Equilibrium Prices in the Presence of Delegated Portfolio Management*

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Abstract

This paper analyzes the asset pricing implications of commonly-used portfolio management contracts linking the compensation of fund managers to the excess return of the managed portfolio over a benchmark portfolio. The contract parameters, the extent of delegation and equilibrium prices are all determined endogenously within the model we consider. Symmetric ("fulcrum") performance fees distort the allocation of managed portfolios in a way that induces a significant and unambiguous positive effect on the prices of the assets included in the benchmark and a negative effect on the Sharpe ratios. Asymmetric performance fees have more complex effects on equilibrium prices and Sharpe ratios, with the signs of these effects fluctuating stochastically over time in response to variations in the funds' excess performance.

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1 Introduction

In modern economies, a significant share of financial wealth is delegated to professional portfolio managers rather than managed directly by the owners, creating an agency relationship. In the U.S., as of 2004, mutual funds managed assets in excess of $8 trillion, hedge funds managed about $1 trillion and pension funds more than $12 trillion. In other industrialized countries, the percentage of financial assets managed through portfolio managers is even larger than in the U.S. (see, e.g., Bank for International Settlements (2003)).

While the theoretical literature on optimal compensation of portfolio managers in dynamic settings points to contracts that are likely to have very complicated path dependencies,[1] the industry practice seems to favor relatively simple compensation schemes that typically include a component that depends linearly on the value of the managed assets plus a component that is linearly or non-linearly related to the excess performance of the managed portfolio over a benchmark.

In 1970, the U.S. Congress amended the Investment Advisers Act of 1940 so as to allow contracts with registered investment companies to include performance-based compensation, provided that this compensation is of the "fulcrum" type, that is, provided that it includes penalties for underperforming the chosen benchmark that are symmetric to the bonuses for exceeding it. In 1985, the SEC approved the use of performance-based fees in contracts in which the client has either at least $500,000 under management or a net worth of at least $1 million. Performance-based fees were also approved by the Department of Labor in August 1986 for ERISA-governed pension funds. As of 2004, 50% of U.S. corporate pension funds with assets above $5 billion, 35% of all U.S. pension funds and 9% of all U.S. mutual funds used performance-based fees.[2] Furthermore, Brown, Harlow and Starks (1996), Chevalier and Ellison (1997) and Sirri and Tufano (1998) have documented that, even when mutual fund managers do not receive explicit incentive fees, an implicit nonlinear performance-based compensation still arises with periodic proportional fees as a result of the fact that the net investment flow into mutual funds varies in a convex fashion as a function of recent performance.[3]

Given the size of the portfolio management industry, studying the implications of this delegation and of the fee structures commonly used in the industry on equilibrium asset prices appears to be a critical task. The importance of models addressing the implications of agency for asset pricing was emphasized by Allen (2001): *"In the standard asset-pricing*

[1]A distinctive feature of the agency problem arising from portfolio management is that the agent's actions (the investment strategy and possibly the effort spent acquiring information about securities' returns) affect both the drift and the volatility of the relevant state variable (the value of the managed portfolio), although realistically the drift and the volatility cannot be chosen independently. This makes the problem significantly more complex than the one considered in the classic paper by Holstrom and Milgrom (1987) and its extensions. With a couple of exceptions, as noted by Stracca (2006) in his recent survey of the literature on delegated portfolio managent, *"the literature has reached more negative rather than constructive results, and the search for an optimal contract has proved to be inconclusive even in the most simple settings."*

[2]The use is concentrated in larger funds: the percentages of assets under management controlled by mutual funds charging performance fees out of funds managing assets of $0.25-1 billion, $1-5 billion, $5-10 billion, and above $10 billion were 2.8%, 4.4%, 9.2%, and 14.2% respectively (data obtained from Greenwich Associates and the Investment Company Institute).

[3]Lynch and Musto (2003) and Berk and Green (2004) provide models in which this convex relationship between flows and performance arises endogenously.

paradigm it is assumed investors directly invest their wealth in markets. While this was an appropriate assumption for the U.S. in the 1950 when individuals directly held over 90% of corporate equities, or even in 1970 when the figure was 68%, it has become increasingly less appropriate as time has progressed [...] For actively managed funds, the people that make the ultimate investment decisions are not the owners. If the people making the investment decisions obtain a high reward when things go well and a limited penalty if they go badly they will be willing to pay more than the discounted cash flow for an asset. This is the type of incentive scheme that many financial institutions give to investment managers."

Existing theoretical research on delegated portfolio management has been primarily restricted to partial equilibrium settings and has focused on two main areas. The first examines the agency problem that arises between investors and portfolio managers, studying how compensation contracts should be structured: it includes Bhattacharya and Pfleiderer (1985), Starks (1987), Kihlstrom (1988), Stoughton (1993), Heinkel and Stoughton (1994), Admati and Pfleiderer (1997), Das and Sundaram (2002), Palomino and Prat (2003), Ou-Yang (2003), Larsen (2005), Liu (2005), Dybvig, Farnsworth and Carpenter (2006), Cadenillas, Cvitanić and Zapatero (2007) and Cvitanić, Wan and Zhang (2007). The second examines how commonly-observed incentive contracts impact managers' decisions: it includes Grinblatt and Titman (1989), Roll (1992), Carpenter (2000), Chen and Pennacchi (2005), Hugonnier and Kaniel (2006) and Basak, Pavlova and Shapiro (2007).

We complement this literature by considering a different problem. As in the literature on optimal behavior of portfolio managers, we take the parametric class of contracts as exogenously given, motivated by commonly observed fee structures. However, we carry the analysis beyond partial equilibrium by studying how the behavior of portfolio managers affects equilibrium prices when the extent of portfolio delegation and the parameters of the management contract are all determined endogenously.

A first step in studying the implications of delegated portfolio management on asset returns was made by Brennan (1993), who considered a static mean-variance economy with two types of investors: individual investors (assumed to be standard mean-variance optimizers) and "agency investors" (assumed to be concerned with the mean and the variance of the difference between the return on their portfolio and the return on a benchmark portfolio). Equilibrium expected returns were shown to be characterized by a two-factor model, with the two factors being the market and the benchmark portfolio. Closely-related mean-variance models have appeared in Gómez and Zapatero (2003) and Cornell and Roll (2005).[4]

To our knowledge, the only general equilibrium analyses of portfolio delegation in dynamic settings are in two recent papers by Kapur and Timmermann (2005) and Arora, Ju and Ou-Yang (2006). Kapur and Timmermann consider a restricted version of our model with mean-variance preferences, normal returns and fulcrum performance fees, while Arora, Ju and Ou-Yang assume CARA utilities and normal dividends and do not endogenize the extent of portfolio delegation: as a result of these assumptions, fulcrum performance fees are optimal in their model.[5] More importantly, both papers consider settings with a single

[4]Brennan (1993) found mixed empirical support for the two-factor model over the period 1931–1991, while Gómez and Zapatero (2003) found stronger support over the period 1983–1997.

[5]In the model of Kapur and Timmerann, performance fees do not dominate fees depending only on the terminal value of the assets under managent.

risky asset. A key shortcoming of models with a single risky asset (or of static models) is that they are unable to capture the shifting risk incentives of portfolio managers receiving implicit or explicit performance fees (and hence the impact of these incentives on portfolio choices and equilibrium prices), as extensively described in both the theoretical and empirical literature.[6]

In contrast to the papers mentioned above, we study the asset pricing implications of delegated portfolio management in the context of a dynamic (continuous-time) model with multiple risky assets and endogenous portfolio delegation. Specifically, we consider an economy with a continuum of three types of agents: "active investors", "fund investors" and "fund managers". Active investors, who trade on their own account, choose a dynamic trading strategy so as to maximize the expected utility of the terminal value of their portfolio. Fund investors, who implicitly face higher trading or information costs, invest in equities only through mutual funds: therefore, their investment choices are limited to how much to delegate to fund managers, with the rest of their portfolio being invested in riskless assets. Fund managers, who are assumed not to have any private wealth, select a dynamic trading strategy so as to maximize the expected utility of their compensation.

The compensation contracts we consider are restricted to a parametric structure that replicates the contracts typically observed in practice, consisting of a combination of the following components: a flat fee, a proportional fee depending on the total value of the assets under management, and a performance fee depending in a piecewise-linear manner on the differential between the return of the managed portfolio and that of a benchmark portfolio.[7]

Departing from the traditional formulation of principal-agent problems, we assume that individual fund investors are unable to make "take it or leave it" contract offers to fund managers and thus to extract the entire surplus from the agency relation: instead, we assume that the market for fund investors is competitive, so that individual investors take the fee structure as given when deciding what fraction of their wealth to delegate.[8] Similarly, we assume competition on the market for portfolio managers. The contract parameters are selected in our model so that they are constrained Pareto efficient, i.e., so that there is no other contract within our parametric class that provides both fund investors and fund managers with higher welfares.

As shown in Section 5, even when fund investors and fund managers have identical preferences, the principle of preference similarity (Ross (1973)) does not apply in our setting and asymmetric performance contracts Pareto-dominate purely proportional contracts (as well as fulcrum performance contracts): intuitively, convex performance fees are a way to incentivise fund managers to select portfolio strategies having higher overall stock allocations, benefiting fund investors who have direct access to riskless investment opportunities.

[6] Clearly, in the presence of performance fees, tracking error volatility directly affects the reward of portfolio managers and this volatility can be dynamically controlled by varying the composition of the managed portfolio.

[7] While our framework allows for both "fulcrum" and "asymmetric" performance fees, it does not allow for "high water mark" fees (occasionally used by hedge funds and discussed by Goetzmann, Ingersoll and Ross (2003)) in which the benchmark equals the lagged maximum value of the managed portfolio.

[8] As noted by Das and Sundaram (2002), the existence of regulation, such as the Investment Advisors Act of 1940, meant to protect fund investors through restrictions on the allowable compensation contracts can be viewed as tacit recognition that these investors do not dictate the form of the compensation contracts.

Because of this incentive role of performance fees, the optimal benchmark typically differs from the market portfolio.[9]

Portfolio delegation can have a substantial impact on equilibrium prices. With fulcrum fees, the presence of a penalty for underperforming the benchmark portfolio leads risk-averse fund managers to be overinvested in the stocks included in the benchmark portfolio and underinvested in the stocks excluded from this portfolio. The bias of managed portfolios in favor of the stocks included in the benchmark portfolio results in the equilibrium expected returns and Sharpe ratios of these stocks being lower than those of comparable stocks not in the benchmark and in their price/dividend ratios being higher. At the same time, stocks in the benchmark portfolio tend to have lower equilibrium volatilities than those of comparable stocks not in the benchmark: this is due to the fact that, as the price of benchmark stocks starts to rise, the tilt of managed portfolios toward these stocks increases, lowering their equilibrium price/dividend ratios and hence moderating the price increase. Therefore, consistent with empirical evidence, our model implies that, if fund managers are mostly compensated with fulcrum fees, a change in the composition of widely-used benchmark portfolios (such as the S&P 500 portfolio) should be accompanied by a permanent increase in the prices and volatilities of the stocks being added to the index and a corresponding permanent decrease in the prices and volatilities of the stocks being dropped from the index. With asymmetric performance fees, the signs of these changes become ambiguous, depending on the current average excess performance of managed portfolios relative to the benchmark.

The remainder of the paper is organized as follows. The economic setup is described in Section 2. Section 3 provides a characterization of the optimal investment strategies. Section 4 focuses on the characterization of equilibria. Section 5 discusses the optimality of performance contracts in our model. Sections 6 provides a detailed numerical analysis of equilibrium under asymmetric and fulcrum performance fees. Section 7 concludes. The Appendix contains all the proofs.

2 The Economy

We consider a continuous-time economy on the finite time span $[0, T]$, modeled as follows.

Securities. The investment opportunities are represented by a riskless bond and two risky stocks (or stock portfolios). The bond is a claim to a riskless payoff $B > 0$. The interest rate is normalized to zero (i.e., the bond price is normalized to B).

Stock j ($j = 1, 2$) is a claim to an exogenous liquidation dividend D^j_T at time T, where

$$D^j_t = D^j_0 + \int_0^t \mu^j_D(D^j_s, s)\, ds + \int_0^t \sigma^j_D(D^j_s, s)\, dw^j_s, \tag{1}$$

for some functions μ^j_D, σ^j_D satisfying appropriate Lipschitz and growth conditions and two Brownian motions w^j with instantaneous correlation coefficient $\rho \in (-1, 1)$. Since dividends are paid only at the terminal date T, without loss off generality we take $\mu^j_D \equiv 0$, so that D^j_t can be interpreted as the conditional expectation at time t of stock j's liquidation dividend.

[9] This is in contrast to the existing mean-variance equilibrium models of portfolio delegation, in which the optimal benchmark is the market portfolio and performance fees are dominated by linear contracts.

We let $D_T = (B, D_T^1, D_T^2)$ denote the vector of terminal asset payoffs and denote by $S_t = (B, S_t^1, S_t^2)$ the vector of asset prices at time t. The aggregate supply of each asset is normalized to one share and we denote by $\bar{\theta} = (1, 1, 1)$ the aggregate supply vector.

Security trading takes place continuously. A dynamic trading strategy is a three-dimensional process θ, specifying the number of shares held of each of the traded securities, such that the corresponding wealth process $W = \theta \cdot S$ satisfies the dynamic budget constraint

$$W_t = W_0 + \int_0^t \theta_s \cdot dS_s \tag{2}$$

and $W_t \geq 0$ for all $t \in [0, T]$. We denote by Θ the set of dynamic trading strategies.[10]

Agents. The economy is populated by three types of agents: active investors, fund investors and fund managers. We assume that there is a continuum of agents of each type and denote by $\lambda \in (0, 1)$ the mass of fund investors in the economy and by $1 - \lambda$ the mass of active investors. Without loss of generality, we assume that the mass of fund managers also equals λ: this is merely a normalization, since the aggregate wealth managed by fund managers is determined endogenously by the portfolio choices of fund investors, as described later in this section.

Active investors receive an endowment of one share of each traded asset at time 0, so that their initial wealth equals $W_0^a = \bar{\theta} \cdot S_0$. They choose a dynamic trading strategy $\theta^a \in \Theta$ so as to maximize the expected utility

$$\mathrm{E}[u^a(W_T^a)]$$

of the terminal value of their portfolio $W_T^a = \theta_T^a \cdot S_T$, taking equilibrium prices as given.

Fund investors also receive an endowment of one share of each asset, so that their initial wealth is $W_0^f = W_0^a = \bar{\theta} \cdot S_0$. However, because of higher trading or information costs (which we do not model explicitly), they do not hold stocks directly and instead delegate the choice of a dynamic trading strategy to fund managers: at time 0 they simply choose to invest an amount $\theta_0^f B \leq W_0^f$ in the riskless asset and invest the rest of their wealth in mutual funds.[11]

Fund managers receive an initial endowment $W_0^m = W_0^f - \theta_0^f B$ from fund investors, which they then manage on the fund investors' behalf by selecting a dynamic trading strategy $\theta^m \in \Theta$. For this, they are compensated at time T with a management fee F_T which is a function of the terminal value of the fund portfolio, $W_T^m = \theta_T^m \cdot S_T$, and of the terminal value of a given benchmark portfolio $W_T^b = \theta_T^b \cdot S_T$, where $\theta^b \in \Theta$.[12] Specifically, we assume

[10]Implicit in the definition of Θ is the requirement that the stochastic integral in equation (2) is well defined.

[11]Because fund investors in our model do not trade dynamically and are not assumed to know the return distribution of the individual assets (only knowledge of the the return distribution of the fund they invest in being assumed), their behavior could be rationalized with a combination of trading and information costs. Of course, in reality there is a wide range of investors with different trading and information costs: the assumption that investors are either "active", with full information and costless access to stock trading, or "passive", requiring the intermediation of mutual funds to obtain exposure to risky assets, is clearly a simplifying one and is made for tractability.

[12]Letting the benchmark to be the terminal value of a dynamic (not necessarily buy-and-hold) trading strategy allows for the possibility of changes in the composition of the benchmark portfolio.

that

$$\begin{aligned} F_T &= F(W_T^m, W_T^b) \\ &= \alpha + \beta W_T^m - \gamma_1 W_0^m \left(\frac{W_T^m}{W_0^m} - \frac{W_T^b}{W_0^b} \right)^- + \gamma_2 W_0^m \left(\frac{W_T^m}{W_0^m} - \frac{W_T^b}{W_0^b} \right)^+ \\ &= \alpha + \beta W_T^m - \gamma_1 (W_T^m - \delta W_T^b)^- + \gamma_2 (W_T^m - \delta W_T^b)^+, \end{aligned} \tag{3}$$

where α, β, γ_1 and γ_2 are given parameters, $\delta = W_0^m / W_0^b$ and $x^+ = \max(0, x)$ (respectively, $x^- = \max(0, -x)$) denotes the positive part (respectively, the negative part) of the real number x.

