \epsilon_{i,t}$$
(25)

The estimated beta is then:

$$\beta_{i,X} = \beta_{i,0} + \beta_{i,1} + \beta_{i,2}$$

Literature has shown that stock level beta estimators are quite noisy and not persistent. In our case, the problem is exacerbated, since we are using a short period to estimate $\beta_{VM,t}^{I}$, and the VM returns are very volatile. Table (2) presents cross-sectional predictive regression results of next month $\beta_{VM,t+1}^{I}$ on various variables estimated in the previous six months. Every month t, we measure the next month $\beta_{VM,t+1}^{I}$ using the daily returns of month t + 1. On average, a cross-section regression of $\beta_{VM,t+1}^{I}$ on $\beta_{VM,t}^{I}$ yields an R^2 of just 0.02.

To improve the predicting ability of $\beta_{VM,t+1}^{I}$, we start by adding volatility a stock cannot have high exposure to portfolio V without being volatile itself. The reverse argument, however, is not necessarily true. Stocks with high volatility can have low exposure to volatile stocks, for example, in the event that their volatility is purely idiosyncratic and does not relate to any other firm. Nevertheless, the formation period volatility has a significant predictive power for the next period β_{VM}^{I} . In fact, as a stand alone predictor, it is not inferior to $\beta_{VM,t+1}^{I}$, judging by the average R^{2} which is practically the same between the two. Table (2) also shows that the measure of idiosyncratic risk using the last month daily returns has significant predicting ability for $\beta_{VM,t+1}^{I}$.

If a certain industry is extremely volatile at a certain point in time, the V portfolio is likely to contain a high proportion of stocks associated with that industry. In such an event, we know that if a certain stock belongs to that industry, it is likely to have high exposure to V. To quantify this intuition, we measure the percentage proportion of every industry i (4 digit SIC code) in the market (denote by m_i) and the percentage proportion of every industry i (4 digit SIC code) in the V portfolio (denote by v_i). We construct a measure of industry affiliation to the V portfolio by:

$$\phi_i = v_i - m_i \tag{26}$$

A stock of industry *i* that has the same proportion of market cap in the V portfolio and in the MKT portfolio, will have a neutral affiliation of $\phi_i = 0$. In the period of July 1999 to December 1999, the most volatile industry was SIC 7370, "Services-Computer Programming, Data" with a MKT proportion of 3.67% and a V porportion of 31.29%. Table (2) shows that higher ϕ_i is positively correlated with $\beta_{VM,t+1}^I$.

Table (2) shows that all three variables are significant in a multiple regression of $\beta_{VM,t+1}^{I}$, and so we use all in our predictive regression:

$$\hat{\beta}_{VM,i,t+1}^{I} = C_0 + C_1 \beta_{VM,i,t}^{I} + C_2 \sigma_{i,t} + C_3 \phi_{i,t}$$
(27)

Each month, we run 240 monthly cross sectional regressions over the previous 20 years and estimate the coefficients in (27) as the average of the cross-sectional values. It is important to update the predictive regression as the relationship between the variables can change over time. For example, SIC codes has become more accurate and informative with time and indeed we can see that C_3 is increasing with time.

4.1.3 Portfolios Statistics

Table (3) depicts statistics for the 25 portfolios. TO BE COMPLETED

4.2 Time Series Analysis

To test a factor model, previous studies form portfolios using pre-formation criteria, but examine **post-ranking** factor loading that are computed over the full sample. To provide a convincing factor risk explanation, I need to show that the portfolios also exhibit persistent loadings on portfolio V over the same period used to compute the alphas. The first two panels in Table (4) depicts the post-formation VM betas of the 25 portfolios and their t-statistics. Indeed, in each and every row the post-formation coefficients on VM follows the pre-formation coefficients in terms of ranking.

To examine whether other asset pricing models overprice assets with high β_{VM}^{I} , we examine the Jensen alpha obtained by our portfolios. The second pair of panels in table (4) show the CAPM alphas for the 25 portfolios. Indeed, for each quintile, we see that alphas are decreasing with β_{VM}^{I} . The effect is most pronounced for the sub-quintile with the highest β_{MF}^{I} . TO BE COMPLETED

An important empirical issue is whether our results coincide with those of Ang et al (2006), who found that stocks with high idiosyncratic volatility relative to the Fama and French (1993) model have abysmally low average returns. Disentangling these two effects is not trivial and we start addressing this by sorting the IVOL quintiles into two sub-quintiles based on β_{VM}^I . Tables 5 and 6 show the results of this analysis. The decile with higher β_{VM}^I has significantly lower simple returns and alphas obtained using the CAPM, Fama-French 3 factor model and the Carhart model. A caveat is that the two variables are correlated and therefore soring by β_{VM}^I creates a distinguishable difference in IVOL.

4.3 Fama-MacBeth

Our asset pricing model predicts expected return follows:

$$E(r_i - r_f) = \beta^I_{i,mf} \lambda^I_{mf} + \beta^I_{i,vm} \lambda^I_{vm}$$
(28)

Or in the bivariate form:

$$E(r_i - r_f) = \beta_{i,mf}^{II} \lambda_{mf}^{II} + \beta_{i,vm}^{I} \lambda_{vm}^{II}$$
⁽²⁹⁾

The model also has a sharp prediction regarding the sign of the univariate premiums. The premium on VM is negative, while the premium on MF is positive. Assuming that the correlation between MF and VM is positive, which is supported by the data, we also have the prediction that:

$$\lambda_{vm}^{II} > \lambda_{vm}^{I}$$
$$\lambda_{mf}^{II} < \lambda_{mf}^{I}$$

And naturally, the model suggests that the bivariate premium on each factor equals its expected returns. The Fama-MacBeth [18] methodology is a convenient framework to examine our predictions. In addition, it allows us to take a close look into the relationship between IVOL and β_{VM} . Following Fama-MacBeth, I first perform time series regressions where I regress the excess portfolio returns on a constant and on various factors - MF, SMB, HML, UMD, VM. In the second step, the excess portfolio returns are regressed on the estimated factor loadings in each sample month. Then, a time series average of the estimated coefficients is taken to arrive at point estimates and statistical significance of the factor premia. To examine the role of IVOL, I also use the time-averaged IVOL value for each portfolio.

