Entrepreneurial Finance and Non-diversifiable Risk*

Hui Chen† Jianjun Miao‡ Neng Wang§

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Abstract

Entrepreneurs face significant non-diversifiable business risks. In a dynamic incomplete-markets model of entrepreneurial finance, we show that such risks have important implications for their interdependent consumption/saving, portfolio choice, external debt/equity and inside equity financing, investment, and endogenous default/cash-out decisions. More risk-averse entrepreneurs default earlier for given debt service, and also choose higher initial leverage for diversification benefits. Non-diversified entrepreneurs demand not only a systematic risk premium but also an idiosyncratic risk premium, which depends on entrepreneurial risk aversion, the project's idiosyncratic volatility and the sensitivity of entrepreneurial value of equity with respect to project's cash flow. External equity and cash-out option improve diversification and raise the private value of firm, but only partially weaken the roles of external risky debt and default option in diversification. After debt is in place, when an entrepreneur chooses among mutually exclusive projects with different idiosyncratic volatilities, the effect of risk aversion tends to dominate the risk-shifting incentives. Even entrepreneurs with relatively low idiosyncratic risk premium do not engage in risk-shifting activities.

Keywords: Default, diversification benefits, entrepreneurial risk aversion, incomplete markets, private equity premium, hedging, capital structure, cash-out option, precautionary saving

JEL Classification: G11, G31, E2

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†MIT Sloan School of Management, 50 Memorial Drive, Cambridge, MA 02142. Email: huichen@mit.edu. Tel.: 617-324-3896.

‡Department of Economics, Boston University, 270 Bay State Road, Boston, MA 02215. Email: miaoj@bu.edu. Tel.: 617-353-6675.

§Columbia Business School and National Bureau of Economic Research. Address: Columbia Business School, 3022 Broadway, Uris Hall 812, New York, NY 10027. Email: neng.wang@columbia.edu. Tel.: 212-854-3869.

1 Introduction

Entrepreneurship plays an important role in fostering innovation and economic growth (Schumpeter (1934)). Entrepreneurial investment activities are quite diverse, ranging from the creation of the state-of-the-art high-tech products to the daily operations of small businesses (e.g., restaurants). While entrepreneurial businesses may differ from one another, they often share one important common feature: their investment opportunities are often illiquid and non-tradable, making entrepreneurs exposed to non-diversifiable risks. For reasons such as incentive alignment and informational asymmetry between the entrepreneur and financiers, the entrepreneur typically holds a significant non-diversified equity position in his business, and thus bear non-diversifiable entrepreneurial business risk.[1] Gentry and Hubbard (2004) report that active businesses on average account for 41.5 percent of entrepreneurs total assets using 1989 survey of consumer finance (SCF). While the share of assets held as a business equity stake varies across entrepreneurs, most entrepreneurs hold a substantial portion of their assets in their business.[2]

The significant lack of diversification invalidates the standard finance textbook valuation analysis designed for firms owned by diversified investors. An entrepreneur's non-diversifiable position in his investment project makes his business decisions (financing and project choice) and household decisions (consumption, saving, and asset allocation) interdependent. This interdependence arises in our model because markets are incomplete for the entrepreneur. As a result, the standard two-step complete-markets (Arrow-Debreu) analysis (i.e. first value maximization and then optimal consumption allocation) no longer applies. This non-separability between value maximization and consumption smoothing has important implications for real economic activities (e.g. investment and capital budgeting) and the valuation of claims that an entrepreneur issues to finance his investment activities.

To the best of our knowledge, this paper provides the first dynamic incomplete-markets model of entrepreneurial investment and financing. We consider an infinitely-lived risk-averse entrepreneur

[1] Bitler, Moskowitz and Vissing-Jorgensen (2005) provide evidence that agency considerations play a key role in explaining why entrepreneurs on average hold large ownership shares.

[2] Using the SCF (1989), Gentry and Hubbard (2004) report that the median portfolio share (relative to assets) is 35.0 percent. The 25th percentile is 14.8 percent, and the 75th percentile is 61.2 percent. Moskowitz and Vissing-Jorgensen (2002) document that about 75 percent of all private equity is owned by households for whom it constitutes at least half of their total net worth. Moreover, households with entrepreneurial equity invest on average more than 70 percent of their private holdings in a single private company in which they have an active management interest. See also Hall and Woodward (2008) on the quantitative analysis for the lack of diversification of venture-capital-backed entrepreneurial firms.

who derives utility from his intertemporal consumption. He has an illiquid non-tradable investment project, which he sets up as a firm with limited liability (the entrepreneurial firm). He can invest his liquid financial wealth in both a risk-free asset and a market portfolio as in a standard consumption-portfolio choice problem (e.g. Merton (1971)). He can also investment part of his financial wealth in the illiquid investment project. The entrepreneurial firm's investment project generates a stream of stochastic cash flow, which are imperfectly correlated with the market portfolio. The entrepreneur can trade dynamically in the market portfolio. While the entrepreneur can hedge the systematic component of his business risks, he cannot fully hedge the idiosyncratic risk component of his business project because the entrepreneur holds a non-diversifiable position in his business and his business risk is not completed spanned by traded securities (i.e. incomplete markets).

To reduce his idiosyncratic risk exposure and equity contribution, the entrepreneur resorts to outside financing (e.g. risky debt and equity). While issuing outside claims allow the entrepreneur to diversify, financing also has costs as in standard corporate finance models. As in standard trade-off models, outside debt potentially induces financial distress and conflicts of interest between the entrepreneur and creditors. Issuing outside equity potentially lowers the firm's expected cash flow growth rate because of mis-alignment of incentives between the entrepreneur and outside equity. After setting the initial financing arrangement with outside debt/equity and inside equity, the entrepreneur starts receiving stochastic uninsurable revenue/profit stream from his business. He can dynamically trade his liquid wealth in the risky asset and risk-free asset to partially hedge the systematic component of his business risk, but not the idiosyncratic component.

Moreover, the entrepreneur has two exit options. By incurring the fixed cost of cashing out (via either an initial public offering (IPO) or a direct sale of the firm to diversified investors), the entrepreneur can achieve full diversification. Our model assumes that cashing out (via either direct placement or IPO) requires the entrepreneur to pay a fixed cost.[3] Moreover, cashing out triggers capital gains. These costs of cashing out generate an option value of waiting, which implies that the entrepreneur will cash out only when the diversification benefit from cashing out is sufficiently high. This feature is consistent the empirical evidence that debt is the primary source of external financing for small businesses (Heaton and Lucas (2004)).

Borrowing on the firm's balance sheet using the illiquid project as collateral gives the entrepreneur an endogenous default option. As in standard tradeoff models of capital structure,

[3] Debt is potentially less information sensitive, and may be preferred in some settings with asymmetric information (Myers and Majluf (1984)). Our model does not explicitly model information asymmetry.

default generates bankruptcy and agency costs. Unlike in these models, the default option here allows the entrepreneur to walk away from his business with limited liability, hence providing *ex ante* insurance to the entrepreneur against the firm's potential poor performance in the future. Moreover, using external debt to finance investment allows the entrepreneur to retain his control rights over his business provided that he does not default on his debt obligation. Hence, the efficiency of business operation is not affected via debt financing, unlike external equity financing (cash-out). This difference between external equity and debt makes the entrepreneur prefer outside debt over equity holding everything else constant.