Thus, the fund managers' compensation at time T can consist of four components: a load fee α which is independent of the managers' performance, a proportional fee βW_T^m which depends on the terminal value of the fund portfolio, a performance bonus $\gamma_2 (W_T^m - \delta W_T^b)^+$ which depends on the performance of the managed portfolio relative to that of the benchmark portfolio, and an underperformance penalty $\gamma_1 (W_T^m - \delta W_T^b)^-$. We assume that $\alpha \geq 0$, $\beta \geq 0$, $\gamma_2 \geq \gamma_1 \geq 0$ and $\beta + \gamma_2 > 0$, so that the fund managers' compensation F is an increasing and convex function of the terminal value of the fund portfolio and a decreasing function of the terminal value of the benchmark portfolio.[13] In addition, we assume that $\alpha + \beta > 0$, so that the fund managers always have at least one feasible investment strategy (buying the benchmark portfolio) that yields a strictly positive fee.

When $\gamma_1 = \gamma_2$, the performance-related component of the managers' compensation is linear in the excess return of the fund over the benchmark. This type of fees are known as *fulcrum performance fees*. As noted in the Introduction, the 1970 Amendment of the Investment Advisers Act of 1940 restricts mutual funds' performance fees to be of the fulcrum type. Hedge funds' performance fees are not subject to the same restriction, and for these funds *asymmetric performance fees* with $\gamma_1 = 0$ and $\gamma_2 > 0$ are the norm.

Fund managers are assumed not to have any private wealth. They therefore act so as to maximize the expected utility

$$\mathrm{E}[u^m(F(W_T^m, W_T^b))]$$

of their management fees, while taking equilibrium prices and the investment choices of fund investors as given. Similarly, fund investors select the amount of portfolio delegation $W_0^f - \theta_0^f B$ so as to maximize the expected utility of their terminal wealth, while taking the equilibrium net-of-fees rate of return on mutual funds

$$R_T = \frac{W_T^m - F(W_T^m, W_T^b)}{W_0^m}, \tag{4}$$

as given, subject to the constraint $W_0^f - \theta_0^f B \geq 0$.

We assume throughout that

$$u^a(W) = u^m(W) = u^f(W) = u(W) = \frac{W^{1-c}}{1-c}$$

[13] We do not necessarily require that $\beta + \gamma_2 < 1$, i.e., that $\frac{\partial}{\partial W^m} F(W^m, W^b) < 1$. In order to guarantee the existence of an equilibrium, we will however later impose a condition that implies that the optimal terminal wealth of fund investors is increasing in aggregate wealth (see equation (23)).

for some $c > 0$, $c \neq 1$.[14]

Equilibrium. An *equilibrium* for the above economy is a price process S for the traded assets and a set $\{\theta^a, \theta^m, \theta_0^f\} \in \Theta \times \Theta \times \mathbb{R}$ of trading strategies such that:

1. the strategy θ^a is optimal for the active investors given the equilibrium stock prices;

2. the strategy θ^m is optimal for the fund managers given the equilibrium stock prices and the fund investors' choice of θ_0^f;

3. the choice θ_0^f is optimal for the fund investors given the equilibrium stock prices and the equilibrium net-of-fees funds' return;

4. the security markets clear:

$$(1-\lambda)\theta_t^a + \lambda\theta_t^m + \lambda(\theta_0^f, 0, 0) = \bar{\theta} \qquad \text{for all } t \in [0, T]. \tag{5}$$

3 Optimal Investment Strategies

Since active investors and fund managers face a dynamically complete market, we use the martingale approach of Cox and Huang (1989) to characterize their optimal investment strategies.

3.1 Active Investors

Given the equilibrium state-price density π_T at time T, the optimal investment problem for the active investors amounts to the choice of a non-negative random variable W_T^a (representing the terminal value of their portfolios) solving the static problem

$$\max_{W_T \geq 0} \; \mathrm{E}\Big[u(W_T)\Big]$$
$$\text{s.t. } \; \mathrm{E}\Big[\pi_T W_T\Big] \leq W_0^a.$$

This implies

$$W_T^a = g^a(\psi^a \pi_T), \tag{6}$$

where $g^a(y) = y^{-\frac{1}{c}}$ denotes the inverse marginal utility function and ψ^a is a Lagrangian multiplier solving

$$\mathrm{E}\Big[\pi_T g^a(\psi^a \pi_T)\Big] = W_0^a = \bar{\theta} \cdot S_0. \tag{7}$$

[14]If $\gamma_1 > 0$, the fee specified in equation (3) can be negative for sufficiently low values of W. However, given our assumption of infinite marginal utility at zero wealth, the fund managers will always optimally act so as to ensure the collection of a strictly positive fee. Thus, exactly the same equilibrium would be obtained if the fee schedule specified above were replaced with the nonnegative fee schedule $F'(W, S) = F(W, S)^+$.

3.2 Fund Managers

Given the equilibrium state-price density π_T and the allocation to mutual funds by fund investors $W_0^m = \bar{\theta}\cdot S_0 - \theta_0^f B$, the optimal investment problem for the fund managers amounts to the choice of a non-negative random variable W_T^m (representing the terminal value of the funds' portfolios) solving the static problem

$$\begin{aligned} \max_{W_T \geq 0} \ & \mathrm{E}\Big[u\Big(F(W_T, W_T^b)\Big)\Big] \\ \text{s.t.} \ & \mathrm{E}\Big[\pi_T W_T\Big] \leq W_0^m. \end{aligned} \tag{8}$$

The added complexity in this case arises from the fact that, unless $\gamma_1 = \gamma_2$, the fund managers' indirect utility function over the terminal portfolio value, $u(F(W, W^b))$ is neither concave nor differentiable in its first argument at the point $W = \delta W^b$ (the critical value at which the performance of the fund's portfolio equals that of the benchmark): see Figure 1. This is a consequence of the convexity and lack of differentiability of the fee function $F(W, W^b)$. Therefore, the optimal choice W_T^m is not necessarily unique and it does not necessarily satisfy the usual first-order condition. Moreover, the non-negativity constraint can be binding in this case (if $\alpha > 0$).

On the other hand, the fact that managers have infinite marginal utility at zero wealth implies that the optimal investment strategy must guarantee that a strictly positive fee is collected at time T, i.e., that $W_T^m > \underline{W}(W_T^b)$, where

$$\underline{W}(W^b) = \inf\Big\{W \geq 0 : F(W, W^b) \geq 0\Big\} = \begin{cases} \left(\frac{\gamma_1 \delta W^b - \alpha}{\beta + \gamma_1}\right)^+ & \text{if } \beta + \gamma_1 \neq 0, \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

Since for any $W^b > 0$ the function $u(F(\cdot, W^b))$ is piecewise concave and piecewise continuously differentiable on the interval $[\,\underline{W}(W^b), \infty)$, we can follow Shapley and Shubik (1965), Aumann and Perles (1965) and Carpenter (2000) in constructing the concavification $v(\cdot, W^b)$ of $u(F(\cdot, W^b))$ (that is, the smallest concave function v satisfying $v(W, W^b) \geq u(F(W, W^b))$ for all $W \geq \underline{W}(W^b)$) and then verifying that the solutions of the non-concave problem (8) can be derived from those of the concave problem

$$\begin{aligned} \max_{W_T \geq 0} \ & \mathrm{E}\Big[v(W_T, W_T^b)\Big] \\ \text{s.t.} \ & \mathrm{E}\Big[\pi_T W_T\Big] \leq W_0^m. \end{aligned} \tag{10}$$

Lemmas 1 and 2 below provide a characterization of the concavifying function v while Proposition 1 and Theorem 1 identify the fund managers' optimal investment policies.

Lemma 1 *Suppose that $\gamma_1 \neq \gamma_2$ and $W^b > 0$. Then there exist unique numbers $W_1(W^b)$ and $W_2(W^b)$ with*

$$\underline{W}(W^b) \leq W_1(W^b) < \delta W^b < W_2(W^b)$$

such that

$$\begin{aligned} u(F(W_1(W^b), W^b)) &= u(F(W_2(W^b), W^b)) \\ &\quad + u'(F(W_2(W^b), W^b))(\beta + \gamma_2)(W_1(W^b) - W_2(W^b)) \end{aligned} \tag{11}$$

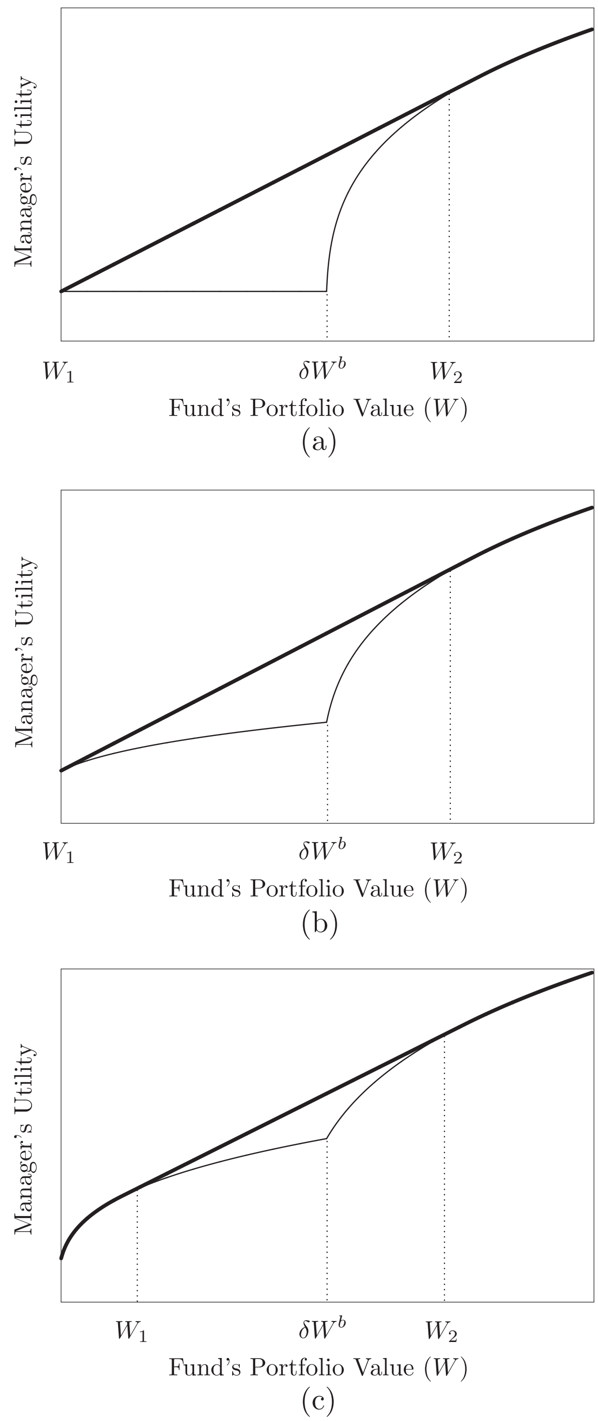


Figure 1: The graph plots three different possible cases for the fund manager's utility function $u(F(W, W^b))$ (lighter solid lines) and the corresponding concavified utility function $v(W, W^b)$ (heavier solid lines). In panel (a) $\beta + \gamma_1 = 0$. In panel (b) $\beta + \gamma_1 > 0$ and $W_1 = 0$. In panel (c) $\beta + \gamma_1 > 0$ and $W_1 > 0$.

and

$$u'(F(W_1(W^b), W^b))(\beta + \gamma_1) \leq u'(F(W_2(W^b), W^b))(\beta + \gamma_2), \tag{12}$$

with equality if $W_1(W^b) > 0$*. In particular, letting* $\eta = \left(\frac{\beta+\gamma_2}{\beta+\gamma_1}\right)^{1-\frac{1}{c}}$,

$$W_1(W^b) = \left(\frac{\left(1 - \frac{\eta}{c}\right) \frac{\alpha - \gamma_1 \delta W^b}{\beta+\gamma_1} - \eta \left(1 - \frac{1}{c}\right) \frac{\alpha - \gamma_2 \delta W^b}{\beta + \gamma_2}}{\eta - 1} \right)^+ \tag{13}$$

if $\beta + \gamma_1 \neq 0$ *and* $W_1(W^b) = 0$ *otherwise. Moreover,*

$$W_2(W^b) = W_1(W^b) + \frac{1}{c} \left(\frac{\alpha - \gamma_1 \delta W^b}{\beta + \gamma_1} - \frac{\alpha - \gamma_2 \delta W^b}{\beta + \gamma_2} \right)$$

if $W_1(W^b) > 0$.

Lemma 2 *Let* $W_1(W^b)$ *and* $W_2(W^b)$ *be as in Lemma 1 if* $\gamma_1 \neq \gamma_2$ *and let* $W_1(W^b) = W_2(W^b) = \delta W^b$ *otherwise. Also, let*

$$A(W^b) = [\,\underline{W}(W^b), W_1(W^b)] \cup [W_2(W^b), \infty),$$

Then the function

$$v(W, W^b) = \begin{cases} u(F(W, W^b)) & \text{if } W^m \in A(W^b), \\ u(F(W_1(W^b), W^b)) \\ \quad + u'(F(W_2(W^b), W^b))(\beta + \gamma_2)(W - W_1(W^b)) & \text{otherwise} \end{cases}$$

is the smallest concave function on $[\,\underline{W}(W^b), \infty)$ *satisfying*

$$v(W, W^b) \geq u(F(W, W^b)) \qquad \forall W \in [\,\underline{W}, \infty).$$

Moreover, $v(W, W^b)$ *is continuously differentiable in* W.

The construction of the concavification $v(W, W^b)$ of $u(F(W, W^b))$ is illustrated in Figure 1. Since $u(F(W, W^b))$ is not concave at $W = \delta W^b$ when $\gamma_1 \neq \gamma_2$, the idea is to replace $u(F(W, W^b))$ with a linear function over an interval $(W_1(W^b), W_2(W^b))$ bracketing δW^b (a linear function is the smallest concave function between two points). The two points $W_1(W^b)$ and $W_2(W^b)$ are uniquely determined by the requirement that the resulting concavified function $v(W, W^b)$ be continuously differentiable and coincide with $u(F(W, W^b))$ at the endpoints of the interval. When $\beta + \gamma_1 = 0$, this requirement necessarily implies $W_1(W^b) = 0$ (as shown in panel (a) of Figure 1): this is the case studied by Carpenter (2000). On the other hand, when $\beta + \gamma_1 > 0$, both the case $W_1(W^b) = 0$ and the case $W_1(W^b) > 0$ are possible (as illustrated in panels (b) and (c) of Figure 1), depending on the sign of the expression in equation (13).

Letting

$$v_W^{-1}(y, W^b) = \left\{ W \in [\,\underline{W}(W^b), \infty) : v_W(W, W^b) = y \right\}$$

be the inverse marginal utility correspondence of the concavified utility v, we then obtain the following characterization of the fund managers' optimal investment policies.

Proposition 1 *A policy W_T^m is optimal for the fund managers if and only if: (i) it satisfies the budget constraint in equation (8) as an equality and (ii) there exists a Lagrangian multiplier $\psi^m > 0$ such that*

$$W_T^m \in v_W^{-1}(\psi^m \pi_T, W_T^b) \cap A(W_T^b) \quad \textit{if } W_T^m > 0, \tag{14}$$

and

$$v_W(W_T^m, W_T^b) \leq \psi^m \pi_T \quad \textit{if } W_T^m = 0.$$

To understand the characterization of optimal policies in Proposition 1, consider first the concavified problem in equation (10). The standard Kuhn-Tucker conditions for this problem are the same as the conditions of Proposition 1, but with equation (14) replaced by

$$W_T^m \in v_W^{-1}(\psi^m \pi_T, W_T^b) \quad \text{if } W_T^m > 0. \tag{15}$$

It follows immediately from Lemma 2 that $v_W(W, W^b)$ is strictly decreasing for $W \in A(W^b)$ and constant and equal to

$$v_W(W_2(W^b), W^b) = u'(F(W_2(W^b), W^b))(\beta + \gamma_2)$$

for $W \in (W_1(W^b), W_2(W^b))$. Thus, the sets $v_W^{-1}(y, W^b)$ are singletons unless

$$y = v_W(W_2(W^b), W^b),$$

in which case $v_W^{-1}(y, W^b) = [W_1(W^b), W_2(W^b)]$. Equation (15) then implies that optimal policies for the concavified problem (10) are uniquely defined for values of the normalized state-price density $\psi^m \pi_T$ different from $v_W(W_2(W_T^b), W_T^b)$, but not for $\psi^m \pi_T = v_W(W_2(W_T^b), W_T^b)$: at this critical value of the normalized state-price density, any wealth level between $W_1(W_T^b)$ and $W_2(W_T^b)$ can be chosen as part of an optimal policy (subject of course to an appropriate adjustment of the Lagrangian multiplier ψ^m), reflecting the fact that $v(\cdot, W_T^b)$ is linear over this range.