T O BE COMPLETED

4.4 GMM - SDF

Our model provides an explicit Stochastic Discount Factor which is linear in MF and VM:

$$M_{t+1} = 1 + \left(\tilde{\mu}' \Sigma^{-1} \tilde{\mu}\right)_t - \left(\iota' \Sigma^{-1} \tilde{\mu}\right)_t r_{MF,t+1} + 2\psi_t(r_{VM,t+1})$$

As Cochrane (2005) [13] notes, when the factors are correlated, one should test whether the SDF-parameter coefficients equals zero to see if factor j helps to price the assets rather than to test whether the factor premium obtained from the Fama-MacBeth methodology equals zero. In our case, the factors are indeed correlated.

I estimate the model E[MR] = 1 using the GMM of Hansen (1982) [22]. In the analysis below, I choose the weighting matrix W to be the asymptotically optimal one, given by the inverse of the covariance matrix of the moment conditions. To examine the role of idiosyncratic volatility, I follow Nyberg (2008) [34], and examine whether idiosyncratic volatility has pricing power beyond the pricing power of the stochastic discount factor. In addition, to examine the role of conditioning information in the stochastic model, I estimate the hedging demands term obtained in the model:

$$\psi = \frac{k-1}{\sigma_v - \sigma_V} \approx \frac{(k-1)\rho_V}{(1-\rho_V)\sigma_V}$$

I use daily returns in the formation period to estimate the average correlation between pairs in portfolio V and to estimate the variance of portfolio V. The test assets are the same 25 portfolio used above.

The moment conditions to test the unconditional asset pricing models are:

$$1 = E\left[(B_0 - B'F_{t+1}) \cdot R_{t+1}\right]$$
(30)

The moment conditions to test the effect of IVOL are:

$$1 = E\left[(B_0 - B'F_{t+1}) \cdot R_{t+1} - \gamma_{IVOL}IVOL_t \right]$$
(31)

And the moment conditions to test the conditional model are:

$$1 = E \left[(B_0 - B_1 R_{MF,t+1} - B_2 (1 + B_3 \psi_t) R_{VM,t+1}) \right) \cdot R_{t+1} - \gamma_{IVOL} IVOL_t \right]$$
(32)

TO BE COMPLETED

5 Appendix

5.1 **Proof For 3.2**

First, we examine the best risky strategy of the laggard, given that he invests some wealth in risky assets. We first observe that the laggard will not invest in more than one stock of the same group. Merging all the wealth invested in some group i into a single stock of i will not change his expected return and his covariance with the leader, however, it will increase his variance. Since the laggard cannot have his expected wealth higher than that of the leader, he is better off investing in a single asset of a single group.

Now, the risky strategy of the laggard can be characterized by a size N vector θ_g , $\theta'_g \iota = 1$, that reflects his investment in a single stock of each group. The risky strategy of the leader can be characterized by a size N vector θ_d , $\theta'_d \iota = 1$, that reflects her investments in an equally-weighted portfolio of the different groups. We can write the laggard problem:

$$Min_{\theta_g} \frac{k \left(w_d \theta'_d \mu + 1 - w_d\right) - \left(w_g \theta'_g \mu + 1 - w_g\right)}{\sqrt{k^2 w_d^2 \theta'_d \hat{\Sigma} \theta_d - 2k w_g w_d \theta'_d \hat{\Sigma} \theta_g + w_g^2 \theta'_g \Sigma \theta_g}}$$
(33)

Note that we used the covariance matrix for groups, Σ to express the variance of the leader, and the covariance matrix for individual stocks, Σ to express the variance of the laggard. It is easy to show that the covariance between group i and group j is equivalent to the covariance between group i and an individual asset of group j, and therefore, we can use the groups covariance matrix in the expression for the covariance between the leader and the laggard.

So, we are left with strategies where the laggard invests across groups, where in each group he invests in a single stock. Now, I will show that the best response of the laggard is to invest in a single stock of a specific group i, that is, $\theta_g = E_i$, where E_i is a vector of zeros except for entry i, which is one. Assume by contradiction that the best response of the laggard is not to invest in a single stock. In this case, there must be two groups i and jwhere he invests a portion of his risky portfolio w_i^* and w_j^* . Due to the short sales constraint, it must be that $0 < w_i^*, w_j^* < 1$. We can examine strategies where the laggard transfers wealth from i to j and invests $w_i = w_i^* + x$ and $w_i = w_j^* - x$ in a single asset of group i and a single asset of group j. Consider the unconstraint problem of the laggard, solving for x. The optimality of w_i^* and w_j^* , and the fact that $0 < w_i^*, w_j^* < 1$, guarantees that x = 0 is a local minimum for this problem. In this point, the value of the objective function of the laggard is positive, as the leader has a higher expected wealth than the laggard. Note that for the unconstraint problem of the laggard, he can increase the weight on the security that has higher expected return and reach negative values for this function. Alternatively, if both securities have the same expected return, he can increase the weight on one of them, taking the denominator to infinity and the value of the objective function to zero. Therefore, the function U(x) must have at least one maximum point. So, U(x) has at least two extremum points. However, the function U(x) is of the form:

$$U(x) = \frac{a+bx}{\sqrt{Ax^2 + Bx + C}} \tag{34}$$

It is easy to show that this function has only one extremum point. Contradiction. Hence, the laggard will invest only in one risky asset. Now, considering his investment between the risky portfolio and the risk free asset, we can use the same "trick" to show that he must invest all-in the risk-free asset or all-in the risky portfolio. In this case, x will be the amount invested in the risky asset, and the objective function U(x) still follows equation (34)

5.2 Proof For (3.3)

This is proof is for one group, and can be easily generalized to many groups, since it is obvious that in a group that the laggard does not invest in, the leader will use an equally weighted portfolio - better from variance point of view, and the same level of expected return and covariance with the laggard.