The entrepreneur chooses the amount of outside risky debt/outside equity, and the timing of cash-out/default to maximize his utility by trading off tax and diversification benefits against bankruptcy and agency costs. We show that this utility maximization problem (with interdependent consumption/saving, portfolio and default decisions) simplifies to a problem of maximizing the entrepreneur's private value of the firm, which is equal to the sum of the entrepreneur's private value of equity and the market value of debt and equity he issues. The entrepreneur's risk attitude and his exposure to the project's idiosyncratic risks have significant effects on the private value of the entrepreneurial firm and its inside equity position, and hence influence the firm's initial capital structure and subsequent default and cash-out decisions.

The private value of the firm also suggests a natural measure of leverage for an entrepreneurial firm: the ratio of the market value of debt to the private value of the firm. We dub this ratio "private leverage" to highlight the impact of entrepreneurial risk aversion and non-diversifiable idiosyncratic risk exposure on the entrepreneurial firm's leverage decision. The measure of private leverage captures the idiosyncratic risk and non-diversification effects on financing and differs from the public leverage ratio in the typical trade-off theory of capital structure for the public firm.

Our main findings are as follows. First, the diversification benefits of debt are large. Even when we turn off the tax benefits of debt, the entrepreneurial firm still issues a significant amount of outside debt for diversification. The more risk-averse the entrepreneur is, the more he values the diversification benefit of risky debt; hence the higher his private leverage. At the first glance, this result might appear counterintuitive: more risk-averse entrepreneurs ought to be less aggressive in terms of financial policies (e.g., a lower leverage ratio). We may reconcile this result as follows. The more risk-averse entrepreneur has a lower subjective valuation of his business project and has a stronger incentive to sell his firm. Since the (diversified) lenders demand no premium for bearing the entrepreneurial firm's idiosyncratic risks, we expect that more risk-averse entrepreneurs sell a higher fraction of their firms via outside debt.

While the effect of risk aversion on leverage is monotonically increasing, there are two opposing effects of risk aversion on debt coupon. On the one hand, the diversification benefits suggest that coupon increases with entrepreneurial risk aversion. On the other hand, the more risk-averse entrepreneur *ex post* defaults earlier for a given level of coupon, lowering the firm's ability to issue debt. We refer to the latter effect as the distance-to-default effect. The net impact of risk aversion on debt coupon is therefore ambiguous.

Second, the entrepreneur's private value of his equity in our incomplete-markets model behaves differently from the market value of equity held by diversified investors. Inspired by important insights from Black and Cox (1976), Leland (1994) initiates strong interests in structural models of credit risk and capital structure. In these complete-markets models, equity value is convex, as in Black and Scholes (1973) and Merton (1973), in that equity is a call option on firm assets. By contrast, our model predicts that the private value of equity is not necessarily globally convex in cash flow. Indeed, the entrepreneur's precautionary saving demand makes his private value of equity potentially concave in cash flow, when precautionary saving motive and/or idiosyncratic volatility is sufficiently high.

This finding has important implications for the entrepreneur's project choice decisions. Jensen and Meckling (1976) point out that managers of public firms have incentives to invest in excessively risky projects after debt is in place because of the convexity feature of equity. In our model, the risk-averse entrepreneur discounts his private value of equity because he bears non-diversifiable idiosyncratic risks. When the degree of risk aversion is high enough, his private value of equity decreases with the idiosyncratic volatility of the project. As a result, he may prefer to invest in a low idiosyncratic volatility project, overturning the asset substitution result of Jensen and Meckling (1976). Our model provides a potential explanation for the lack of empirical and survey evidence on asset substitution and risk-shifting incentives. Even when the implied idiosyncratic risk premium is only 20 basis points, the entrepreneur's aversion to consumption fluctuation induces him to choose projects with low idiosyncratic volatility.

Third, the entrepreneur demands not only a systematic risk premium but also an idiosyncratic risk premium due to the lack of diversification. We derive an analytical formula for the idiosyncratic risk premium whose key determinants are risk aversion, idiosyncratic volatility and the sensitivity of entrepreneurial value of equity with respect to cash flow. When the entrepreneurial firm is close to default, the systematic risk premium approaches infinity, while the idiosyncratic risk premium is still finite. When the entrepreneurial firm is close to going public, the idiosyncratic risk premium goes up.

Finally, we show that the leverage ratio may drop substantially when the entrepreneur has access to external equity (or the cash-out option) to diversify his project's idiosyncratic risks. Intuitively, the incremental value of debt financing is lower when the entrepreneur substitutes some debt with the expected use of outside equity in the future. The entrepreneur maximizes his *ex ante* private value of the firm by trading off the debt issuance/default option against the cash-out option. The more attractive the external equity financing (cash-out option), the lower the firm's private leverage and debt coupon.

We now turn to the related literature. We provide a generalized tradeoff model for the entrepreneurial firm's capital structure under incomplete markets, where the risky outside debt offers an additional diversification benefit over inside equity. Our model includes the structural (complete-markets) credit risk/capital structure models such as the workhorse Leland (1994) model as a special case. We show that precautionary saving demand plays an important role in determining the entrepreneurial firm's leverage and default strategies.

Our model is related to the incomplete-markets consumption smoothing/precautionary saving literature.[4] For analytical tractability reasons, we adopt the expected constant-absolute-risk-averse (CARA) utility specification as in Merton (1971), Caballero (1991), Kimball and Mankiw (1989), and Wang (2006). Our model contributes to this literature by extending the CARA-utility-based precautionary saving problem to allow the entrepreneur to reduce his idiosyncratic risk exposure via exit strategies such as cash-out and default. Because our model incorporates portfolio choice, it also contributes to this literature pioneered by Merton (1971, 1973).

Our paper is also related to the real options literature.[5] The closest paper is by Miao and Wang (2007) that analyze the impact of the entrepreneur's non-diversifiable idiosyncratic risks on his growth option exercising decision. The present paper focuses on the entrepreneurial firm's investment and financing (internal versus external, debt versus equity), and endogenous default and cash-out decisions.

[4]See Hall (1978) for an early contribution and see Deaton (1992) and Attanasio (1999) for recent surveys.

[5]See Brennan and Schwartz (1986), McDonald and Siegel (1986), Abel and Eberly (1994), and Dixit and Pindyck (1994) for example.

Our paper builds on some recent work on entrepreneurial finance, particularly Heaton and Lucas (2004). Heaton and Lucas (2004) emphasize the diversification benefit of risky debt. One key difference is that our model is dynamic and theirs is static. Our dynamic framework models the endogenous default/cash-out decisions as a perpetual American option exercising problem under incomplete markets. The key driving force in our model is the entrepreneur's intertemporal consumption smoothing/precautionary saving motive. In their static model, consumption is equal to terminal wealth and hence risk aversion plays the key role. Other differences between the two papers include the following: We incorporate taxes and operating leverage, while they do not. We parameterize the cost of financial distress as in Leland (1994) and other tradeoff models, while they use adverse selection as the cost of external financing as in Leland and Pyle (1977). We allow the entrepreneur to invest in the risky asset to partially hedge his project risk and hence include the complete-markets setting as a special case, while they assume away hedgable component of risk. Finally, we use CARA utility, while they use constant relative risk aversion (CRRA) utility.