Consider now policies satisfying the stronger condition in equation (14). It can be easily verified from the definitions that

$$v_W^{-1}(y, W^b) \cap A = v_W^{-1}(y, W^b) \quad \text{for } y \neq v_W(W_2(W^b), W^b)$$

and

$$v_W^{-1}(y, W^b) \cap A = \left\{ W_1(W^b), W_2(W^b) \right\} \quad \text{for } y = v_W(W_2(W^b), W^b).$$

Thus, the policies satisfying the conditions of Proposition 1 are a subset of the policies that are optimal for the concavified problem in equation (10): when the normalized state-price density $\psi^m \pi_T$ equals $v_W(W_2(W_T^b), W_T^b)$, they are restricted to take either the value $W_1(W_T^b)$ or the value $W_2(W_T^b)$, rather than any value in the interval $[W_1(W_T^b), W_2(W_T^b)]$. It is easy to see why any such policy W_T^m must be optimal in the original problem in equation (8): since W_T^m is optimal for the concavified problem in equation (10) and takes values in $A(W_T^b)$, it follows that

$$\mathrm{E}[u(F(W_T, W_T^b))] \leq \mathrm{E}[v(W_T, W_T^b)] \leq \mathrm{E}[v(W_T^m, W_T^b)] = \mathrm{E}[u(F(W_T^m, W_T^b))]$$

for any feasible policy W_T, where the first inequality follows from the fact that

$$u(F(W_T, W_T^b)) \leq v(W_T, W_T^b),$$

the second inequality follows from the fact that W_T^m is optimal for the problem in equation (10), and the last equality follows from the fact that $u(F(W, W_T^b)) = v(W, W_T^b)$ for $W \in A(W_T^b)$.

Since managers are indifferent between selecting $W_1(W_T^b)$ or $W_2(W_T^b)$ as the terminal value of the fund's portfolio when the scaled state-price density equals $v_W(W_2(W_T^b), W_T^b)$, we allow them to independently randomize between $W_1(W_T^b)$ and $W_2(W_T^b)$ in this case: in other words, we allow, for this particular value of the scaled state-price density, the optimal policy to be a lottery that selects $W_1(W_T^b)$ with some probability $p \in [0,1]$ and $W_2(W_T^b)$ with probability $(1-p)$, and denote such a lottery by

$$p \circ W_1(W_T^b) + (1-p) \circ W_2(W_T^b).$$

Of course, p can depend on the information available to the agents at time T, i.e., be a random variable. Using the expression for v in Lemma 2 and Proposition 1, we then obtain the following result.

Theorem 1 *Let P_T be a random variable taking values in $[0,1]$ and let*

$$g^m(y, W^b, p) = \begin{cases} \frac{y^{-\frac{1}{c}}}{(\beta+\gamma_2)^{1-\frac{1}{c}}} - \frac{\alpha-\gamma_2\delta W^b}{\beta+\gamma_2} > W_2(W^b) & \text{if } y < v_W(W_2(W^b), W^b), \\ p \circ W_1(W^b) + (1-p) \circ W_2(W^b) & \text{if } y = v_W(W_2(W^b), W^b), \\ \left(\frac{y^{-\frac{1}{c}}}{(\beta+\gamma_1)^{1-\frac{1}{c}}} - \frac{\alpha-\gamma_1\delta W^b}{\beta+\gamma_1}\right)^+ < W_1(W^b) & \text{if } y > v_W(W_2(W^b), W^b) \\ & \text{and } \beta+\gamma_1 \neq 0, \\ 0 & \text{otherwise.} \end{cases} \tag{16}$$

Then the policy

$$W_T^m = g^m(\psi^m \pi_T, W_T^b, P_T), \tag{17}$$

where ψ^m is a Lagrangian multiplier solving

$$\mathrm{E}\Big[\pi_T g^m(\psi^m \pi_T, W_T^b, P_T)\Big] = W_0^m = \bar{\theta} \cdot S_0 - \theta_0^f B,$$

is optimal for the fund managers.

Given the existence of a continuum of mutual funds,[15] the aggregate terminal value of the funds' portfolios equals

$$\lambda \bar{g}^m(\psi^m \pi_T, W_T^b, P_T),$$

[15]Since the fund managers' indirect utility functions are non-convex, our assumption of a continuum of mutual funds is critical to ensure the convexity of the aggregate preferred sets and hence the existence of an equilibrium. Aumann (1966) was the first to prove that, with a continuum of agents, the existence of an equilibrium could be assured even without the usual assumption of convex preferences.

where

$$\overline{g}^m(\psi^m \pi_T, W_T^b, P_T) = \begin{cases} g^m(\psi^m \pi_T, W_T^b, P_T) & \text{if } y \neq v_W(W_2(W_T^b), W_T^b) \\ P_T\, W_1(W_T^b) + (1 - P_T)\, W_2(W_T^b) & \text{otherwise.} \end{cases}$$

As it will become clear in the next section, this in turn implies that the randomizing probabilities P are uniquely determined in equilibrium by market clearing.

3.3 Fund Investors

Fund investors select the allocation θ_0^f to bonds so as to maximize the expected utility of their terminal wealth, while taking the equilibrium net-of-fees rate of return on mutual funds R_T (defined in equation (4)) as given. That is, they solve

$$\max_{\theta_0 \in \mathbb{R}} \mathrm{E}\Big[u^f\Big(\theta_0 B + (W_0^f - \theta_0 B) R_T\Big)\Big]$$
$$\text{s.t. } \theta_0 B \leq W_0^f.$$

Since this is a strictly concave static maximization problem, the optimal choice θ_0^f satisfies the standard Kuhn-Tucker conditions

$$\begin{gathered} \mathrm{E}\Big[(\theta_0^f B + (W_0^f - \theta_0^f B) R_T)^{-c}(1 - R_T)\Big] - \psi^f = 0 \\ \psi^f(W_0^f - \theta_0^f B) = 0 \end{gathered} \tag{18}$$

for some Lagrangian multiplier $\psi^f \geq 0$.

4 Equilibrium: Characterization

In equilibrium the terminal stock prices equal the liquidation dividends, i.e., $S_T = D_T$. Multiplying the market-clearing condition (5) at time T by D_T and using equations (6) and (17), it then follows that the equilibrium state-price density π_T must solve

$$(1 - \lambda) g^a(\psi^a \pi_T) + \lambda \overline{g}^m(\psi^m \pi_T, W_T^b, P_T) + \lambda \theta_0^f B = \bar{\theta} \cdot D_T. \tag{19}$$

This shows that the equilibrium the state-price density π_T and the randomizing probabilities P_T must be deterministic functions of D_T and W_T^b. Letting

$$\Pi(D_T, W_T^b) = \psi^m \pi_T$$

and

$$\varphi = (\psi^a / \psi^m)^{-\frac{1}{c}},$$

substituting in (19), recalling the definitions of g^a and $\overline{g}^m$ and rearranging gives

$$\bar{\theta} \cdot D_T - \lambda \theta_0^f B - (1 - \lambda) \varphi \Pi(D_T, W_T^b)^{-\frac{1}{c}}$$

$$
= \begin{cases} \lambda\left(\frac{\Pi(D_T,W_T^b)^{-\frac{1}{c}}}{(\beta+\gamma_2)^{1-\frac{1}{c}}} - \frac{\alpha-\gamma_2\delta W_T^b}{\beta+\gamma_2}\right) & \text{if } \Pi(D_T,W_T^b) < v_W(W_2(W_T^b),W_T^b), \\ \lambda\left(P_T\, W_1(W_T^b) + (1-P_T)W_2(W_T^b)\right) & \text{if } \Pi(D_T,W_T^b) = v_W(W_2(W_T^b),W_T^b), \\ \lambda\left(\frac{\Pi(D_T,W_T^b)^{-\frac{1}{c}}}{(\beta+\gamma_1)^{1-\frac{1}{c}}} - \frac{\alpha-\gamma_1\delta W_T^b}{\beta+\gamma_1}\right)^+ & \text{if } \Pi(D_T,W_T^b) > v_W(W_2(W_T^b),W_T^b) \\ & \text{and } \beta+\gamma_1 \neq 0, \\ 0 & \text{otherwise.} \end{cases} \tag{20}
$$

Solving the above equation shows that the scaled state-price density $\Pi(D_T, W_T^b)$ can take one of four different functional forms (corresponding to the four different cases on the right-hand side of equation (20)),

$$
\Pi_1(D_T,W_T^b) = \left(\frac{(\beta+\gamma_2)(\bar\theta\cdot D_T - \lambda\theta_0^f B) + \lambda(\alpha-\gamma_2\delta W_T^b)}{\lambda(\beta+\gamma_2)^{\frac{1}{c}} + (1-\lambda)\varphi(\beta+\gamma_2)}\right)^{-c},
$$

$$
\Pi_2(D_T,W_T^b) = v_W(W_2(W_T^b),W_T^b) = F(W_2(W_T^b),W_T^b)^{-c}(\beta+\gamma_2),
$$

$$
\Pi_3(D_T,W_T^b) = \left(\frac{(\beta+\gamma_1)(\bar\theta\cdot D_T - \lambda\theta_0^f B) + \lambda(\alpha-\gamma_1\delta W_T^b)}{\lambda(\beta+\gamma_1)^{\frac{1}{c}} + (1-\lambda)\varphi(\beta+\gamma_1)}\right)^{-c},
$$

$$
\Pi_4(D_T,W_T^b) = \left(\frac{\bar\theta\cdot D_T - \lambda\theta_0^f B}{(1-\lambda)\varphi}\right)^{-c}.
$$

It then follows from the inequality conditions in equation (20) and the fact that equation (16) implies

$$
\begin{gathered}
g^m(\Pi_1(D_T,W_T^b),W_T^b,P_T) > W_2(W_T^b), \\
W_1(W_T^b) \le g^m(\Pi_2(D_T,W_T^b),W_T^b,P_T) \le W_2(W_T^b), \\
0 < g^m(\Pi_3(D_T,W_T^b),W_T^b,P_T) < W_1(W_T^b)
\end{gathered}
$$

that the scaled equilibrium state-price density is given by:

$$
\Pi(D_T,W_T^b) = \begin{cases} \Pi_1(D_T,W_T^b) & \text{if } \Pi_1(D_T,W_T^b) < \Pi_2(D_T,W_T^b), \\ \max[\Pi_2(D_T,W_T^b), & \text{if } \Pi_1(D_T,W_T^b) \ge \Pi_2(D_T,W_T^b), \\ \quad \Pi_3(D_T,W_T^b),\Pi_4(D_T,W_T^b)] & W_1(W^b) > 0 \text{ and } \beta+\gamma_1 \neq 0, \\ \max[\Pi_2(D_T,W_T^b),\Pi_4(D_T,W_T^b)] & \text{otherwise.} \end{cases} \tag{21}
$$

In addition, the equality in equation (20) corresponding to the case

$$
\Pi(D_T,W_T^b) = v_W(W_2(W_T^b),W_T^b) = \Pi_2(D_T,W_T^b)
$$

can be solved for the market-clearing randomizing probabilities P_T, yielding

$$
P_T = P(D_T,W_T^b) = \frac{\lambda W_2(W_T^b) - \bar\theta\cdot D_T + \lambda\theta_0^f B + (1-\lambda)\varphi\Pi_2(D_T,W_T^b)^{-\frac{1}{c}}}{\lambda(W_2(W_T^b) - W_1(W_T^b))}. \tag{22}
$$

Equation (21) provides an explicit expression for the scaled state-price density $\Pi(D_T, W_T^b)$ in terms of three yet-undetermined constants, δ, θ_0^f and φ. In turn, $\delta = W_0^m / W_0^b$ is a known function of θ_0^f and the initial stock prices S_0^1 and S_0^2.

It follows from the shape of the state-price density that the optimal consumption policies are piecewise linear functions of the liquidation dividends. In order to guarantee the existence of an equilibrium, we require that the coefficients of these functions be positive, i.e., that increases in aggregate consumption be shared among the agents. Assuming that $\bar{\theta}^b \geq 0$ (i.e., that the benchmark portfolio does not include short positions), this amounts to the following parameter restriction:

$$(\beta + \gamma_2)\bar{\theta}^j - \lambda\gamma_2\delta\bar{\theta}^{bj} > 0 \qquad \text{for } j = 1, 2, \tag{23}$$

where $\bar{\theta}^j$ (respectively, $\bar{\theta}^{bj}$) denotes the number of shares of stock j in the market (respectively, in the benchmark) portfolio.

The following theorem completes the characterization of equilibria by providing necessary and sufficient conditions for existence, together with an explicit procedure to determine the unknown constants θ_0^f, φ, S_0^1 and S_0^2.

Theorem 2 *Assume that the condition (23) is satisfied. Then an equilibrium exists if and only if there exist constants $(\theta_0^f, \psi^f, \varphi, S_0^1, S_0^2)$ with $\theta_0^f \leq (B + S_0^1 + S_0^2)/B$ and $\psi^f \geq 0$ solving the system of equations*

$$\begin{cases} \mathrm{E}\Big[(\theta_0^f B + (B + S_0^1 + S_0^2 - \theta_0^f B) R_T)^{-c}(1 - R_T)\Big] - \psi^f = 0 \\ \psi^f (B + S_0^1 + S_0^2 - \theta_0^f B) = 0 \\ \mathrm{E}\Big[\Pi(D_T, W_T^b)(\varphi \Pi(D_T, W_T^b)^{-\frac{1}{c}} - \bar{\theta} \cdot D_T)\Big] = 0 \\ \mathrm{E}\Big[\Pi(D_T, W_T^b)(D_T^1 - S_0^1)\Big] = 0 \\ \mathrm{E}\Big[\Pi(D_T, W_T^b)(D_T^2 - S_0^2)\Big] = 0 \end{cases} \tag{24}$$

where

$$\begin{aligned} R_T &= R(D_T, W_T^b) \\ &= \frac{g^m(\Pi(D_T, W_T^b), W_T^b, P(D_T, W_T^b)) - F(g_P^m(\Pi(D_T, W_T^b), W_T^b, P(D_T, W_T^b)), W_T^b)}{B + S_0^1 + S_0^2 - \theta_0^f B} \end{aligned}$$

is the equilibrium net return on mutual funds defined in (4) and g_P^m, Π and P are the functions defined in (16), (21) and (22), respectively.

Given a solution $(\theta_0^f, \psi^f, \varphi, S_0^1, S_0^2)$, the equilibrium state-price density is given by

$$\pi_t = \mathrm{E}_t[\Pi(D_T, W_T^b)] / \psi^m,$$

the equilibrium stock price processes are given by

$$S_t^j = \mathrm{E}_t[\pi_T D_T^j] / \pi_t \quad (j = 1, 2), \tag{25}$$

the optimal investment policy for the fund investors is given by θ_0^f *and the optimal wealth processes for the active investors and fund managers are given by*

$$W_t^a = \mathrm{E}_t[\pi_T g^a(\psi^a \pi_T)]/\pi_t \tag{26}$$

and

$$W_t^m = \mathrm{E}_t[\pi_T g^m(\Pi(D_T, W_T^b), W_T^b, P(D_T, W_T^b))]/\pi_t \tag{27}$$

respectively, where $\psi^a = \psi^m \varphi^{-c}$ *and* $\psi^m = \mathrm{E}[\Pi(D_T, W_T^b)]$.

The five equations in (24) can be easily identified, respectively, with the first-order conditions for the fund investors in equation (18), the budget constraint for the active investors in equation (7) and the two Euler equations that define the initial stock prices.

The next corollary provides an explicit solution for the equilibrium trading strategies in the case in which the performance fees are of the fulcrum type ($\gamma_2 = \gamma_1$) and there are no load fees.

Corollary 1 *If* $\gamma_1 = \gamma_2$ *and* $F(0, W^b) \leq 0$ *for all* $W^b \geq 0$*, then in equilibrium the fund managers' portfolio consists of a combination of a long buy-and-hold position in the market portfolio, a long buy-and-hold position in the benchmark portfolio and a short buy-and-hold position in the riskless asset:*

$$\theta_t^m = \frac{(\beta+\gamma_2)^{\frac{1}{c}}\bar{\theta} + \varphi(1-\lambda)\gamma_2\delta\bar{\theta}_t^b - \left(\lambda(\beta+\gamma_2)^{\frac{1}{c}}\theta_0^f + \varphi(1-\lambda)\frac{\alpha}{B}\right)\bar{\theta}^1}{\lambda(\beta+\gamma_2)^{\frac{1}{c}} + \varphi(1-\lambda)(\beta+\gamma_2)} \tag{28}$$

for all $t \in [0, T]$*, where* $\bar{\theta}^1 = (1, 0, 0)$*. Similarly, the active investors' portfolio consists of a long buy-and-hold position in the market portfolio, a short buy-and-hold position in the benchmark portfolio and a buy-and-hold position in the riskless asset (which can be either long or short):*

$$\theta_t^a = \varphi\frac{(\beta+\gamma_2)\bar{\theta} - \lambda\gamma_2\delta\bar{\theta}_t^b - \lambda\left((\beta+\gamma_2)\theta_0^f - \frac{\alpha}{B}\right)\bar{\theta}^1}{\lambda(\beta+\gamma_2)^{\frac{1}{c}} + \varphi(1-\lambda)(\beta+\gamma_2)}. \tag{29}$$

In addition, if $\gamma_1 \neq 0$ *and* D_T^1 *and* D_T^2 *are identically distributed conditional on the information at time* t*, then* $S_t^1 > S_t^2$ *(respectively,* $S_t^1 < S_t^2$*) if and only if the benchmark portfolio is certain to hold more (respectively, less) shares of stock 1 than of stock 2 at time* T*.*

The above corollary shows that, if performance fees are of the fulcrum type and there are no load fees (or, more generally, if $\alpha \leq \gamma_1\delta W_T^b$), then in equilibrium the fund managers hold more (respectively, fewer) shares of stock 1 than of stock 2 at time t if and only if they are benchmarked to a portfolio holding more (respectively, fewer) shares of stock 1 than of stock 2 at time t. This tilt in the fund portfolios toward the stock more heavily weighted in the benchmark portfolio results in the equilibrium price of this stock being higher, *ceteris paribus*, than the equilibrium price of the other stock. Moreover, if the benchmark portfolio is buy-and-hold, then the equilibrium trading strategies are also buy-and-hold. Thus, in our model, performance fees of the fulcrum type do not increase the fund portfolios' turnover.[16]

[16] Clearly, this buy-and-hold result is due to the fact that fund managers and active investors are assumed to have the same utility function.