Given w_d , the leader is looking for a risky portfolio θ to maximizes:

$$U_n(x) = \frac{1}{n} \sum_{j=1}^n \Phi\left(\frac{k\left(\mu w_d + 1 - w_d\right) - \mu}{\sigma\sqrt{1 - 2kw_d\left(\rho + (1 - \rho)\theta_j\right) + k^2 w_d^2\left(\rho + (1 - \rho)\theta'\theta\right)}}\right)$$
(35)

We will prove this proposition in several steps:

Lemma 5.1. The function $U_n(x)$ has a maximum

Proof. Since this function represents a probability it is positive and bounded from above by 1. Since the leader always pursues strategies that give her a higher expected wealth than the laggard, the function is bounded from below by 0.5. In addition, the function asymptotically goes to 0.5 as the distance of portfolios from the equally weighted portfolio goes to infinity. All we need to conclude that there is a maximum point is continuity.

The denominator of the argument in Φ is the standard deviation of the wealth difference between the players at the end of the game. It could be zero in case $kw_d = 1$. In this case the leader matches not only the portfolio of the laggard, but also the dollar amount invested in the asset. We augment the function by setting these points (we have n of them) to be the value of the limit at this point, which is obtained just by replacing $\Phi(\frac{A}{0})$ in 1. The augmented function is continuous and so we have a maximum point.

Lemma 5.2. The points that maximize the function $U_n(x)$ take the following form:

$$\theta = x \frac{1}{n} \iota + (1 - x) E_j; \ j \in \{1, ..., n\}$$
(36)

That is, they are a linear combination of the equally weighted portfolio and a vector with 1 in a single entry and zero in the rest.

Proof. Two "hand-waving" arguments at this point:

- 1. The leader response reflects the tradeoff between variance and covariance. If the leader wants to minimize variance, she should invest in the equally weighted portfolio. However, if she wants to maximize the realized covariance, in case she bets successfully on the laggard's stock, she is better off investing in a single asset. The solution reflects this tradeoff and so it is a linear combination between the equally weighted portfolio and a vector with 1 in a single entry and zero in the rest.
- 2. Since this problem is symmetric, we expect to see symmetry in the solutions. The equally weighted portfolio is an obvious candidate since it is symmetric in the *n* entries. A combination between the E_j vector and the equally weighted portfolio is symmetric in n-1 entries. Say we have a solution that is symmetric in n-2 entries. This means that the order of number of solutions is $O(n^2)$, which is not likely for this problem.

We need a rigorous proof.

Lemma 5.3. There exists an n_0 such that for every $n > n_0$, the equally weighted point is a local maximum

Proof. Now the leader solves for the right combination x between the equally weighted and , WLOG, E_1 :

$$U_{n}(x) = \frac{n-1}{n} \Phi \left(\frac{k\left(\mu w_{d} + 1 - w_{d}\right) - \mu}{\sigma \sqrt{1 - 2kw_{d}\left(\rho + (1-\rho)\left[\frac{x}{n}\right]\right) + k^{2}w_{d}^{2}\left(\rho + (1-\rho)\left[\frac{x(2-x)}{n} + (1-x)^{2}\right]\right)}}}\right) + \frac{1}{(37)} \Phi \left(\frac{k\left(\mu w_{d} + 1 - w_{d}\right) - \mu}{\sigma \sqrt{1 - 2kw_{d}\left(\rho + (1-\rho)\left[\frac{x}{n} + (1-x)\right]\right) + k^{2}w_{d}^{2}\left(\rho + (1-\rho)\left[\frac{x(2-x)}{n} + (1-x)^{2}\right]\right)}}}\right)$$

Economizing notation, we write the problem as:

$$U_n(x) = \frac{n-1}{n} \Phi\left(\frac{A}{\sqrt{f_n(x)}}\right) + \frac{1}{n} \Phi\left(\frac{A}{\sqrt{g_n(x)}}\right)$$
(38)

Where the variance of the wealth difference between the players in the event that the laggard chose an asset different than 1 is expressed as a function of x, the weight invested in the equally weighted portfolio:

$$f_n(x) = 1 - 2kw_d \left(\rho + (1-\rho) \left[\frac{x}{n}\right]\right) + k^2 w_d^2 \left(\rho + (1-\rho) \left[\frac{x(2-x)}{n} + (1-x)^2\right]\right)$$
(39)

And where the variance of the wealth difference between the players in the event that the laggard chose asset 1:

$$g_n(x) = f_n(x) - 2kw_d(1-\rho)(1-x)$$
(40)

We check whether the F.O.C holds for the equally weighted portfolio, that is, x = 1:

$$U'_{n}(x) = (n-1)\frac{f'_{n}(x)\Phi'(\frac{A}{\sqrt{f_{n}(x)}})}{f_{n}(x)^{\frac{3}{2}}} + \frac{g'_{n}(x)\Phi'(\frac{A}{\sqrt{g_{n}(x)}})}{g_{n}(x)^{\frac{3}{2}}} = 0$$
(41)

We find the first derivatives of $f_n(x)$ and $g_n(x)$:

$$f'_{n}(x) = \frac{2k(1-\rho)w(-1+k(n-1)w(x-1))}{n}$$
(42)

$$g'_n(x) = f'_n(x) + 2kw_d(1-\rho)$$
(43)

