

# Learning, Confidence and Option Prices

Ivan Shaliastovich \*

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Comments Welcome

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\*Ivan Shaliastovich (email: [ivan.shaliastovich@duke.edu](mailto:ivan.shaliastovich@duke.edu)) is at the Department of Economics, Duke University. I would like to thank Ravi Bansal, George Tauchen, Adriano Rampini and David Hsieh, and the participants of Financial Econometrics Duke Workshop and Fuqua Brown Bag Seminar for their comments.

# Learning, Confidence and Option Prices

## Abstract

Out-of-the-money index put options appear overpriced, so that the insurance for large downward moves in underlying asset prices is expensive relative to standard models. These findings indicate that investors are concerned with large, negative moves in underlying prices, which occur approximately once a year in the data. This evidence is puzzling as in the data there are no jumps in consumption at such frequencies. I present a long-run risks type model where consumption shocks are Gaussian, and the agent learns about unobserved expected growth from the cross-section of signals. The uncertainty about expected growth (confidence measure), as in the data, is time-varying and subject to jump-like risks. I show that confidence jump risk channel can quantitatively account for the cross-section of option prices and large moves evidence in asset prices, without hard-wiring jumps into consumption. Based on the two estimation approaches, the model is not rejected in the data and provides a good fit to the option price, confidence measure, returns and consumption data.

# Introduction

Option markets are important, as they can provide significant information about the risks that investors perceive in financial markets. One of the central puzzles in option data is that deep out-of-the-money index put options appear overpriced, so that the insurance for large downward movements in the stock market is too expensive relative to standard models (see e.g. Rubenstein, 1994). These findings indicate that investors are concerned with large, negative moves in underlying prices, which occur approximately once a year in the data.<sup>1</sup> However, there is no persuasive evidence in the data for large contemporaneous moves in the real economy at the considered frequencies, which presents a challenge for economic explanation of option markets. In this paper, I show that fluctuating confidence of investors about unobserved expected growth can quantitatively explain asset-price anomalies in derivative markets and account for the observed large moves in returns, while keeping fundamental consumption dynamics smooth Gaussian. Based on two estimation approaches, I find that the model with learning and confidence jump risks delivers plausible preference and model parameters and provides a good fit to option prices, investor confidence, returns, and consumption in the data.

Earlier structural works which address issues in option markets typically introduce jumps into the fundamental consumption process: see Eraker and Shaliastovich (2008), Drechsler and Yaron (2008), Santa-Clara and Yan (2008), Gabaix (2007), Bates (2006), Benzoni, Collin-Dufresne, and Goldstein (2005), Liu, Pan, and Wang (2005). In this paper, I do not entertain the possibility of jumps in consumption, and instead show that learning and fluctuating confidence about expected growth can account for the key features of option and equity data. The economy setup closely follows Bansal and Shaliastovich (2008a) and, as in standard long-run risks model of Bansal and Yaron (2004), features Gaussian dynamics of true consumption growth with a persistent expected growth component, time-varying consumption volatility, and the recursive utility of Epstein and Zin (1989) and Weil (1989). However, unlike the standard long-run risks model, expected growth is not directly observable, and investors learn about it using a cross-section of signals. The quality of signals determines the uncertainty of investors about their estimate of expected growth. This uncertainty, referred to as "confidence measure," is time-varying and contains large positive shocks. Due to imperfect information and learning, the confidence measure affects the beliefs of investors about future consumption and impacts equilibrium asset prices in the economy.

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<sup>1</sup>Recent empirical work featuring jumps in prices includes Bakshi, Cao, and Chen (1997), Pan (2002), Andersen, Benzoni, and Lund (2002), Eraker, Johannes, and Polson (2003), Eraker (2004), Broadie, Chernov, and Johannes (2007), Santa-Clara and Yan (2008). For a nonparametric analysis of high-frequency data, refer also to Barndorff-Nielsen and Shephard (2006) and Andersen, Bollerslev, Diebold, and Vega (2003).

As in a standard long-run risks specification, investors in the model demand compensation for short-run, long-run and consumption volatility risks. The novel contribution of the model is that the confidence risks are also priced in equilibrium, so that when agents have a preference for early resolution of uncertainty, states with higher uncertainty about expected growth are discounted more heavily. Notably, confidence jump shocks receive risk compensation although there are no jump risks in consumption. Learning and confidence jump risk channels can explain the key features of option price data. Out-of-the-money put options hedge jump risks in confidence and thus appear expensive relative to models with no jump risks. This can account for the smirk pattern in option prices, where Black-Scholes implied volatilities are decreasing in the strike price of the contract. Further, endogenous jumps in equilibrium prices due to positive jumps in uncertainty about future growth can account for large downward moves in asset prices, and a negatively skewed and heavy-tailed unconditional distribution of returns.

The key economic mechanism in this paper, such as learning about expected growth, is featured in a number of asset-pricing models. In the class of learning models considered by David (1997), Veronesi (1999), Ai (2007), unobserved drift is modeled as a regime-shift process, so that investor's uncertainty about the estimate is stochastic and is related to fundamental shocks in the economy. David and Veronesi (2002) show that this channel endogenously generates the correlation between equilibrium returns and return volatility which can explain time-variation in option-implied volatility and the skewness and kurtosis premium in option prices. The model of Buraschi and Jitsov (2006) features heterogeneous agents and learning about the dividend growth rate and can explain option prices and the dynamics of option volume<sup>2</sup>. Alternative learning models are presented in Hansen and Sargent (2006), who specify model-selection rules which capture investors' concerns about robustness and potential model misspecification, and Piazzesi and Schneider (2007), who use survey data to characterize and study the subjective beliefs of agents in the economy. Relative to the models in the literature, the novel dimension of this paper is the time-variation in the quality of signals available to the investors and the ensuing confidence jump risks in asset markets. Fluctuations in confidence measure are consistent with theoretical model of Veldkamp (2006) and Van Nieuwerburgh and Veldkamp (2006), where information flow about the unobserved economic state endogenously varies with the level of economic activity.

The main target in this paper is to quantitatively explain option pricing puzzles and at the same time account for the key dimensions of consumption, returns and confidence measure in the data. I use the cross-section of forecasts of next-quarter GDP from the Survey of Professional Forecasters and construct empirical confidence measure as a cross-sectional variance of the average forecast, consistent with the the-

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<sup>2</sup>Rational learning is also featured in Detemple (1986), Gennotte (1986), Brennan (1998), Veronesi (2000), Brennan and Xia (2001), David and Veronesi (2008), Croce, Lettau, and Ludvigson (2006).

oretical specification. I show that in the data, confidence measure contains significant information about Black-Scholes volatilities in the option market. The option volatilities across all strikes and maturities are about 7% higher in quarters when uncertainty is high, relative to quarters when uncertainty about future growth is low. Further, in projections of option-implied volatilities 2 and 3 quarters ahead, the slope coefficient on confidence measure is large and significant at all strikes, while the slope coefficient on the current value of option volatility is small and is typically decreasing with the horizon. In addition, the empirical confidence measure exhibits large positive moves, whose frequencies and magnitudes are plausible to account for the jump features of option and asset market data. Indeed, using formal econometric analysis, Bansal and Shaliastovich (2008a) find significant evidence for jump-like shocks in confidence measure at frequencies of about 4 months. These large moves in confidence measure are related to large moves in returns and in variance of returns implied by option markets. On the other hand, there is no persuasive evidence in the data for the link between large moves in returns and large moves in the real economy at the considered frequencies of about 1 year.

I use two econometric approaches to estimate and test the model. For GMM estimation, I consider moments of confidence measure and equity returns, which characterize non-Gaussian features of the distribution, as well as the information in interest rates and option-implied volatilities in the data. I also employ the latent-factor MLE approach, where I treat confidence measure as well as consumption volatility and expected growth state as latent factors and back them out from the option, return and consumption data, similar to Duffie and Singleton (1997), Pan (2002) and Santa-Clara and Yan (2008). The quantitative implications from the two estimation approaches are very similar and provide empirical support for the long-run risks model with learning, fluctuating investor confidence and jump-like confidence risks. I obtain plausible preference parameters, which indicate that investors have a preference for early resolution of uncertainty. The estimated model parameters suggest that the confidence measure significantly fluctuates over time; moreover, nearly all the variation in the series is driven by Poisson jumps. Large moves in uncertainty about future growth can quantitatively explain over-pricing of out-of-the-money put options and produce the implied volatility curve comparable to the data. Using the backed out confidence measure and consumption volatility states from the MLE estimation, I show that these states account for more than 95% of the total variation in option volatilities. Due to jumps and higher persistence, shocks in confidence measure are more important for out-of-the-money and longer maturities contracts.

Based on GMM estimation, the estimated frequency of jumps in asset prices, driven endogenously by jumps in confidence measure, is one every 5 months, and the average jump in returns is  $-3.3\%$ , monthly. Using MLE estimates, the frequency of large moves in returns is about once every 9 months, while average jump in return is  $-7.5\%$ , monthly. The frequency of moves in returns of such magnitude in the data is

consistent with the model; for example, in my sample monthly returns fall below the cutoff of  $-3.3\%$  once every 6 months. Confidence jump risks contribute about 2% to the total equity premium of 6%, while expected growth shocks account for 3%. The estimates of the jump risk premium in the economy is consistent with Pan (2002) and Broadie et al. (2007), who find that jump risks account for about one-third of the total equity premium in the economy.

Based on GMM estimation, the model with confidence jumps is not rejected in the data, with a  $p$ -value of 0.3. The in-sample and out-of-sample tests suggest that the model can account for the cross-section of option prices and distribution of confidence measure and returns in the data. The dynamics of consumption and confidence measure from the two estimations are consistent with features of the data based on a long historical sample. On the other hand, the restricted model with no jump risks in confidence is rejected both in sample and out of sample, as it fails to capture the over-pricing of out-of-the-money put options and non-Gaussian features of returns and confidence measure in the data. Overall, the empirical results strongly indicate that confidence jumps risk is a key channel to empirically explain option and equity prices in the data without introducing jumps into the fundamental consumption process.

Earlier structural models which aim to explain option prices and large moves in returns typically hardwire jumps into consumption fundamentals. Eraker and Shaliastovich (2008) show that when investors have preference for the timing of resolution of uncertainty, jumps in consumption fundamentals are priced in equilibrium and affect asset valuations and returns. In particular, positive jumps in consumption volatility endogenously translate into negative jumps in equilibrium prices, which can capture the shape of the implied volatility curve in option prices. Benzoni et al. (2005) consider jumps in expected consumption, which they show can also rationalize the volatility smirk observed in the data. Eraker (2007) and Drechsler and Yaron (2008) further argue that jumps in conditional moments of consumption can account for some key features of the risk-neutral variance of returns implied by the cross-section of option prices in the data. In a related literature, Liu et al. (2005) introduce rare jumps into the endowment dynamics and argue that concerns for model uncertainty can explain the over-pricing of out-of-the money puts and the smirk pattern of option prices in the data. This implied volatility pattern can also be generated in a rare disaster model with a time-varying probability of a crash, as discussed by Gabaix (2007). In a similar vein, Santa-Clara and Yan (2008) estimate risks of investors implied from the option markets and argue for substantial *Peso* issues in measuring jumps from the stock market data alone. Bates (2006) studies the equilibrium implications of the model which features jump news in dividends and crash-averse investors with heterogeneous attitudes towards crash risk. In an earlier study, Naik and Lee (1990) analyze general-equilibrium option pricing when the underlying dividend follows a jump-diffusion process. Relative to the above literature, I do not entertain the possibility of jumps in consumption; rather, I show that learning and fluctuating

confidence of investors about expected growth can account for the empirical jump evidence in option and equity data.

Other approaches which incorporate learning and option prices include Campbell and Li (1999), who consider learning about volatility regimes, and Guidolin and Timmermann (2003), who study Bayesian learning implications for option pricing in context of the equilibrium model. A number of papers highlight the importance of information in option prices to learn about the risks in financial markets. The empirical evidence presented in Bollerslev, Tauchen, and Zhou (2008), Todorov (2007), Buraschi and Jackwerth (2001), Bakshi and Kapadia (2003), as well as from parametric models of asset prices, suggest that the risk premia in options cannot be explained by compensation for diffusive stock market risk alone. A number of papers also use option market data to study the characteristics of investor preferences; these works include Brown and Jackwerth (2004), Bondarenko (2003), Garcia, Luger, and Renault (2003), Ait-Sahalia and Lo (2000), Jackwerth (2000).

The rest of the paper is organized as follows. In the next section I set up the model and discuss preferences of the representative agent and dynamics of the economy given the information set of investors. Solutions to the discount factor and asset and option prices are shown in Section 2. Section 3 describes the data and empirical evidence on the option pricing puzzles. I present GMM estimation results and implications to option prices and equity premium in Section 4, while the MLE estimation is discussed in Section 5, followed by the Conclusion.

# 1 Model Setup

## 1.1 Preferences

I consider a discrete-time real endowment economy. Investor's preferences over the uncertain consumption stream  $C_t$  are described by the Kreps and Porteus (1978) recursive utility function of Epstein and Zin (1989):

$$U_t = \left\{ (1 - \delta)C_t^{\frac{1-\gamma}{\theta}} + \delta(E_t[U_{t+1}^{1-\gamma}])^{1/\theta} \right\}^{\frac{\theta}{1-\gamma}}, \quad (1.1)$$

where  $\gamma$  is a measure of a local risk aversion of the agent,  $\psi$  is the intertemporal elasticity of substitution and  $\delta \in (0, 1)$  is the subjective discount factor. The conditional expectation is taken with respect to date- $t$  information set of the agent, which is discussed later in the paper. For notational simplicity, parameter  $\theta$  is defined as

$$\theta = \frac{1 - \gamma}{1 - \frac{1}{\psi}}. \quad (1.2)$$

When  $\theta = 1$ , that is,  $\gamma = 1/\psi$ , the above recursive preferences collapse to standard expected utility. As is pointed out in Epstein and Zin (1989), in this case the agent is indifferent to the timing of resolution of uncertainty in the consumption path. When risk aversion exceeds the reciprocal of the intertemporal elasticity of substitution, investors prefer early resolution of uncertainty; otherwise they prefer late resolution of uncertainty. Preference for the timing of the resolution of uncertainty has important implications for risk channels and equilibrium asset-prices in the economy. In the long-run risk model agents prefer early resolution of uncertainty in the consumption path.

As shown in Epstein and Zin (1989), the logarithm of the intertemporal marginal rate of substitution for these preferences is given by

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1}, \quad (1.3)$$

where  $\Delta c_{t+1} = \log(C_{t+1}/C_t)$  is the log growth rate of aggregate consumption and  $r_{c,t+1}$  is the log of the return (i.e., continuous return) on an asset which delivers aggregate consumption as its dividends. This return is not observable in the data. It is different from the observed return on the market portfolio as the levels of market dividends and consumption are not equal: aggregate consumption is much larger than aggregate dividends. To solve the model, I assume an exogenous process for consumption growth and use a standard asset pricing restriction

$$E_t[\exp(m_{t+1} + r_{t+1})] = 1, \quad (1.4)$$

which holds for any log return  $r_{t+1} = \log(R_{t+1})$  to calculate asset prices in the economy.

The dynamics of the real economy and agent's information set is described in the next sections.

## 1.2 Real Economy

Following Bansal and Yaron (2004), the true dynamics for log consumption growth  $\Delta c_{t+1}$  incorporates a time-varying mean  $x_t$  and stochastic volatility  $\sigma_t^2$ :

$$\Delta c_{t+1} = \mu + x_t + \sigma_t \eta_{t+1}, \quad (1.5)$$

$$x_{t+1} = \rho x_t + \varphi_e \sigma_t \epsilon_{t+1}, \quad (1.6)$$

$$\sigma_{t+1}^2 = \sigma^2 + \nu_c (\sigma_t^2 - \sigma^2) + \varphi_w \sigma_t w_{c,t+1}, \quad (1.7)$$

where  $\eta_t$ ,  $\epsilon_t$  and  $w_{c,t+1}$  are independent standard Normal shocks which capture short-run, long-run and volatility risks in consumption, respectively. Parameters  $\rho$  and  $\nu_c$



determine the persistence of the conditional mean and variance of the consumption growth rate, while  $\varphi_e$  and  $\varphi_w$  govern their scale. The empirical motivation for the time-variation in the conditional moments of the consumption process comes from the long-run risks literature, see e.g. Bansal and Yaron (2004), Hansen, Heaton, and Li (2008) and Bansal, Kiku, and Yaron (2007b).