It can be easily verified that for this case the parameter restrictions in equation (23) are equivalent to the requirement that the fixed shares holdings in equations (28) and (29) are strictly positive.

5 Optimality of Performance Contracts

While our objective in this paper is to understand the impact of commonly observed performance contracts on equilibrium returns, we briefly address in this section the rationale for performance contracts within our model.

Given that in our model investors and managers have utilities with linear risk tolerance and identical cautiousness, the principle of preference similarity of Ross (1973) would seem to imply that a linear fee should be optimal. Specifically, Ross showed that, under the stated assumption on preferences, a linear fee achieves first best. However, two distinctive features of our model are that fund investors have direct access to riskless investment opportunities and that they take the fee structure as given when formulating their investment decisions: this is in contrast to standard models of delegated portfolio management, in which the principal is assumed to delegate the management of his entire portfolio and to be able to dictate the fee structure (subject only to the managers' participation constraint).

To see why these features negate the optimality of a linear fee, recall from Ross (1973) or Cadenillas, Cvitanić and Zapatero (2007) that the first-order condition for a linear fee $F(W) = \alpha + \beta W$ to achieve first best is

$$u'(\theta_0^f B + W_T^m - F(W_T^m)) = \omega^{-c}\, u'(F(W_T^m), \tag{30}$$

where ω is a positive constant that depends on the managers' reservation utility: equation (30) states that the fee should make the marginal utility for the principal proportional to that of the manager, ensuring Pareto-optimal risk sharing. This condition is satisfied if and only if $\alpha = \theta_0^f B/(1+\omega) \geq 0$ and $\beta = 1/(1+\omega) > 0$. Thus, in order for a linear fee to achieve first best, the load component α must depend on the portfolio allocation chosen by fund investors (in particular, $\alpha = 0$ if the fund investors delegate their entire portfolios). If fund investors were able to choose the managers' compensation contract while committing to delegating the amount $W_0^f - \theta_0^f B$, this fee would indeed be optimal. However, since in our model the fund investors choose their portfolio allocation taking the fee as given, equation (30) being satisfied becomes equivalent to the investors choosing $\theta_0^f B = \alpha/\beta$ ex post when confronted with a fee $F(W) = \alpha + \beta W$ with $\alpha \geq 0$ and $\beta > 0$. It is immediate to see that this would not be the case when $\alpha = 0$, as having $\theta_0^f = 0$ (that is, delegating the entire portfolio) is clearly suboptimal if $\beta > 0$. More generally, whenever there is some portfolio delegation (i.e., $\theta_0^f B < W_0^f$), θ_0^f satisfies the first-order condition in equation (18), which can in this case be written as

$$\mathrm{E}\Big[u'(\theta_0^f B + W_T^m - F(W_T^m))(W_T^m - F(W_T^m) - W_0^m)\Big] = 0. \tag{31}$$

Equations (30) and (31) imply that the fund investors choosing ex-post the level of delegation that ensures that a linear fee achieves first best is equivalent to having

$$\mathrm{E}\Big[u'(F(W_T^m))(W_T^m - F(W_T^m) - W_0^m)\Big] = 0. \tag{32}$$

However, since $v(W, W^b) = u(F(W))$ and $A(W^b) = [0, \infty)$ with linear fees, it follows from equation (14) that $\beta u'(F(W_T^m)) = \psi^m \pi_T$. Substituting in equation (32) and using the fact that $\pi_t W_t^m$ and π_t are martingales gives:

$$\mathrm{E}\Big[u'(F(W_T^m))(W_T^m - F(W_T^m) - W_0^m)\Big] = -\frac{\psi^m}{\beta}(\alpha + \beta W_0^m) < 0.$$

Hence, given a linear fee, individual fund investors would not choose ex post the level of delegation that ensures that a linear fee achieves first best: since individual investors do not internalize the fact that fees will have to increase if they "underinvest" in mutual funds in order to continue to guarantee a given certainty equivalent to fund managers, they will always invest less than the amount needed to achieve efficient risk sharing.[17]

What types of compensation contracts could dominate linear fees? Since investors do not pay management fees on bonds they hold directly, but do pay fees on bonds they hold indirectly through mutual funds, it is in their best interest to select compensation contracts that induce portfolio managers to hold portfolios with high equity exposures. Contracts with convex payoffs provide a possible way to incentivise managers to increase the overall equity exposure, although, as noted by Ross (2004), this incentive does not necessarily hold over the entire state space.[18]

Figure 2 plots the Pareto frontiers for purely proportional contracts and for contracts including asymmetric or load fees.[19] For low managers' reservation utilities neither asymmetric performance fees nor load fees generate significant Pareto improvements over purely proportional contracts: this is consistent with the above discussion, as when fees are low fund investors optimally choose to delegate almost their entire portfolio and the "underinvestment" relative to the amount ensuring efficient risk sharing with proportional fees is minimal. However, for high managers' reservation utilities both load fees and asymmetric performance fees Pareto-dominate purely proportional contracts, with performance fees in turn dominating load fees.[20,21] This is again what could be expected from the above discussion: when fund investors hold a significant amount of bonds in their private accounts, linear contracts with positive load fees dominate purely proportional contracts. On the other hand, any positive load fee α is, given the ex post allocation chosen by fund investors, always higher that what would be needed to ensure optimal risk sharing, resulting

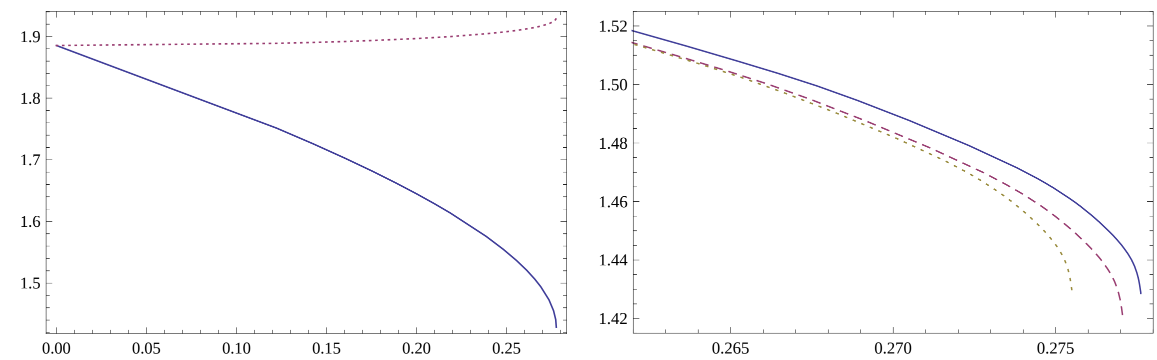
[17]For simplicity, in the sequel we will refer to the utility (or certainty equivalent) provided to managers by their compensation as the managers' *reservation* utility (or certainty equivalent), although as noted this utility is not necessarily that required by a binding participation constraint, but possibly the equilibrium outcome of some more complicated bargaining game.

[18]Simply restricting fund managers to trade only equity is suboptimal if the allocation to mutual funds cannot be continuously rebalanced, as it leads to significant variations over time in fund investors' effective portfolio mix of bonds and equity.

[19]The managers' certainty equivalent (CE^m) is defined by $u^m(CE^m) = \mathrm{E}\Big[u^m\Big(F(W_T^m, W_T^b)\Big)\Big]$. Similarly, the fund investors' certainty equivalent (CE^f) is defined by $u^f(CE^f) = \mathrm{E}\Big[u^f\Big(\theta_0^f B + (W_0^f - \theta_0^f B)R_T\Big)\Big]$, where R_T is the net return on mutual funds defined in equation (4).

[20]When the managers' reservation certainty equivalent is 0.15 (respectively, 0.275), the percentage increase in the fund investors' certainty equivalent when asymmetric performance contracts are used instead of purely proportional contracts is two basis points (respectively, 140 basis points).

[21]Some empirical support for the existence of welfare gains associated with the use of performance fees is provided by Coles, Suay and Woodbury (2000), who find that closed-end funds that use performance fees tend to command a premium that is about 8% larger than similar funds that do not use these fees.



(a) (b)

Figure 2: The graph plots the Pareto frontier at time $t = 0$ with asymmetric performance fees (solid line). The managers' certainty equivalent is on the horizontal axis and the fund investors' certainty equivalent is on the vertical axis. Panel (a) also plots the active investors' certainty equivalent (dotted line). Panel (b) also plots the Pareto frontiers with load fees (dashed line) and proportional fees (dotted line): the plot range is reduced so that the difference between the different frontiers is clearly visible.

in significant utility losses for fund investors in states in which their terminal wealth is low. This creates the potential for performance fees to dominate load fees, although, as it will become clear in Section 6.3, performance fees have a negative impact in terms of portfolio diversification. It is worthwhile to point out that in Figure 2 the benchmark portfolio is exogenously fixed to coincide with the first stock: thus, selecting the benchmark optimally would lead to an even stronger dominance of performance fees over load fees. We will return to this point in Section 6.4.

Adding asymmetric performance fees to a purely proportional contract (that is, letting γ_2 increase from zero while keeping β fixed) intially increases the welfares of both fund investors (due to the higher allocation to equity by mutual funds) and fund managers (due to higher overall fees). After a given level, however, the welfare of fund managers becomes decreasing in γ_2, due to the increasing risk of their compensation and the decreasing extent of delegation by fund investors. The contracts along the efficient frontier are characterized by levels of the performance sensitivity parameter γ_2 that are beyond the point at which the utility of fund managers starts being decreasing in γ_2. Thus, moving along the efficient frontier with asymmetric performance contracts, increases in the fund managers' certainty equivalent are associated with simultaneous increases in both the proportional fee component and the in asymmetric performance fee component: the optimal proportional fee parameter β increases from 0% to 37.26%, while the optimal performance sensitivity parameter γ_2 increases from 0% to 27.07%. In particular, the model implies a positive correlation between managers' overall compensation and contract performance sensitivity, which appears to be consistent with anecdotal empirical evidence.

As shown in panel (b) of Figure 2, within our model, fulcrum fees never generate a Pareto improvement over purely proportional fees.[22] Adding a fulcrum fee to a proportional

[22]Das and Sundaram (2002) find that asymmetric performance fees also dominate fulcrum fees in a signaling model with fund managers of different skills, although this dominance arises in their model for a

contract increases the welfare of fund investors (since fulcrum fees also induce an increase in the equity allocation chosen by fund managers) but strongly decreases the welfare of fund managers, due to the utility losses in states in which the excess return of the managed portfolio over the benchmark is negative.

6 Analysis of Equilibrium

This section contains a numerical analysis of the asset pricing implications of delegated portfolio management. The dividend processes in equation (1) are taken to be geometric Brownian motions with $D_0^1 = D_0^2 = 1$. Through most of the section, we assume that $\sigma_D^1(D,t) = \sigma_D^2(D,t) = 0.2D$, implying that the two liquidation dividends are unconditionally identically distributed. Since the only ex ante difference between the two stocks (portfolios) arises from their weighting in the benchmark portfolio, this assumption enables us to clearly isolate the price effects arising from benchmarking when comparing the equilibrium price processes of the two stocks. We assume that stock (portfolio) 1 is the benchmark portfolio, i.e., that $\bar{\theta}^b = (0,1,0)$: we address in Subsection 6.4 the robustness to our results to alternative choices of the benchmark (including optimal choice). The correlation between the two Brownian motions is set to $\rho = 0.9$: clearly, a lower correlation would allow for larger pricing differences across the two stocks to emerge in equilibrium as a result of benchmarking.

We set the fraction λ of fund investors in the economy to 0.5 and the investors' relative risk aversion coefficient c to 10. Based on evidence reported by Del Guercio and Tkac (1998) that 42.6% of the pension fund sponsors who use performance-based fees to compensate managers rely on a 3-5 years investment horizon to measure performance and that the median holding period among the mutual fund investors who totally redeem their shares is 5 years, we set $T = 5$.[23] Finally, the aggregate supply of the bond is set to $B = 0.72$: this value implies an equilibrium stocks-to-bonds ratio at time $t = 0$ of about 1.

Our goal is to understand the asset pricing implications of commonly observed performance contracts. We consider two performance fee structures: fulcrum fees ($\gamma_1 = \gamma_2 > 0$) and asymmetric performance fees ($\gamma_1 = 0$, $\gamma_2 > 0$). In both cases the performance compensation is added on top of a proportional fee ($\beta > 0$), as is typically done in practice.

6.1 Benchmark Economy and Proportional Fees

Before moving to economies in which fund managers receive performance fees, it is useful to review equilibrium prices in the version of our economy in which all agents have direct costless access to the equity market (i.e., $\lambda = 0$ or equivalently $\alpha = \beta = \gamma_1 = \gamma_2 = 0$) and in the version in which purely proportional management contracts are used (i.e., $\alpha = \gamma_1 = \gamma_2 = 0$). Figure 3 plots key equilibrium quantities at the midpoint of our time horizon ($t = T/2 = 2.5$) as a function of the second stock's dividend share, $D_t^2/(D_t^1 + D_t^2)$, for a value

completely different reason: the ability of fund managers to more easily signal their skill, and thus to extract a higher surplus, in the presence of fulcrum fees. Ou-Yang (2003) provides a model in which investors are assumed to delegate the management of their entire portfolio and fulcrum fees are optimal.

[23] The section of the working paper comparing investment horizons in the pension fund and mutual fund industry does not appear in the published article (Del Guercio and Tkac (2002)).

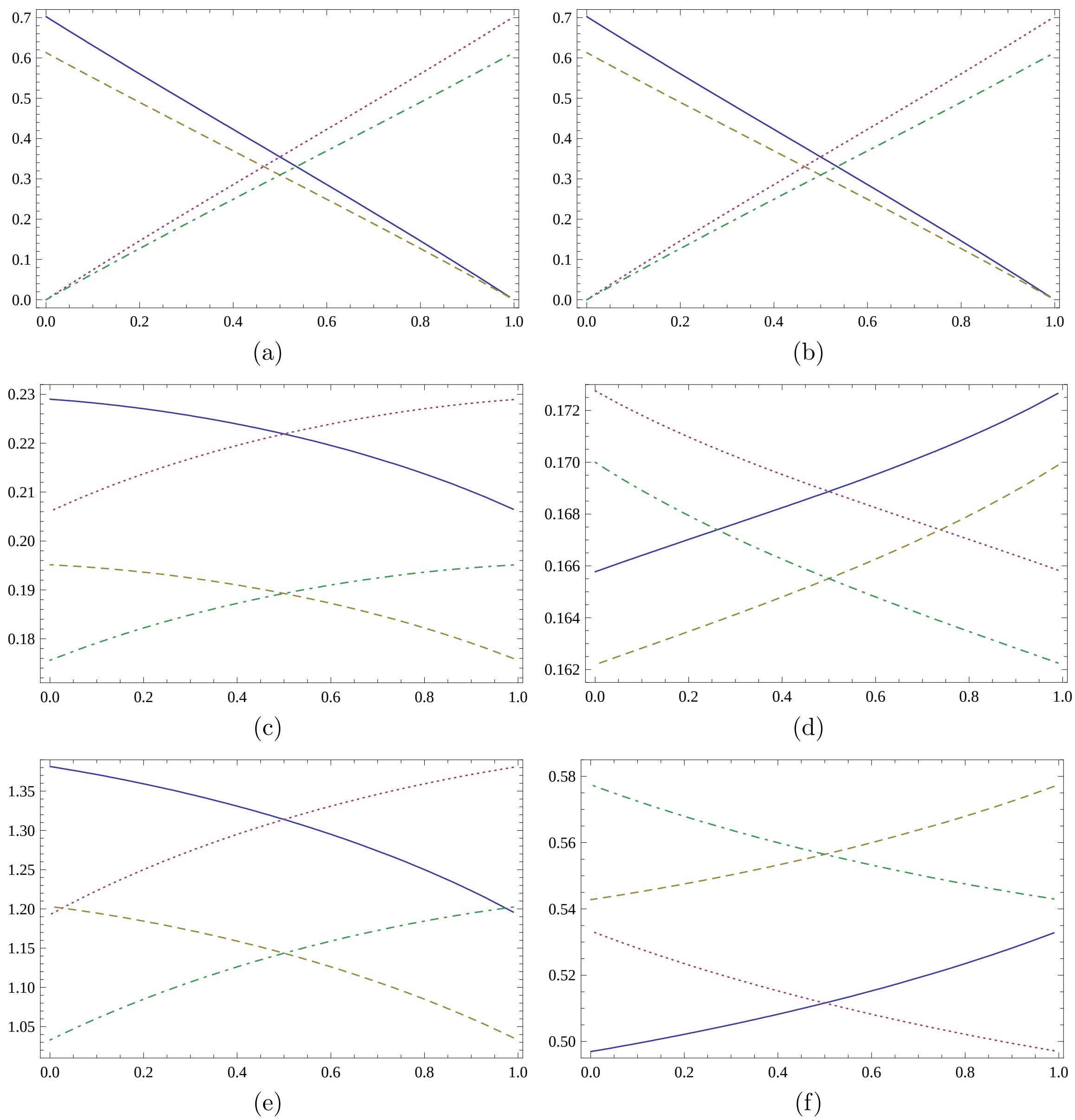


Figure 3: The graph plots key equilibrium quantities at time $t = T/2$ with proportional fees as a function of the second stock's dividend share. The proportional fee parameter β is set at 19.02%. For comparison, the corresponding values in the benchmark economy ($\beta = 0$) are also plotted. The quantities plotted are: the funds' portfolio weights (panel (a)), the active investors' portfolio weights (panel (b)), the stock instantaneous expected returns (panel (c)), the stock volatilities (panel (d)), the stock Sharpe ratios (panel (e)) and the stock price/dividend ratios (panel (f)). The solid (respectively, dotted) line refers to first stock (respectively, the second stock) with proportional fees, while the dashed (respectively, dot-dashed) line refers to the first stock (respectively, the second stock) in the benchmark economy.