2 Model setup

Consider an infinitely-lived risk-averse entrepreneur's decision problem in a continuous-time setting. The entrepreneur derives utility from a consumption process $\{c_t : t \geq 0\}$ according to the following time-additive utility function:

$$\mathbb{E}\left[\int_0^\infty e^{-\delta t} u\left(c_t\right) dt\right], \tag{1}$$

where $\delta > 0$ is the entrepreneur's subjective discount rate and $u(\cdot)$ is an increasing and concave function. The entrepreneur has standard financial investment opportunities as in Merton (1971), in that he allocates his liquid financial wealth between a risk-free asset which pays a constant rate of interest r and a market portfolio (the risky asset) with returns R_t satisfying:

$$dR_t = \mu_p dt + \sigma_p dB_t, \tag{2}$$

where μ_p is the expected return on the risky asset and σ_p is the return volatility. Let

$$\eta = \frac{\mu_p - r}{\sigma_p} \tag{3}$$

denote the after-tax Sharpe ratio of the market portfolio. Let $\{x_t : t \geq 0\}$ denote the entrepreneur's liquid (financial) wealth process. The entrepreneur invests the amount ϕ_t in the market portfolio (the risky asset) and the remaining amount $x_t - \phi_t$ in the risk-free asset.

In addition to his liquid wealth, the entrepreneur also has a take-it-or-leave-it project at time 0. If the entrepreneur chooses to start the project, he sets up a separate entity such as a limited liability company (LLC) or an S corporation to run the project. The LLC or the S corporation allows the entrepreneur to face single-layer taxation for his business income and also achieves limited liability. If the entrepreneur chooses not to start the project, he receives his outside option value, the alternative generating zero cash flows. While we can endogenize the entrepreneur's career choice and provide an endogenous opportunity cost of taking on the entrepreneurial project, this normalization does not change key economics of our paper in any significant way.

If the entrepreneur starts the project, he finances the initial one-time lump-sum cost I via his own funds (internal financing) and external (debt and equity) financing. One natural interpretation of external debt is bank loan. The entrepreneur uses the firm's assets as collateral to borrow. That is, debt is secured. The limited-liability feature of LLC implies that debt is also non-recourse. In addition to bank debt, the entrepreneur also issues external equity at time 0. External equity provides diversification benefits, but potentially lowers the expected growth rate for revenue $\{y_t : t \geq 0\}$. Intuitively, the more concentrated the entrepreneur's ownership ψ is, the better incentive alignment (Berle and Means (1931) and Jensen and Meckling (1976)). Let ψ denote the fraction of equity that the entrepreneur retains and hence $1 - \psi$ denote the fraction of external equity. Let μ denote the expected growth rate of revenue. We capture the relation between the expected revenue growth μ and the entrepreneur's ownership ψ via an increasing and concave function $\mu(\psi)$ (i.e. $\mu'(\psi) > 0$ and $\mu''(\psi) < 0$). Intuitively, the concavity relation suggests that the incremental value from incentive alignment at higher concentrated ownership is lower, *ceteris paribus*. For analytical convenience, we assume that the entrepreneur's debt and equity issuance are made at time 0 and remain unchanged until the entrepreneur exits. We may further extend the model to allow for dynamic adjustment at the cost of further complicating the model.

Assume that the stochastic revenue process $\{y_t : t \geq 0\}$ follows a geometric Brownian motion given by:

$$dy_t = \mu(\psi) y_t dt + \omega y_t dB_t + \epsilon y_t dZ_t, \; y_0 \text{ given,} \tag{4}$$

where $\mu(\psi)$ is the ownership-dependent expected growth rate of the revenue, ω and ϵ are the corresponding volatility parameters, and B and Z are independent standard Brownian motions driving the (market) systematic and idiosyncratic risks, respectively. We may interpret $\omega > 0$ and $\epsilon \geq 0$ as systematic and idiosyncratic volatility parameters of the revenue growth. The project's total revenue growth volatility σ is then given by

$$\sigma = \sqrt{\omega^2 + \epsilon^2}. \tag{5}$$

We will show that these volatility parameters ω, ϵ, and σ have different effects on the entrepreneurial decision making. The project incurs the flow operating cost at the constant rate of w, provided that the project is not liquidated. Time-t operating profit is given by $(y_t - w)$. Note that the operating cost w generates operating leverage and hence the abandonment option value is positive.

The entrepreneur can dynamically trade the market portfolio (the risky asset) to hedge against his business risks. In general, dynamic trading in our entrepreneurial setting does not necessarily complete the market unlike the standard Black-Scholes-Merton (option pricing) paradigm. That is, when the entrepreneur bears non-diversifiable idiosyncratic risks from owning the project (i.e. when the revenue process (4) and the market portfolio return (2) are not perfectly correlated (i.e. $\epsilon > 0$)), the standard law-of-one-price valuation paradigm is no longer applicable.

In addition to having access to external equity, the entrepreneur can also incur the fixed cost to completely cash out (via either public securities markets or direct equity sale of inside equity to diversified investors). Cashing out involves fixed costs such as transaction costs for initial public offering (IPO) or brokerage fees, which particularly discourage small firms from using this diversification channel.[6] Indeed, empirically, entrepreneurial firms mostly use bank debt as the primary form of external financing, particularly for smaller ones where their business serves as a collateral with sufficient liquidation value such as commercial real estate and railroads.[7] We abstract away from other frictions such as borrowing constraints that the entrepreneur may face. Introducing financial constraints does not change the key economic mechanism of our model: the effect of non-diversifiable risk on entrepreneurial financing decisions.

The entrepreneurial firm pays taxes for both his business profits and also capital gains when the entrepreneur sells his business. Let τ_e and τ_g denote the respective tax rates on the flow business profit and capital gains upon sale. Taxes have economically interesting implications on the timing decision of entrepreneurial exit. On one hand, deferring the cash-out decision delays tax liability and lowers the present value of the fixed cash-out cost, but also lowers diversification benefits.

Provided that the fixed cost of accessing external equity K is sufficiently large, the entrepreneur

[6] While we do not formally model dilution costs of information-sensitive claims (such as external equity) due to adverse selection (Myers and Majluf (1984)), we may potentially link and relate the fixed costs to these frictions.

[7] See Vissing-Jorgensen and Moskowitz (2001), Gentry and Hubbard (2004), and Heaton and Lucas (2004)), among others.

rationally defers cashing out into the future for the standard option value argument. Therefore, at time 0, the model predicts that the entrepreneur uses external debt and equity to reduce his exposure and partially to diversify his idiosyncratic project risks as in Heaton and Lucas (2004). This diversification benefit argument does not apply to firms held by owners who have diversified portfolios as in the standard asset-pricing theory. Note that diversification benefits of debt for the entrepreneur exits in our model because debt is risky (also see this point in the static model of Heaton and Lucas (2004)).