As in Bansal and Shaliastovich (2008a), I assume that the agent knows the structure and parameters of the model and can observe consumption volatility  $\sigma_t^2$ . However, the true expected growth factor  $x_t$  is not directly observable and has to be inferred from the data, which includes history of consumption and cross-section of signals about future growth. These signals, together with consumption data, provide all the information about the expected growth state in the economy.

Specifically, I assume that agents receive  $n$  signals about the expected growth  $x_{i,t}$ , for  $i = 1, 2, \dots, n$ . Each signal deviates from the true state  $x_t$  by a random noise  $\xi_{i,t}$ ,

$$x_{i,t} = x_t + \xi_{i,t}, \quad (1.8)$$

where the errors  $\xi_{i,t}$  are randomly drawn from a Normal distribution and are uncorrelated with fundamental shocks in the economy.

The date- $t$  imprecision in signal  $i$  is captured by  $V_{i,t}$  :

$$\xi_{i,t} \sim N(0, V_{i,t}). \quad (1.9)$$

In general, the imprecision in the signal can be different across signals and can vary across time, hence the subscripts  $i$  and  $t$ . For simplicity, I further assume that all the signals are ex-ante identical, so that at each date  $t$  the uncertainty in each signal is the same and denote  $V_{0,t} \equiv V_{i,t}$  for all  $i$ .

As all the signals come from the same distribution and are ex-ante equally informative, the investor assigns same weight to each of them. That is, in the end the average signal is a sufficient statistic for the cross-section of all the individual ones. Define the average signal  $\bar{x}_t$ , which corresponds to the sample average of the individual signals. Then, using (1.8),

$$\bar{x}_t \equiv \frac{1}{n} \sum x_{i,t} = x_t + \xi_t, \quad (1.10)$$

where the cross-sectional uncertainty in average signal  $V_t$  and the average signal error are given by

$$V_t = \frac{1}{n} V_{0,t}, \quad \xi_t = \frac{1}{n} \sum \xi_{i,t}, \quad (1.11)$$

so that

$$\xi_t \sim N(0, V_t). \quad (1.12)$$

The uncertainty  $V_t$  determines the confidence of investors about their estimate of expected growth; as in Bansal and Shaliastovich (2008a), I also refer to it as confidence measure. In the model, confidence measure is assumed to be observable to investors. It can be estimated in the data from the cross-section of individual signals. Indeed, the signal equation (1.8) implies that

$$E \left( \frac{1}{n-1} \sum_{i=1}^n (x_{i,t} - \bar{x}_t)^2 \right) = E \left( \frac{1}{n-1} \sum_{i=1}^n (\xi_{i,t} - \xi_t)^2 \right) = V_{0,t}, \quad (1.13)$$

so that the cross-sectional variance of the signals adjusted by the number of signals  $n$  can provide an estimate of the confidence measure  $V_t$  in the data.

The time-variation in the quality of the signals and ensuing confidence risks is a novel dimension of this model. Standard learning models, see for example David (1997), Veronesi (2000), Brennan and Xia (2001), Ai (2007) and David and Veronesi (2008), feature a constant level of imprecision in observed signals. Hansen and Sargent (2006) consider constant variance as well and specify alternative learning rules robust to model misspecification.

### 1.3 Confidence Dynamics

The specification of the time-series fluctuation of confidence measure  $V_t$  is a key ingredient of the model. As in Bansal and Shaliastovich (2008a), I consider a discrete-time jump-diffusion model for confidence measure, which features persistence and both Gaussian and jump-like innovations:

$$V_{t+1} = \sigma_v^2 + \nu(V_t - \sigma_v^2) + \sigma_w \sqrt{V_t} w_{t+1} + Q_{t+1}. \quad (1.14)$$

The parameters  $\sigma_v^2$  is the mean value of  $V_t$ ,  $\nu$  captures its persistence while  $\sigma_w$  determines the volatility of the smooth Gaussian shock  $w_{t+1}$ . The non-Gaussian innovation in confidence process is denoted by  $Q_{t+1}$ . I model it as a compound Poisson jump,

$$Q_{t+1} = \sum_{i=1}^{N_{t+1}} J_{i,t+1} - \mu_j \lambda_t, \quad (1.15)$$

where  $N_{t+1}$  is the Poisson process, whose intensity  $\lambda_t \equiv E_t N_{t+1}$  corresponds to the probability of having one jump in continuous-time model, while  $J_{i,t+1}$  determines the distribution of the size of the jump. Parameter  $\mu_j$  is the unconditional mean of jump size, so subtracting  $\mu_j \lambda_t$  on the right-hand side of the above equation ensures that the jump innovation  $Q_{t+1}$  is conditionally mean zero<sup>3</sup>. In estimation of the model,

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<sup>3</sup>Indeed,

$$E_t(Q_{t+1}) = E_t(E_t(Q_{t+1}|N_{t+1})) = E_t(\mu_j N_{t+1}) - \mu_j \lambda_t = 0.$$

I consider an exponential distribution for jumps, which is convenient as it is fully described by a single parameter  $\mu_j$ .

To capture the dependence of jump probability on the level of variance, I assume that the arrival intensity  $\lambda_t$  is linear in  $V_t$ ,

$$\lambda = \lambda_0 + \lambda_1 V_t. \quad (1.16)$$

When  $\lambda_1 > 0$ , the probability of jumps increases in the level of the confidence measure, so jumps are more frequent when the uncertainty about expected growth is high.

This specification of the time-series evolution of the uncertainty about future growth is very similar to the models for the variance process in continuous time considered in Eraker (2004), Broadie et al. (2007) and Eraker and Shaliastovich (2008).

## 1.4 Filtering Dynamics

At each point in time, the agent estimates expected consumption growth given the information set  $\mathcal{I}_t$ , which includes the history of consumption, consumption volatility, signals and confidence measure:

$$\mathcal{I}_t = \left\{ \left\{ \Delta c_{t-j}, \sigma_{t-j}^2, \{x_{i,t-j}\}_{i=1,2,\dots}, V_{t-j} \right\}_{j=0,1,\dots} \right\}. \quad (1.17)$$

Let  $\hat{x}_t$  stand for investors' estimate of the expected growth,

$$\hat{x}_t = E(x_t | \mathcal{I}_t), \quad (1.18)$$

and denote  $\omega_t^2$  the variance of the filtering error which corresponds to the estimate  $\hat{x}_t$ :

$$\omega_t^2 = E((x_t - \hat{x}_t)^2 | \mathcal{I}_t). \quad (1.19)$$

Appendix A.1 shows that the filtering problem of the agent has a one-step ahead innovation representation, where the expectations about future growth are updated using the observed consumption and average signal data. The optimal weights given to consumption and signal news are time-varying and reflect the relative quality of consumption and signal information, that is,  $\sigma_t^2$  versus  $V_t$ . In general, solutions to the optimal signal  $\hat{x}_t$  and filtering uncertainty  $\omega_t^2$  are complicated non-linear functions of the whole history of consumption and signal data. To simplify the solution to the model, I follow Bansal and Shaliastovich (2008a) and consider an approximate specification where the Kalman Filter weight on consumption news is 0, and that on the signal news is set to a constant steady-state value: a positive number slightly less than 1. This approximation is exact in a complete information case when average

signal perfectly reveals the true state, that is, when  $V_t = 0$ . The approximation is very accurate when the uncertainty in the average signal is much smaller than the consumption variance. I verify that at the considered model parameter values the time-series correlation of the filtered expected growth states from the approximate and exact Kalman Filter specification is in excess of 0.99, and utility losses from the considered approximate setup are small.

The approximate solution to the agents filtering problem implies that the evolution of the economy given the information of the agent is given by,

$$\Delta c_{t+1} = \mu + \hat{x}_t + a_{c,t+1}, \quad (1.20)$$

$$\bar{x}_{t+1} = \rho \hat{x}_t + a_{x,t+1}, \quad (1.21)$$

$$\hat{x}_{t+1} = \rho \hat{x}_t + K_2 a_{x,t+1}. \quad (1.22)$$

The immediate filtered consumption innovations are given by  $a_{c,t+1}$ , while  $a_{x,t+1}$  denotes the filtered news about the average signal. As shown in (1.22), the agents update their expectations about the true expected growth based on the filtered news about the average signal  $a_{x,t+1}$ , so that the estimate of the expected state can also be written as a weighted average of the expected value of the state as of last period and current average signal:

$$\hat{x}_{t+1} = (1 - K_2) \rho \hat{x}_t + K_2 \bar{x}_{t+1}. \quad (1.23)$$

The weight on the average signal news  $K_2$  is constant and is given by the steady-state solution to the Kalman Filter problem of the agent (see Appendix A.1).

Investor's uncertainty about the estimate of expected growth  $\omega_t^2$  is directly related to the confidence measure from the cross-section of signals:

$$\omega_t^2 = K_2 V_t. \quad (1.24)$$

If uncertainty about future growth is constant, a standard Kalman Filter result obtains that the steady-state variance of the filtering error is constant. On the other hand, when investor confidence is stochastic, the variance of the filtering error fluctuates one-to-one with the uncertainty about future growth. Learning models considered by David (1997) and Veronesi (1999) use regime-shift specification for expected growth component and feature alternative time-varying dynamics of the filtering uncertainty.

The innovations into consumption and average signal contain fundamental short and long-run consumption shocks and filtering errors; in general, the three cannot be separately identified based on the information set of the agent:

$$\begin{aligned} a_{c,t+1} &= x_t - \hat{x}_t + \sigma_t \eta_{t+1}, \\ a_{x,t+1} &= \rho(x_t - \hat{x}_t) + \varphi_e \sigma_t \epsilon_{t+1} + (\bar{x}_{t+1} - x_t). \end{aligned} \quad (1.25)$$

In a complete information setting, investors observe the true expected growth, so the two innovations above collapse to standard short-run and long-run consumption shocks. On the other hand, with imperfect information, the confidence of investors about their estimate of expected growth affects their beliefs about the distribution of future consumption. Even if the fundamental consumption volatility is constant, the variance of consumption growth tomorrow given the available information of investors is time-varying due to the variation in the precision of the signals, and lower confidence of investors (high  $V_t$ ) implies higher uncertainty about future consumption.

The equations (1.20) - (1.22), together with the time-series model for the confidence measure in (1.14) and aggregate consumption volatility in (1.7) fully describe the evolution of the economy given agent's period- $t$  information. In the next section, I incorporate preferences and solve the equilibrium asset prices in the economy.

## 2 Model Solution

### 2.1 Discount Factor

To solve the model, I first use the dynamics of the economy given the information set of the agent and Euler equation (1.4) to calculate the price of the consumption claim. The equilibrium price-consumption ratio is linear in the expected growth state, aggregate consumption volatility, and the confidence level of the investors:

$$pc_t = B_0 + B_x \hat{x}_t + B_v V_t + B_\sigma \sigma_t^2, \quad (2.1)$$

where the expressions for the loadings are provided in Appendix A.

The loading  $B_x$  measures the sensitivity of the price-consumption ratio to expected growth. It is positive for  $\psi > 1$ , so that when the substitution effect dominates the income effect, prices rise following positive news about expected consumption, similar to a standard long-run risks model. The loadings  $B_v$  and  $B_\sigma$  capture the effects of confidence measure and consumption volatility on asset valuations. When the agent has a preference for early resolution of uncertainty ( $\gamma > 1/\psi$ ), these loadings are negative. In this case, lack of confidence about the expected growth state and high aggregate uncertainty decrease equilibrium asset valuations and the utility of the agent.

The relative magnitudes of the loadings of the price-consumption ratio on the aggregate volatility and confidence measure depend on the quality of signal information about expected growth. In the complete information case, the true expected state is known and the consumption volatility factor  $\sigma_t^2$  alone determines the conditional variation of short-run and long-run consumption shocks. On the other hand, with

learning, the volatilities of these shocks are now driven by two factors,  $\sigma_t^2$  and  $V_t$  (see equation (1.25)), so that the volatility channel is now represented by consumption volatility and confidence measure states. This reduces the price of consumption volatility risks and the risk compensation for consumption volatility shocks relative to the complete information case.

Using the equilibrium solution to the consumption asset, I can express the discount factor in (1.3) in terms of the underlying states and shocks in the economy. The equilibrium solution to the discount factor and the Euler equation (1.4) can then be used to directly obtain equity, bond and option prices in the economy. In equilibrium, the log discount factor is equal to,

$$m_{t+1} = m_0 + m_x \hat{x}_t + m_v V_t + m_\sigma \sigma_t - \gamma a_{c,t+1} - \lambda_x K_2 a_{x,t+1} - \lambda_v \left( \sigma_w \sqrt{V_t} w_{t+1} + Q_{t+1} \right) - \lambda_\sigma \varphi_w \sigma_t w_{c,t+1}, \quad (2.2)$$

where the expressions for the discount factor loadings and prices of risks are pinned down by the model and preference parameters of the investors. Their expressions are provided in Appendix A.

Innovations in the discount factor determine the risks that investors face in the economy. As in standard long-run risks model with complete information, short-run, long-run and consumption volatility risks are priced. The novel dimension of the model is that confidence shocks also receive risk compensation; in particular, confidence jump risks  $Q_{t+1}$  are priced even though there are no jumps in fundamental consumption. Due to learning, the magnitudes of risk prices change relative to a standard model. As investors cannot observe the true long-run risks shocks, the price of long-run risk decreases, while the price of short-run consumption risk increases relative to complete information; this is consistent with Croce et al. (2006). In addition, the risk compensation for consumption volatility shocks also decreases relative to a standard long-run risks model.

Using the solution for the discount factor, I can derive the expressions for the equilibrium risk-free rates in the economy. Real interest rates with  $n$  periods to maturity are linear in the expected growth state, investor confidence and consumption variance:

$$r f_{t,n} = -F_{0,n} - F_{x,n} \hat{x}_t - F_{v,n} V_t - F_{\sigma,n} \sigma_t^2. \quad (2.3)$$

where the bond coefficients are given in the Appendix A. In particular, real yields increase in the expected growth state, and decrease with positive shocks to confidence measure or aggregate volatility.

## 2.2 Risk-Neutral Probability

The evolution of the consumption process in (1.20)-(1.22), confidence measure in (1.14) and consumption volatility in (1.7) is specified under the objective probability measure in the data. The economy dynamics can also be written under the risk-neutral probability, which is characterized by the condition that the price of any payoff  $R_{t+1}$  can be computed by taking the expectation of its payoff under the risk-neutral measure discounted by the risk-free rate:

$$E_t(M_{t+1}R_{t+1}) = e^{-r_{ft}} E_t^q R_{t+1}, \quad (2.4)$$

where  $E_t^q$  refers to the expectation of the payoff tomorrow under the risk-neutral measure.