of the proportional fee component β equal to 19.02%.[24] Variations in the dividend share are obtained by simultaneously varying the two state variables D_t^1 and D_t^2 by equal amounts in opposite directions so as to keep the sum $D_t^1 + D_t^2$ constant at its expected level. For comparison, the figure also plots the corresponding equilibrium values with costless access to the equity market, an economy we will refer to in the sequel as the *benchmark economy*.[25]

Starting from the benchmark economy with costless portfolio delegation, since fund managers and active investors have identical preferences and wealths in this case (as fund investors optimally delegate the management of their entire endowment), they must each hold a constant number of shares in equilibrium. Hence, when a stock's dividend share increases, resulting in an increase in the price of that stock relative to the price of the other stock, investors must be induced to allocate a higher fraction of their portfolios to that stock in order for the market to clear, as shown in panels (a) and (b) of Figure 3. This equilibrium incentive takes the form of a higher instantaneous expected return (which compensates investors for the higher correlation between the stock's dividend and aggregate consumption) and a higher Sharpe ratio (panels (c) and (e)). Stock volatility also falls in response to an increased dividend share, although this effect is small (panel (d)). Finally, reflecting the increasing expected returns, price/dividend ratios are monotonically decreasing functions of the dividend shares (panel (f)).

Moving from the benchmark economy to an economy with costly portfolio delegation and proportional fees, the allocation to mutual funds by fund investors decreases to less that 100% and the allocation to bonds becomes strictly positive. As a result, a lower fraction of aggregate wealth is available for investment in the equity market and, in order to restore market clearing, fund managers and active investors must be induced to increase their equity holdings relative to the benchmark economy, as shown in panels (a) and (b) of Figure 3. To ensure this, expected returns and Sharpe ratios increase relative to the benchmark economy, while price/dividend ratios decrease (Figure 3, panels (c), (e) and (f)). Stock volatilities slightly rise relative to the benchmark economy reflecting a small increase in the variance of the price/dividend ratios (panel (d)). Clearly, in the absence of benchmarking, delegated portfolio management has a symmetric price impact on the two stocks.

Although not shown, the deviations between stock expected returns, volatilities, Sharpe ratios and price/dividend ratios in the presence of proportional fees and the corresponding values in the the benchmark economy are monotonic in the proportional fee parameter β. The qualitative pricing effects are identical in the presence of load fees.

6.2 Fulcrum Fees

As discussed in Section 5, within our model fulcrum fees are suboptimal. However, for completeness and given that the 1970 Amendment of the Investment Advisers Act of 1940 restricts mutual fund performance fees to be of the fulcrum type, we examine next their impact on equilibrium. Figure 4 plots the key equilibrium quantities at time $t = T/2$

[24] The value $\beta = 19.02\%$ is chosen so as to match that used in Figures 6 and 8. A proportional management fee of 19.02% over 5 years is equivalent to an annual fee of 4.13%.

[25] Our benchmark economy is similar to the two-trees Lucas economy studied in Cochrane, Longstaff and Santa-Clara (2007), the differences being that Cochrane, Longstaff and Santa-Clara assume logarithmic utility, infinite horizon, intertemporal consumption and bonds in zero net supply, while we assume general CRRA utilities, finite horizon, consumption at the terminal date only and bonds in positive net supply.

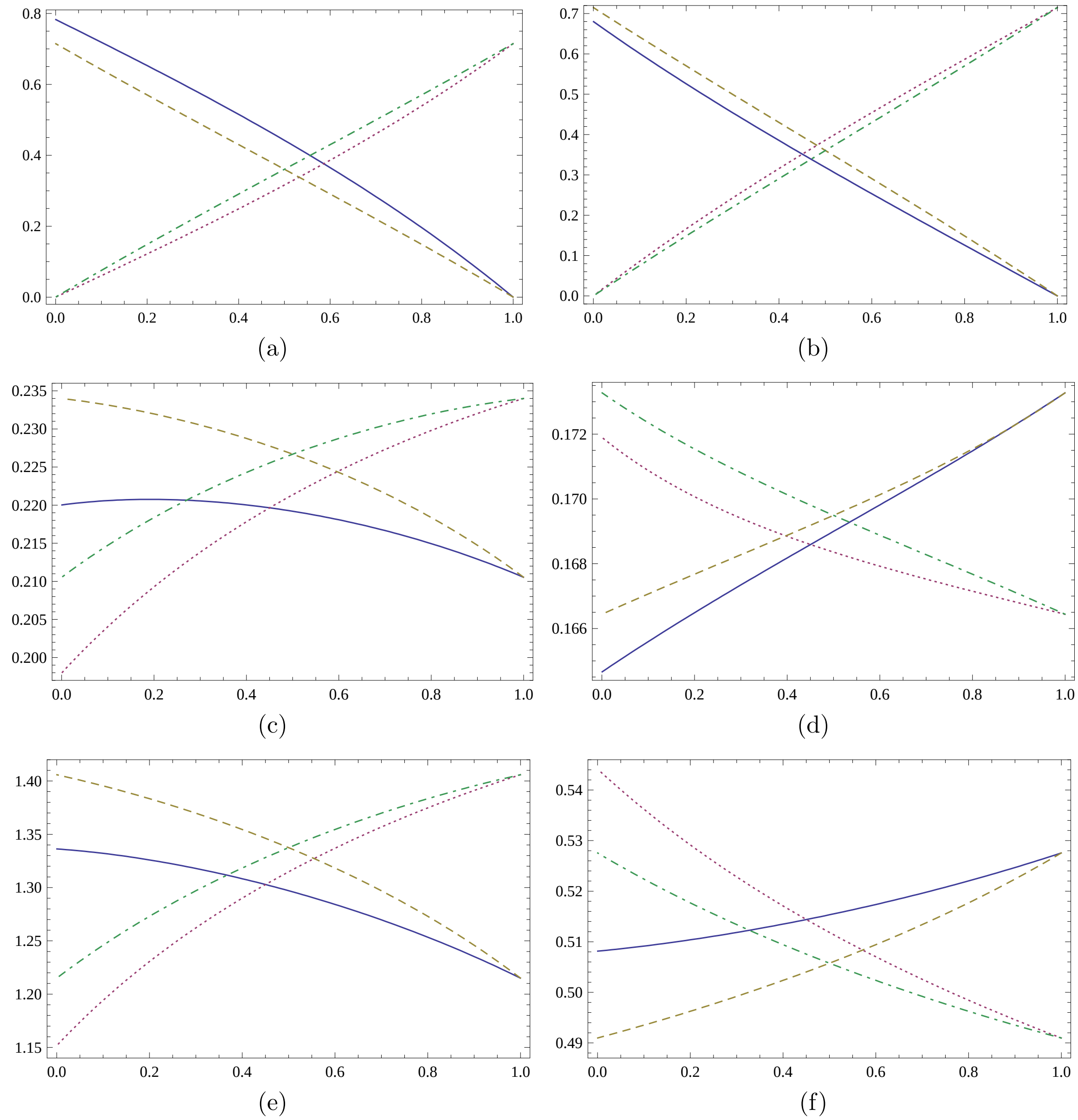


Figure 4: The graph plots key equilibrium quantities at time $t = T/2$ with fulcrum performance fees as a function of the second stock's dividend share. The contract parameters ($\beta = 21.00\%$, $\gamma_1 = \gamma_2 = 3.91\%$) are chosen so that the resulting contract provides the fund managers a certainty equivalent of 0.22. For comparison, the corresponding values with proportional fees ($\gamma_1 = \gamma 2 = 0$) are also plotted. The quantities plotted are: the fund managers' portfolio weights (panel (a)), the active investors' portfolio weights (panel (b)), the stock instantaneous expected returns (panel (c)), the stock volatilities (panel (d)), the stock Sharpe ratios (panel (e)) and the stock price/dividend ratios (panel (f)). The solid (respectively, dotted) line refers to first stock (respectively, the second stock) with asymmetric fees, while the dashed (respectively, dot-dashed) line refers to the first stock (respectively, the second stock) with proportional fees.

as a function of the dividend share of the second (non-benchmark) stock, assuming $\gamma_1 = \gamma_2 = 3.91\%$ and $\beta = 21.00\%$.[26] For comparison, the Figure 4 also plots the corresponding equilibrium values in an economy with purely proportional fees.[27]

As shown in Corollary 1, the presence of a penalty for underperforming the benchmark portfolio implicit in fulcrum fees leads fund managers to tilt their portfolios toward the benchmark stock, while holding a constant number of shares of each stock.[28] In addition, the overall equity allocation by fund managers is higher than with purely proportional fees. Thus, in order to ensure market clearing, the active investors must be induced to hold portfolios that have lower equity allocations and are tilted toward the non-benchmark stock over the entire state space. Given constant share holdings, the proportional tilt toward the non-benchmark stock must be larger when the price of the benchmark stock is large relative to that of the non-benchmark stock, i.e., when the second stock's dividend share is low, as shown in panel (b) of Figure 4. As a result, the equilibrium price distortions relative to an economy with purely proportional fees are larger when the second stock's dividend share is low, having otherwise the expected signs. Specifically, as shown in panels (c) and (e) of Figure 4, expected returns and Sharpe ratios are lower than in an economy with purely proportional fees (in order to induce a lower overall equity allocation by active investors), with the difference being more pronounced for the benchmark stock (in order to induce active investors to tilt their portfolios toward the other stock). Correspondingly, price/dividend ratios are higher than with purely proportional fees, with the difference being once again more pronounced for the benchmark stock (Figure 4, panel (f)). As a result of the lower variability in price/dividend ratios, stock volatilities slightly decline, with the change being once again more pronounced for the benchmark stock (Figure 4, panel (d)).

Harris (1989) has documented a small positive average difference in daily return standard deviations between stocks in the S&P 500 index (the most commonly used benchmark) and a matched set of stocks not in the S&P 500 over the period 1983–1987. Since this difference was insignificant over the pre-1983 period, Harris attributed it to the growth in index derivatives trading, noticing that the contemporaneous growth in index funds was unlikely to be a possible alternative explanation, as "it seems unlikely that volatility should increase when stock is placed under passive management". Yet, our model delivers this implication concerning volatilities: integrating the volatilities plotted in panels (d) of Figure 4 over the distribution of the dividend share, gives an unconditional expected volatility at time $t = T/2$ of 16.90% for the benchmark stock and 16.83% for the other stock.[29]

To further relate the implications of our model to the available empirical evidence con-

[26]While in the case of asymmetric fees discussed in the next subsection the fee parameters are chosen so as to guarantee that the resulting contract is efficient and provides the managers a given certainty equivalent, the suboptimality of fulcrum fees in our setting implies that there is no natural way to endogenously determine the contract specification. Therefore, we fix the performance sensitivity parameter γ_2 to the same level that is efficient in the case of asymmetric fees (as described in the next subsection) and adjust the proportional component β so as to provide fund managers the same equilibrium certainty equivalent. The qualitative results are insensitive to the specific choice of the fee parameters.

[27]These values are slightly different from those plotted in Figure 3 since the proportional fee coefficient β is slightly different (21.00% versus 19.02%).

[28]With a sufficiently large fulcrum fee component, the fund essentially behaves as an index fund, with a 100% allocation to the benchmark stock.

[29]Increasing the performance sensitivity parameter $\gamma_1 = \gamma_2$ (which, as already noted, would make the funds in our model behave more like index funds) would increase this volatility differential.

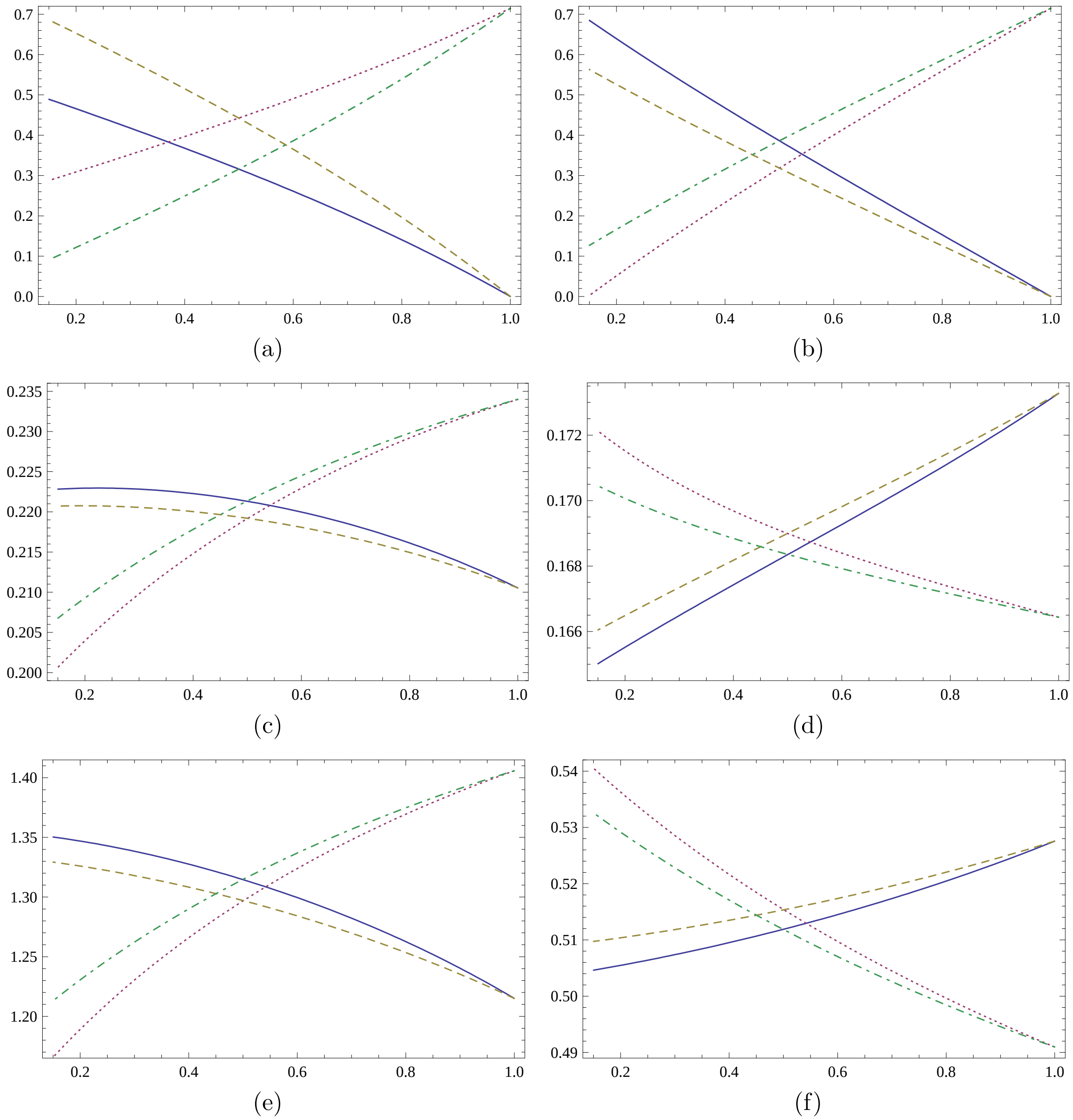


Figure 5: The graph plots key equilibrium quantities at time $t = T/2$ with fulcrum performance fees as a function of the second stock's dividend share, immediately following an unanticipated change of the benchmark from stock 1 to stock 2. The contract parameters ($\beta = 21.00\%, \gamma_1 = \gamma_2 = 3.91\%$) are chosen so that the resulting contract provides the fund managers a certainty equivalent of 0.22. For comparison, the corresponding values prior to the benchmark recomposition are also plotted. The quantities plotted are: the fund managers' portfolio weights (panel (a)), the active investors' portfolio weights (panel (b)), the stock instantaneous expected returns (panel (c)), the stock volatilities (panel (d)), the stock Sharpe ratios (panel (e)) and the stock price/dividend ratios (panel (f)). The solid (respectively, dotted) line refers to first stock (respectively, the second stock) after the benchmark recomposition, while the dashed (respectively, dot-dashed) line refers to the first stock (respectively, the second stock) before the benchmark recomposition.

cerning the equilibrium pricing effects of benchmarking, we examine next how equilibrium quantities change in our model in response to changes in the composition of the benchmark portfolio. In particular, we consider an unanticipated change in the composition of the benchmark at time $t = T/2$ from 100% stock 1 to 100% stock 2. Figure 5 plots the key equilibrium quantities immediately prior to and immediately following the announcement of the benchmark recomposition.