We assume that debt is issued at par and is an interest-only debt for analytical tractability reasons as in Leland (1994) and Duffie and Lando (2001). Let b denote the coupon payment of debt and F_0 denote the par value of debt. The remaining cash flow from operation after debt service and tax payments (i.e. $(1-\tau_e)(y-w-b)$) accrues to the entrepreneur. The terms of debt issuance are determined at time 0. After debt is in place, at each point in time $t > 0$, the entrepreneur continues his project until he decides either to default on his outstanding debt which will lead to the liquidation of his firm, or to cash out by selling his firm to a diversified buyer by paying the fixed cash-out transaction cost and triggering the capital gains taxes. Both default and cash-out decisions are one-time irreversible decisions. Default and cash-out resemble American-style put and call options on the underlying non-tradeable entrepreneurial firm. By American style, we mean that the entrepreneur can endogenously time his default and cash-out decisions, both of which depend on the firm's operating performance. Neither default nor cash-out options are tradeable and cannot be evaluated by the standard dynamic replication argument (Black-Scholes-Merton) because the entrepreneurial firm's risk is not spanned by the market portfolio. The entrepreneur chooses the default and the cash-out timing policies to maximize his own utility after he chooses the time-0 debt level for the firm. The entrepreneur's default and cash-out timing strategies are not contractible at time 0. There is an inevitable conflict of interest between financiers and the entrepreneur (e.g. Jensen and Meckling (1976)).

If the entrepreneur defaults on the firm's debt at his endogenously chosen (stochastic) time T_d, the firm goes into bankruptcy. After bankruptcy, the lender takes control of the firm and liquidates the firm or sells the firm to the potential investors. Bankruptcy *ex post* is costly as in standard tradeoff models of capital structure. Assume that the liquidation/sale value of the firm is equal to a fraction α of an all equity-value (i.e. unlevered) public firm. Let $A(y)$ denote the after-tax unlevered market value of the firm. The remaining fraction $(1-\alpha)$ accounts for bankruptcy costs.

While there is potential room for the lender and the entrepreneur to renegotiate *ex post*, we abstract from this issue given the focus of our paper.

Intuitively, the entrepreneur defaults when the firm does sufficiently poorly to walk away from his liability. Assume that absolute priority is enforced. The limited liability and the entrepreneur's voluntary default imply that the entrepreneur and outside equity receive nothing upon default and the lender collects the proceeds from liquidation. Since inside and outside equity have pro rata cash flow rights, they will also contribute their pro rata equity injection when the firm's profits are insufficient to serve debt (in flow terms). Outside equity investors are willing to accept this contract because they price their equity claims in competitive markets. After liquidation, the entrepreneur is no longer exposed to the firm's idiosyncratic risk. Outside equity value is zero, i.e. $E_0(y_d) = 0$. In our generalized tradeoff model, the entrepreneur trades off diversification benefits against the default and agency costs of debt. The tax implication for the entrepreneurial firm is different from that for the public firm due to the single-layer taxation feature of the entrepreneurial firm.

When the firm does sufficiently well, the entrepreneur may find it optimal to cash out by incurring the fixed cash-out cost K. The entrepreneur then triggers capital gains taxes. As we will show below, capital gains taxes have potentially important effects on the firm's cash-out timing decision. As in the real world, the entrepreneur needs to retire the firm's debt obligation at par F_0 and to buy out exiting outside equity in order to cash out in our model. The proceeds that the entrepreneur obtains from selling his firm are fairly priced by the competitive markets. Since new owners are well diversified and do not demand the idiosyncratic risk premium, we will use the complete-markets solution to obtain the sale value of the firm. The new owners will optimally relever the firm by issuing a perpetual debt with a different coupon level b^* as in the complete-markets model of Leland (1994). Let τ_m denote the effective marginal tax rate for the public firm after the entrepreneur sells his firm. Unlike the entrepreneurial firm, the public firm is potentially subject to double taxation (at the corporation and the individual's levels) and hence we will interpret τ_m as an effective rate accordingly (Miller (1977)). The entrepreneur and the market have rational expectations and hence the entrepreneur also benefits from this releverage upon the firm's exit. To summarize, at the endogenously chosen stochastic cash-out time T_u, the entrepreneur pays the fixed transaction cost K, retires debt at par F_0, buys out the existing external equity at fair market value $E_0(y_u)$, and pays capital gains taxes to capitalize on the value of the project.

After the entrepreneur exits from his business by either defaulting on his debt or cashing out his business, he retires and has no other non-financial income. He then solves a standard complete-markets consumption and portfolio choice problem as in Merton (1971). Extending our model to allow for sequential rounds of entrepreneurial activities will complicate our analysis. We leave this extension for future research.

We finally close the model by allowing the entrepreneur to initially choose one among many mutually exclusive projects with different idiosyncratic volatilities ϵ, after he borrows F_0 from the bank and issues outside equity. The bank and outside equity have rational expectations and anticipates the entrepreneur's *ex post* choice of the project's ϵ and hence charges the corresponding fair prices *ex ante* to make zero economic profits. The time inconsistency nature of the entrepreneur's project choice induces interesting predictions on entrepreneurial decision making, idiosyncratic risk premium, and debt pricing. The heterogeneity in idiosyncratic volatility across projects allows us to analyze the standard risk shifting/asset substitution incentive (Jensen and Meckling (1976)) for non-diversified risk-averse decision makers such as entrepreneurs. Our model provides a potential reconciliation with the empirical findings that the risk-shifting problem is quantitatively a second-order issue when decision makers are not able to diversify their idiosyncratic risks.

3 Model solution

We solve the entrepreneur's decision problem as follows. First, we summarize the complete-markets solution for firm value and financing decisions when the firm is owned by diversified investors. This complete-markets solution gives the cash-out value for the entrepreneur when he sells his firm. Then, we analyze the entrepreneur's interdependent consumption/saving, portfolio choice, cash-out/default, and initial investment and financing decisions.

3.1 Complete-markets firm valuation and financing policy

Consider a public firm owned by diversified investors. Because equityholders internalize the benefits and costs of debt issuance, they will choose the firm's debt policy to maximize *ex ante* firm value by trading off the tax benefits of debt against bankruptcy and agency costs. For analytical convenience, as in Leland (1994), we assume that there is no re-adjustment of debt after initial debt issuance.[8]

[8] We abstract away from the dynamic capital structure decisions after the entrepreneur cashes out to keep the analysis tractable and also analogous to our treatment before the entrepreneur exits. While extending the model by allowing for dynamic financing adjustments will enrich the model, it complicates our analysis without changing the key economic tradeoff that we focus: the impact of idiosyncratic risk on entrepreneurial financing decisions. We leave extensions along the line of Goldstein, Ju, and Leland (2001) for future research.

See Appendix A for details of the complete-markets analysis, which essentially follows from Leland (1994), Goldstein, Ju, and Leland (2001), and Miao (2005).