Since in the equally weighted portfolio f(x) = g(x), we are left with:

$$U'_{n}(1) = (n-1) f'_{n}(1) + g'_{n}(1) = (n-1) f'_{n}(1) + f'_{n}(1) + 2kw_{d}(1-\rho) = (44)$$
$$-2kw_{d}(1-\rho) + 2kw_{d}(1-\rho) = 0$$

So, we conclude that the equally weighted is an extremum point. We will examine whether it is a minimum or maximum by taking the second derivative. First we take the second derivative of f(x) and g(x):

$$g_n''(x) = f_n''(x) = \frac{2k^2 w^2 (n-1) (1-\rho)}{n}$$
(45)

And to the second derivative of the objective function:

$$U_{n}''(x) = \frac{3Ag_{n}'(x)^{2}\Phi'(\frac{A}{\sqrt{g_{n}(x)}})}{4g_{n}(x)^{\frac{5}{2}}} - \frac{A\Phi'(\frac{A}{\sqrt{g_{n}(x)}})g_{n}''(x)}{2g_{n}(x)^{\frac{3}{2}}} + \frac{A^{2}g_{n}'(x)^{2}\Phi''(\frac{A}{\sqrt{g_{n}(x)}})}{4g_{n}(x)^{3}} + (46)$$

$$(n-1)\left(\frac{3Af_{n}'(x)^{2}\Phi'(\frac{A}{\sqrt{f_{n}(x)}})}{4f_{n}(x)^{\frac{5}{2}}} - \frac{A\Phi'(\frac{A}{\sqrt{f_{n}(x)}})f_{n}''(x)}{2f_{n}(x)^{\frac{3}{2}}} + \frac{A^{2}f_{n}'(x)^{2}\Phi''(\frac{A}{\sqrt{f_{n}(x)}})}{4f_{n}(x)^{3}}\right)$$

The terms $g'_n(1), g''_n(1)$ are of order one, and when we take this term to the limit as n goes to infinity (eliminating subscripts to denote the limit):

$$U''(x) = \lim_{n \to \infty} n \left(\frac{3Af'(x)^2 \Phi'(\frac{A}{\sqrt{f(x)}})}{4f(x)^{\frac{5}{2}}} - \frac{A\Phi'(\frac{A}{\sqrt{f(x)}})f''(x)}{2f(x)^{\frac{3}{2}}} + \frac{A^2f'(x)^2 \Phi''(\frac{A}{\sqrt{f(x)}})}{4f(x)^3} \right)$$
(47)

We see that f'(1) is of the order of $O\left(\frac{1}{n}\right)$, f(1) is bounded from below, and f''(1) is a positive constant, and so:

$$U''(1) = -\lim_{n \to \infty} n\left(\frac{A\Phi'(\frac{A}{\sqrt{f(1)}})f''(1)}{2f(1)^{\frac{3}{2}}}\right) = -\infty$$
(48)

We conclude that there is an n_0 such that for all $n > n_0$, the equally weighted is a local maximum.

Lemma 5.4. There exists an n_0 such that for every $n > n_0$, the equally weighted point is a **global** maximum

Proof. Given that the solution for this problem is of the form of (36), we are going to show that there could not be $\tilde{x} \neq 1$ that is a maximum (note that we are not concerned by the kw = 1 case, because we can show that it is neither a minimum nor a maximum point). We assume by contradiction that there is a another local maximum point \tilde{x} of the form of (36). If \tilde{x} is a local maximum, and we know that the equally-weighted point is a local maximum, then it means that there must be a point \hat{x} that takes the form of (36) and is a local minimum. However, we next show that any point x that satisfies the F.O.C is a maximum point for a large enough n.

We use the F.O.C relation of (44) in the second order equation (46) to show that for large enough n, any point that satisfies the F.O.C is a maximum point. Similarly to (49), the limit of the second derivative of the solution is minus infinity:

$$U''(x) = -\lim_{n \to \infty} n \left(\frac{A\Phi'(\frac{A}{\sqrt{f(1)}})f''(x)}{2f(x)^{\frac{3}{2}}} \right) = -\infty$$
(49)

This is a contradiction.

This completes the proof of proposition (3.3).

5.3 Proof for Theorem 3.4

Denoting $A = \tilde{\mu}' \hat{\Sigma}^{-1} \tilde{\mu}$, we can simplify the following terms:

$$\theta'\tilde{\mu} = xA + y\tilde{\mu}_1$$

$$\theta'\hat{\Sigma}\theta = x^2A + 2xy\tilde{\mu}_1 + y^2\hat{\sigma}_1^2$$
$$\theta'\hat{\Sigma}E_1 = x\tilde{\mu}_1 + y\hat{\sigma}_1^2$$

So, we can write the leader's problem as:

$$Max_{(x,y)} \frac{k\left([xA+y\tilde{\mu}_{1}]+1\right)-(\tilde{\mu}_{1}+1)}{\sqrt{k^{2}[x^{2}A+2xy\tilde{\mu}_{1}+y^{2}\hat{\sigma}_{1}^{2}]-2k\left[x\tilde{\mu}_{1}+y\hat{\sigma}_{1}^{2}\right]+\sigma_{1}^{2}}}$$
(50)

Taking the first order conditions for x and y and equating both to zero leads to the solution:

$$x = \frac{\sigma_1^2 - \hat{\sigma}_1^2}{k(k-1)}$$
$$y = \frac{1}{k}$$

So the leader's risky portfolio over groups is:

$$\theta = \frac{\sigma_1^2 - \hat{\sigma}_1^2}{k(k-1)} \Sigma^{-1} \tilde{\mu} + \frac{1}{k} E_1$$
(51)

Given the leader's strategy, we revisit the problem of the laggard and find the conditions required to keep the laggard investing in group 1. Since the laggard invests only in a single stock or in the risk free asset, we examine his utility from investing in asset j (note that the laggard is interested in minimizing U_j):

$$U_j = \frac{k \left(\theta' \tilde{\mu} + 1\right) - \left(\tilde{\mu}_j + 1\right)}{\sqrt{k^2 \theta' \hat{\Sigma} \theta - 2k \theta' \hat{\Sigma} E_j + \sigma_j^2}}$$
(52)