As shown in panel (a), the weight in the funds' portfolio of the stock that is added to (respectively, deleted from) the benchmark increases (decreases). This is consistent with the evidence reported in Pruitt and Wei (1989) that changes in the portfolios of institutional investors are positively correlated with changes in the composition of the S&P 500 index. As a result, the price of the stock that is added to the benchmark portfolio increases following the announcement, while its Sharpe ratio decreases (panels (e) and (f)). The changes in the price and Sharpe ratio of the stock that is deleted from the benchmark have the opposite sign, although the effects are asymmetric, with the expected absolute percentage changes in both prices and Sharpe ratios being larger for the stock that is added to the benchmark than for the stock that is dropped. Shleifer (1986), Harris and Gurel (1986), Beneish and Whaley (1996), Lynch and Mendenhall (1997) and Wurgler and Zhuravskaya (2002), among others, have documented a positive permanent price effect of about 3%–5% associated with the inclusion of a stock in the S&P 500 index. Chen, Noronha and Singal (2004) have reported that the price effect is asymmetric for additions and deletions, with the permanent price impact of deletions being smaller in absolute value.[30]

6.3 Asymmetric Performance Fees

Moving now to equilibria in the presence of asymmetric performance fees, Figure 6 plots key equilibrium quantities at time $t = T/2$ as a function of the second stock's dividend share. For the analysis in Figure 6, we select the contract parameters ($\beta = 19.02\%$ and $\gamma_2 = 3.91\%$) to correspond to those of the contract on the Pareto frontier in Figure 2 giving the managers a certainty equivalent of 0.22. For comparison, Figure 6 also plots the corresponding equilibrium values in an economy with purely proportional fees. It is is useful to keep in mind for the following discussion that an increase in the dividend share of the second (non-benchmark) stock is associated with an increase in the difference between the price of the non-benchmark stock and the price of the benchmark stock: thus, the funds' excess return over the benchmark portfolio, $(W_t^m - \delta S_t^1)/W_0^m$, is a monotonically increasing function of the second stock's dividend share, with the excess return being zero in Figure 6 at a dividend share of about 58%.

While fulcrum fees unambiguously induce fund managers to tilt their portfolios toward the benchmark stock, asymmetric performance fees can induce risk-averse fund managers either to select portfolios having high correlation with the benchmark in an attempt to hedge their compensation, or to select portfolios having low correlation with the benchmark in an attempt to maximize the variance of the excess return of the managed portfolio over the

[30] Chen, Noronha and Singal interpret the asymmetry of the price effect as evidence against the hypothesis that the effect is due to downward-sloping demand curves and in favor of the alternative hypothesis that the effect is due to increased investors' awareness. Our analysis implies that portfolio delegation and benchmarking is an alternative possible explanation for the asymmetry of the effect.

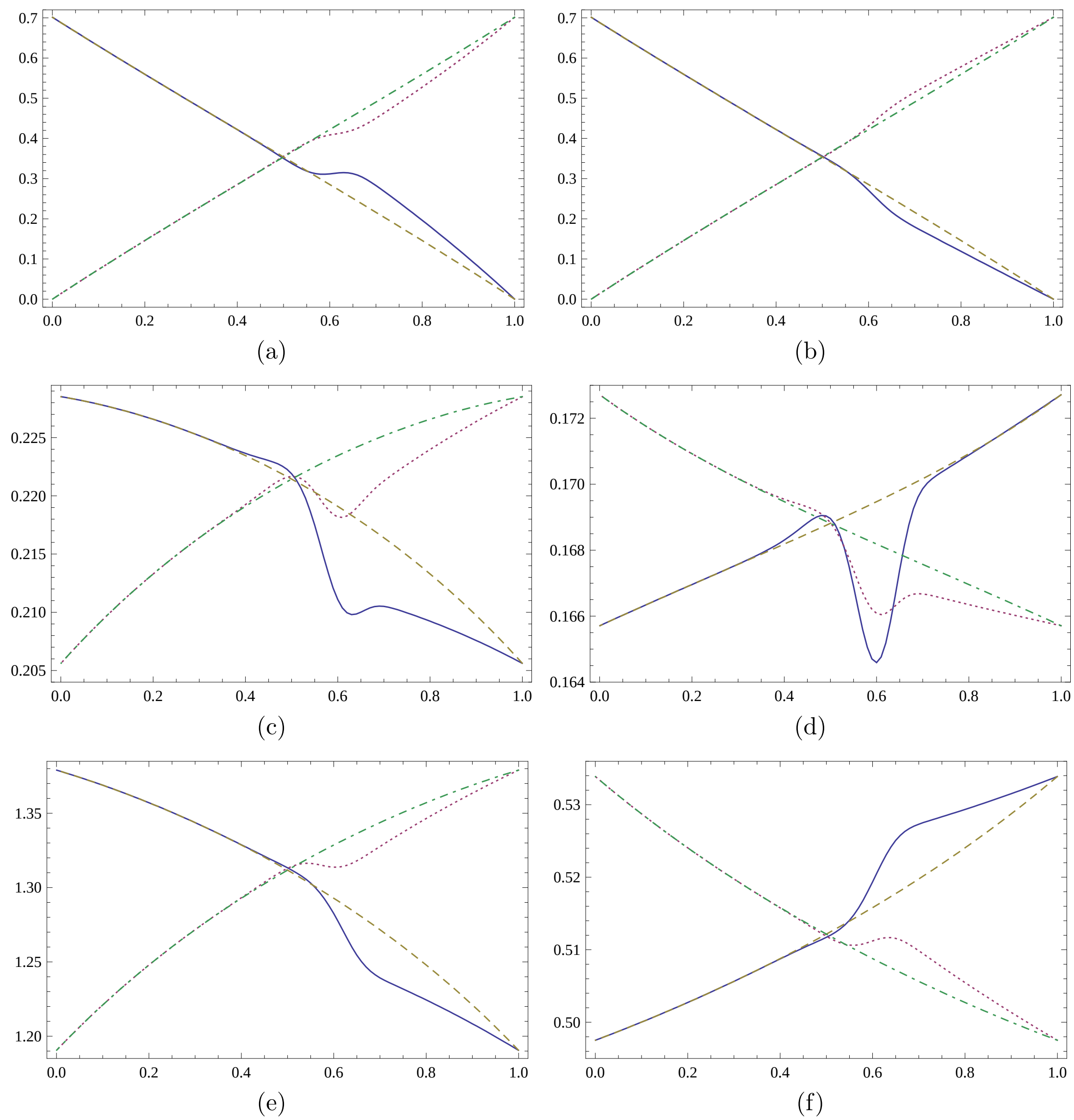
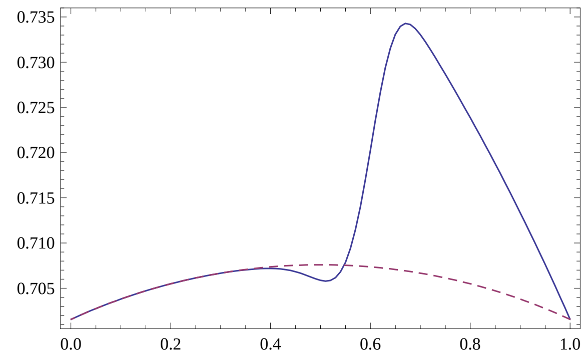
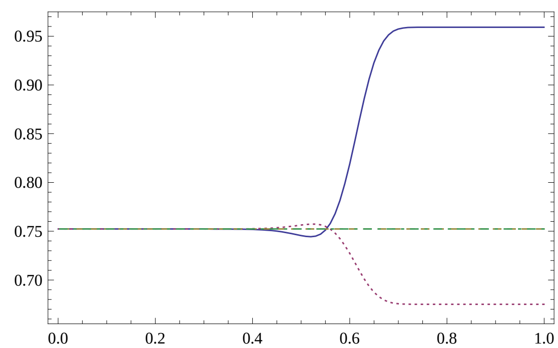


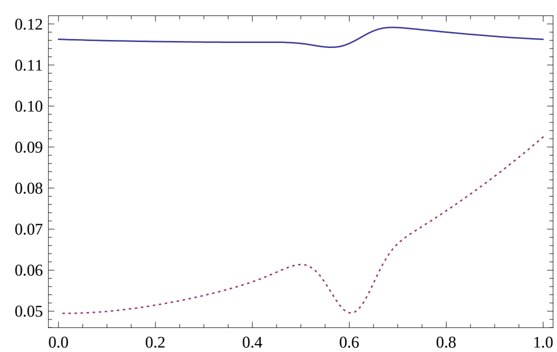
Figure 6: The graph plots key equilibrium quantities at time $t = T/2$ with asymmetric performance fees as a function of the second stock's dividend share. The contract parameters ($\beta = 19.02\%$, $\gamma_2 = 3.91\%$) are chosen so that the resulting contract is efficient and provides the fund managers a certainty equivalent of 0.22. For comparison, the corresponding values with proportional fees ($\gamma_2 = 0$) are also plotted. The quantities plotted are: the funds' portfolio weights (panel (a)), the active investors' portfolio weights (panel (b)), the stock instantaneous expected returns (panel (c)), the stock volatilities (panel (d)), the stock Sharpe ratios (panel (e)) and the stock price/dividend ratios (panel (f)). The solid (respectively, dotted) line refers to first stock (respectively, the second stock) with asymmetric fees, while the dashed (respectively, dot-dashed) line refers to the first stock (respectively, the second stock) with proportional fees.



(a)



(b)



(c)

Figure 7: The graph plots the funds' overall equity portfolio weight (panel (a), solid line), the funds' shares holdings (panel (b), solid and dotted lines) and the funds' return and tracking error volatilities (panel (c), solid and dotted lines) with asymmetric performance fees as a function of the second stock's dividend share. The contract parameters ($\beta = 19.02\%$, $\gamma_2 = 3.91\%$) are chosen so that the resulting contract is efficient and provides the fund managers a certainty equivalent of 0.22. For comparison, in panels (a) and (b) the corresponding values with proportional fees ($\gamma_2 = 0$) are also plotted.

benchmark (and hence the expected value of their performance fees, which are a convex function of this excess return). These shifting risk incentives are evident in panel (a) of Figure 6, which plots the funds' portfolio weights, or, even more clearly, in panel (a) of Figure 7, which plots the funds' share holdings. As shown in the figures, the first incentive dominates (inducing fund managers to tilt the fund portfolios toward the benchmark stock) when the fund's excess return is positive or moderately negative (in Figures 6 and 7, when the second stock's dividend share is larger than 56%, corresponding to an excess return above -6.2%), while the second incentive dominates (inducing fund managers to tilt the fund portfolios toward the non-benchmark stock) when the fund's excess return is sufficiently negative (below -6.2%).[31] When the fund's excess return is strongly negative, so that the probability of earning positive performance fees is negligible, the fund managers essentially behave as if they were receiving a purely proportional fee and the difference between the holdings of the two stocks is close to zero. As shown in panel (b) of Figure 7, the overall equity allocation by mutual funds exceeds the overall allocation that would have been chosen with purely proportional fees when the excess return is positive, has a small dip below that level when the excess return is moderately negative, and converges to that level as the excess return become more and more negative.

As a result of the described portfolio strategy, the funds' return volatility and tracking error volatility are non-linear functions of the funds' excess return, as shown in panel (c) of Figure 7. In particular, while tracking error volatility is decreasing in the fund's excess return in the region where the funds' excess return is between -22% and +6%, tracking error volatility is increasing in excess return outside of this region. This non-linear behavior is consistent with the evidence on the risk-taking by mutual funds reported by Chevalier and Ellison (1997) and Basak, Pavlova and Shapiro (2007). It is also interesting to note that the fund's total return volatility is increasing in excess return in the region where the funds' excess return is between -60% and +32%, although the range is small (between 11.43% and 11.92%). This is consistent with the finding of a positive (although not statistically significant) relationship by Busse (2001) and Basak, Pavlova and Shapiro (2007).[32]

The behavior of expected returns and Sharpe ratios in panels (c) and (e) of Figure 6 reflects the behavior that could be expected to ensure market clearing given the fund managers' portfolio strategy described above. In particular, the non-benchmark stock's expected return and Sharpe ratio are above (respectively, below) those of the benchmark stock in the region in which the fund managers' portfolios are tilted toward the benchmark (respectively, the non-benchmark) stock. Moreover, both stock's expected returns and Sharpe ratios are below (respectively, above) the corresponding quantities in an economy with purely propor-

[31] Clearly, which incentive dominates at a given excess return critically depends on both the managers' risk aversion and the time horizon T. A less risk averse manager has stronger incentives to pick portfolios with low correlation with the benchmark, in an attempt to maximize the variance of excess return over the benchmark. A shorter time horizon has a similar effect.

[32] With fulcrum fees, the tracking error volatility is monotonically increasing in the excess return, while the return volatility is monotonically decreasing. Thus, a convex performance-based compensation is needed in order to generate the non-linearities described in the text. However, Brown, Harlow and Starks (1996), Chevalier and Ellison (1997) and Sirri and Tufano (1998) have documented that, even when mutual fund managers do not receive explicit incentive fees, an implicit nonlinear performance-based compensation still arises with periodic proportional fees as a result of the fact that the net investment flow into mutual funds varies in a convex fashion as a function of recent performance.

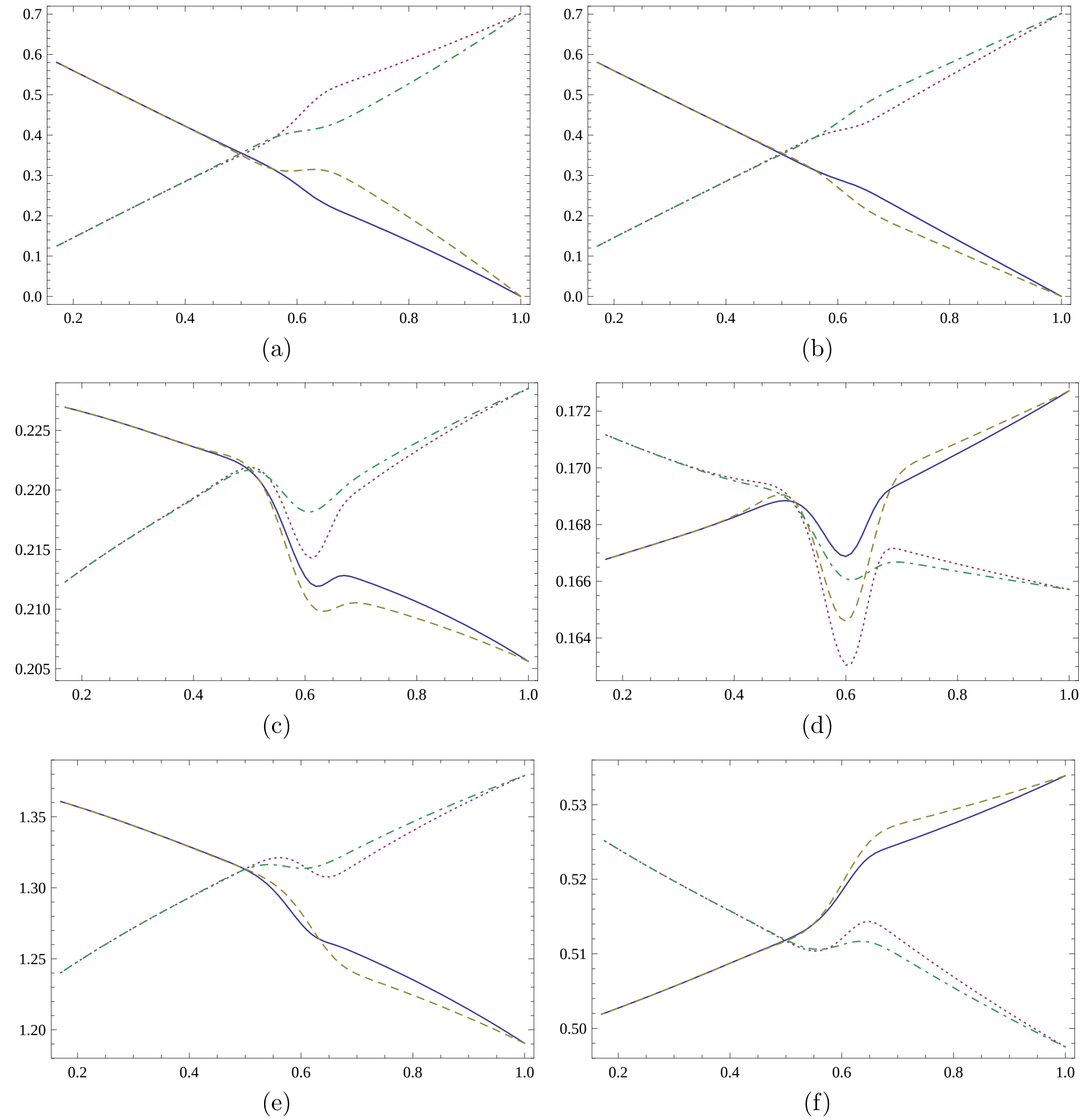


Figure 8: The graph plots key equilibrium quantities at time t = T/2 with asymmetric performance fees as a function of the second stock's dividend share, immediately following an unanticipated change in the benchmark from stock 1 to stock 2. The contract parameters ($\beta = 19.02\%, \gamma_2 = 3.91\%$) are chosen so that the resulting contract provides the fund managers a certainty equivalent of 0.22. For comparison, the corresponding values prior to the benchmark recomposition are also plotted. The quantities plotted are: the fund managers' portfolio weights (panel (a)), the active investors' portfolio weights (panel (b)), the stock instantaneous expected returns (panel (c)), the stock volatilities (panel (d)), the stock Sharpe ratios (panel (e)) and the stock price/dividend ratios (panel (f)). The solid (respectively, dotted) line refers to first stock (respectively, the second stock) after the benchmark recomposition, while the dashed (respectively, dot-dashed) line refers to the first stock (respectively, the second stock) before the benchmark recomposition.

tional fees in the region in which the overall equity allocation by fund managers is above (respectively, below) that with purely proportional fees. Price/dividend ratios also have their expected behavior given their inverse relation to expected returns (panel (f)).