Given the solution to the discount factor, the dynamics of the states under the risk-neutral measure is given by,

$$\Delta c_{t+1} = \mu + \hat{x}_t - \gamma Var_{t+1}(a_{c,t+1}) - \lambda_x Cov_{t+1}(a_{c,t+1}, K_2 a_{x,t+1}) + a_{c,t+1}^q, \quad (2.5)$$

$$\hat{x}_{t+1} = \rho \hat{x}_t - \lambda_x Var_{t+1}(K_2 a_{x,t+1}) - \gamma Cov_{t+1}(a_{c,t+1}, K_2 a_{x,t+1}) + K_2 a_{x,t+1}^q, \quad (2.6)$$

$$\sigma_{t+1}^2 = \sigma^2 + \nu_c(\sigma_t^2 - \sigma^2) - \lambda_\sigma \varphi_w^2 \sigma_t^2 + \varphi_w \sigma_t w_{c,t+1}^q, \quad (2.7)$$

$$V_{t+1} = \sigma_v^2 + \nu(V_t - \sigma_v^2) - \lambda_{vx} \sigma_w^2 V_t + \sigma_w \sqrt{V_t} w_{t+1}^q + Q_{t+1}^q. \quad (2.8)$$

The risk-neutral transformation of the probability measure is standard and reflects risk compensation for the underlying shocks in the economy. The drifts of consumption growth and expected consumption in (2.5) and (2.6) are adjusted by the risk prices multiplied by the variance-covariance of the corresponding shocks  $a_{c,t+1}^q$  and  $a_{x,t+1}^q$ , while the conditionally Gaussian distributions of these shocks are unchanged. Further, under the objective measure, expected growth shocks  $a_{x,t+1}$  are Gaussian given current volatility states and next-period confidence measure  $V_{t+1}$ :

$$Var_{t+1}(a_{x,t+1} | \mathcal{I}_t, V_{t+1}) = \rho^2 K_2 V_t + \varphi_e^2 \sigma_t^2 + V_{t+1}, \quad (2.9)$$

so  $Var_{t+1}(a_{x,t+1})$  depends on  $V_{t+1}$  (see Appendix A.1). Then, as is evident from expression (2.6), the total innovation into expected growth under the risk-neutral measure incorporates confidence shocks in  $V_{t+1}$ . Hence, under the risk-neutral measure investors' estimate of expected growth state exhibits large moves, as positive jumps in confidence measure (high uncertainty) cause negative jumps in the expected growth magnified by the price of risk parameter  $\lambda_x$ . In contrast, under the objective measure, shocks in expected growth and investors' confidence are uncorrelated. Due to the negative correlation of shocks into expected state and confidence measure, the risk adjustment of confidence shocks,  $\lambda_{vx} = \lambda_v - \frac{1}{2} \lambda_x^2 K_2^2$ , depends on the price of confidence risks  $\lambda_v$  and risk compensation for shocks to expected growth  $\lambda_x$ .

Confidence jump shocks are compound Poisson both under the objective and risk-neutral measures,

$$Q_{t+1}^q = \sum_{i=1}^{N_{t+1}^q} \tilde{J}_{i,t+1} - \mu_j \lambda_t, \quad (2.10)$$

but the frequency and distribution of jumps are different under the two measures. When investors prefer an early resolution of uncertainty, they dislike positive shocks to  $V$ , ( $\lambda_{vx} < 0$ ), so that the jump component in the confidence measure is magnified under the risk-neutral measure. Indeed, relative to objective measure, jumps are expected to arrive more frequently,

$$\lambda_t^q \equiv E_t^Q N_{t+1}^q = \frac{\lambda_t}{1 + \mu_j \lambda_{vx}} > \lambda_t, \quad (2.11)$$

and their size is larger,

$$\mu_j^q = \frac{\mu_j}{1 + \mu_j \lambda_{vx}} > \mu_j, \quad (2.12)$$

under the risk neutral measure.

## 2.3 Equity Prices

To obtain implications for equity prices, I consider a dividend process of the form

$$\Delta d_{t+1} = \mu_d + \phi(\Delta c_{t+1} - \mu) + \varphi_d \sigma_t \eta_{d,t+1}, \quad (2.13)$$

where  $\eta_{d,t+1}$  is a dividend shock independent from all other innovations in the economy. I continue to maintain the assumption that the average signal data is much more informative about the expected growth than consumption or dividend data, so investors learn about the expected state only from the average signals (see specification (1.20)-(1.22)).

The equilibrium price-dividend ratio is linear in the expected growth state and the confidence level of the investors:

$$pd_t = H_0 + H_x \hat{x}_t + H_v V_t + H_\sigma \sigma_t^2, \quad (2.14)$$

where solutions for the loadings are provided in Appendix A. Similar to the valuation of consumption asset, equity prices increase in expected growth factor and decrease when the confidence of investors is low or the aggregate volatility is high. In particular, large positive moves in  $V_t$  endogenously translate into large jumps in asset valuations and returns. Indeed, the equilibrium log return on the dividend asset satisfies

$$\begin{aligned} r_{d,t+1} = & \mu_r + b_x \hat{x}_t + b_v V_t + b_\sigma \sigma_t^2 + \phi_{a,c,t+1} + \kappa_{d,1} H_x K_2 a_{x,t+1} \\ & + \kappa_{d,1} H_v \left( \sigma_w \sqrt{V_t} w_{t+1} + Q_{t+1} \right) + \kappa_{d,1} H_\sigma \varphi_w \sigma_t w_{c,t+1} + \varphi_d \sigma_t \eta_{d,t+1}, \end{aligned} \quad (2.15)$$



for certain loadings  $b_x, b_v$  and  $b_\sigma$ . As the return beta to confidence measure is negative ( $H_v < 0$ ), large positive shocks in confidence measure translate into negative moves in returns, magnified by the loading  $H_v$ . This channel plays an important role to empirically explain large moves in asset prices and over-pricing of out-of-the-money put options, keeping the consumption dynamics smooth as in the data.

The dynamics of returns under the objective measure in (2.15) and the evolution of the states under the risk-neutral measure in (2.5)-(2.8) can be also used to characterize the variation in returns under both probability measures. The conditional variance of returns under the two measures is linear in confidence measure and consumption variance. Hence, positive jumps in confidence measure endogenously translate into simultaneous positive jumps in conditional variance of returns and negative jumps in prices. In a related model, Eraker and Shaliastovich (2008) show that positive jumps in aggregate volatility of consumption  $\sigma_t^2$  can also lead to negative jumps in equilibrium returns and positive jumps in the conditional variance of returns.

## 2.4 Option Prices

The equilibrium asset-pricing framework can be used to compute prices of options written on the dividend claim. In Appendix A.4 I show that price  $C_t(K/P_t, n)$  of a put option contract with moneyness  $K/P_t$  and maturity  $n$  depends on the underlying expected growth, confidence measure and aggregate volatility states:

$$\frac{C_t(K/P_t, n)}{P_t} = \frac{1}{2\pi} \frac{K}{P_t} \int_{iz_i - \infty}^{iz_i + \infty} \frac{e^{G_{0,n} + G_{x,n}\hat{x}_t + G_{v,n}V_t + G_{\sigma,n}\sigma_t^2 + iz \log(K/P_t)}}{iz - z^2} dz, \quad (2.16)$$

where  $z_i \equiv \text{Im}(z) < 0$ , and complex-valued loadings  $G$  depend on the model and preference parameters. The option price can be easily computed numerically for given states and parameters of the economy.

I convert theoretical option prices  $C_t(K/P_t, n)$  into Black-Scholes implied volatility units  $\sigma_{BS,t}^2$  using model-implied interest rate  $rf_{t,n}$  and log price-dividend ratio  $pd_t$  (see expressions (2.3) and (2.14), respectively). This transformation is convenient, as the implied volatilities are easier to interpret than the original option prices. Indeed, implied volatilities are directly comparable across strikes and maturities; in fact, the observed differences in implied volatilities constitute major puzzles in the option pricing literature. In addition, while the price of the option in (2.16) can in principle depend on all the expected growth, confidence and aggregate volatility states, in numerical simulations I verify that Black-Scholes implied volatilities are driven nearly entirely by the confidence measure and aggregate volatility alone. This is not surprising, as the variance of market returns under physical and risk-neutral measure depends linearly on  $V_t$  and  $\sigma_t^2$ , so the expected growth state is expected to have an insignificant effect on the volatilities implied in the option contracts. This

insight proves very useful in the MLE estimation of the model, as it allows me to back out the confidence measure and aggregate volatility states directly from the implied volatilities, while inferring about the expected growth state using macroeconomic and asset price data.

In addition, as option-implied volatilities in the model are driven nearly entirely by the confidence measure and consumption volatility, positive jumps in confidence measure endogenously translate into positive jumps in the option-implied variance. The timing of these moves corresponds to negative jumps in returns.

## 3 Empirical Evidence

### 3.1 Data

I collect monthly data on European S&P 500 index option prices for the period of January 1996 to June 2007 from the OptionMetrics database. The dataset also includes index price level, zero coupon yields at different maturities and dividend yield implied from the put-call parity relationship in the option market. The option contracts typically expire at the end of every third week of the month. As the theoretical model is specified on a monthly frequency, I use Wednesday prices every third week of the month to ensure that the time to expiration is an integer. Specifically, I use options with maturities of 1 and 2 months and moneyness closest to 0.9, 0.95, 1.00, 1.05 and 1.10, which are among the most actively traded contracts on the exchange. To mitigate microstructure problems, I exclude all observations with option prices less than one eights of a dollar, as well as those with no trading volume or with open interest less than 100 contracts. In the last step, I check for basic arbitrage violations in the option markets. For estimation, I consider put option prices only, as they are more actively traded than call options and the latter would be redundant given the put-call parity relationship.

Using the interpolated zero coupon rates and price-dividend ratios, I convert option prices into Black-Scholes implied volatility units. That is, I solve for the implied Black-Scholes volatility of the put contract given the observed option price, its strike price, time to maturity, current index level and the interest rate and log price-dividend ratio in the data. As discussed in the previous section, implied volatilities are easier to interpret than the original option prices; further, focusing on implied volatilities forces the estimation to directly address the key option pricing puzzles.

I obtain the data on real consumption growth rate, monthly, for the same period of 1996 to 2007 from the BEA Tables. Additionally, I construct an empirical measure of the confidence of investors as an estimate of the cross-sectional variance of the average

forecast of real GDP from the Survey of Professional Forecasts. The calculations follow Bansal and Shaliastovich (2008a), and the details are provided in Appendix B.

The key features of consumption and return data, shown in Table 1, are comparable to the standard estimates in the literature. Mean log return is 7% and mean inflation-adjusted interest rate is 1.6%, so the average excess return in the sample is 5.4%. Interest rates are quite persistent, with autocorrelation coefficient of 0.97 and annualized volatility of 0.5%. Consumption growth averages 2% and has a standard deviation of just below 1%. A well-known feature of consumption growth data at monthly frequency is negative autocorrelation. In my sample, the estimated persistence coefficient is  $-0.37$ , while the persistence of consumption growth is reliably positive at lower frequencies and longer historical sample; see Table 5. To deal with the data issues in monthly consumption, I introduce a measurement noise in log consumption level in the MLE estimation of the model.

I discuss the option-price evidence and related dimensions of return and macroeconomic data in the next section.

## 3.2 Option Pricing Puzzles

One of the key puzzles in option markets is that out-of-the-money put options appear overpriced, so that the insurance for large downward movements in asset prices is too expensive relative to standard models (see e.g. Rubenstein, 1994). According to the Black-Scholes model, the option-implied volatilities across all strikes and maturities should be equal to the volatility of the underlying asset. Table 1 reports that in the data, the average volatility of out-of-the money options of 21.4% exceeds the at-the-money volatility of 17.7% by nearly 4%. In fact, this difference ('volatility smirk') is always positive in the sample and ranges between 2% and 7%, as shown in the second panel of Figure 1. Similar results obtain for a broader range of put option strikes and for longer maturities (see Table 2). The empirical evidence of over-pricing of out-of-the-money put options suggests that the cross-section of option prices cannot be explained by standard Gaussian models and points to the jump risk factors in the economy.<sup>4</sup>

Consistent with this evidence, option and asset prices exhibit large moves (jumps) in the data. The unconditional distribution of returns is characterized by negative skewness of  $-0.7$  and high kurtosis of 4.5 — for Normal distribution, these statistics are 0 and 3, respectively. Excess kurtosis and negative skewness are indicative of large negative moves in returns. Similarly, positive skewness in implied variance indicates the presence of large positive movements in the series. Sizeable variation across

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<sup>4</sup>Statistical evidence on the importance of jumps for option prices is discussed in Bakshi et al. (1997), Bates (2000), Pan (2002), Broadie et al. (2007), Santa-Clara and Yan (2008).

time and occasional large positive spikes are apparent on the plot of option-implied volatility on Figure 1. Direct evidence for large moves can be obtained by isolating abnormal movements in prices. Specifically, I identify large move as a two standard deviation or higher innovation based on the AR(1)-GARCH(1,1) fit. In the data, the frequency of identified large moves in returns and implied variance is the same, once every 17 months. 75% of the identified large moves in returns are negative, while all of the large moves in implied variance are positive. The timing of large moves in implied variance and returns is highly related, as 5 out of 8 identified large moves in the two series occur at the same time. These findings on large moves in asset prices are broadly consistent with jump evidence from the parametric models of asset-prices discussed in Singleton (2006), and with empirical results in Tauchen and Todorov (2008), who present strong evidence for common jumps in stock price and implied volatility from the option markets based on the high-frequency data.

While there is strong support for large common moves in asset and option prices, there is no evidence for large moves in the real economy that can economically account for the jump features of financial data at the considered frequencies. In my sample, none of the large moves in financial prices can be explained by a simultaneous large jump in real consumption. The estimated conditional mean and variance of consumption growth are even smoother than the underlying series and also show no large moves that could explain jumps in prices. Similar evidence is presented in Bansal and Shaliastovich (2008a), who document that there is no link in the data between large moves in equity returns and moves in a variety of macroeconomic variables, while Bansal and Shaliastovich (2008b) argue that years with daily jumps in returns are not predictable by the level of the real economy.

While there is no direct empirical evidence for jump risks in consumption, measures of investors' uncertainty about future growth exhibit substantial fluctuations and large moves in the data, which can potentially explain the cross-section of option prices and the time-series dynamics of asset and option prices. Indeed, confidence measure has significant information about the option price volatilities in the data. As shown in Table 2, option volatilities across all strikes and maturities are about 7% higher in quarters where the uncertainty is high, relative to quarters where the uncertainty about future growth is low. Further, the confidence measure in the data has significant information about future option-implied volatilities, even controlling for the current implied volatility of the contract. Table 3 documents that, in projections of option-implied volatilities 2 and 3 quarters ahead, the slope coefficient on confidence measure is large and significant at all strikes, while the slope coefficient on the current value of option volatility is small and is typically decreasing with horizon. (Beyond 3 quarters, both slopes become insignificant.) This evidence is consistent with Buraschi and Jitsov (2006), who show that the cross-sectional dispersion of forecasts from the Survey of Professional Forecasts and the Consumer Confidence Survey

has information about the level and slope of the option smile and future realized volatility of returns.

The empirical confidence measure exhibits large positive moves, whose frequencies and magnitudes are plausible to account for the jump features of option and asset market data. As shown in the last panel of Table 1, the unconditional distribution of the confidence measure in the data is very heavy-tailed and positively skewed, especially for the full period from 1968 to 2008. The large positive spikes in the series depicted on Figure 2 indicate the possibility of large positive shocks to the uncertainty about future growth. Indeed, using formal econometric analysis, Bansal and Shaliastovich (2008a) find significant evidence for a jump-like component in the confidence measure and document that large moves in confidence measure in the data are related to large moves in returns and in variance of returns implied from the option markets.

Hence, based on the empirical evidence, a measure of investors' uncertainty about future growth provides a channel to explain option prices puzzles and the jump features of financial market data, without relying on jumps in consumption. I formally assess the ability of the model with time-varying confidence jump risks to explain option, return and macroeconomic data in the next section.

## 4 GMM Model Estimation

### 4.1 Consumption Calibration

I calibrate the parameters of consumption dynamics and estimate preference, confidence measure and dividend parameters using a GMM approach.

The consumption dynamics parameters are calibrated on monthly frequency. The baseline values for the parameters, which are reported in Table 4, are very similar to those used in the long-run risks literature (see e.g. Bansal and Yaron, 2004). Specifically, the annualized consumption growth rate is set at 2%. The persistence in the expected growth  $\rho$  of 0.978 and the magnitudes of the scale parameters  $\varphi_e$  and  $\sigma$  ensure that the model can match historic volatility of consumption growth of 2% and its persistence of about 0.45. Similar to Bansal and Yaron (2004), I set the persistence of the consumption variance to  $\nu_c = 0.987$ , and calibrate the volatility of volatility parameter to  $\varphi_w = 5.72 \times 10^{-4}$ . I calibrate the model on monthly frequency and then time-aggregate to an annual horizon. Table 5 shows that the model can successfully match mean, volatility, auto-correlations and variance ratios of the consumption dynamics over the long period in the data.