Let b^* denote time-0 firm-value-maximizing coupon and $V^*(y)$ denote the corresponding levered market value of the firm as a function of the (initial) revenue y, in that

$$V^*(y) = \max_b V\left(y; y_d^*\right), \tag{6}$$

where $V\left(y; y_d\right)$ is *ex ante* firm value given by (A.17). The default threshold y_d^* in (6), which is given by (A.13), is optimally chosen by equityholders after debt is in place. Because equityholders have a default option after debt is in place, they choose y_d^* to maximize their equity value and hence may walk away from their liabilities, reflecting the conflicts of interest between equityholders and debtholders.

It is worth pointing out that $V^*(y)$ will serve as the exit value for the entrepreneur when he cashes out by selling his firm to well diversified outside investors. The new owners optimally relever the firm by choosing b^* as the above optimization problem states.

Next, we turn to analyze the entrepreneur's decision problem before he exits from his business.

3.2 Entrepreneur's decision making

We solve the entrepreneur's decision making in three steps by backward induction. First, we briefly summarize the entrepreneur's consumption/saving and portfolio choice problem after he retires from his business via either cashing out or defaulting on debt. This optimization problem is the same as in Merton (1971), a dynamic complete-markets consumption/portfolio choice problem. Second, we solve the entrepreneur's joint consumption/saving, portfolio choice, default, and cash-out decisions when the entrepreneur runs his private business. Finally, we solve the entrepreneur's initial investment and financing decision at time 0.

Conceptually, our model setup applies to any utility function $u(c)$ under technical regularity conditions. For analytical tractability, we adopt the CARA utility from now on.[9] That is, let $u(c) = -e^{-\gamma c}/\gamma$, where $\gamma > 0$ is coefficient of absolute risk aversion, which also measures precautionary motive.[10] While CARA utility does not capture wealth effects, we emphasize that the main results

[9] The CARA utility specification proves tractable in incomplete-markets consumption-saving problems with labor income. Kimball and Mankiw (1989), Caballero (1991), Svensson and Werner (1993), and Wang (2006) have all adopted this utility specification in various precautionary saving models. Miao and Wang (2007) use this utility specification to analyze a real option exercising problem when the decision maker faces uninsurable idiosyncratic risk from his investment opportunity.

[10] Precautionary saving is motivated by the consumption Euler equation analysis to focus on uncertainty of income

and insights of our paper (the effect of non-diversifiable idiosyncratic shocks on investment timing) do not rely on the choice of this utility function. As we show below, the driving force of our results is the precautionary savings effect, which is captured by utility functions with convex marginal utility such as CARA.[11] While power utility is more commonly used and more appealing for quantitative analysis, this utility specification will substantially complicate our analysis since it will lead to a numerically much harder two-dimensional double-barrier free-boundary problem.

Consumption/saving and portfolio choice after exit. After he exits from his business (via either default or cash-out), the entrepreneur retires, no longer has business income and lives on his financial wealth. The entrepreneur's optimization problem becomes the standard complete-market consumption and portfolio choice problem (e.g. Merton (1971)). We summarize the results in the appendix. Next, we turn to the entrepreneur's optimization problem before he exits.

Entrepreneur's decision making while running the firm. While running his private business, the entrepreneur faces an incomplete-markets environment. He cannot fully diversify his business risk. Before stating the main result for the entrepreneur's decision making and subjective valuation of his equity, we first describe the entrepreneur's tax implications when he cashes out. We will show that taxes has important implications on the entrepreneur's exit decision.

When the entrepreneur cashes out, he needs to retire the existing debt and buys out the existing shareholders. The new owners will relever the firm. Anticipating that the new investors will relever the firm, the existing external equityholders will demand their pro rata share of firm value, i.e. we have $E_0(y_u) = V^*(y_u)$ holds at the cash-out boundary y_u. The tax liability at cash-out is given by $\tau_g\left(\psi V^*\left(y_u\right) - K - \left(I - \left(1-\psi\right)E_0\right)\right)$.[12] The entrepreneur optimally incorporates the tax implications when making his cash-out decision. We summarize the solution for consumption/saving,

rather than on risk attitude towards wealth and consumption. Leland (1968) is among the earliest studies on precautionary saving models. Kimball (1990) links the degree of precautionary saving to the convexity of the marginal utility function $u'(c)$. By drawing an analogy to the theory of risk aversion, Kimball (1990) defines $-u'''(c)/u''(c)$ as the coefficient of absolute prudence. For CARA utility, we have $-u'''(c)/u''(c) = \gamma$.

[11] A well-known implication of CARA-utility-based models is that consumption and wealth may sometimes turn negative (see e.g. Merton (1971), Grossman (1976) and Wang (1993)). Cox and Huang (1989) provide analytical formulae for consumption under complete markets for CARA utility with non-negativity constraints. However, in our incomplete-markets setting, imposing non-negativity constraints substantially complicates the analysis. Intuitively, requiring consumption to be positive increases the entrepreneur's demand for precautionary saving because he will increase his saving today to avoid hitting the constraints in the future. The induced stronger precautionary saving demand in turn makes our results (such as diversification benefits of outside risky debt) stronger.

[12] First, note that the tax base established at time 0 is $(I - F_0 - (1-\psi)E_0)$. Second, the exit value of equity upon cash-out is given by $V^*(y_u) - F_0 - (1-\psi)E_0(y_u) - K$. Using $E_0(y_u) = V^*(y_u)$ and simplifying the formula, we obtain the tax bill upon cash-out.

portfolio choice, default trigger y_d, and cash-out trigger y_u in the following theorem.

Theorem 1 *The entrepreneur exits from his business when the revenue process $\{y_t : t \geq 0\}$ reaches either the default threshold y_d or the cash-out threshold y_u, whichever occurring the first. When the entrepreneur runs his firm, he chooses his consumption and portfolio rules as follows:*

$$\begin{aligned} \overline{c}\,(x,y) &= r\left(x+\psi G\,(y)+\frac{\eta^2}{2\gamma r^2}+\frac{\delta-r}{\gamma r^2}\right), &(7)\\ \overline{\phi}\,(x,y) &= \frac{\eta}{\gamma r \sigma_p}-\frac{\psi\omega}{\sigma_p} y G'\,(y)\,, &(8)\end{aligned}$$

where $(G(\,\cdot\,), y_d, y_u)$ solves the free boundary problem given by the differential equation:

$$rG(y) = (1-\tau_e)\,(y-b-w)+\nu y G'(y)+\frac{\sigma^2 y^2}{2}G''(y)-\frac{\psi\gamma r\epsilon^2 y^2}{2}G'(y)^2, \tag{9}$$

subject to the following (free) boundary conditions:

$$\begin{aligned} G(y_d) &= 0, &(10)\\ G'(y_d) &= 0\,, &(11)\\ \psi G(y_u) &= \psi V^*\,(y_u)-F_0-K-\tau_g\left(\psi V^*\,(y_u)-K-(I-(1-\psi)\,E_0)\right), &(12)\\ \psi G'(y_u) &= (1-\tau_g)\psi V^{*\prime}\,(y_u) &(13)\end{aligned}$$

where complete-markets firm value $V^*(y)$ is defined in (6), the value of external debt $F_0 = F(y_0)$ is given in (C.6), and the value of external equity $E_0(y)$ is given by (C.10).