Plugging the investment of the leader, where

$$A = \tilde{\mu}' \hat{\Sigma}^{-1} \tilde{\mu}$$
$$x = \frac{\sigma_1^2 - \hat{\sigma}_1^2}{k(k-1)}$$

We get:

$$U_{j} = \frac{k\left(xA + \frac{\tilde{\mu}_{1}}{k} + 1\right) - (\tilde{\mu}_{j} + 1)}{\sqrt{k^{2}\left[x^{2}A + 2x\frac{\tilde{\mu}_{1}}{k} + \frac{\hat{\sigma}_{1}^{2}}{k^{2}}\right] - 2k\left[x\tilde{\mu}_{j} + \frac{\hat{\sigma}_{1,j}}{k}\right] + \sigma_{j}^{2}}}$$
(53)

We examine the conditions to guarantee that $U_j > U_1$ for all j. The condition is algebraically involved, but we can find sufficient conditions to satisfy this inequality:

- 1. $\hat{\sigma}_{1,j} \geq 0 \ \forall j$
- 2. $\sigma_1 \ge \sqrt{2}\sigma_j \ \forall j$
- 3. $\sigma_1 \geq 2\hat{\sigma}_1$

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Factor	Mean	StdDev	Corr(i,mktrf)	$\operatorname{Corr}(i, \operatorname{smb})$	Corr(i,hml)	$\operatorname{Corr}(i, \operatorname{umd})$	$\operatorname{Corr}(i, VM)$
mktrf	0.38	4.45	1.00	0.30	-0.38	-0.08	0.52
smb	0.24	3.19	0.30	1.00	-0.26	0.01	0.69
hml	0.43	2.89	-0.38	-0.26	1.00	-0.13	-0.53
umd	0.86	4.03	-0.08	0.01	-0.13	1.00	-0.06
vm	-0.86	8.22	0.52	0.69	-0.53	-0.06	1.00

Table 1: Factors Statistics The table reports the means, standard deviations and correlations of VM and various factors. The factors MKTRF, SMB, HML are the Fama and French (1993) factors, the momentum factor UMD is constructed by Kenneth French. Sample period is July 1963 to December 2008, and the estimation values relate to monthly returns. VM is constructed every month using the following procedure. I consider only stocks with daily returns for every trading day in the previous six months. Next, I filter stocks with the lowest decile in terms of dollar volume (volume times price). Then, I estimate the monthly return volatility of stocks using the six months daily returns corrected for one-lag autocorrelation as in FSS (87). I form a value-weighted portfolio out of the highest decile of stocks sorted by total volatility and obtain the monthly return for the consecutive month. Finally, I substract the Fama and French MKT return to obtain the monthly return of VM.

const	$\beta^{I}_{VM,i,t}$	$\sigma_{i,t}^2$	$\phi_{i,t}$	$IVOL_{i,t}$	\bar{R}^2
0.35	0.38				0.02
(9.15)	(11.31)				
0.11		0.04			0.02
(2.77)		(7.09)			
0.56			0.05		0.01
(8.79)			(14.71)		
0.34				0.09	0.01
(7.05)				(5.96)	
0.16	0.22	0.02	0.03		0.03
(4.32)	(11.78)	(6.23)	(9.30)		

Table 2: **Predictive Regressions of** $\beta_{VM,i,t+1}^{I}$ - the table summarizes the results of firm-level cross-sectional predictive regressions of $\beta_{VM,i,t+1}^{I}$ on various variables. $\beta_{VM,i,t+1}^{I}$ is estimated using the daily returns in month (t+1) as in regression (25). The independent variables $\beta_{VM,i,t}^{I}$, $\sigma_{i,t}^{2}$, and $\phi_{i,t}$ are estimated using daily returns in six month prior to month (t+1). $\beta_{VM,i,t}^{I}$ is estimated by a univariate regression of stock daily returns on VM as in regression (25). $\sigma_{i,t}^{2}$ is estimated using the prior six month daily returns corrected for one-lag autocorrelation as in FSS (87) and in equation 24. $\phi_{i,t}$ measures the affiliation of the 4 digits SIC industry of stock *i* with portfolio V, formed using the prior six month daily returns of all stocks, as in equation (26). IVOL is estimated following Ang et al (2006) using the daily returns of the month prior to (t+1). I ran the cross sectional regressions each month from July 1963 to December 2008. Robust NeweyWest t-stats obtained from the time series of estimated coefficients, and reported in parenthesis. The column $\overline{R^2}$ represents the average R^2 across time.

	Low	2	3	4	High	Low	2	3	4	High
		Me	ean Ret	urn			Stc	l.dev Re	turn	
Low β^{I}_{MF}	0.89	0.79	1.00	0.96	0.95	3.28	3.62	4.50	5.38	7.31
2	0.94	1.05	1.09	0.95	0.81	3.63	4.22	4.54	5.20	6.66
3	1.08	1.06	1.09	0.95	0.78	4.21	4.79	5.41	6.17	7.54
4	1.01	0.96	1.01	1.07	0.90	5.29	5.81	6.52	7.29	8.35
High β^{I}_{MF}	1.09	1.10	0.89	0.66	0.37	6.67	8.06	8.44	9.20	10.58
		Mε	rket Sl	nare			Boo	ok To M	arket	
Low β^{I}_{MF}	0.09	0.05	0.03	0.02	0.01	0.66	0.57	0.59	0.63	0.63
2	0.12	0.06	0.04	0.02	0.01	0.55	0.54	0.58	0.59	0.59
3	0.13	0.05	0.03	0.02	0.01	0.51	0.55	0.57	0.57	0.54
4	0.10	0.04	0.03	0.02	0.01	0.53	0.54	0.56	0.55	0.52
High β^{I}_{MF}	0.05	0.02	0.02	0.01	0.01	0.54	0.55	0.54	0.54	0.54
		IV	OL (da	uily)		Volatility (Monthly)				
1	0.88	1.17	1.48	1.91	2.85	4.88	6.26	7.71	9.59	13.01
2	0.95	1.13	1.33	1.62	2.24	5.70	6.65	7.60	8.92	11.62
3	1.04	1.26	1.46	1.74	2.24	6.54	7.53	8.57	9.87	11.99
4	1.21	1.47	1.70	1.97	2.37	7.65	8.88	9.94	11.24	12.78
5	1.59	1.95	2.21	2.53	3.18	9.69	11.46	12.66	14.06	16.21