## 4.2 GMM Estimation

I estimate preference, confidence measure and dividend parameters using standard GMM approach. In particular, I bring in quarterly observations on confidence measure, based on real GDP forecasts from the Survey of Professional Forecasts, and corresponding observations on monthly returns, interest rates and volatilities implied from option contracts with moneyness closest to 0.95, 1.00 and 1.05 and maturities of 1 and 2 months, for the period from 1996 to mid 2007. As the confidence measure in the data is based on quarterly forecasts of real GDP, I adjust the confidence measure data to account for scale and time-aggregation issues, as discussed in Appendix B.

In estimation, I set up four blocks of orthogonality conditions which correspond to the moments of confidence measure, real excess return, interest rate and option prices. For the confidence measure, I choose to match its first four unconditional moments and quarter-ahead auto-covariance, which characterize the level, variation, persistence, and non-Gaussian properties of the series. For the stock market data, I also use the first four moments of excess equity returns, its auto-covariance as well as the covariance of excess returns with confidence measure. Further, I construct orthogonality conditions based on the level, variation and persistence of interest rates, and the levels of 6 option volatilities which span three moneyness categories (0.95, 1.00 and 1.05) and two maturities (1 and 2 months). Further details of moment conditions are provided in the Appendix C.

Let  $g_t(\Theta)$  stand for the overall vector of orthogonality conditions based on confidence measure, equity return, interest rate and option price data,  $\Theta$  is the vector of estimated parameter and  $\bar{g}(\Theta)$  denotes the sample average of moment conditions across time. Then, the GMM objective function is given by,

$$Q_{GMM}(\Theta) = T\bar{g}(\Theta)'\Sigma_T^{-1}\bar{g}(\Theta). \quad (4.1)$$

As usual, the estimation proceeds in two steps, and the optimal weighting matrix  $\Sigma_T^{-1}$  corresponds to the inverse of the Newey-West estimate of the variance of the moment conditions based on the first-step parameter estimates. The computation of standard errors and hypothesis testing follows standard asymptotic results for GMM, as discussed in Hansen (1982).

## 4.3 GMM Parameter Estimates

The estimation uses 20 moment conditions to estimate 10 parameters of investor preferences, confidence measure, and dividend growth rate. The consumption dynamics parameters are calibrated as discussed in the Section 4.1. For identification reasons, I also fix the subjective discount factor at 0.999. Bansal, Gallant, and Tauchen (2007a)

discuss that it is difficult to separately identify the subjective discount factor and the inter-temporal elasticity of substitution.

The estimated parameters of the full model, which features time-varying consumption volatility and fluctuating confidence driven by Normal and jump-like innovations, are presented in the left column of Table 6. The estimated preference parameters are quite standard in the long-run risk literature: the inter-temporal elasticity of substitution is 1.36, while the risk aversion is 11.03. This parameter configuration implies that agents prefer early resolution of uncertainty, so that they dislike negative shocks to expected consumption and positive shocks to consumption volatility and confidence measure.

The leverage of dividend growth to expected consumption is estimated at  $\phi = 3.7$ , and the dividend volatility scale is  $\varphi_d = 4.9$ . These values are consistent with the literature (see e.g. Bansal and Yaron, 2004), and imply the annualized dividend volatility of 11% and the persistence of 0.4, which agree well with the historical data.

The estimated parameters of the confidence dynamics are presented in the lower panel of Table 6. The unconditional level of confidence measure is 15 times smaller than the level of consumption volatility, which is consistent with the evidence in the data. The level of the confidence measure is directly related to the uncertainty that investors face about their estimate of expected growth. Indeed, as shown in equation (1.24), the variance of the filtering error  $\omega_t^2$  due to Kalman Filter learning about the state is proportional to the confidence measure, where the proportionality coefficient  $K_2$  (Kalman Filter weight on the forecast innovations) is equal to 0.5, based on the estimates of the model. Hence, the model-implied two standard deviations band around the investors' estimate of expected consumption growth is on average  $\pm 0.25\%$ , annualized. The average uncertainty about expected growth is quite small; for comparison, the calibrated standard deviation of the consumption growth is 2% on annual basis.

The estimated model parameters indicate that the confidence measure significantly fluctuates over time. Indeed, the persistence of confidence shocks is 0.91, which translates into a half-life of 7 months. The model for the confidence measure in (1.14) allows both for Gaussian and jump-like confidence shocks, however, the estimation results suggest that nearly all the variation in the series is driven by Poisson jumps. The Gaussian volatility parameter  $\sigma_w$  is very small and insignificant, while all the jump parameters  $\mu_j, \lambda_0, \lambda_1$  are highly significant individually. Further, the Wald test for their joint significance overwhelmingly rejects the null that jump parameters are zero with  $p$ -value well below 1%. The average frequency of jumps in confidence measure is one every 5 months, which agrees with the estimate in Bansal and Shaliastovich (2008a), who estimate the univariate model for confidence measure by MLE and find jumps occurring once every 4 months in the long sample from 1968 to 2008. Finally, the results indicate that the probability of confidence jump



is increasing when the confidence measure is high (high uncertainty): the intensity parameter  $\lambda_1$  is estimated positive and statistically significant.

The right column of Table 6 presents the estimation results for the restricted model with no jumps in the confidence measure. To generate enough variation in the confidence measure, the volatility of Gaussian shocks increases thirtyfold relative to the model with jumps, while the persistence of shocks goes up to 0.94. The estimation results suggest that the fit of the model with no confidence jumps substantially worsens, as the Likelihood Ratio test based on the difference in GMM objective functions of full and restricted models overwhelmingly rejects the restrictions that jump parameters are zero, consistent with an earlier result on the joint significance of the jump parameters based on the Wald Test.

## 4.4 Option Pricing Implication

Confidence jump risks play an important role to explain the cross-section of option prices and jump-like features of asset market data. Indeed, as shown on Figure 3, the model with confidence jump risks can quantitatively explain overpricing of the out-of-the-money put options in the data. On the other hand, in the restricted model with no confidence jumps, the implied volatility curve is virtually flat. This result is consistent with Pan (2002), Bates (2000), Bakshi et al. (1997), who show that standard models with Gaussian shocks cannot account for the cross-section of option prices in the data.

The full model with confidence jumps can deliver an implied volatility smile similar to the data. While the model somewhat under-predicts the volatilities of the deep out-of-the-money options, the magnitudes of option pricing errors are comparable to other studies. On average, the difference between option volatility in the data and in the model are less than 2% in Black-Scholes volatility units for put contract used in the estimation. For comparison, Santa-Clara and Yan (2008) document root-square errors of 2%, while Pan (2002) reports absolute pricing errors in the range from 1% to 3%. The underlying tension in the estimation is between fitting the tails of return and confidence measure distributions and the slope of the option smile. Indeed, higher jump contribution can substantially increase the deep out-of-the-money option volatilities; however, it will adversely impact the fit of return and confidence measure by making their distribution more heavy-tailed.

In the model with confidence jump risks, out-of-the-money put options hedge large positive moves in the uncertainty about future growth, which explains the overpricing of these contracts relative to standard models. As discussed in Section 2.3, large positive jumps in confidence measure endogenously translate into negative moves in returns and positive contemporaneous moves in the conditional variance of returns.



This can account for the heavy tails of the unconditional distribution of returns and implied option volatility, as well as the evidence on common large moves in these series in the data. The model with confidence jumps delivers the kurtosis of the unconditional distribution of excess returns equal to 6 and negative skewness of -0.2. These values are broadly consistent with statistics in the data, shown in Table 1. For comparison, in the model with no confidence jumps, kurtosis of return distribution is 3.8 and skewness is 0. The plots of the unconditional distributions of excess returns from the two model specifications and from the data are shown on Figure 4. These plots visually indicate that the model with jumps provides a better fit to the return distribution in the data than the model with no confidence jumps. To focus on tail properties of returns, on Figure 5 I show the QQ plot of quantiles of return distribution in the model versus the data. In a model with confidence jumps, the points cluster along the 45-degree line, which indicates a close fit of the model-implied distribution of returns to the data. On the other hand, the model with no jumps cannot account for the left tail of the distribution of return in the data, as in absence of jump risks the model cannot generate negative skewness of returns.

In the model, jumps in the asset prices are endogenously driven by large moves in investor’s confidence about future growth. At the estimated parameters, the average frequency of jumps is about one every 5 months. An average jump in returns implied from the model is  $-3.3\%$ , monthly. The moves of such magnitudes are quite common in the data; indeed, in my sample monthly returns fall below the cutoff of  $-3.3\%$  once every 6 month. As shown in Section 2.2, preference for early resolution of uncertainty implies that the contribution of confidence jump risk is magnified under the risk-neutral measure, with jumps being larger and more frequent relative to the objective measure. Indeed, the mean jump in return is  $-3.6\%$ , and the average frequency of jumps is 4 months under the risk-neutral measure.

I discuss the implications of the confidence jump risks for the total premium in the economy in the next section.

## 4.5 Equity Premium

Table 8 shows the magnitude of the model-implied equity premium and its decomposition due to sources of risks in the economy. For a benchmark specification, I consider a standard, complete information long-run risks model where the uncertainty about expected growth is zero (see Bansal and Yaron, 2004). Based on the calibration of consumption dynamics and the estimated dividend and preference parameters from the full model, the implied equity premium in the model with no learning is 6.3%. Most of the compensation (3.3%) is due to investors’ exposure to the long-run risks; consumption volatility risks demand 2.1%, while immediate consumption growth shocks — about 1.4%. Learning and fluctuating confidence channels change

the compensation for the fundamental risks in the economy relative to the benchmark case. As investors cannot separate true short-run and long-run consumption innovations, the compensation for the expected growth risks decreases to 3.2% while the compensation for immediate consumption shocks increases slightly; this is consistent with Croce et al. (2006). In addition, as discussed in Section 2.1, fluctuating confidence of investors diminishes the importance of consumption volatility channel and reduces risk compensation for consumption volatility risks, compared to the complete information setup. Based on the parameter estimates of the full model, consumption volatility risks now contribute less than 0.2% to the total equity premium of 6.1%. On the other hand, confidence risk is the second most important risk channel in the economy, adding 1.7% to the equity premium. As nearly all of the confidence risks are jump risks, the compensation for confidence risks of almost 2% determines the overall contribution of jump risks to the equity premium in the economy. This magnitude is consistent with other studies. For example, Broadie et al. (2007) and Pan (2002) estimate the jump risk premium between 2%–3% and 3.5%, respectively, or one third of the total equity premium in the sample, which agrees with the estimate provided in this paper. Finally, in the model with no jumps in confidence measure, the estimated equity premium drops to 4.9%, as the compensation for purely Gaussian confidence shocks decreases to 0.3%.

I also verify that the model with confidence jump risks delivers sensible implications for the bond markets. The model-implied 1 month interest rate is 2%. Its volatility is 0.4%, annualized, and the persistence is 0.91. These values broadly agree with the statistics for interest rate reported in Table 1. The model-implied real term-structure is nearly flat, and the real interest rate at 5 year maturity is 1%.

## 4.6 GMM Model Fit

As shown in Hansen (1982), under the null hypothesis that the model is correctly specified the GMM criterion function in (4.1) has a  $\chi^2$  distribution, which can be used for an overall goodness-of-fit test of the model. The test does not reject the null that the orthogonality conditions based on the full model with confidence jump risks are equal to zero: the  $p$ -value for the test is 0.30. In addition, I follow Eichenbaum, Hansen, and Singleton (1988) to test the blocks of moment equations corresponding to the confidence measure, equity returns, interest rate and option-price data, respectively; see Appendix C for details. These moment restrictions are not rejected in the data: the  $p$ -value for the test that confidence measure moments are zero is 0.60, it is 0.8 for the return moments, 0.5 for the moments of interest rate and 0.3 for the orthogonality conditions based on the option price data.

In the estimation, I do not incorporate deep out- and in-the-money options with moneyness closest to 0.9 and 1.1, respectively, at 1 and 2 months to maturity; these

options can be used now for an out-of-sample test of the model. The orthogonality conditions for the levels of these option volatilities are not rejected at 5% significance level; in Section 4.4 I showed that the model somewhat under-predicts the implied volatility for deep out-of-the-money options, especially for longer maturity. Similarly, I can use the available data from 1968 to 1996 for an out-of-sample test of the orthogonality restrictions on the confidence measure. The average uncertainty is higher in the earlier period, so that the moment condition for the level of the series is rejected at 1% significance level. However, the joint test of all the higher moment restrictions which characterize the variation and non-Gaussian features of confidence measure cannot be rejected at 5%.

The goodness of fit of the model substantially deteriorates when confidence shocks do not include a jump-like component. The blocks of moment restrictions corresponding to confidence measure, return and option volatility data are rejected in sample at a 1% significance level. Further, the out-of-sample tests of moment conditions based on the additional option price data for moneyness 0.9 and 1.1 and the confidence data from 1968 to 1996 are also rejected with p-value below 1%.

## 5 Latent State MLE Model Estimation

### 5.1 Econometric Method

In the second approach, I treat confidence measure, as well as consumption volatility and expected growth state, as latent factors and estimate the model using the maximum likelihood method. The estimation framework allows me to use monthly observations on real consumption growth, equity returns, interest rates and the cross-section of 6 option prices to recover preference, consumption and confidence dynamics parameters and back out the unobserved states.

The main idea in the estimation is that in the model, option-implied volatilities are driven almost entirely by the confidence measure and aggregate volatility states. This allows me to back out volatility states from the implied volatilities alone and to estimate expected growth using the data on consumption, risk-free rate and equity return. The method is similar to Pan (2002), who inverts the option price to solve for the latent market volatility, and Santa-Clara and Yan (2008), who use two option prices to solve for the unobserved variance and stochastic intensity states. More generally, this approach is motivated by the literature on estimation of affine dynamic term structure models, see for example Duffie and Singleton (1997) and Duffee (2002). I outline the estimation approach below, and provide further details in Appendix D.

Denote  $\Theta$  the parameters of the model, and let a vector  $Z_t = [rf_t^{data} \quad r_{d,t} \quad \Delta c_{t+1}^{data}]'$  contain period- $t$  observations on interest rate, log real return and log consumption growth. I allow for i.i.d. Normal measurement errors in the log consumption level and interest rate to deal with measurement issues of these series in the data.

Each period, given the parameters  $\Theta$ , I solve for the unobserved confidence measure and consumption volatility states  $V_t(\Theta)$  and  $\sigma_t^2(\Theta)$  to directly match Black-Scholes volatilities of out- and at-the-money short-term put options in the model with moneyness of 0.95 and 1, respectively, and maturity of 1 month to their counterparts in the data. Using the model for confidence measure and consumption volatility specified in (1.14) and (1.8), I can compute the conditional likelihood of the two implied option volatilities  $l_t(\sigma_{BS,t}^2)$  which are used to invert the states. Further, given the history of observed macro and return data and the current implied confidence and consumption volatility states, the conditional distribution of asset market and consumption data  $Z_t$  is Normal. Therefore, I can apply standard Kalman Filter methods to write down the likelihood of the observed data  $l_t(Z_t)$  and the evolution of the estimate of the expected state  $\tilde{x}_t(\Theta)$ . To bring into the estimation the implied volatilities of the remaining put options, I assume that the pricing errors  $\xi_{bs,t}$ , that is, the difference between the model predicted and observed implied volatility for these contracts, are pure measurement errors which are Normal and independent from each other and fundamental shocks in the economy. Hence, their conditional likelihood  $l_t(\xi_{bs,t})$  is Normal.

I can combine the information from the consumption and asset-price data and the cross-section of option volatilities into the period- $t$  log likelihood:

$$l_t(Z_t, \sigma_{BS,t}^2, \xi_{bs,t}) = l_t(Z_t) + l_t(\sigma_{BS,t}^2) + l_t(\xi_{bs,t}). \quad (5.1)$$

The total log likelihood function of the sample is given by,

$$L(\Theta) = \sum_{t=1}^T l_t(Z_t, \sigma_{BS,t}^2, \xi_{bs,t}). \quad (5.2)$$

The optimal parameter value maximizes the sample likelihood function  $L(\Theta)$  given that the solution to the model and the implied states exist, and that the implied confidence measure  $V_t(\Theta)$  and consumption variance  $\sigma_t^2(\Theta)$  are greater than zero. I use parametric bootstrap method to compute standard errors on the estimated parameters.