Perhaps surprisingly, equilibrium stock volatilities in the presence of asymmetric performance fees tend to be generally smaller than those in an economy with purely proportional fees, in spite of the higher portfolio turnover.[33] The reduction in volatilities is particularly strong in the region in which the funds' performance is moderately positive (Figure 6, panel (d)): in fact, in this region the volatility of the first stock is not only lower than the corresponding volatility in an economy with purely proportional fees, but also lower than than in the benchmark economy. The source of this volatility reduction can be understood by noticing that, in the presence of asymmetric performance fees, the turnover by mutual funds is concentrated in the region in which their excess return is moderately positive and is associated with a portfolio reallocation away from the stock whose dividend share (and, hence, whose price) is rising (Figure 7, panel (a)): to ensure market clearing (that is, to induce active investors to hold even more of the stock whose dividend share is rising), the price of stock must rise less than what it would have otherwise, lowering its volatility. The small increase in volatilities over their values in an economy with purely proportional fees can be explained by a symmetric argument.

Turning now to the effect of changes in the composition of the benchmark portfolio, Figure 8 plots the key equilibrium quantities immediately before and after an unanticipated change at time $t = T/2$ in the benchmark from 100% stock 1 to 100% stock 2.

With asymmetric performance fees the effect of a benchmark change depends on funds' excess return, reflecting the fund managers' portfolio allocation decisions described above. While it is difficult to distinguish this in panel (f) of Figure 8, in the region where funds' excess return is negative (respectively, below -12.4%), the price of the stock added to (respectively, deleted from) the index slightly decreases (respectively, increases), while the opposite behavior prevails for larger excess returns. These price effects are however much smaller than with fulcrum fees, the absolute percentage price change not exceeding 55 basis points in the calibration in Figure 8. Sharpe ratios exhibit a similar pattern, with typically an opposite sign of the differential between pre- and post-recomposition values than the one for prices.

Figures 6–8 were plotted for a single contract on the Pareto frontier in Figure 2. By contrast, Figure 9 shows how the key equilibrium quantities at time 0 vary as we consider different contracts on the Pareto frontier. As noted in Section 5, along this frontier higher performance sensitivities γ_2 are associated with higher managers' certainty equivalents. Apart for the expected monotonic widening of the differentials across the two stocks associated with larger performance sensitivities, the relationship between equilibrium quantities at time 0 plotted in Figure 9 and the performance sensitivity depends critically on the endogenously-chosen allocation to mutual funds by fund investors in response to the varying cost of portfolio management (as measured by the fund managers' certainty equivalent). As the optimal performance sensitivity increases from 0% to 27%, reflecting an increase in the fund managers' equilibrium certainty equivalent from 0 to 0.28, the fund investors' allocation to mutual funds decreases from 100% to 37% of their endowment. As

[33] Equilibrium strategies are buy-and-hold in the presence of purely proportional fees, while, as shown in panel (a) of Figure 7, there is positive turnover in the presence of asymmetric performance fees.

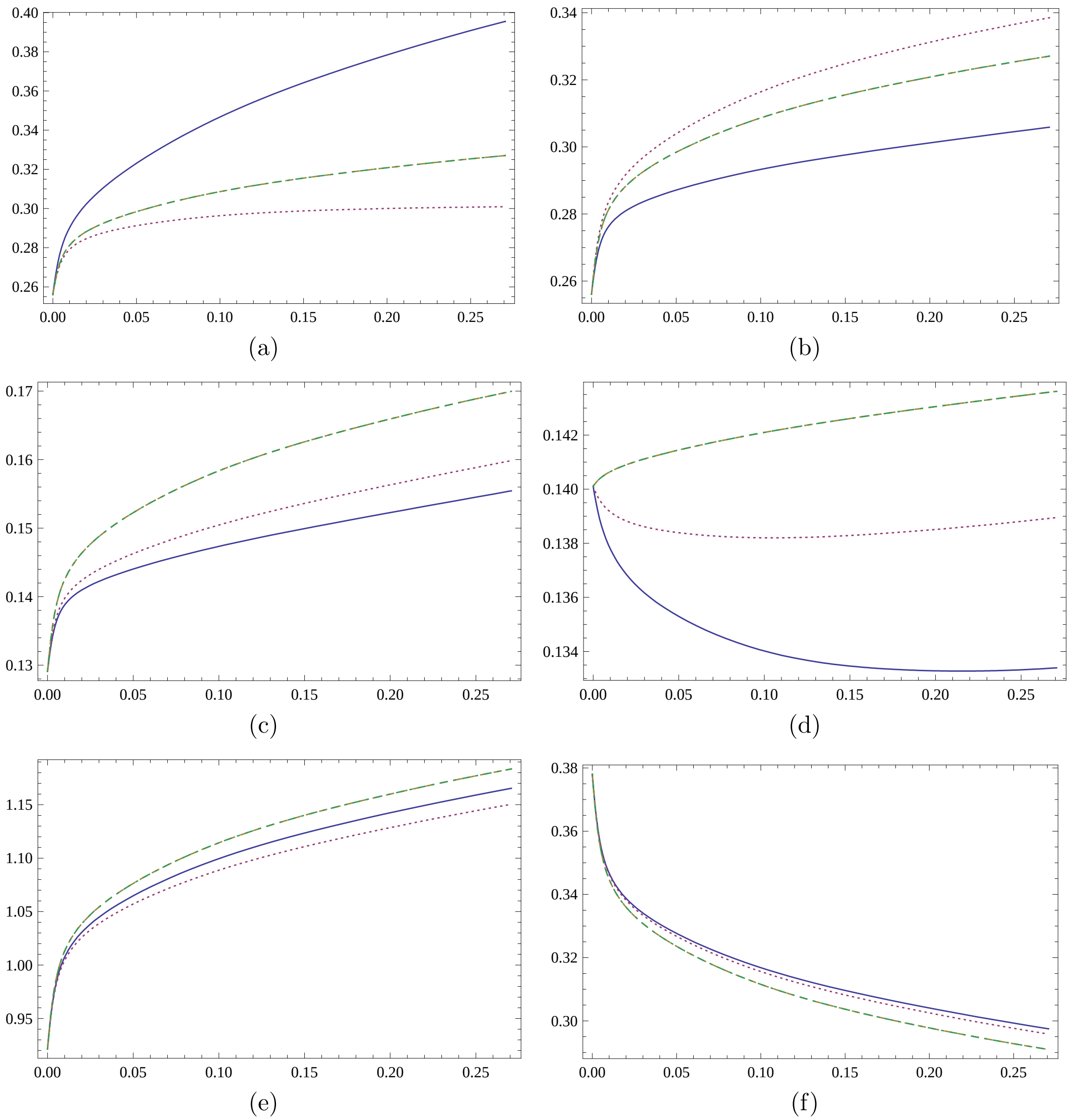


Figure 9: The graph plots key equilibrium quantities at time $t = 0$ with asymmetric performance fees as a function of the performance fee parameter γ_2. The proportional fee parameter β is adjusted so that the resulting contracts are efficient. For comparison, the corresponding values with proportional fees ($\gamma_2 = 0$) are also plotted. The quantities plotted are: the fund managers' portfolio weights (panel (a)), the active investors' portfolio weights (panel (b)), the stock instantaneous expected returns (panel (c)), the stock volatilities (panel (d)), the stock Sharpe ratios (panel (e)) and the stock price/dividend ratios (panel (f)). The solid (respectively, dotted) line refers to first stock (respectively, the second stock) with asymmetric fees, while the dashed (respectively, dot-dashed) line refers to the first stock (respectively, the second stock) with proportional fees.

a result of this quickly decreasing allocation to mutual funds, the number of shares of both stocks held by mutual funds is a monotonically decreasing function of γ_2 in spite of the increasing proportional allocation to stocks by mutual funds. Hence, the active investors' proportional allocation to both stocks must be an increasing function of γ_2, as shown in panel (b) of Figure 9, leading to increasing expected returns and Sharpe ratios (panels (c) and (e)).

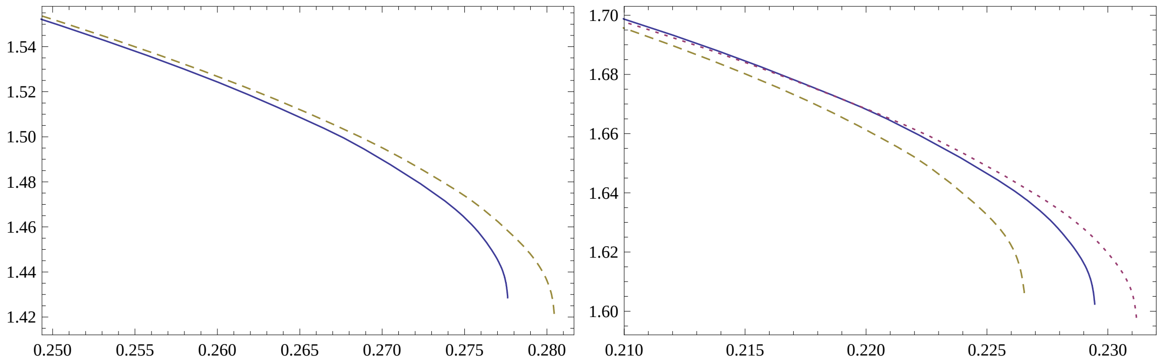
As shown in panel (a) of Figure 9, at time 0, when excess return is null, fund managers are in the region in which they find it optimal to tilt their portfolios toward the benchmark stock. This tilt is monotonic in the performance sensitivity parameter γ_2. Therefore, equilibrium prices must adjust so that active investors are induced to tilt their portfolios toward the non-benchmark stock. As shown in panels (c)–(e), while the equilibrium expected return for the non-benchmark stock exceeds that of the benchmark stock, this higher expected return is offset by a higher volatility, resulting in the Sharpe ratio of the non-benchmark stock being below that of the other stock. Therefore, at time 0, the equilibrium incentive for active investors to tilt their portfolios toward the non-benchmark stock does not arise from the benchmark stock having a higher Sharpe ratio or a lower volatility than the non-benchmark stock, but from hedging considerations. To understand the source of this hedging demand, it is useful to refer back to Figures 6 and 7. Around the point corresponding to zero excess performance for mutual funds (a dividend share around 58%), the total equity allocation by mutual funds is an increasing function of the dividend share and, to ensure market clearing, the Sharpe ratios of both stocks are decreasing functions of the dividend share. Thus, in this region, active investors have positive hedging demand for the non-benchmark stock, whose price (being increasing in the stock's dividend share) is negatively correlated with the stocks' Sharpe ratios and hence with the quality of the invest opportunity set.

6.4 Optimal Benchmarking

In order to clearly isolate the impact of delegated portfolio management on equilibrium prices, our numerical analysis has focused on the case in which the two liquidation dividends are identically distributed and the benchmark portfolio is exogenously specified to be one of the two stocks. However, in equilibrium the composition of the benchmark portfolio should be determined endogenously and so we briefly address the robustness of our results to optimal benchmarking in the context of asymmetric performance fees.[34]

When optimally determining the composition of the benchmark, fund investors seek to maximize the overall equity allocation by fund managers while at the same time minimizing the investment distortions induced by benchmarking. In general, the portfolio held by fund managers is a (dynamically-rebalanced) combination of the market portfolio, the benchmark portfolio and the riskless asset. Therefore, when the benchmark portfolio coincides with the market portfolio of risky assets, the fund managers' equilibrium trading strategy consists of a combination of this portfolio and the riskless asset, eliminating the distortions in the allocation to risky assets induced by benchmarking and benefiting fund investors. On the other hand, because it is suboptimal in this case for fund managers to increase the tracking error volatility of their portfolio (and hence the expected value of their performance fees) by

[34]Asymmetric performance fees dominate other fee structures within our piecewise linear class for all benchmark choices.



(a) (b)

Figure 10: The graph plots the Pareto frontier at time $t = 0$ with asymmetric performance fees and three different benchmark portfolios: stock 1 (solid line), stock 2 (dotted line) and the market portfolio of risky assets (dashed line). Panel (a) is for the case $\sigma_D^1 = \sigma_D^2 = 0.2D$, while panel (b) is for the case $\sigma_D^1 = 0.3D$ and $\sigma_D^2 = 0.1D$. The managers' certainty equivalent is on the horizontal axis and the fund investors' certainty equivalent is on the vertical axis.

holding a portfolio of risky assets that deviates from the benchmark portfolio, they optimally choose to reduce the risk of their compensation by holding a portfolio characterized by a lower overall equity allocation. Clearly, this latter effect is detrimental to fund investors.

When the two liquidation dividends are identically distributed and the aggregate supplies of both stocks are the same, it is easy to see that the optimal benchmark portfolio must coincide with the market portfolio of risky assets, as the equilibrium must then be fully symmetric in the two stocks. As a result, all cross-sectional price distortions are eliminated in this case, although delegated portfolio management still has an impact on equilibrium risk premia and volatilities. This is confirmed by panel (a) of Figure 10, which compares the Pareto frontiers when stock 1 is used as the benchmark and when the market portfolio of risky assets is used as benchmark, assuming identical dividend distributions and aggregate supplies.

The above is however a knife-edge case: if the two liquidation dividends are not identically distributed, the market portfolio of risky assets is not the optimal benchmark. This is shown in panel (b) of Figure 10 for the case $\sigma_D^1(D,t) = 0.3D$ and $\sigma_D^2(D,t) = 0.1D$: interestingly, in this case benchmarking fund managers to either asset results in welfare gains over benchmarking them to the market portfolio of risky assets. In general, the optimal benchmark is tilted toward the asset with lower volatility, as this induces the opposite tilt in the portfolio allocation chosen by fund managers, increasing the volatility, and hence the systematic risk and the expected return, of the fund's portfolio. As it should be clear, the optimal benchmark not being the market portfolio of risky assets is all that is needed for the qualitative results described in the previous section to hold.

7 Conclusion

We have examined the impact of delegated portfolio management on equilibrium prices within a dynamic general-equilibrium setting in which the parameters of the management contract, the extent of delegated portfolio management and the returns of both benchmark and non-benchmark securities are all determined endogenously.

When fund managers receive performance fees of the fulcrum type, they optimally choose to tilt their portfolios towards stocks that are part of the benchmark: in equilibrium, this results in a significant positive price effect associated with the addition of a stock to the benchmark and a smaller negative price effect associated a deletion. These implications are consistent with empirical evidence regarding changes in the composition of the S&P 500 index (the most widely used benchmark portfolio). Everything else being the same, we also find that benchmark stocks have lower expected returns, lower Sharpe ratios and higher volatilities than similar non-benchmark stocks.

With asymmetric performance fees, the composition of the portfolios selected by fund managers depends critically on the funds' excess return. As a result, cross-sectional differences between benchmark and non-benchmark stocks can have either sign, depending on the funds' performance relative to the benchmark. Interestingly, the presence of portfolio managers receiving asymmetric performance fees can stabilize prices by decreasing the equilibrium stock volatilities of both benchmark and non-benchmark stocks, although portfolio turnovers are higher with asymmetric fees.

Previous literature has typically taken asset returns and the level of delegated portfolio management as given, analyzing managers' portfolio choice decisions and deriving optimal contracts. We have complemented this literature by analyzing the asset pricing implications of a prevalent parametric class of existing contracts, when the level of delegated portfolio management is determined endogenously. We have also demonstrated that when full delegation is not exogenously imposed, performance contracts may be welfare-improving, even when there is no asymmetric information and investors and fund managers have CRRA preferences with the same risk aversion coefficient.