Univariate β_{VM}^{I} Quintiles

Table 3: Statistics for 5x5 portfolios - I form 25 value-weighted portfolios, sorting first on univariate MF beta β_{mf}^{I} , and then sorting by univariate VM beta (β_{vm}^{I}) . I form portfolio VM as described in Table (1). Then, for every stock with more than 12 days and more than 75% of trading days in each month in the past six month, I run a univariate regression on MF to obtain β_{mf}^{I} , and a univariate regression on VM to obtain β_{vm}^{I} , as in equation (25). Stocks are then sorted to quintiles according to β_{mf}^{I} , and within each equintile, they are sorted to 5 sub quintiles according to β_{vm}^{I} . The statistics in the first row panels labeled Raw Returns Mean and Std.Dev. are measured in monthly percentage terms and apply to total simple returns. The panel Market Share is in percentage and represent the average market share of each portfolio. The values in the panels BM, IVOL, and Volatility are calculated in each formation period for each portfolio using a valueweighted average across stocks, and then averaged across time. The panel BM represents the book-to-market ratio within each portfolio, calculated following Fama-French. IVOL is measured following Ang et al using the last month daily returns. Volatility is measured using the six months daily returns corrected for one-lag autocorrelation as in FSS (87). univariate β_{VM}^{I} , univariate β_{MKT}^{I} , and bivariate β_{MKT}^{II} report the value-weighted average of pre-formation regressions of stock returns on VM (portfolio V - the market), the market, and a bivariate regression of stock returns on the market and VM, averaged across the entire sample. The pre-formation regressions use Dimson one-lag correction. The period is from January 1945 to December 2008.

	Univariate β_{VM}^I Quintiles											
	Low	2	3	4	High	HL	Low	2	3	4	High	HL
-		F	Post-For	m β_{VM}^I					${ m t}(eta)$	$_{VM}^{l})$		
Low β^{I}_{MF}	0.04	0.11	0.21	0.32	0.64	0.60	1.15	2.37	4.06	5.67	9.45	10.00
2	0.08	0.19	0.22	0.30	0.54	0.46	1.40	3.76	3.48	4.98	8.21	10.72
3	0.18	0.25	0.36	0.47	0.70	0.52	3.51	3.90	5.71	6.98	11.51	15.07
4	0.34	0.42	0.53	0.70	0.77	0.44	6.24	6.73	8.97	17.90	8.11	7.37
High β_{MF}^{I}	0.55	0.76	0.85	1.01	1.21	0.66	10.04	16.71	18.22	15.99	14.53	11.16
_	Post-Form α_{CAPM}								$t(\alpha_{C})$	$_{APM})$		
Low β_{MF}^{I}	0.22	0.05	0.14	0.03	-0.13	-0.35	2.46	0.47	1.24	0.20	-0.70	-1.66
2	0.19	0.19	0.19	-0.03	-0.28	-0.48	2.48	2.28	2.32	-0.30	-2.02	-2.69
3	0.21	0.11	0.06	-0.14	-0.41	-0.62	3.17	1.56	0.78	-1.44	-2.53	-3.31
4	0.00	-0.12	-0.14	-0.12	-0.39	-0.39	-0.03	-1.15	-1.46	-0.79	-2.15	-1.90
High β^{I}_{MF}	-0.07	-0.18	-0.42	-0.69	-1.08	-1.01	-0.57	-1.02	-2.40	-3.18	-4.11	-4.19
$P(\alpha = 0)$	0.0014											
_	Post-Form α_{FF3}								$t(\alpha)$	FF3)		
Low β^{I}_{MF}	0.07	-0.09	-0.01	-0.14	-0.17	-0.25	0.92	-0.95	-0.14	-1.12	-1.08	-1.41
2	0.09	0.09	0.08	-0.15	-0.30	-0.38	1.25	1.19	1.13	-1.85	-2.43	-2.79
3	0.16	0.06	0.04	-0.14	-0.36	-0.52	2.54	0.77	0.53	-1.55	-2.52	-3.38
4	0.05	-0.04	-0.05	-0.01	-0.32	-0.37	0.68	-0.45	-0.49	-0.06	-1.92	-2.05
High β^{I}_{MF}	0.05	0.02	-0.24	-0.50	-0.84	-0.90	0.47	0.13	-1.76	-2.79	-3.58	-3.60
$P(\alpha = 0)$	0.0001											
		Pe	ost-Form	n α_{CAR}	4				$t(\alpha_C$	(AR4)		
Low β^{I}_{MF}	0.14	-0.04	0.00	-0.15	-0.20	-0.34	1.45	-0.41	0.02	-1.08	-1.11	-1.64
2	0.14	0.14	0.13	-0.15	-0.25	-0.39	1.71	1.80	1.59	-1.66	-1.99	-2.62
3	0.17	0.13	0.13	-0.09	-0.23	-0.40	2.81	1.87	1.44	-0.98	-1.32	-2.23
4	0.09	0.01	0.05	0.05	-0.24	-0.33	0.98	0.07	0.44	0.38	-1.43	-1.83
High β_{MF}^{I}	0.09	0.04	-0.19	-0.38	-0.75	-0.84	0.70	0.30	-1.28	-2.17	-2.81	-3.04
$P(\alpha = 0)$	0.0001											
		Pos	t-Form	α_{MF+V}	M				$t(\alpha_{MH})$	F+VM)		
Low β^{I}_{MF}	0.06	-0.08	0.08	0.08	0.25	0.19	0.69	-0.89	0.78	0.53	1.45	1.03
2	0.02	0.09	0.11	-0.05	-0.06	-0.08	0.24	1.15	1.40	-0.54	-0.48	-0.53
3	0.10	0.04	0.09	-0.01	-0.03	-0.12	1.48	0.53	1.08	-0.09	-0.17	-0.73
4	0.00	-0.04	0.03	0.26	0.03	0.03	0.00	-0.37	0.36	2.08	0.17	0.16
High β^{I}_{MF}	0.13	0.23	0.08	0.00	-0.16	-0.29	1.13	1.68	0.58	0.01	-1.01	-1.51
$P(\alpha = 0)$	0.0953											