## 5.2 MLE Estimation Results

Table 7 presents parameter estimates for the full model with fluctuating confidence and time-varying consumption volatility. As before, I fix the subjective discount

factor at 0.999. In addition, I set the mean of consumption and dividend growth to be equal to 2%. This is a standard estimate in the data; indeed, Tables 1 and Table 5 report the average consumption growth of 2%, both for the recent and long historical samples. Finally, to stabilize estimation I also fix the dividend leverage parameter  $\phi$  to 3.5, which is a common value in the literature (see e.g. Bansal and Yaron, 2004).

The estimation results for the model parameters are comparable to the ones from unconditional GMM. The estimated risk aversion coefficient is almost 10 and the intertemporal elasticity of substitution is 2.4; this indicates preference for early resolution of uncertainty. Expected consumption growth is very persistent,  $\rho = 0.976$ , so that the half-life of expected growth shocks is almost 2.5 years. At the same time, the variation in expected growth is very small and accounts for less than 5% of the total variation in consumption. Estimated consumption volatility is on average 2%, and it is moderately persistent, with an autoregression coefficient of 0.85. As for the estimates of confidence dynamics, its average level, in volatility units, is about 30 times lower than the unconditional volatility of consumption growth. Confidence shocks are quite persistent, with an autocorrelation coefficient of 0.92. Most of the fluctuation in the confidence measure is driven by non-Gaussian shocks, as jumps account for three quarters of the conditional variance of the series. Though the consumption and confidence measure parameters are estimated using a recent sample from 1996 to 2007, the implied dynamics of the series is consistent with a long historical sample from 1930-2006 for consumption and from 1968 to 2008 for confidence measure. I simulate the model and verify that the key features of their distribution, such as volatility, skewness and kurtosis of confidence measure and the ratio of confidence over the time-series volatility, match very well their counterparts in the data. The details are omitted in the interest of space.

The confidence measure and consumption volatility states implied from option prices are plotted in Figure 6. Confidence measure exhibits substantial variation over time, with occasional large positive spikes, while nearly hitting a zero boundary in the late period of the sample. At the quarterly frequency, the correlation of confidence measure implied from option prices and confidence measure in the data constructed from the professional forecasts of future GDP is 0.4. The estimated expected growth state is depicted in Figure 7. The extracted drift component significantly predicts next-period consumption with  $R^2$  of 2%.

### 5.3 Option and Asset Prices

The model with confidence jump risks can quantitatively explain the cross-section of option prices and the variation in option-implied volatilities. As shown in Table 9, the absolute pricing errors for implied volatilities are 1.2% for 1 month in-the-money options and less than 0.8% for those with 2 months to maturity, so that the

option fit of the model improves relative to GMM. In the right panel of Table 9 I quantify the amount of variation in implied volatilities that can be explained by the confidence and aggregate variance states. In linear projections, these states account for more than 95% of the total variation in option volatilities. Due to jumps and higher persistence, confidence measure is more important for out-of-the-money and longer maturity contracts. For a graphical representation of the model fit, on Figure 9 I show the implied volatility curves for 9 representative days with low, medium and high values for confidence measure and consumption volatility. Overall, the results suggest that the model with confidence jumps provide an adequate fit to the cross-section of option prices.

The model implications for the distribution of return and implied volatility and jumps are similar to the ones based on GMM fit. Relative to GMM, the frequency of large moves in returns decreases to one every 9 months, while average jump in return is higher,  $-7.5\%$ , monthly. The jump dimensions of return distribution are consistent with the data.

The model-implied equity premium is  $5.2\%$  (see Table 8) which is close to the estimate in the data of  $5.5\%$ . Expected growth shocks and confidence shocks contribute about  $2.4\%$  to the total equity premium, so that the relative contribution of confidence jump risks somewhat increases relative to the unconditional GMM parameter fit. The time-series of the fitted equity premium, as well as the premia due to expected growth and confidence shocks, are shown on Figure 8. The implied equity premium in the economy shows substantial variation over time, driven by the compensation for the expected growth and confidence jump risks.

A model without confidence jumps cannot explain the cross-section of option prices in the data. For robustness checks, I also estimate the model without time-varying consumption volatility. Most of the results are qualitatively similar; however, the cross-sectional fit of option prices deteriorates.

## 6 Conclusion

I present a long-run risks type model which features learning and fluctuating investor confidence about their estimate of unobserved expected growth. Uncertainty about expected growth (confidence measure) is time-varying and subject to jump-like risks. This confidence jump risk channel can quantitatively account for the cross-section of option prices and large moves in asset prices, without hard-wiring jumps into consumption. Out-of-the-money put options hedge jump risks in confidence and thus appear expensive relative to models with no jump risks. Positive jumps in confidence measure endogenously translate into negative jumps in equilibrium prices,

which can account for large downward moves and negatively skewed and heavy-tailed unconditional distribution of returns.

I provide empirical evidence that confidence measure in the data contains significant information about current and future implied volatilities. Further, I use two econometric approaches to formally estimate and test the model. The empirical results provide a strong support for a long-run risks model with learning, fluctuating confidence of investors and jump-like confidence risks. The model is not rejected in the data and provides a good fit to the option price, confidence measure, returns, and consumption data. Overall, empirical results strongly indicate that confidence jump risk plays an important role to explain option and equity prices in the data without introducing jumps into fundamental consumption.

# A Model Solution

## A.1 Kalman Filter

Given the dynamics of the underlying economy in (1.5)-(1.6) and the specification of signals in (1.8), the distribution of the states given the current information set and next-period confidence measure is conditionally Normal:

$$\begin{bmatrix} x_{t+1} \\ \Delta c_{t+1} \\ \bar{x}_{t+1} \end{bmatrix} \mid \mathcal{I}_t, V_{t+1} \sim N \left( \begin{bmatrix} \rho \hat{x}_t \\ \mu + \hat{x}_t \\ \rho \hat{x}_t \end{bmatrix}, \Sigma_{t+1} \right), \quad (\text{A.1})$$

where the variance-covariance matrix is given by,

$$\Sigma_{t+1} = \begin{bmatrix} \rho^2 \omega_t^2 + \varphi_e^2 \sigma_t^2 & \rho \omega_t^2 & \rho^2 \omega_t^2 + \varphi_e^2 \sigma_t^2 \\ \rho \omega_t^2 & \omega_t^2 + \sigma_t^2 & \rho \omega_t^2 \\ \rho^2 \omega_t^2 + \varphi_e^2 \sigma_t^2 & \rho \omega_t^2 & \rho^2 \omega_t^2 + \varphi_e^2 \sigma_t^2 + V_{t+1} \end{bmatrix}. \quad (\text{A.2})$$

The innovation representation of the system can then be written in the following way:

$$\Delta c_{t+1} = \mu + \hat{x}_t + a_{c,t+1}, \quad (\text{A.3})$$

$$\bar{x}_{t+1} = \rho \hat{x}_t + a_{x,t+1}, \quad (\text{A.4})$$

$$\hat{x}_{t+1} = \rho \hat{x}_t + K_{1,t+1} a_{c,t+1} + K_{2,t+1} a_{x,t+1}, \quad (\text{A.5})$$

where the Kalman Filter weights and the update for the filtering variance  $\omega_t^2$  satisfy standard equations

$$\begin{aligned} K_{t+1} &= \Sigma_{t+1}^{12} (\Sigma_{t+1}^{22})^{-1}, \\ \omega_{t+1}^2 &= \Sigma_{t+1}^{11} - \Sigma_{t+1}^{12} (\Sigma_{t+1}^{22})^{-1} \Sigma_{t+1}^{21}, \end{aligned} \quad (\text{A.6})$$

where the superscripts refer to the partitioning of  $\Sigma_{t+1}$  into four blocks, such that  $\Sigma_{t+1}^{11}$  is the (1,1) element of the matrix,  $\Sigma_{t+1}^{12}$  contain the elements from the first row and second and third columns, etc. The explicit solutions for the Kalman Filter weights satisfy

$$K_{1,t+1} = \frac{\rho \omega_t^2 V_{t+1}}{(\omega_t^2 + \sigma^2) V_{t+1} + (\varphi_e^2 \sigma^2 + (\varphi_e^2 + \rho^2) \omega_t^2) \sigma^2}, \quad (\text{A.7})$$

$$K_{2,t+1} = \frac{(\varphi_e^2 \sigma^2 + (\varphi_e^2 + \rho^2) \omega_t^2) \sigma^2}{(\omega_t^2 + \sigma^2) V_{t+1} + (\varphi_e^2 \sigma^2 + (\varphi_e^2 + \rho^2) \omega_t^2) \sigma^2}, \quad (\text{A.8})$$

while the evolution of the variance of the filtering error is given by

$$\omega_{t+1}^2 = V_{t+1} K_{2,t+1}. \quad (\text{A.9})$$

In the preferred specification, the Kalman Filter weights in the innovations representation of the system are constant. When investors do not look at consumption data and only update based on the average forecast,  $K_1 = 0$  and  $K_2$  is a steady-state solution to

$$K_{2,t+1} = \frac{\rho^2 \omega_t^2 + \varphi_e^2 \sigma_t^2}{\rho^2 \omega_t^2 + \varphi_e^2 \sigma_t^2 + V_{t+1}}. \quad (\text{A.10})$$



To solve for the steady state of the system, I plug the solution for filtering uncertainty in  $w_t^2 = K_2 V_t$  into the above equation and solve a quadratic equation for the constant value of  $K_2$  when the volatility processes  $V_t$  and  $\sigma_t^2$  are set to their unconditional means.

## A.2 Discount Factor

The aggregate consumption volatility  $\sigma_t^2$  follows a square-root process specified in (1.7), while the dynamics of confidence measure is given by a discrete-time jump-diffusion specification outlined in (1.14). The distribution of jump size  $J_{i,t+1}$  is defined by its moment generating function,

$$l(y) \equiv E e^{y J_i}. \quad (\text{A.11})$$

For example, when jump size follows exponential distribution with mean jump  $\mu_j$ ,

$$l(y) = (1 - \mu_j y)^{-1}. \quad (\text{A.12})$$

The conditional variance-covariance of consumption and expected growth shocks is given by,

$$\Sigma_{cx,t+1} = \text{Var} \left( \begin{bmatrix} a_{c,t+1} \\ a_{x,t+1} \end{bmatrix} \right) = \begin{bmatrix} K_2 V_t + \sigma_t^2 & \rho K_2 V_t \\ \rho K_2 V_t & \rho^2 K_2 V_t + \varphi_e^2 \sigma_t^2 + V_{t+1} \end{bmatrix}. \quad (\text{A.13})$$

The log price-to-consumption ratio  $pc_t$  is linear in the states of the economy:

$$pc_t = B_0 + B_x \hat{x}_t + B_v V_t + B_\sigma \sigma_t^2. \quad (\text{A.14})$$

Using Euler equation (1.4), I can directly solve for the loading  $B_x$  :

$$B_x = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho}. \quad (\text{A.15})$$

The loading on the confidence measure  $B_v$  satisfies non-linear equation

$$\frac{1}{2} \theta \kappa_1^2 \sigma_w^2 z^2 - (1 - \kappa_1 (\nu - \lambda_1 \mu_j)) z + \frac{1}{2} \theta B_x^2 K_2 ((1 - (1 - K_2) \kappa_1 \rho)^2 + \kappa_1 K_2) + \frac{\lambda_1}{\theta} (l(\theta \kappa_1 z) - 1) = 0, \quad (\text{A.16})$$

for  $z = B_v + \frac{1}{2} \theta \kappa_1 B_x^2 K_2^2$ , while  $B_\sigma$  solves a quadratic equation

$$\frac{1}{2} \theta \kappa_1^2 \varphi_w^2 B_\sigma^2 - (1 - \nu_c \kappa_1) B_\sigma + \frac{1}{2} \theta \left( \left(1 - \frac{1}{\psi}\right)^2 + \kappa_1^2 B_x^2 K_2^2 \varphi_e^2 \right) = 0. \quad (\text{A.17})$$

Finally, the log-linearization parameter, which is pinned down by the equilibrium level of the price-consumption ratio, satisfies the following non-linear equation:

$$\begin{aligned} \log \kappa_1 = \log \delta + \left(1 - \frac{1}{\psi}\right) \mu + B_\sigma (1 - \kappa_1 \nu_c) \sigma^2 \\ + (B_v (1 - \kappa_1) + \kappa_1 (1 - \nu) z) \sigma_v^2 + \frac{\lambda_0}{\theta} (l(\theta \kappa_1 z) - \theta \kappa_1 z \mu_j - 1). \end{aligned} \quad (\text{A.18})$$

As in Eraker and Shaliastovich (2008), in case of multiple roots for  $B_\sigma$  and  $B_v$  I choose the solution which is non-explosive as the variation in  $V_t$  or  $\sigma_t^2$  is approaching zero.

Using the equilibrium solution to the price-consumption ratio, I can write down the expression for the discount factor in the following way:

$$m_{t+1} = m_0 + m_x x_t + m_v V_t + m_\sigma \sigma_t^2 - \lambda_c a_{c,t+1} - \lambda_x K_2 a_{x,t+1} - \lambda_v \left( \sigma_w \sqrt{V_t} w_{t+1} + Q_{t+1} \right) - \lambda_\sigma \varphi_w \sigma_t w_{c,t+1}, \quad (\text{A.19})$$

where the discount factor loadings and the prices of risks are pinned down by the dynamics of factors and preference parameters of the investors. Their solutions are given by,

$$\begin{aligned} m_x &= -\frac{1}{\psi}, \quad m_v = (1 - \theta) B_v (1 - \kappa_1 \nu), \quad m_\sigma = (1 - \theta) B_\sigma (1 - \kappa_1 \nu_c), \\ m_0 &= \theta \log \delta + (1 - \theta) \log \kappa_1 - \gamma \mu - m_v \sigma_v^2 - m_\sigma \sigma^2, \end{aligned} \quad (\text{A.20})$$

and

$$\lambda_x = (1 - \theta) \kappa_1 B_x, \quad \lambda_\sigma = (1 - \theta) \kappa_1 B_\sigma, \quad \lambda_v = (1 - \theta) \kappa_1 B_v. \quad (\text{A.21})$$

### A.3 Asset Prices

Consider a log payoff tomorrow expressed as,

$$p_{n-1,t+1} = F_{0,n-1} + F_{x,n-1} \hat{x}_{t+1} + F_{v,n-1} V_{t+1} + F_{\sigma,n-1} \sigma_{t+1}^2 + F_{g,n-1} \Delta c_{t+1} + F_{d,n-1} \sigma_t \eta_{d,t+1}. \quad (\text{A.22})$$

Then, the solution for the coefficients in its log price today  $p_{n,t}$  satisfies

$$\begin{aligned} F_{g,n} &= F_{d,n} = 0, \\ F_{x,n} &= m_x + F_{x,n-1} \rho + F_{g,n-1}, \\ F_{\sigma,n} &= m_\sigma + F_{\sigma,n-1} \nu_c + \frac{1}{2} \left( (F_{g,n-1} - \lambda_c)^2 + \varphi_e^2 (F_{x,n-1} - \lambda_x)^2 K_2^2 + \varphi_w^2 (F_{\sigma,n-1} - \lambda_\sigma)^2 + F_{d,n-1}^2 \right), \\ F_{v,n} &= m_v + \frac{1}{2} (F_{g,n-1} - \lambda_c + \rho (F_{x,n-1} - \lambda_x) K_2)^2 K_2 + (q_{vx} + \lambda_v) \nu + \frac{1}{2} q_{vx}^2 \sigma_w^2 + \lambda_1 (l(q_{vx}) - q_{vx} \mu_j - 1), \\ F_{0,n} &= m_0 + F_{0,n-1} + F_{g,n-1} \mu + F_{\sigma,n-1} \sigma^2 (1 - \nu_c) + (q_{vx} + \lambda_v) \sigma_v^2 (1 - \nu) + \lambda_0 (l(q_{vx}) - q_{vx} \mu_j - 1) \end{aligned} \quad (\text{A.23})$$

for  $q_{vx} = F_{v,n-1} - \lambda_v + \frac{1}{2} (F_{x,n-1} - \lambda_x)^2 K_2^2$ .

Setting  $F_{0,n-1} = F_{x,n-1} = F_{v,n-1} = F_{\sigma,n-1} = F_{g,n-1} = F_{d,n-1} = 0$  in the above recursion, I can obtain the solution to  $n$ -period real risk-free rate.