The differential equation (9) provides a valuation equation for the certainty equivalent wealth $G(y)$ from the entrepreneur's perspective. Unlike the expected revenue growth rate is μ, As in standard asset pricing models evaluating corporate claims, there will be a risk premium correction to account for the systematic risk. This explains the change of revenue growth rate from μ to ν, which is the risk-adjusted expected growth rate of revenue and is given by

$$\nu = \mu - \omega\eta. \tag{14}$$

As in the standard CAPM model, only the systematic risk demands a risk premium under the complete-markets setting. In our dynamic setting, the systematic risk premium for the revenue component is $\omega\eta$. The last nonlinear term captures the intuition behind the discount due to the non-diversifiable idiosyncratic risk. Intuitively, a higher risk aversion parameter γ, a higher idiosyncratic volatility ϵ, or a more concentrated inside equity position ψ, a larger discount on $G(y)$ due to the non-diversifiable idiosyncratic risk. The next section provides more detailed analysis on the impact of idiosyncratic risk on $G(y)$.

Equation (10) comes from the value-matching condition for the entrepreneur's default decision. It states that the private value of equity $G(y)$ upon default is equal to zero. Equation (11) follows from the smooth-pasting condition. It can be interpreted as the optimality condition from the maximization of $G(y)$.

Now we turn to the cash-out boundary. Because the entrepreneur pays the fixed cost K and triggers capital gains when cashing out, he naturally has incentives to wait before cashing out. However, waiting to cash out reduces his diversification benefits *ceteris paribus*. The entrepreneur optimally trades off tax implications, diversification benefits, and transaction costs when choosing the timing of cashing out. The value-matching condition (12) at the cash-out boundary states that the private value of equity upon the firm's cashing out is equal to the after-tax value of the public firm value after the entrepreneur pays the fixed costs K, retires outstanding debt at par F_0, buys out external equity, and pays capital gains taxes. The smooth-pasting condition (13) states that the cash-out decision is optimally chosen.

Initial financing and investment decisions. Theorem 1 characterizes the entrepreneur's decisions after debt is in place.

Next, we complete the model solution by endogenizing the entrepreneur's initial investment and financing decision. The entrepreneurial firm has three financial claimants: inside equity (entrepreneur), diversified outside equity investors and outside creditors. The entrepreneur holds a non-diversifiable equity fraction ψ with a certainty equivalent value $\psi G(y)$. Diversified lenders price debt in a competitive capital markets at $F(y)$, and diversified outside equity investors hold $(1-\psi)$ fraction of total equity in the firm, priced competitively at $(1-\psi)E_0(y)$. Note that neither $F(y)$ nor $E_0(y)$ contains idiosyncratic risk premium adjustment because outside investors are diversified and demand systematic risk premium (i.e. CAPM holds for them in our model). The above analysis suggests the following natural valuation metric for the entrepreneurial firm:

$$S(y) = \psi G(y) + (1-\psi)E_0(y) + F(y). \tag{15}$$

We dub $S(y)$ as the private value of the entrepreneurial firm, and may interpret $S(y)$ as the fair price that an investor needs to pay in order to acquire the entrepreneurial firm by paying $\psi G(y)$ from the entrepreneur, $(1-\psi)E_0(y)$ and $F(y)$ from diversified outside investors.

At time 0, the entrepreneur chooses debt coupon b and initial ownership ψ to maximize the private value of the firm $S(y)$, in that the entrepreneur solves the following time-0 optimization problem:

$$\max_{b,\psi} \quad S(y_0) \,. \tag{16}$$

Intuitively, the entrepreneur internalizes the benefits and costs of external financing. In the appendix, we show that (16) indeed arises from the entrepreneur's utility maximization problem stated in (B.17). Note the conflicts of interest between the entrepreneur and external financiers. After external financing is in place, the entrepreneur chooses default and cash-out thresholds to maximize his private value of equity $G(y)$. Unlike publicly held firms (as in Leland (1994) or other structural default/credit risk models), the optimal coupon b and inside ownership ψ in our model maximizes the private value $S(y_0)$, not the market value $V(y_0) = E(y_0) + F(y_0)$ because of the non-diversifiable risk that the entrepreneur faces.

We may interpret our model's implication on capital structure as a generalized tradeoff model of capital structure for the entrepreneurial firm, where the entrepreneur trades off the benefits of outside debt/equity financing (diversification and potential tax implications) against the costs of external financing (bankruptcy and agency conflicts between the entrepreneur and outside lenders, and potential lower expected revenue growth). The objective function in our model for outside financing choice is private value of firm $S(y)$, the sum of private value of equity $\psi G(y)$, public value of equity $(1-\psi)E_0(y)$, and public value of debt $F(y)$. The natural measure of leverage (based on our optimization problem) is the ratio between public value of debt $F(y)$ and the private value of firm $S(y)$, in that

$$L(y) = \frac{G(y)}{S(y)}. \tag{17}$$

We label $L(y)$ as private leverage to reflect the impact of idiosyncratic risk on the leverage choice. Note that the entrepreneur's preference such as risk aversion influences the firm's capital structure. The standard arguments that shareholders can diversify themselves and hence diversification has no role for capital structure decisions of public firms are no longer valid for entrepreneurial firms.

So far, we have focused on the interesting parameter regions where the entrepreneur first establishes his firm as a private business and finances its operation via an optimal mix of external and internal financing. There are two special cases. The first is the one where the cost of cashing out is sufficiently small so that it is optimal for the entrepreneur to sell the firm immediately to diversified investors. That is, the cash-out option is immediately worth exercising at time 0 ($y_u < y_0$). The other special case is the one where asset recovery rate is sufficiently high or the value discount from outside equity financing is sufficiently low, or the entrepreneur is sufficiently risk averse. Then, the entrepreneur raises as much outside financing as possible (i.e. 100% sale to the lender or 100% sale to outside investors, or a mixture of external financing with outside equity and debt, whichever option is the cheapest). In our analysis below, we will consider parameter values that rule out these special cases. We next turn to the model's predictions and results.

4 Risky debt, endogenous default, and diversification

In this section, we focus on the analysis of diversification benefits of risky debt and the default option. This setting is a special case of the optimization problem characterized in Theorem 1, when the cash-out cost is infinitely large and no equity issuance at time 0.[13]

We use the following (annualized) baseline parameter values: risk-free interest rate $r = 0.03$, expected growth rate of revenue $\mu = 0.04$, systematic volatility of growth rate $\omega = 0.1$, idiosyncratic volatility $\varepsilon = 0.2$, market price of risk $\eta = 0.4$, and asset recovery rate $\alpha = 0.6$. We set the effective marginal Miller tax rate τ_m to 11.29% as in Graham (2000) and Hackbarth, Hennessy and Leland (2007).[14] In our baseline parametrization, we set $\tau_e = 0$, which reflects the fact that the entrepreneur can avoid taxes on his business income completely by deducting various expenses. Shutting down tax benefits allows us to focus on the role of the diversification benefits of debt. Later, we also consider the case with $\tau_e = \tau_m$, so that we can compare our entrepreneurial model with the complete-markets model of Section 3.1. We consider four values of the risk aversion parameter $\gamma \in \{0, 1, 2, 4\}$. In the baseline model, we also set the operating cost $w = 0$, which simplifies our numerical solution. In Section D. 2, we analyze the case with positive operating cost $w > 0$ and study the role of operating leverage. Finally, we set the initial level of revenue $y_0 = 1$.