Table 4: Post-Formation Regressoins - January 1945 to December 2008 - For the 5x5 portfolios described in Table 3, the table depicts resluts of various post-formation monthly regressions. The left panels depict point estimations and the right panels depict robust Newey-West t-stats. The table shows 5 values for each portfolio. The first is the post-formation β_{VM}^I estimated using a regression of portfolio monthly returns on VM monthly returns. The next three values are Jensens alphas with respect to the CAPM, the FamaFrench (1993) three-factor model, and the Fama-French-Carhart four-factor model. The last value is the alpha obtained from a regression of portfolio return on MF and VM. In each panel, the last row is the p value obtained for a joint test for the alphas equal to zero. The test is conducted by first estimating all 25 portfolio simultaneously using GMM with robust Heteroskedasticity and Autocorrelation Consistent Covariance Matrix, and then using a Wald test.

	IVOL Quintiles												
	beta	2	3	4	High	Low	2	3	4	High			
		Me	ean Ret	urn			Std	l.dev Re	turn				
Low β_{VM}^{I}	0.85	0.94	0.90	0.86	0.31	3.55	4.40	5.21	6.29	7.47			
High β_{VM}^{I}	0.96	0.98	1.06	0.69	-0.39	4.91	6.26	7.59	9.43	10.67			
		Ma	rket Sł	nare			Boo	k To M	arket				
Low β_{VM}^{I}	0.38	0.15	0.07	0.03	0.01	0.53	0.56	0.58	0.62	0.72			
High β_{VM}^{I}	0.19	0.09	0.05	0.02	0.01	0.50	0.52	0.53	0.55	0.64			
		IV	OL (da	ily)			Volatility (Monthly)						
Low β_{VM}^{I}	0.88	1.47	2.04	2.80	4.39	6.07	7.84	9.51	11.52	14.23			
High β_{VM}^{I}	1.01	1.52	2.09	2.88	4.98	7.95	10.26	12.74	15.49	18.79			

TVOL Outstil

Table 5: β_{VM}^I and IVOL - We form quinile portfolios sorted by IVOL following Ang et al (2006). Then, we sort each quintile into two sub-quintiles according to β_{VM}^I . The statistics correspond to the ones in table 3. The sample period is from July 1963 to December 2008.

		IVOL Quintiles													
	Low	2	3	4	High	HL	Low	2	3	4	High	HL			
			Post-Fo	rm β_{VM}	[$t(eta_{VM})$.							
Base	0.14	0.29	0.48	0.71	0.89	0.75	3.43	5.23	8.94	16.99	19.19	23.34			
Low β_{VM}	0.08	0.17	0.33	0.49	0.63	0.55	1.94	3.02	6.56	10.81	11.62	12.30			
High β_{VM}	0.26	0.47	0.68	0.95	1.16	0.90	5.58	8.35	11.76	21.12	23.07	30.49			
HL β_{VM}	0.19	0.30	0.34	0.46	0.53	0.35	13.85	11.90	12.48	23.03	13.56	8.87			
	Post-Form α_{CAPM}								α_{CA}	APM	5 10 / 178				
Base	0.12	0.06	0.01	-0.22	-1.07	-1.20	2.14	1.32	0.09	-1.35	-5.19	-4.78			
Low β_{VM}	0.13	0.13	0.02	-0.08	-0.67	-0.80	1.69	1.86	0.25	-0.60	-3.77	-3.55			
High β_{VM}	0.11	0.02	0.02	-0.44	-1.55	-1.65	1.61	0.21	0.14	-2.01	-5.37	-5.36			
HL β_{VM}	-0.02	-0.11	0.00	-0.36	-0.87	-0.86	-0.21	-0.79	0.01	-2.01	-3.66	-3.78			
]	Post-For	$m \alpha_{FF}$	3			α_{FF3}							
Base	0.10	0.06	0.03	-0.16	-1.14	-1.23	2.18	1.15	0.49	-1.27	-7.60	-6.95			
Low β_{VM}	0.06	0.04	-0.05	-0.17	-0.86	-0.92	1.03	0.71	-0.62	-1.62	-6.34	-5.46			
High β_{VM}	0.13	0.13	0.12	-0.26	-1.52	-1.65	2.25	1.38	1.15	-1.50	-6.95	-7.22			
HL β_{VM}	0.07	0.08	0.17	-0.10	-0.66	-0.73	0.77	0.76	1.29	-0.56	-2.89	-3.15			
		P	ost-For	m α_{CAF}	84				α_C	AR4					
Base	0.08	0.12	0.10	-0.07	-0.88	-0.96	1.54	2.14	1.54	-0.60	-5.45	-4.88			
Low β_{VM}	0.05	0.06	-0.02	-0.07	-0.67	-0.72	0.77	0.98	-0.29	-0.66	-4.41	-3.83			
High β_{VM}	0.10	0.20	0.18	-0.16	-1.21	-1.31	1.51	2.08	1.71	-0.90	-4.74	-4.80			
HL β_{VM}	0.05	0.14	0.20	-0.09	-0.54	-0.59	0.62	1.12	1.56	-0.45	-1.97	-2.17			

Table 6: β_{VM}^{I} and IVOL - We form quinile portfolios sorted by IVOL following Ang et al (2006). Then, we sort each quintile into two sub-quintiles according to β_{VM}^{I} . The statistics correspond to the ones in table 4. The sample period is from July 1963 to December 2008.