On the other hand, the price-dividend ratio is given by,

$$pd_t = H_0 + H_x \hat{x}_t + H_v V_t + H_\sigma \sigma_t^2, \quad (\text{A.24})$$

where the loadings satisfy the following equations:

$$\begin{aligned}
H_x &= m_x + \kappa_{d,1} \rho H_x + \phi, \\
H_\sigma &= m_\sigma + \kappa_{d,1} H_\sigma \nu_c + \frac{1}{2} ((\phi - \lambda_c)^2 + \varphi_e^2 (\kappa_{d,1} H_x - \lambda_x)^2 K_2^2 + \varphi_w^2 (\kappa_{d,1} H_\sigma - \lambda_\sigma)^2 + \varphi_d^2), \\
H_v &= m_v + \frac{1}{2} (\phi - \lambda_c + \rho (\kappa_{d,1} H_x - \lambda_x) K_2)^2 K_2 + (q_{vx} + \lambda_v) \nu + \frac{1}{2} q_{vx}^2 \sigma_w^2 + \lambda_1 (l(q_{vx}) - q_{vx} \mu_j - 1),
\end{aligned} \tag{A.25}$$

for  $q_{vx} = \kappa_{d,1} H_v - \lambda_v + \frac{1}{2} (\kappa_{d,1} H_x - \lambda_x)^2 K_2^2$ , and the log-linearization parameter

$$\begin{aligned}
\log \kappa_{d,1} &= m_0 + \mu_d + \left( H_v (1 - \kappa_{d,1} \nu) + \frac{1}{2} (\kappa_{d,1} H_x - \lambda_x)^2 K_2^2 (1 - \nu) \right) \sigma_v^2 \\
&\quad + H_\sigma (1 - \kappa_{d,1} \nu_c) \sigma^2 + \lambda_0 (l(q_{vx}) - q_{vx} \mu_j - 1).
\end{aligned} \tag{A.26}$$

## A.4 Option Prices

The option prices are computed using the approach in Lewis (2000). Unlike other methods in the literature, it relies on a single integration along the complex line, which reduces computational burden (see Eraker and Shaliastovich, 2008).

The option price with strike  $K$  and maturity  $n$  is given by,

$$\begin{aligned}
C_t(K/P_t, n) &= E_t [M_{t,t+n} \max(e^{p_{t+n}} - K, 0)] \\
&= \frac{1}{2\pi} \left[ \int_{iz_i - \infty}^{iz_i + \infty} E_t (M_{t,t+n} e^{-izp_{t+n}}) \hat{w}(z) dz \right],
\end{aligned} \tag{A.27}$$

where  $M_{t,t+n}$  is the discount factor which can be used to price  $n$ -period ahead payoffs,  $p_t$  is the log equity price and  $\hat{w}(z)$  is the generalized Fourier transform of the payoff function of the option equal to,

$$\begin{aligned}
\hat{w}(z) &= \int_{-\infty}^{\infty} e^{izx} (e^x - K)^+ dx \\
&= -\frac{K^{iz+1}}{z^2 - iz}.
\end{aligned} \tag{A.28}$$

The integration region is parallel to the real line in the complex plane, and  $z_i \equiv \text{Im}(z) > 1$  for call options and  $z_i < 0$  for put options.

Using the equilibrium solution to the discount factor and asset valuations, the expectation inside the integral in (A.27) is given by

$$\log E_t e^{m_{t+n} - izp_{t+n}} = G_{0,n} + G_{x,n} \hat{x}_t + G_{v,n} V_t + G_{\sigma,n} \sigma_t^2 - izp_t, \tag{A.29}$$

where complex-valued loadings  $G_{0,n}, G_{x,n}, G_{v,n}$  and  $G_{\sigma,n}$  satisfy recursive equations similar to those computed in Appendix A.3.

Hence, the equilibrium put option price normalized by the equity price satisfies

$$\frac{C_t(K/P_t, n)}{P_t} = \frac{1}{2\pi} \frac{K}{P_t} \left[ \int_{iz_i - \infty}^{iz_i + \infty} \frac{e^{G_{0,n} + G_{x,n} \hat{x}_t + G_{v,n} V_t + G_{\sigma,n} \sigma_t^2 + iz \log(K/P_t)}}{iz - z^2} dz \right] \tag{A.30}$$

## B Confidence Measure

I use the cross-section of individual forecasts from the Survey of Professional Forecasts to calculate the average (consensus) forecast and its standard error for the next-quarter GDP for the period of 1996 to 2007. The survey started in the last quarter of 1968 as a joint project of the American Statistical Association and the National Bureau of Economic Research; in 1990 it was taken by the Federal Reserve Bank of Philadelphia. The data set contains quarterly forecasts on a variety of macroeconomic and financial variables made by the professional forecasters who largely come from the business world and Wall Street, see Croushore (1993) for details and Zarnowitz and Braun (1993) for a comprehensive study of the survey.

I use forecasts of nominal GDP and price index and to back out the average forecast and uncertainty in average forecast for real GDP. Specifically, for each quarter  $t$  let  $NGDP_{i,t}$  and  $P_{i,t}$  denote the next quarter forecasts of nominal GDP and price level of forecaster  $i$ . If  $n_t$  is the number of available forecasts, then the average forecast for the log real GDP ( $RGDP$ ) growth rate is

$$\overline{\Delta \log(RGDP)}_t = \frac{1}{n_t} \sum_{i=1}^{n_t} \left( \log \frac{NGDP_{i,t}}{NGDP_t} - \log \frac{P_{i,t}}{P_t} \right), \quad (\text{B.1})$$

where  $NGDP_t$  and  $P_t$  are the current values of the series known to the forecasters. For my sample, the average number of forecasts each period is about 40.

The cross-sectional variance of forecasts divided by the number of forecasts then determines the uncertainty (variance) in the average forecast:

$$\begin{aligned} V_t &= \frac{1}{n_t} \text{Var} \left( \log \frac{RGDP_{i,t}}{RGDP_t} \right) \\ &= \frac{1}{n_t} \left( \frac{1}{n_t - 1} \sum_{i=1}^{n_t} \left( \log \frac{NGDP_{i,t}}{NGDP_t} - \log \frac{P_{i,t}}{P_t} - \overline{\Delta \log(RGDP)}_t \right)^2 \right). \end{aligned} \quad (\text{B.2})$$

The cross-sectional variance of the forecasts adjusted by the number of forecasts provides an unbiased estimate for the confidence measure in the model. To make the inference robust to possible outliers and errors, I delete observations which are more than two standard deviations from the sample mean. The above calculations of the confidence measure in the data follow Bansal and Shaliastovich (2008a); David and Veronesi (2008) and Buraschi and Jitsov (2006) use similar approach to compute the uncertainty measures which rely on the cross-sectional dispersion in signals.

The confidence measure in the data is based on quarterly forecasts of real GDP, however, in the model the signals are for the next month expected consumption growth. To deal with the difference in scale, I adjust confidence measure in the data by the ratio of the calibrated unconditional consumption variance  $\sigma^2$  over the variance of realized GDP over the sample period (its standard deviation is 1%, annualized). Indeed, the properties of

the scaled confidence measure are very similar across different macroeconomic series in the data; Bansal and Shaliastovich (2008a) document that the ratio of confidence measure over conditional variance behaves very similarly for real GDP and industrial production forecasts.

I next deal with the time-aggregation in the uncertainty in forecasts using the approximation of multi-period consumption growth dynamics developed in Bansal et al. (2007b). Using Taylor expansion, log quarterly consumption is approximately equal to,

$$\log(C_{t+1} + C_{t+2} + C_{t+3}) \approx \log(3C_t) + \Delta c_{t+1} + \frac{2}{3}\Delta c_{t+2} + \frac{1}{3}\Delta c_{t+3}. \quad (\text{B.3})$$

Using the specification of the consumption dynamics, the true expected quarterly consumption can be written as

$$E_t^{true} \log(C_{t+1} + C_{t+2} + C_{t+3}) \approx \log(3C_t) + 2\mu + (1 + \frac{2}{3}\rho + \frac{1}{3}\rho^2)x_t. \quad (\text{B.4})$$

The true expected quarterly consumption growth is proportional to the true expected monthly consumption growth, up to known constants and consumption levels. Hence, the uncertainty in the estimate of expected quarterly consumption is proportional to the uncertainty in the estimate of expected monthly consumption, with the proportionality coefficient of  $(1 + \frac{2}{3}\rho + \frac{1}{3}\rho^2)^2$ .

I use the above scaling factor to further adjust the confidence measure based on quarter-ahead forecasts to obtain its counterpart for the next month expected growth.

## C GMM Estimation

The unconditional moments of  $V$  are given by,

$$\begin{aligned} E(V_t) &= \sigma_v^2, \\ E(V_t - \sigma_v^2)^2 &= \frac{\sigma_w^2 \sigma_v^2 + 2\mu_j^2(\lambda_0 + \lambda_1 \sigma_v^2)}{1 - \nu^2}, \\ E(V_t - \sigma_v^2)^3 &= \frac{3}{1 - \nu^3} (\nu(\sigma_w^2 + 2\mu_j^2 \lambda_1)E(V_t - \sigma_v^2)^2 + \mu_j^3(\lambda_0 + \lambda_1 \sigma_v^2)), \\ E(V_t - \sigma_v^2)^4 &= \frac{1}{1 - \nu^4} (6\nu^2(\sigma_w^2 + 2\mu_j^2 \lambda_1)E(V_t - \sigma_v^2)^3 \\ &\quad + (6\nu^2(\sigma_v^2 \sigma_w^2 + 2\mu_j^2(\lambda_0 + \lambda_1 \sigma_v^2)) + 3\sigma_w^4 + 10\mu_j^2 \sigma_w^2 \lambda_1 + 12\mu_j^3 \nu \lambda_1)E(V_t - \sigma_v^2)^2 \\ &\quad + (3\sigma_w^4 + 10\mu_j^2 \sigma_w^2 \lambda_1) \sigma_v^4 + (10\mu_j^2 \sigma_w^2 \lambda_0 + 16\mu_j^4 \lambda_1) \sigma_v^2 + 16\mu_j^4 \lambda_0), \\ E(V_{t+m} V_t) &= \sigma_v^4 + \nu^m E(V_t - \sigma_v^2)^2. \end{aligned} \quad (\text{C.1})$$

Moments of consumption volatility follow directly from above by setting jump parameters to zero. The moments of excess returns can be computed in a similar way using the model dynamics for returns; the results are available upon request.

Denote  $g_t^v(\Theta)$  the vector of orthogonality condition based on the confidence data:

$$g_t^v(\Theta) = [V_t \quad V_t^2 \quad V_t^3 \quad V_t^4 \quad V_t V_{t-3}]' - m_v(\Theta),$$

where  $\Theta$  are the estimated parameters and  $m_v(\Theta)$  is the corresponding vector of unconditional moments implied by the model.

Denote  $er_{t+1} = r_{d,t+1} - rf_t$  the excess return in the data. Then, the orthogonality conditions based on the return data can be expressed in the following way:

$$g_t^{er}(\Theta) = [er_t \quad er_t^2 \quad er_t^3 \quad er_t^4 \quad er_t er_{t-3} \quad er_t V_t]' - m_r(\Theta), \quad (C.2)$$

where  $m_r(\Theta)$  is the vector of unconditional moments calculated based on the return dynamics in Section 2.3. Further, I use the information in the level, variation and persistent of the interest rate, so I construct the orthogonality conditions based on the interest rate data,

$$g_t^{rf}(\Theta) = [rf_t \quad rf_t^2 \quad rf_t rf_{t-3}]' - m_{rf}(\Theta), \quad (C.3)$$

and I use the level of 6 option price volatilities, stacked in vector  $\sigma_{BS,t}$ , which span three moneyness categories (0.95, 1.00 and 1.05) and two maturities (1 and 2 months). The orthogonality conditions based on the option price data are then,

$$g_t^{BS}(\Theta) = \sigma_{BS,t} - m_{BS}(\Theta), \quad (C.4)$$

where  $m_{BS}(\Theta)$  are the corresponding implied volatilities from the model.

Let  $g_t(\Theta)$  stand for the overall vector of orthogonality conditions based on confidence measure, equity return, interest rate and option price data:

$$g_t(\Theta) = [g_t^v(\Theta) \quad g_t^{er}(\Theta) \quad g_t^{rf}(\Theta) \quad g_t^{BS}(\Theta)], \quad (C.5)$$

and  $\bar{g}$  denote its sample average across time. Then, the GMM objective function is given by,

$$Q_{GMM}(\Theta) = T\bar{g}(\Theta)' \Sigma_T^{-1} \bar{g}(\Theta). \quad (C.6)$$

As usual, the estimation is proceeded in two steps, and the optimal weighting matrix  $\Sigma_T^{-1}$  corresponds to the inverse of the Newey-West estimate of the variance of the moment conditions based on the first-step parameter estimates. The computation of standard errors and hypothesis testing follows from the standard asymptotic results for GMM.

Hansen (1982) shows that, under the null hypothesis that the model is correctly specified the GMM criterion function in (C.6) has a  $\chi^2$  distribution with 10 degrees of freedom, which can be used for an overall goodness-of-fit test of the model. I follow Eichenbaum et al. (1988) to test individual blocks of moment conditions (see also Singleton, 2006). For instance,

under the null hypothesis that moment conditions for the confidence measure are satisfied, a statistics

$$T \min_{\Theta} \bar{g}(\Theta)' \Sigma_T^{-1} \bar{g}(\Theta) - T \min_{\Theta} \begin{bmatrix} g_t^{er}(\Theta) & g_t^{rf}(\Theta) & g_t^{iv}(\Theta) \end{bmatrix}' \Sigma_{T,2}^{-1} \begin{bmatrix} g_t^{er}(\Theta) & g_t^{rf}(\Theta) & g_t^{iv}(\Theta) \end{bmatrix}$$

follows a  $\chi^2$  distribution with degrees of freedom equal to the number of moment restrictions in confidence measure block, while the weighting matrix  $\Sigma_{T,2}^{-1}$  can be constructed using the corresponding lower-right block of the estimate of the variance of the vector of moment conditions under the null. Then LR test that jump parameters are zero is conducted in a similar way by comparing the GMM objective functions of the restricted and unrestricted specifications.

For an out-of-the-sample test of moment conditions corresponding to the deep out- and in-the-money put options, I construct moment conditions for the levels of these volatilities  $\bar{g}^{IV}$  as in (C.4). Then, the statistics

$$T \bar{g}^{IV}(\Theta)' \Sigma_{T,IV}^{-1} \bar{g}^{IV}(\Theta) \sim \chi^2(4),$$

where  $\Sigma_{T,IV}^{-1}$  refers to the appropriate partition of the variance-covariance matrix  $\Sigma_T^{-1}$ , is distributed  $\chi^2$  with 4 degrees of freedom under the null that orthogonality conditions are satisfied. Similar test procedure is used for the orthogonality conditions for the confidence data from 1968 to 1996.

## D Latent Factor MLE Estimation

Given the dynamics of the economy, the conditional log-likelihood for the implied confidence measure and consumption volatility states is given by,

$$\begin{aligned} l_t(V_t(\Theta), \sigma_t^2(\Theta)) &= l(V_t(\Theta), \sigma_t^2(\Theta) \mid \{Z_{t-j}, V_{t-j}(\Theta), \sigma_{t-j}^2(\Theta)\}_{j=1,2,\dots}) = \\ &= l_t(V_t(\Theta) \mid V_{t-1}(\Theta)) + l_t(\sigma_t^2(\Theta) \mid \sigma_{t-1}^2(\Theta)). \end{aligned} \quad (\text{D.1})$$

This decomposition reflects the assumption that the two processes are independent. As the consumption variance follows a square-root process, its conditional distribution is Gaussian. Confidence measure follows a mixture of Normal-Gamma, and the details of the computation of its likelihood are provided below in Appendix D.1.