4.1 Private value of equity $G(y)$ and default threshold

Figure 1 plots private value of equity $G(y)$ and its derivative $G'(y)$ as functions of y. The top and the bottom panels are for $\tau_e = 0$ and $\tau_e = \tau_m$, respectively. When $\tau_e = 0$, the entrepreneur with very low risk aversion ($\gamma \to 0$, effectively complete-markets) issues no debt, because there

[13]In this special case, there is only one endogenous (lower) default boundary. The upper boundary is replaced by the transversality condition: $\lim_{T\to\infty} E\left[e^{-\delta T} J^s\left(w_T, y_T\right)\right] = 0$.

[14]We may interpret τ_m as the effective Miller tax rate which integrates the corporate income tax, individual's equity and interest income tax. Using the Miller's formula for the effective tax rate , and setting the interest income tax at 0.30, corporate income tax at 0.31, and the individual's long-term equity (distribution) tax at 0.10, we obtain an effective tax rate of 11.29%.

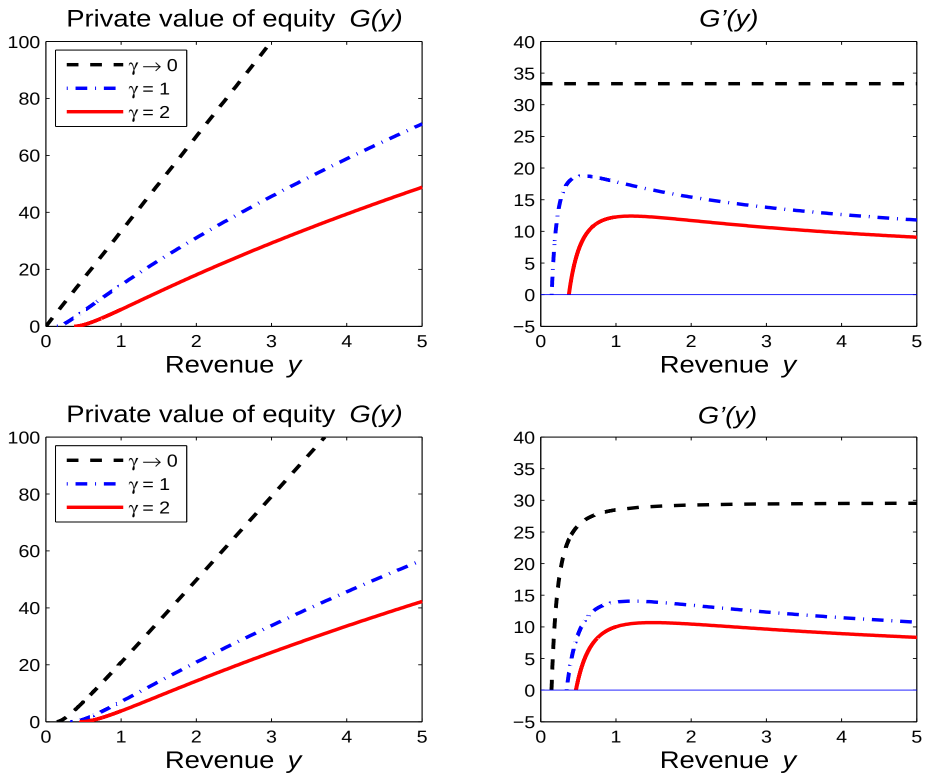

Figure 1: **Private value of equity $G(y)$: Debt financing only.** The top and bottom panels plot $G(y)$ and its first derivative $G'(y)$ for $\tau_e = 0$ and $\tau_e = \tau_m$, respectively. We plot the results for two levels of risk aversion ($\gamma = 1, 2$) and the benchmark complete-market solution ($\gamma \to 0$). The remaining parameters are: $r = \delta = 0.03$, $\eta = 0.4$, $\mu = 0.04$, $\omega = 0.1$, $\varepsilon = 0.2$, $\alpha = 0.6$, and $\tau_m = 11.29\%$.

are neither tax benefits ($\tau_e = 0$) nor diversification benefits ($\gamma \to 0$). Equity value is equal to the present discounted value of future cash flows (the straight line shown in the top-left panel). A risk-averse entrepreneur has incentives to issue debt in order to diversify idiosyncratic risks. The entrepreneur defaults when y falls to y_d, the point where $G(y_d) = G'(y_d) = 0$. When $\tau_e = \tau_m$, the entrepreneurial firm issues debt to take advantage of tax benefits in addition to diversification benefits. The bottom two panels of Figure 1 plot this case.

The derivative $G'(y)$ measures the sensitivity of private value of equity $G(y)$ with respect to revenue y. As expected, private value of equity $G(y)$ increases with revenue y, i.e. $G'(y) > 0$. Analogous to Black-Scholes-Merton's observation that firm equity is a call option on firm assets, the entrepreneur's private equity $G(y)$ also has a call option feature. For example, in the bottom panels of Figure 1 ($\tau_e = \tau_m$), when γ approaches 0 (complete markets case), equity value is convex in revenue y, reflecting its call option feature.

Unlike the standard Black-Scholes-Merton paradigm, neither the entrepreneurial equity nor the firm is tradable. When the risk-averse entrepreneur cannot fully diversify his project's idiosyncratic risks, the global convexity of $G(y)$ no longer holds, as shown in Figure 1 for cases where $\gamma > 0$. The entrepreneur now has precautionary saving demand to partially buffer the project's non-diversifiable idiosyncratic shocks. This precautionary saving effect introduces concavity in $G(y)$. The effect is stronger when the idiosyncratic volatility ϵy is larger (i.e. either ϵ or revenue y is larger). Moreover, the option (convexity) effect is smaller when the revenue y is higher (i.e. the default option is further out of the money). Thus, the precautionary saving effect dominates the option effect for sufficiently high, making $G(y)$ concave in y. When y is low, idiosyncratic volatility ϵy is low, while the default option is closer to being in the money. For sufficiently low y, the convexity effect of the default option dominates the concavity effect of precautionary saving demand, making $G(y)$ convex in y. This tradeoff explains the convexity of $G(y)$ for low y and concavity of $G(y)$ for high y.

Now, we turn to the effect of the entrepreneur's risk aversion γ on his subjective valuation and default threshold y_d. A more risk-averse entrepreneur discounts the cash flows with idiosyncratic risks more heavily, and has a stronger incentive to diversify idiosyncratic risks by retaining less of the firm. Both effects contribute to a lower private value of equity $G(y)$, as illustrated in Figure 1.

There are two effects of risk aversion on the default threshold y_d. First, the more risk-averse entrepreneur has stronger incentives to issue debt, which in turn calls for a larger coupon b and hence a higher default threshold, *ceteris paribus*. Second, the more risk-averse entrepreneur discounts future cash flows more heavily, which makes him exercise the default option earlier given the same level of coupon b. This effect also leads to a higher default threshold. Figure 1 confirms that the default threshold y_d increases in risk aversion γ.