	с	λ_{mkt}	λ_{smb}	λ_{hml}	λ_{umd}	λ_{vm}	IVOL	R^2	$Adj.R^2$
1	0.68	-0.24						0.16	0.13
	(2.76)	(-0.80)							
2	0.56					-0.41		0.28	0.25
	(3.51)					(-1.05)			
3	0.77						-0.20	0.37	0.34
	(3.86)						(-1.34)		
4	-0.54	1.67^{I}				-2.47^{I}		0.61	0.58
	(-1.67)	(3.31)				(-3.64)			
5	-0.54	0.97				-0.86		0.61	0.58
	(-1.67)	(2.68)				(-2.22)			
6	0.70	0.18					-0.28	0.39	0.34
	(2.84)	(0.54)					(-2.14)		
7	-0.50	1.63^{I}				-2.39^{I}	-0.02	0.61	0.56
	(-1.48)	(3.51)				(-2.99)	(-0.11)		
8	-0.50	0.96				-0.82	-0.02	0.61	0.56
	(-1.48)	(2.88)				(-1.48)	(-0.11)		
9	0.24	0.60				-0.82^{\perp}	-0.27	0.61	0.56
	(0.55)	(1.35)				(-1.48)	(-2.11)		
10	-0.54	0.97				-0.86	-0.02^{\perp}	0.61	0.56
	(-1.67)	(2.68)				(-2.22)	(-0.11)		
11	-0.33	0.75	-0.39	0.36				0.37	0.28
	(-0.99)	(2.01)	(-1.86)	(1.26)					
12	-0.02	0.49	-0.10	-0.48		-1.02		0.69	0.63
	(-0.06)	(1.28)	(-0.45)	(-1.42)		(-2.57)			
13	-0.20	0.75	-0.40	0.14	1.85			0.47	0.36
	(-0.59)	(1.98)	(-1.92)	(0.47)	(2.15)				
14	0.05	0.50	-0.13	-0.57	1.62	-1.11		0.73	0.66
	(0.13)	(1.32)	(-0.58)	(-1.66)	(1.90)	(-2.75)			

Table 7: **Fama-MacBeth Analysis** This table shows the estimated Fama-MacBeth (1973) factor premiums on 25 portfolios sorted first by β_{MF}^{I} and then by β_{VM}^{I} . MKT is the market factor, SMB and HML are the Fama-French (1993) factors and UMD is a momentum factor. VM is constructed as explained in Table 1. IVOL is the value weighted average of idiosyncratic risk as calculated by Ang et al (2006) measured using the previous month daily returns. Fama-MacBeth (1973) t-values are shown in brackets. A notation I means that I used the beta obtained from a univariate time series regression. The notation \perp for a certain variable means that I first orthogonalized the variable with respect to the other independent variables in the regression. The sample period is from July 1963 to December 2008.

	С	b_{MKT}	b_{SMB}	b_{HML}	b_{UMD}	b_{VM}	γ_{IVOL}	ψ	ТJ	P-Val
1	0.99	-0.16							41.82	0.01
	(195.93)	(-0.13)								
2	1.00	3.25					-0.005		36.05	0.03
	(140.32)	(2.39)					(-4.55)			
3	1.10	10.64				-5.34			29.47	0.13
	(38.93)	(4.91)				(-4.86)				
4	1.10	11.20				-6.19		127.62	25.88	0.21
	(36.63)	(4.84)				(-3.66)		(1.13)		
5	1.08	9.59				-4.21	-0.002	. ,	29.41	0.10
	(36.99)	(4.27)				(-3.26)	(-1.54)			
6	1.04	8.35	-8.62	6.77		· /	· /		35.47	0.03
	(29.27)	(2.92)	(-3.23)	(1.46)						
7	1.09	8.84	3.48	13.08			-0.007		28.83	0.09
	(23.48)	(3.33)	(0.92)	(2.71)			(-3.62)			
8	1.10	9.63	3.83	-2.50		-6.15	· /		29.79	0.07
	(26.86)	(3.49)	(0.99)	(-0.48)		(-3.59)				
9	1.12	6.71	-8.55	2.16	12.19	· /			31.85	0.04
	(20.02)	(2.51)	(-3.22)	(0.49)	(2.68)					
10	1.18	7.97	3.55	10.08	11.55		-0.007		27.76	0.09
	(16.95)	(2.38)	(0.84)	(1.63)	(2.07)		(-3.07)			
11	1.09	4.44	4.24	-16.28	7.47	-7.14	. ,		29.18	0.06
	(19.21)	(1.62)	(0.97)	(-2.86)	(1.64)	(-3.23)				

Table 8: **SDF/GMM Tests** This table shows results from estimating the stochastic discount factor M = CF, using the moment conditions E[MR] = 1. MKT is the market factor, SMB and HML are the Fama-French (1993) factors and UMD is a momentum factor. IVOL is a measure of idiosyncratic volatility as used by Ang et al. It is measured in the last month of the formation period. The parameter estimates are obtained from minimizing the GMM-criterion function where the weighting matrix of moment conditions is the asymptotically optimal one. TJ is Hansen.s (1982) test of overidentifying restrictions , and P-Val the corresponding p-value. The sample period is from July 1963 to December 2008.