Given the likelihood of these states, I can write down the conditional likelihood of the two implied volatilities that are used to invert the confidence and consumption variance states:

$$\begin{aligned} l_t(\sigma_{BS,t}^2) &= l_t(\sigma_{BS,t}^2 \mid \{Z_{t-j}, V_{t-j}(\Theta), \sigma_{t-j}^2(\Theta)\}_{j=1,2,\dots}, V_t(\Theta), \sigma_t^2(\Theta)) \\ &= l_t(V_t(\Theta), \sigma_t^2(\Theta)) - \log(\text{abs}(J_t)), \end{aligned} \quad (\text{D.2})$$

where  $J_t$  is the Jacobian of the transformation of confidence and consumption variance into the two implied variance states. As the option variances are nearly linear in the

confidence measure and consumption variance, the Jacobian is computed numerically from the projection of implied option variances on the two variance states.

As for the observed consumption and interest rate data, I assume that one-period risk-free rate and log consumption level are observed with a measurement error:

$$\begin{aligned} r f_t^{data} &= r f_t + \xi_{rf,t}, \\ &= -F_0 - F_x \hat{x}_t - F_v V_t - F_\sigma \sigma_t^2 + \xi_{rf,t}. \end{aligned} \quad (D.3)$$

and

$$c_t^{data} = c_t + \xi_{c,t}, \quad (D.4)$$

so that

$$\Delta c_{t+1}^{data} = \mu + \hat{x}_t + a_{c,t+1} + \xi_{c,t+1} - \xi_{c,t}. \quad (D.5)$$

Indeed, real interest rates are not observed in the data, so  $\xi_{rf,t}$  captures measurement errors due to inflation adjustment of nominal yields, interpolation and other data issues. The measurement error in observed consumption  $\xi_{c,t}$  can account for a negative autocorrelation of monthly consumption growth rate. For simplicity, the measurement errors  $\xi_{rf,t}$  and  $\xi_{c,t}$  are assumed to be Normal, homoscedastic and independent from each other and all the other shocks in the economy.

In Appendix D.2 I show that the conditional distribution of observed asset market and consumption data  $Z_t$  given the history of  $Z_t$  and past and current values of implied states  $\sigma_t^2(\Theta)$  and  $V_t(\Theta)$  is Normal. Therefore, I can apply standard Kalman Filter methods to write down the conditionally Normal likelihood of the observed data,

$$l_t(Z_t) = l(Z_t \mid \{Z_{t-j}, V_{t-j}(\Theta), \sigma_{t-j}^2(\Theta)\}_{j=1,2,\dots}, V_t(\Theta), \sigma_t^2(\Theta)) \quad (D.6)$$

and the evolution of the estimate of the expected state  $\tilde{x}_t(\Theta)$  in a recursive way as a function of the observed macro and asset-price data and the implied confidence and volatility states.

## D.1 Likelihood for Confidence Measure

To simplify the exposition, I drop the dependence of the implied state  $V_t(\Theta)$  on model parameters  $\Theta$ .

The dynamics for confidence measure is given by,

$$V_{t+1} = \sigma_v^2 + \nu(V_t - \sigma_v^2) + \sigma_w \sqrt{V_t} w_{t+1} + \sum_{i=1}^{N_{t+1}} J_{i,t+1} - \mu_j \lambda_t, \quad (D.7)$$

where  $w_{t+1}$  is Gaussian shock,  $N_{t+1}$  is the Poisson process with stochastic intensity  $\lambda_t = \lambda_0 + \lambda_1 V_t$ , and  $J_{i,t+1}$  is jump size, whose distribution is i.i.d exponential with mean  $\mu_j$ .



To simplify the notations, denote the total variance shock  $\xi_{t+1}$ ,

$$\xi_{t+1} = V_{t+1} - (\sigma_v^2 + \nu(V_t - \sigma_v^2) - \mu_j \lambda_t),$$

which is equal to the sum of conditionally Normal shock  $\sigma_w \sqrt{V_t} w_{t+1}$  and jump shocks  $\sum_{i=1}^{N_{t+1}} J_{i,t+1}$ .

To find the conditional likelihood of variance shocks  $f(\xi_{t+1}|V_t)$  I can first condition on the number of jumps  $N_{t+1}$ , so that

$$f(\xi_{t+1}|V_t) = \sum_{N=0}^{\infty} f(\xi_{t+1}|V_t, N_{t+1} = N) Pr(N_{t+1} = N),$$

where the conditional probability function of the Poisson distribution is given by

$$Pr(N_{t+1} = N) = \lambda_t^N \exp(-\lambda_t) / N!.$$

As the sum of  $N$  exponentially distributed variables follows Gamma distribution, given previous state  $V_t$  and the number of jumps  $N_{t+1}$  jump shock has Gamma distribution  $Gamma(N, \mu_j)$ , so that the distribution of the total variance shock  $\xi_{t+1}$  is Normal-Gamma. For  $N = 0$ , its conditional likelihood is Normal; for  $N > 1$  it can be written in the following way:

$$f(\xi_{t+1}|V_t, N) = \frac{(\sigma_w \sqrt{V_t})^{N-1}}{\sqrt{2\pi} \Gamma(N) \mu_j^N} \exp\left(-\frac{\xi_{t+1}}{\mu_j} + \frac{\sigma_w^2 V_t}{2\mu_j^2}\right) \int_{b_t}^{\infty} (t - b_t)^{N-1} \exp\left(-\frac{t^2}{2}\right) dt,$$

for

$$b_t = -\frac{\xi_{t+1}}{\sigma_w \sqrt{V_t}} + \frac{\sigma_w \sqrt{V_t}}{\mu_j}.$$

To calculate the integral on the right-hand side, I use the fact the representation

$$\int_{b_t}^{\infty} t^{N-1} \exp\left(-\frac{t^2}{2}\right) dt = 2^{\frac{N-2}{2}} \Gamma(N/2) (1 - \text{sgn}(b_t)^N F_{\chi^2}(b_t^2|N)),$$

where  $F_{\chi^2}(\cdot|N)$  is the cdf of  $\chi$ -square distribution with  $N$  degrees of freedom, and  $\text{sgn}(b_t)$  gives the sign of  $b_t$ .

## D.2 Likelihood for Consumption and Asset-Price Data

To simplify the exposition, I drop the dependence of the implied states  $V_t(\Theta)$  and  $\sigma_t^2(\Theta)$  on model parameters  $\Theta$ .

Denote by  $\mathcal{H}_t$  the period- $t$  history of data and implied volatility states observed by econometrician:

$$\mathcal{H}_t = \{Z_{t-j}, V_{t-j}, \sigma_{t-j}^2\}_{j=0,1,\dots}, \quad (\text{D.8})$$

and let  $\tilde{x}_t$  and  $\tilde{\xi}_{c,t}$  denote the filtered value of unobserved expected growth state and measurement error in consumption, respectively, while  $\omega_{xc,t}^2$  stand for the variance of the filtering error of the econometrician,

$$\begin{bmatrix} \tilde{x}_t \\ \tilde{\xi}_{c,t} \end{bmatrix} = E \left( \begin{bmatrix} \hat{x}_t \\ \hat{\xi}_{c,t} \end{bmatrix} \mid \mathcal{H}_t \right), \quad \omega_{xc,t}^2 = E \left( \begin{bmatrix} \tilde{x}_t - \hat{x}_t \\ \tilde{\xi}_{c,t} - \hat{\xi}_{c,t} \end{bmatrix}^2 \mid \mathcal{H}_t \right). \quad (\text{D.9})$$

The one-step ahead conditional distribution of unobserved states and consumption and asset-price data is Normal,

$$\begin{bmatrix} \hat{x}_{t+1} \\ \hat{\xi}_{c,t+1} \\ Z_{t+1} \end{bmatrix} \mid \mathcal{H}_t, V_{t+1}, \sigma_{t+1}^2 \sim N \left( \begin{bmatrix} \rho \tilde{x}_t \\ 0 \\ \mu Z_t \end{bmatrix}, \Omega_{t+1} \right), \quad (\text{D.10})$$

where the drift is given by

$$\mu_{Zt} = \begin{bmatrix} -F_0 - F_x \rho \tilde{x}_t - F_v V_{t+1} - F_\sigma \sigma_{t+1}^2 \\ r_0 + \frac{1}{\psi} \tilde{x}_t + H_v (\kappa_{d,1} V_{t+1} - V_t) + H_\sigma (\kappa_{d,1} \sigma_{t+1}^2 - \sigma_t^2) \\ \mu + \tilde{x}_t - \tilde{\xi}_{c,t} \end{bmatrix}, \quad (\text{D.11})$$

$$r_0 = \mu - \log \kappa_{d,1} + (1 - \kappa_{d,1})(H_v \sigma_v^2 + H_\sigma \sigma^2),$$

and the variance-covariance matrix satisfies

$$\Omega_{t+1} = N \times \text{diag} \begin{bmatrix} \Sigma_{cx,t+1} \\ \omega_{xc,t}^2 \\ \sigma_{rf}^2 \\ \sigma_{\xi,c}^2 \\ \sigma_t^2 \end{bmatrix} \times N', \quad N = \begin{bmatrix} 0 & K_2 & \rho & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -F_x K_2 & -F_x \rho & 0 & 1 & 0 & 0 \\ \phi & \kappa_1 H_x K_2 & \frac{1}{\psi} & 0 & 0 & 0 & \varphi_d \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}. \quad (\text{D.12})$$

The updates for the filtered states and the variance of the filtering error satisfy standard Kalman Filter recursions

$$\begin{bmatrix} \tilde{x}_{t+1} \\ \tilde{\xi}_{c,t+1} \end{bmatrix} = \begin{bmatrix} \rho \tilde{x}_t \\ 0 \end{bmatrix} + K_{t+1} (Z_{t+1} - \mu_{Zt})$$

$$\omega_{xc,t+1}^2 = \Omega_{t+1}^{11} - \Omega_{t+1}^{12} (\Omega_{t+1}^{22})^{-1} \Omega_{t+1}^{21}, \quad (\text{D.13})$$

where the Kalman Filter Gain is given by

$$K_{t+1} = \Omega_{t+1}^{12} (\Omega_{t+1}^{22})^{-1}. \quad (\text{D.14})$$

The superscripts refer to the partitioning of  $\Omega_{t+1}$  into four blocks, such that  $\Omega_{t+1}^{11}$  is the two by two upper corner of the matrix,  $\Omega_{t+1}^{12}$  contain the elements from first two rows and third to fourth columns, etc. The conditional distribution of the observed asset-price and macro data in  $Z_{t+1}$  is Normal with mean  $\mu_{Zt}$  and variance-covariance matrix given by  $\Omega_{t+1}^{22}$  which allows me write down the conditional likelihood  $l_t(Z_t)$ .

# Tables and Figures

Table 1: **Summary Statistics**

	Mean	AR(1)	Std. Dev.	Skew	Kurt
Implied volatility	17.66 (0.97)	0.78 (0.04)	5.85 (0.52)	0.78 (0.23)	3.41 (0.59)
Smirk	3.72 (0.12)	0.46 (0.09)	0.86 (0.08)	0.50 (0.31)	3.95 (0.71)
Consumption growth	2.00 (0.20)	-0.37 (0.07)	0.93 (0.09)	-0.11 (0.21)	4.00 (0.45)
Interest rate	1.56 (0.33)	0.97 (0.04)	0.51 (0.02)	-0.29 (0.29)	1.71 (0.25)
Log return	6.99 (4.36)	-0.06 (0.08)	15.17 (1.41)	-0.69 (0.25)	4.51 (0.71)
Confidence Measure					
1996-2007	0.07 (0.01)	0.45 (0.11)	0.02 (0.002)	1.55 (0.27)	5.77 (1.42)
1968-2007	0.11 (0.01)	0.53 (0.07)	0.06 (0.01)	4.36 (0.69)	30.80 (9.67)

Summary statistics for implied volatility and smirk (top pane), real consumption growth, inflation-adjusted interest rate and log real returns (middle panel) and confidence measure for the overlapping and long sample (bottom panel). Implied volatility is computed for the short-term at-the-money option (time to expiration of 1 month and moneyness closest to  $K/P = 1$ ), while smirk is given by the difference between the implied volatility of out-of-the-money and at-the-money short-term put options (moneyness of 0.95 and 1, respectively and maturity of 1 month). Mean and volatility for confidence measure are computed for the square root of the confidence measure in the data. Statistics are annualized, in percent. Newey-West standard errors with 4 lags. Monthly observations from January 1996 to June 2007.

Table 2: **Option-Implied Volatility**

Moneyiness	0.90	0.95	1.00	1.05	1.10
<i>1 month to maturity:</i>					
Average implied vol	25.79	21.38	17.66	15.68	15.74
	(0.94)	(0.52)	(0.97)	(0.88)	(0.76)
Low uncertainty $V$	20.56	15.70	11.85	10.29	11.45
High uncertainty $V$	28.01	23.61	19.71	17.40	17.19
<i>2 months to maturity:</i>					
Average implied vol	24.15	21.14	18.42	16.97	16.64
	(0.96)	(0.96)	(0.95)	(0.89)	(0.80)
Low uncertainty $V$	18.62	15.54	12.66	11.39	11.67
High uncertainty $V$	26.30	23.22	20.42	18.71	18.37

Black-Scholes implied volatilities for 1 and 2 months to maturity options. Average volatilities are based on monthly observations of put option prices from January 1996 to June 2007. Low and high uncertainty  $V$  panels report average implied volatilities for the quarters with high and low confidence measure; high and low values correspond to above 75% and below 25% percentile of the unconditional distribution of confidence measure in the data, respectively.

Table 3: **Option Price Predictability by Confidence Measure**

	Option Variance	Confidence Measure	$R^2$
<i>Out-of-the-Money</i>			
1 quarter ahead	0.52** (0.08)	0.11 (0.11)	0.31
2 quarters ahead	0.27 (0.15)	0.43* (0.18)	0.29
3 quarters ahead	0.31 (0.17)	0.51** (0.18)	0.39
<i>At-the-Money</i>			
1 quarter ahead	0.42** (0.10)	0.09 (0.07)	0.40
2 quarters ahead	0.33** (0.08)	0.24** (0.11)	0.38
3 quarters ahead	0.13 (0.10)	0.30** (0.10)	0.23
<i>In-the-Money</i>			
1 quarter ahead	0.27** (0.09)	0.12 (0.08)	0.26
2 quarters ahead	0.30** (0.06)	0.19** (0.08)	0.37
3 quarters ahead	0.08 (0.06)	0.24** (0.10)	0.18

Projection of future option-implied variance on current option variance and current confidence measure. Table reports slope coefficients on the two regressors and the  $R^2$ . Option variance and confidence measure are standardized. Put options contracts are 1 month to maturity. Standard errors are Newey-West adjusted with 3 lags. One and two stars indicate significance at 5% and 1%, respectively.

Table 4: **Calibration of Consumption Dynamics**

Parameter	Value
$\mu$	0.0017
$\rho$	0.978
$\sigma$	0.0058
$\nu_c$	0.987
$\varphi_w$	5.72e-04
$\varphi_e$	0.044

Calibrated parameter values, monthly frequency.

Table 5: **Consumption Dynamics: Data and Model Calibration**

	Data		Model		
	Estimate	S.E.	5%	Median	95%
Mean	1.95	(0.32)	1.13	1.97	2.73
Vol	2.13	(0.52)	1.51	2.08	2.71
AR(1)	0.44	(0.13)	0.22	0.47	0.68
AR(2)	0.16	(0.18)	-0.05	0.21	0.49
AR(5)	-0.01	(0.10)	-0.18	0.06	0.28
VR(2)	1.58	(0.18)	1.24	1.47	1.66
VR(5)	2.23	(0.86)	1.39	2.12	3.04

Calibration of consumption dynamics. Data is annual real consumption growth for 1930-2006. Model is based on 100 simulations of 80 years of monthly consumption data aggregated to annual horizon, based on the full specification with fluctuating confidence and consumption volatility.

Table 6: **GMM Parameter Estimates**

	Full Model		No Jumps	
	Estimate	S.E.	Estimate	S.E.
<i>Preference:</i>				
$\delta$	0.999		0.999	
$\psi$	1.36	(0.11)	1.43	(0.15)
$\gamma$	11.03	(1.30)	10.56	(5.09)
<i>Dividend:</i>				
$\phi$	3.68	(0.85)	4.01	(2.00)
$\varphi_d$	4.88	(1.49)	5.52	(3.03)
<i>Confidence Measure:</i>				
$\sigma_v \times 10^4$	3.73	(0.11)	3.07	(0.11)
$\nu$	0.91	(0.01)	0.94	(0.03)
$\sigma_w \times 10^5$	0.34	(1.15)	10.34	(0.60)
$\lambda_0 \times 10^2$	6.37	(2.76)		
$\lambda_1 \times 10^{-6}$	1.00	(0.51)		
$\mu_j \times 10^7$	1.95	(0.25)		

GMM Parameter Estimates of the full model (left panel) and the model with no jumps in confidence measure (right panel).