Appendix

To simplify the notation, throughout this Appendix we frequently suppress the explicit dependence of quantities such as $\underline{W}$, W_1 and W_2 on W^b.

Proof of Lemma 1. Suppose first that $\beta + \gamma_1 \neq 0$ and consider the system of equations

$$\begin{cases} u'(\alpha + \beta W_1^* + \gamma_1(W_1^* - \delta W^b))(\beta + \gamma_1) \\ \qquad = u'(\alpha + \beta W_2^* + \gamma_2(W_2^* - \delta W^b))(\beta + \gamma_2) \\ u(\alpha + \beta W_1^* + \gamma_1(W_1^* - \delta W^b)) \\ \qquad = u(\alpha + \beta W_2^* + \gamma_2(W_2^* - \delta W^b)) \\ \qquad\quad + u'(\alpha + \beta W_2^* + \gamma_2(W_2^* - \delta W^b))(\beta + \gamma_2)(W_1^* - W_2^*) \end{cases} \tag{33}$$

in the unknowns (W_1^*, W_2^*). Direct computation shows that the system has the unique solution

$$\begin{cases} W_1^* = \dfrac{\left(1 - \frac{\eta}{c}\right) \frac{\alpha - \gamma_1 \delta W^b}{\beta + \gamma_1} - \eta \left(1 - \frac{1}{c}\right) \frac{\alpha - \gamma_2 \delta W^b}{\beta + \gamma_2}}{\eta - 1} \\ W_2^* = W_1^* + \dfrac{1}{c}\left(\dfrac{\alpha - \gamma_1 \delta W^b}{\beta + \gamma_1} - \dfrac{\alpha - \gamma_2 \delta W^b}{\beta + \gamma_2}\right), \end{cases} \tag{34}$$

where $\eta = \left(\frac{\beta + \gamma_2}{\beta + \gamma_1}\right)^{1 - \frac{1}{c}}$. Moreover,

$$\frac{\gamma_1 \delta W^b - \alpha}{\beta + \gamma_1} < W_1^* < \delta W^b < W_2^*. \tag{35}$$

Letting $\underline{W}(W^b) = \left(\frac{\gamma_1 \delta W^b - \alpha}{\beta + \gamma_1}\right)^+$ (as in equation (9)) and $W_1(W^b) = (W_1^*)^+$ (as in equation (13)), it follows from the above inequality that $\underline{W} \leq W_1 < \delta W^b$.

If $W_1(W^b) > 0$, it then immediately follows from the system (33) that equations (11) and (12) are satisfied, with equality in equation (12)) and $W_2(W^b) = W_2^*$, establishing the lemma for this case.

If, on the other hand, $W_1(W^b) = 0$, then $W_1^* \leq 0$ and it follows from equation (35) that $\alpha - \gamma_1 \delta W^b > 0$. Therefore, the function

$$\begin{aligned} f(W) &= u(\alpha - \gamma_1 \delta W^b) - u(\alpha + \beta W + \gamma_2(W - \delta W^b)) \\ &\quad + u'(\alpha + \beta W + \gamma_2(W - \delta W^b))(\beta + \gamma_2) W \end{aligned} \tag{36}$$

is well defined for $W \geq W^b$ and the existence of a solution $W_2(W^b) > \delta W^b$ to equation (11) is equivalent to the existence of a zero of f in $(\delta W^b, \infty)$. Clearly, f is continuous and strictly decreasing on $[\delta W^b, \infty)$. Moreover,

$$\begin{aligned} f(\delta W^b) &= u(\alpha - \gamma_1 \delta W^b) - u(\alpha + \beta \delta W^b) + u'(\alpha + \beta \delta W^b)(\beta + \gamma_2) \delta W^b \\ &> u(\alpha - \gamma_1 \delta W^b) - u(\alpha + \beta \delta W^b) \\ &\quad + u'(\alpha + \beta W_2^* + \gamma_2(W_2^* - \delta W^b))(\beta + \gamma_2) \delta W^b \end{aligned}$$

$$
\begin{aligned}
&= u(\alpha - \gamma_1 \delta W^b) - u(\alpha + \beta \delta W^b) \\
&\qquad + u'(\alpha + \beta W_1^* + \gamma_1 (W_1^* - \delta W^b))(\beta + \gamma_1)\delta W^b \\
&\geq u(\alpha - \gamma_1 \delta W^b) - u(\alpha + \beta \delta W^b) + u'(\alpha - \gamma_1 \delta W^b)(\beta + \gamma_1)\delta W^b \\
&> 0,
\end{aligned}
$$

where the second (in)equality follows from the fact that u' is decreasing and $W_2^* > \delta W^b$, the third from the first equation in the system (33), the fourth from the fact that u' is decreasing and $W_1^* < 0$ and the last from the fact that the function $u(\alpha + \beta W + \gamma_1(W - \delta W^b))$ is concave in W. Similarly,

$$
\begin{aligned}
f(W_2^*) &= u(\alpha - \gamma_1 \delta W^b) - u(\alpha + \beta W_2^* + \gamma_2 (W_2^* - \delta W^b)) \\
&\qquad + u'(\alpha + \beta W_2^* + \gamma_2 (W_2^* - \delta W^b))(\beta + \gamma_2) W_2^* \\
&= u(\alpha - \gamma_1 \delta W^b) - u(\alpha + \beta W_1^* + \gamma_1 (W_1^* - \delta W^b)) \\
&\qquad + u'(\alpha + \beta W_1^* + \gamma_1 (W_1^* - \delta W^b))(\beta + \gamma_1) W_1^* \\
&\leq 0,
\end{aligned}
$$

where the second equality follows from the second equation in the system (33), while the inequality follows from the concavity of the function $u(\alpha + \beta W + \gamma_1(W - \delta W^b))$. Therefore, there exists a unique $W_2(W^b) \in (\delta W^b, W_2^*)$ such that $f(W_2(W^b)) = 0$, thus establishing the existence of a unique solution to equation (11). The inequality in equation (12) follows from the fact that in this case we have

$$
\begin{aligned}
u'(F(W_1(W^b), W^b))(\beta + \gamma_1) &\leq u'(F(W_1^*, W^b))(\beta + \gamma_1) \\
&= u'(F(W_2^*, W^b))(\beta + \gamma_2) \\
&< u'(F(W_2(W^b), W^b))(\beta + \gamma_2),
\end{aligned}
$$

where the first (in)equality follows from the fact that u' is decreasing and $W_1(W^b) \geq W_1^*$, the second from the first equation in the system (33), and the last from the fact that u' is decreasing and $W_2(W^b) < W_2^*$.

Finally, suppose that $\beta + \gamma_1 = 0$ and let $\underline{W}(W^b) = 0$ (as in equation (9)) and $W_1(W^b) = 0$. With $f(W)$ defined as in equation (36), we then have

$$
f(\delta W^b) = u'(\alpha)\gamma_2 \delta W^b > 0.
$$

Since

$$
\frac{2(\alpha + \gamma_2(W - \delta W^b))\delta W^b}{\gamma_2 (W - \delta W^b)^2} \to 0 \quad \text{as } W \to +\infty
$$

and $-Wu''(W)/u'(W) = c$ for all W, there exists a $W_3^* > \delta W^b$ such that

$$
\begin{aligned}
\frac{2(\alpha + \gamma_2(W_3^* - \delta W^b))\delta W^b}{\gamma_2 (W_3^* - \delta W^b)^2} &< c \\
&= -\frac{(\alpha + \gamma_2(W_3^* - \delta W^b))u''(\alpha + \gamma_2(W_3^* - \delta W^b))}{u'(\alpha + \gamma_2(W_3^* - \delta W^b))},
\end{aligned}
$$

or

$$
u'(\alpha + \gamma_2(W_3^* - \delta W^b))\delta W^b + \frac{1}{2} u''(\alpha + \gamma_2(W_3^* - \delta W^b))\gamma_2 (W_3^* - \delta W^b)^2 < 0. \tag{37}
$$

Moreover, since $u''' > 0$, it follows from a second-order Taylor expansion of the function $u(\alpha + \gamma_2(W - \delta W^b))$ around the point W_3^* that

$$u(\alpha) < u(\alpha + \gamma_2(W_3^* - \delta W^b)) - u'(\alpha + \gamma_2(W_3^* - \delta W^b))\gamma_2(W_3^* - \delta W^b) + \frac{1}{2}u''(\alpha + \gamma_2(W_3^* - \delta W^b))\gamma_2^2(W_3^* - \delta W^b)^2. \tag{38}$$

Hence,

$$\begin{aligned} f(W_3^*) &= u(\alpha) - u(\alpha + \gamma_2(W_3^* - \delta W^b)) + u'(\alpha + \gamma_2(W_3^* - \delta W^b))\gamma_2 W_3^* \\ &< u'(\alpha + \gamma_2(W_3^* - \delta W^b))\gamma_2 \delta W^b + \frac{1}{2}u''(\alpha + \gamma_2(W_3^* - \delta W^b))\gamma_2^2(W_3^* - \delta W^b)^2 \\ &< 0, \end{aligned}$$

where the first inequality follows from equation (38), while the second follows from equation (37). Therefore, f has a zero on $(\delta W^b, W_3^*)$, thus establishing the existence of a solution to equation (11) in this case. The inequality in equation (12) is trivially satisfied, since the left-hand side equals zero in this case. □

Proof of Lemma 2. Equation (11) shows that $v(\cdot, W^b)$ is continuous at $W_2(W^b)$, while equation (12) shows that $v(\cdot, W^b)$ is continuously differentiable at $W_1(W^b)$ if $W_1(W^b) > 0$ (that is, if $W_1(W^b) > \underline{W}(W^b)$). It then immediately follows from the definition of v that $v(\cdot, W^b)$ is continuously differentiable and concave on $[\underline{W}(W^b), \infty)$. Moreover, since $v(W_1(W^b), W^b) = u(F(W_1(W^b), W^b))$ and $u(F(\cdot, W^b))$ is strictly concave on the interval $(W_1(W^b), \delta W^b]$ while $v(\cdot, W^b)$ is linear, it follows that $v(\cdot, W^b) > u(F(\cdot, W^b))$ on that interval. A similar argument implies that $v(\cdot, W^b) > u(F(\cdot, W^b))$ on the interval $[\delta W^b, W_2(W^b))$. Since $v(\cdot, W^b) = u(F(\cdot, W^b))$ on $A(W^b)$, it follows that $v(W, W^b) \geq u(F(W, W^b))$ on the interval $[\underline{W}(W^b), \infty)$.

Finally, if $\hat{v}(\cdot, W^b)$ is any other concave function with $\hat{v}(\cdot, W^b) \geq u(F(\cdot, W^b))$ on the interval $[\underline{W}(W^b), \infty)$, then $\hat{v}(\cdot, W^b) \geq u(F(\cdot, W^b)) = v(\cdot, W^b)$ on $A(W^b)$. Moreover, $\hat{v}(\cdot, W^b) \geq v(\cdot, W^b)$ on $(W_1(W^b), W_2(W^b))$ since $\hat{v}(W_1(W^b), W^b) \geq v(W_1(W^b), W^b)$, $\hat{v}(\cdot, W^b)$ is concave and $v(\cdot, W^b)$ is linear on that interval. Therefore, v is the smallest concave function $\hat{v}(\cdot, W^b)$ with $\hat{v}(\cdot, W^b) \geq u(F(\cdot, W^b))$ □

Proof of Proposition 1. Standard optimization theory implies that a policy W_T is optimal in the concavified problem (10) if and only if W_T satisfies the conditions of Proposition 1 with equation (14) replaced by the weaker condition

$$W_T^m \in v_W^{-1}(\psi^m \pi_T, W_T^b) \quad \text{if } W_T^m > 0.$$

Thus, a policy W_T^m satisfying the conditions of Proposition 1 is optimal in the concavified problem (14). The fact that W_T^m takes values in $A(W_T^b)$ implies $u(F(W_T^m, W_T^b)) = v(W_T^m, W_T^b)$. Since $u(F(\cdot, W_T^b)) \leq v(\cdot, W_T^b)$, optimality of W_T^m in the concavified problem (10) then implies optimality in the original problem (8). This proves that the conditions in Proposition 1 are sufficient for optimality.

Necessity follows immediately by noticing that if W_T is any other feasible policy, then

$$\begin{aligned} \mathrm{E}\Big[u\Big(F(W_T, W_T^b)\Big)\Big] &\leq \mathrm{E}\Big[v\Big(W_T, W_T^b\Big)\Big] \\ &\leq \mathrm{E}\Big[v\Big(W_T^m, W_T^b\Big)\Big] \\ &= \mathrm{E}\Big[u\Big(F(W_T^m, W_T^b)\Big)\Big], \end{aligned}$$

with the first inequality being strict if $W_T \notin A(W_T^b)$, and the second inequality being strict otherwise. □

Proof of Theorem 1. It can be easily verified that $g^m(y, W^b, p) = \Big(v_W^{-1}(y, W^b)\Big)^+$ for $y \neq v_W(W_2, W^b)$. The claim then immediately follows from Proposition 1.

Proof of Theorem 2. The proof of Theorem 2 is straightforward and is thus omitted. □

Proof of Corollary 1. Under the stated assumptions, $\gamma_1 = \gamma_2$ and

$$\alpha - \gamma_1 \delta W_T^b \leq 0,$$

so that

$$\Pi_1(D_T, W_T^b) = \Pi_3(D_T, W_T^b) \geq \Pi_4(D_T, W_T^b).$$

It then follows from equation (21), using the fact that $\beta + \gamma_1 = \beta + \gamma_2 > 0$, that

$$\psi^m \pi_T = \Pi(D_T, W_T^b) = \Pi_1(D_T, W_T^b).$$

In addition, since by Lemma 2 $W_1 = W_2$ when $\gamma_1 = \gamma_2$, it follows from (16) and (17) that

$$W_T^m = g^m(\psi^m \pi_T, W_T^b, P_T) \;= \frac{\Pi_1(D_T, W_T^b)^{-\frac{1}{c}}}{(\beta + \gamma_2)^{1-\frac{1}{c}}} - \frac{\alpha - \gamma_2 \delta W_T^b}{\beta + \gamma_2} = \theta_T^m \cdot D_T,$$

where

$$\theta_T^m = \frac{\varphi(1-\lambda)\gamma_2 \delta \bar{\theta}_T^b + (\beta+\gamma_2)^{\frac{1}{c}} \bar{\theta} - \Big(\lambda(\beta+\gamma_2)^{\frac{1}{c}} \theta_0^f + \varphi(1-\lambda)\frac{\alpha}{B}\Big)\bar{\theta}^1}{\lambda(\beta+\gamma_2)^{\frac{1}{c}} + \varphi(1-\lambda)(\beta+\gamma_2)}$$

It then immediately follows that the optimal trading strategy for the fund managers is as given in equation (28) for all $t \in [0, T]$. The optimal trading strategy for the active investors follows from the market-clearing conditions.

Next, equation (25) implies that $S_t^1 - S_t^2 \geq 0$ if and only if

$$\mathrm{E}_t\left[\Pi_1(D_T, W_T^b)(D_T^1 - D_T^2)\right] \geq 0.$$

Let $\hat{D}_T$ be two-dimensional random variable obtained from D_T by exchanging the values of D_T^1 and D_T^2. Since D_T^1 and D_T^2 are identically distributed conditional on the information at time t, we have

$$\begin{aligned} &\mathrm{E}_t\left[\Pi_1(D_T, W_T^b)(D_T^1 - D_T^2)\right] \\ &\quad = \mathrm{E}_t\left[\Pi_1(D_T, W_T^b)(D_T^1 - D_T^2)1_{\{D_T^1 > D_T^2\}}\right] + \mathrm{E}_t\left[\Pi_1(D_T, W_T^b)(D_T^1 - D_T^2)1_{\{D_T^2 > D_T^1\}}\right] \\ &\quad = \mathrm{E}_t\left[\Pi_1(D_T, W_T^b)(D_T^1 - D_T^2)1_{\{D_T^1 > D_T^2\}}\right] + \mathrm{E}_t\left[\Pi_1(\hat{D}_T, W_T^b)(D_T^2 - D_T^1)1_{\{D_T^1 > D_T^2\}}\right] \\ &\quad = \mathrm{E}_t\left[\Big(\Pi_1(D_T, W_T^b) - \Pi_1(\hat{D}_T, W_T^b)\Big)(D_T^1 - D_T^2)1_{\{D_T^1 > D_T^2\}}\right]. \end{aligned}$$

The claim regarding the equilibrium stock prices then easily follows from the fact that, when $\gamma_2 \neq 0$ and $D_T^1 > D_T^2$, $\Pi_1(D_T, \theta_T^b) > \Pi_1(\hat{D}_T, \theta_T^b)$ (respectively, $\Pi_1(D_T, \theta_T^b) < \Pi_1(\hat{D}_T, \theta_T^b)$) if and only if the benchmark portfolio θ_T^b holds more (respectively, less) of stock 1 than of stock 2. $\square$

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