Table 7: MLE Parameter Estimates

	Full Model	
	Estimate	S.E.
<i>Preference:</i>		
$\delta$	0.999	
$\psi$	2.44	(0.78)
$\gamma$	9.66	(1.23)
<i>Consumption:</i>		
$\mu \times 10^3$	1.66	
$\rho$	0.976	(0.003)
$\varphi_e \times 10^2$	4.50	(0.71)
<i>Dividend:</i>		
$\phi$	3.5	
$\varphi_d$	6.22	(0.41)
<i>Confidence Measure:</i>		
$\sigma_v \times 10^4$	2.01	
$\nu$	0.92	(0.01)
$\sigma_w \times 10^5$	9.81	(1.07)
$\lambda_0 \times 10^2$	3.47	(0.40)
$\lambda_1 \times 10^{-6}$	1.96	(0.29)
$\mu_j \times 10^7$	1.06	(0.13)
<i>Aggregate Volatility:</i>		
$\sigma \times 10^3$	5.95	(0.47)
$\nu_c$	0.86	(0.03)
$\varphi_w \times 10^3$	2.07	(0.13)

MLE parameter estimates of the model



Table 8: **Equity Premium Decomposition**

	Expected Growth	Consumption Growth	Confidence Measure	Aggregate Volatility	Total
No Learning	3.32	1.35	0.00	2.07	6.27
GMM - Full Model	3.19	1.36	1.73	0.17	6.11
GMM - No Jumps	3.52	1.37	0.28	0.21	4.87
Latent Factor MLE	2.36	0.95	2.44	0.02	5.18

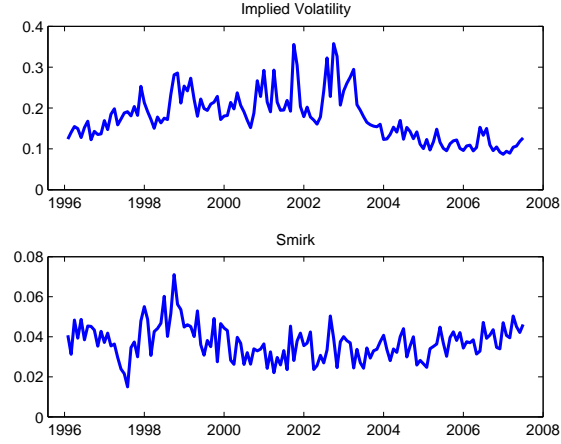
Decomposition of equity risk premium into contributions from expected growth, consumption, confidence and aggregate volatility innovations. No Learning refers to standard complete information setup with no learning (perfect confidence). GMM - Full Model and GMM - No Jumps panels show premium decomposition based on GMM estimation of the full model with confidence jump risks and model with no jumps in confidence, respectively. Calculations in Latent Factor MLE are based on the MLE estimation of the model. Equity premium is annualized, in percent.

Table 9: **Implied Volatilities Fit from MLE**

$K/P$	Pricing error		Explained Variation		
	RMSE	Abs. Mean	$V$	$\sigma^2$	Total
<i>1 month</i>					
0.95			0.83	0.17	1.00
1.00			0.70	0.30	1.00
1.05	1.33	1.15	0.52	0.43	0.95
<i>2 months</i>					
0.95	0.90	0.75	0.83	0.14	0.97
1.00	0.86	0.68	0.78	0.20	0.98
1.05	0.89	0.75	0.69	0.28	0.96

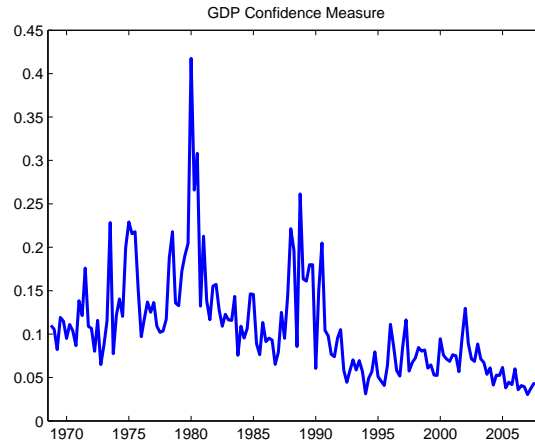
Implied volatility fit based on the MLE estimation of the model. The left panel shows the standard deviation of the measurement error in implied volatility, annualized, percent. The right panel shows the total explained variation in implied variance and the contributions of confidence measure and aggregate volatility, based on the linear projection.

Figure 1: **Time Series of Implied Volatility**



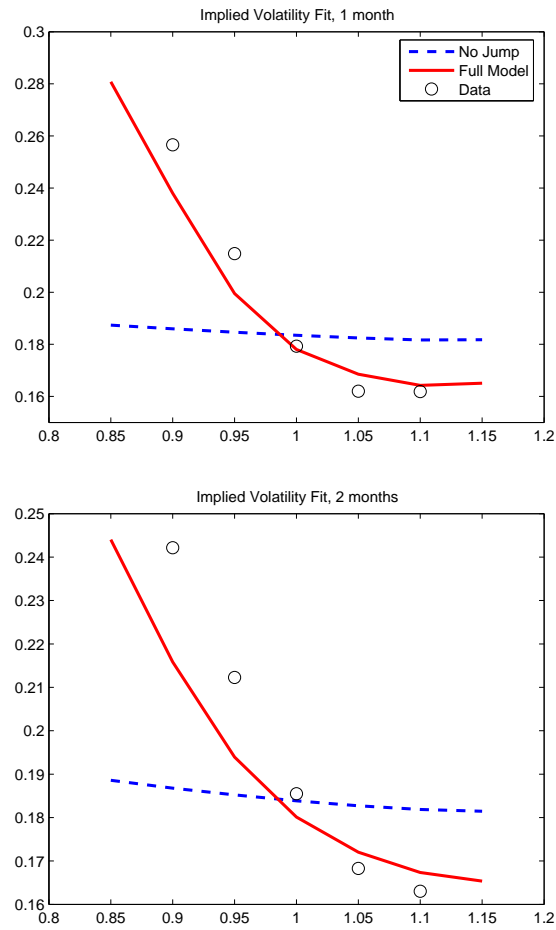
Top panel depicts the implied volatility of at-the-money, short-term options (time to expiration of 1 month and moneyness closest to  $K/P = 1$ ). Second panel plots the difference between the implied volatility of out-of-the-money and at-the-money short-term put options (moneyness of 0.95 and 1, respectively and maturity of 1 month). Monthly observations from January 1996 to June 2007.

Figure 2: **Confidence Measure**



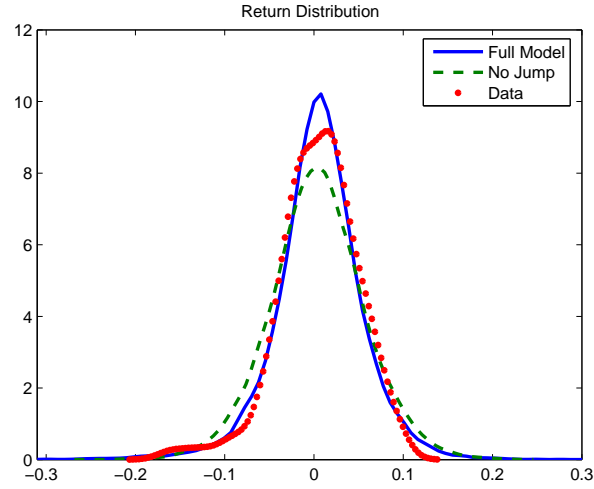
Confidence measure based on forecasts of next-quarter real GDP, annualized in percent.

Figure 3: Implied Volatility Smile



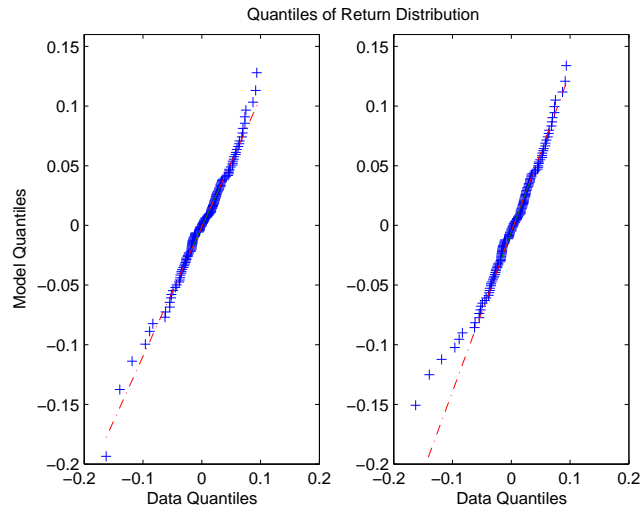
Implied volatility smile based on the GMM estimations of the full model (solid line) and model with no jumps (dashed line). Data (circles) are based on quarterly observations from January 1996 to June 2007. Top and bottom panel refer to option contracts expiring in 1 and 2 months, respectively.

Figure 4: **Unconditional Distribution of Returns**



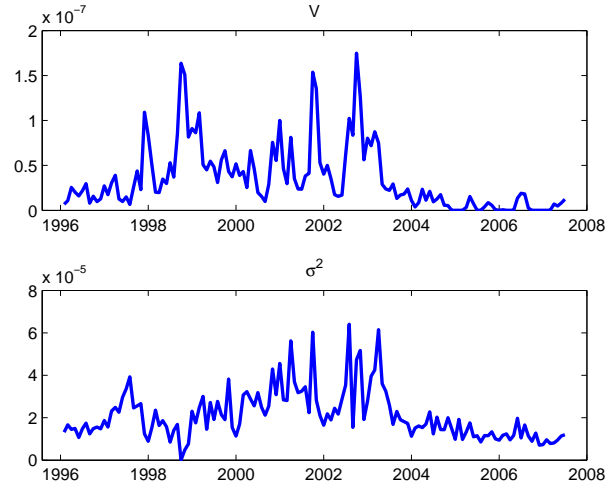
Unconditional distribution of excess returns based on the GMM estimations of the full model (solid line) and model with no jumps (dashed line). Return data (circles) is from January 1996 to June 2007.

Figure 5: **QQ Plot of Return Distribution**



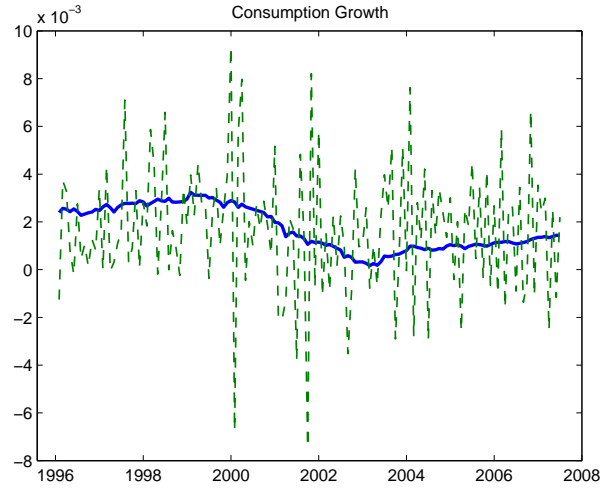
Plot of quantiles of unconditional excess return distribution in the data versus full model (left panel) and model with no confidence jumps (right panel), based on the GMM estimation. Return data is from January 1996 to June 2007.

Figure 6: **Model-Implied Confidence Measure and Consumption Volatility**



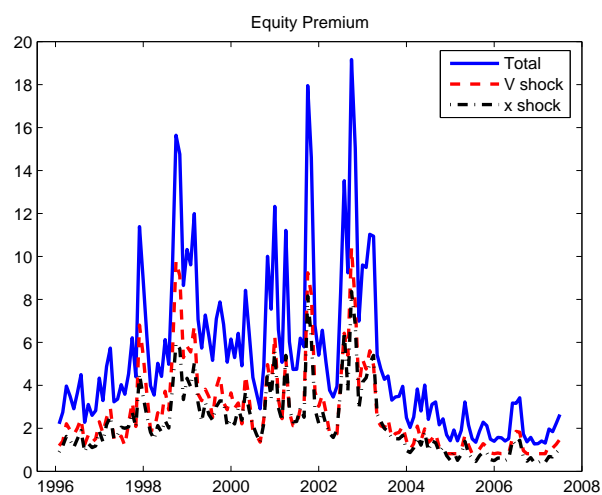
Times-series of implied confidence measure (top panel) and aggregate volatility (bottom panel). Monthly data from January 1996 to June 2007.

Figure 7: **Model-Implied Expected Consumption**



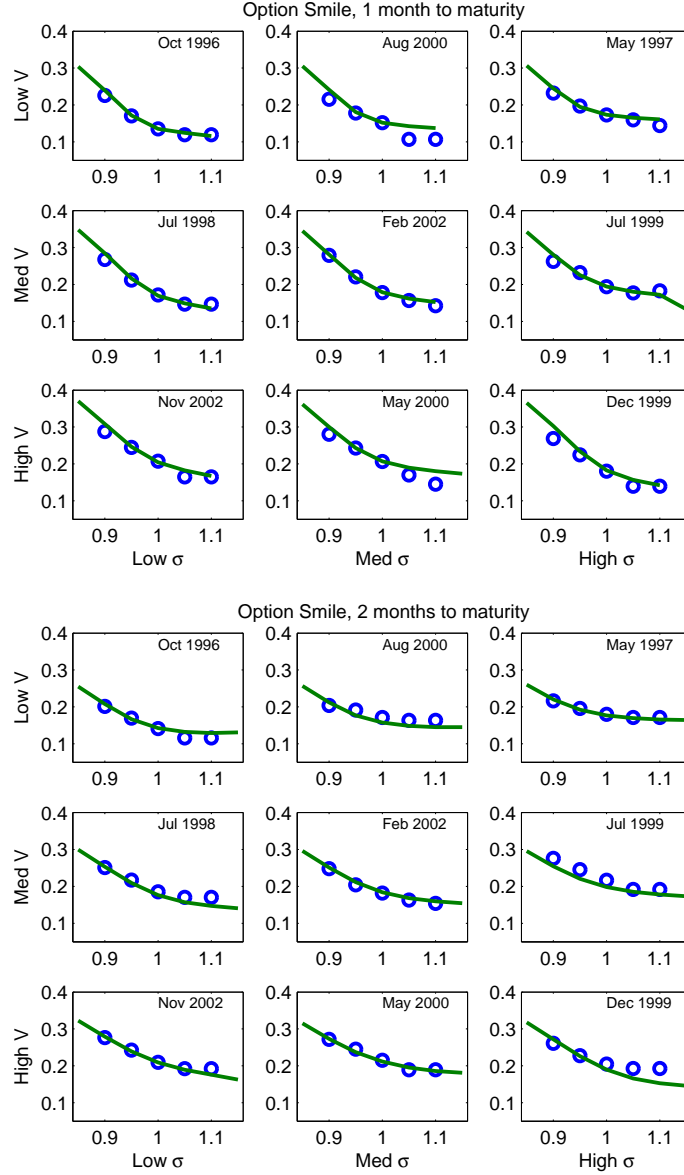
Times-series of expected consumption growth (solid line) and realized monthly consumption (dashed line). Monthly data from January 1996 to June 2007.

Figure 8: **Equity Risk Premium**



Times-series of total equity risk premium (solid line) and the premia for expected growth (dashed line) and confidence risks (dash-dotted line) implied from the MLE estimation of the model. Monthly data from January 1996 to June 2007.

Figure 9: **Option-Implied Volatility Smile**



Implied volatility from put options on 9 representative days for low, medium and high values of confidence level and consumption volatility. Circles represent actual data points and solid line is a smile based on the MLE estimates of full model with jump confidence risks. Top and bottom panels are for contracts for 1 and 2 months to maturity.

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