4.2 Capital structure for entrepreneurial firms

Next, we analyze the impact of non-diversifiable idiosyncratic risks on the entrepreneurial firm's capital structure. To highlight the role of idiosyncratic risks in a simplest possible way, we consider two scenarios. We first consider the special case where debt has no tax benefits for the entrepreneur (i.e. $\tau_e = 0$), and then incorporate the tax benefits of debt into our analysis.

The top panel in Table 1 provides results for the entrepreneurial firm's capital structure when $\tau_e = 0$. If the entrepreneur is very close to being risk neutral ($\gamma \to 0$), the model's prediction is essentially the same as the complete-market benchmark. In this case, the standard tradeoff theory

Table 1: Capital Structure of Entrepreneurial Firms: Debt financing only.

This table reports the results for the setting where the entrepreneur only has access to debt financing and hence has a subsequent default option. The parameters are: $r = \delta = 0.03$, $\eta = 0.4$, $\mu = 0.04$, $\omega = 0.1$, $\varepsilon = 0.2$, and $\alpha = 0.6$. These parameters are annualized when applicable. The initial revenue is $y_0 = 1$. We report results for two business income tax rates ($\tau_e = 0, \tau_m(11.29\%)$) and three levels of risk aversion. The case "$\gamma \to 0$" corresponds to the complete-markets (Leland) model.

	coupon b	public debt F_0	private equity G_0	private firm S_0	private leverage (%) L_0	credit spread (bp) CS	10-yr default probability (%) $p_d(10)$
				$\tau_e = 0$			
$\gamma \to 0$	0.00	0.00	33.33	33.33	0.0	0	0.0
$\gamma = 1$	0.31	8.28	14.39	22.68	36.5	72	0.4
$\gamma = 2$	0.68	14.66	5.89	20.55	71.3	166	12.1
				$\tau_e = \tau_m$			
$\gamma \to 0$	0.35	9.29	20.83	30.12	30.9	75	0.3
$\gamma = 1$	0.68	14.85	7.02	21.86	67.9	159	9.5
$\gamma = 2$	0.85	16.50	3.77	20.27	81.4	213	22.3

of capital structure implies that the entrepreneurial firm will entirely financed by equity. The risk-neutral entrepreneur values the firm at its market value 33.33.

For $\gamma = 1$, the entrepreneur borrows $F_0 = 8.28$ in market value with coupon $b = 0.31$, and values his non-tradable equity G_0 at 14.39, giving the private value of the firm $S_0 = 22.68$. Note the substantial 32% drop of S_0 from 33.33 to 22.68. This drop of S_0 is not primarily due to the default risk premium of the risky debt, since the 10-year cumulative default probability is only 0.4% and the implied credit spread on the perpetual debt is 72 basis points. Instead, the significant drop is mainly due to the entrepreneur's subjective valuation discount of his non-tradable equity position for bearing non-diversifiable idiosyncratic risks.

The natural measure of leverage for entrepreneurial firms is private leverage L_0, which is given by the ratio of public debt value F_0 and private value of the firm S_0. As we have discussed, L_0 captures the entrepreneur's tradeoff between private value of equity and public value of debt in choosing debt coupon policy. For $\gamma = 1$, the private leverage ratio is about 36.5%.

With a higher risk aversion level $\gamma = 2$, the entrepreneur borrows more ($F_0 = 14.66$) with a higher coupon ($b = 0.68$).[15] He values his remaining non-tradable equity at $G_0 = 5.89$, and the implied private leverage ratio $L_0 = 71.3\%$ is much higher than 36.5%, the value for $\gamma = 1$. The more risk-averse entrepreneur takes on more leverage, which is consistent with the diversification benefits argument: the more risk-averse entrepreneur has incentives to sell more of the firm. The high leverage ratio (i.e. 71.3%) gives rise to a higher credit spread (166 basis points over the risk-free rate), and a higher 10-year cumulative default probability (12.1%), i.e. non-investment-grade debt. Despite the substantially higher default risk and lower equity value, the private value of the firm S_0 for $\gamma = 2$ is 20.55, only about 9% lower than 22.68, the value for $\gamma = 1$. The reason is that the entrepreneur with $\gamma = 2$ takes on more debt, and thus the increase in the market value of debt partially offsets the decrease in private equity value.

Next, we incorporate the effect of tax benefits for the entrepreneur into our generalized tradeoff model of capital structure for entrepreneurial firms. To compare with the complete-markets benchmark, we set $\tau_e = \tau_m = 11.29\%$. Therefore, the only difference between an entrepreneurial firm and a public firm is that the entrepreneur faces non-diversifiable idiosyncratic risks.

The second panel of Table 1 reports the results. The first row in this panel gives the results for the complete-market benchmark. Facing positive corporate tax rates, the public firm has incentives to issue debt, but is also concerned with bankruptcy costs. By the standard tradeoff theory, the public firm optimally issues debt at the market value $F_0 = 9.29$ with coupon $b = 0.35$, which gives 30.9% initial leverage and 0.3% 10-year cumulative default probability.

Similar to the case with $\tau_e = 0$, an entrepreneur facing non-diversifiable idiosyncratic risks has incentives to issue more risky debt to diversify these risks. The second panel of Table 1 shows that the entrepreneur with $\gamma = 1$ borrows an amount 14.85 (with the coupon rate $b = 0.68$), higher than the level $b = 0.35$ for the public firm. The private leverage more than doubles to 67.9%. As a result, the entrepreneur faces a higher default probability and the credit spread of his debt is also higher. With $\gamma = 2$, the amount of debt rises to 16.50, private leverage to 81.4%.

4.3 Determinants of capital structure decisions

We conduct two numerical experiments in Table 2 to further demonstrate the important role of idiosyncratic risks in determining the capital structure of entrepreneurial firms. For comparison, we include in the first and the last row of this table the results for the entrepreneurial firm with

[15] This result does not always hold. We will return to this point when we analyze the setting when $\tau_e = \tau_m$. Two effects: 1. Sell more debt to diversify; 2. Higher default boundary reduces the entrepreneur's ability to issue outside risky debt, which leads to a lower coupon.

Table 2: DECOMPOSITION OF PRIVATE LEVERAGE FOR ENTREPRENEURIAL FIRMS

This table compares a private firm owned by a risk-averse entrepreneur with a public firm that has the same amount of debt outstanding (coupon is fixed at $b = 0.85$). There is no option to cash out. The model parameters are: $r = \delta = 0.03$, $\eta = 0.4$, $\mu = 0.04$, $\omega = 0.1$, $\varepsilon = 0.2$, $\alpha = 0.6$ and $\tau_e = \tau_m$. These parameters are annualized when applicable. All the results are for initial revenue $y_0 = 1$.

	10-yr default probability (%) $p_d(10)$	public debt F_0	equity value G_0	firm value S_0	financial leverage (%) L_0	credit spread (bp) CS
$\gamma = 2$ ($b = 0.85$, $y_d = 0.47$)	22.3	16.50	3.77	20.27	81.4	213
Public ($b = 0.85$, $y_d = 0.47$)	22.3	16.50	11.10	27.60	59.8	213
Public ($b = 0.85$, $y_d = 0.35$)	9.8	17.71	11.56	29.26	60.5	178
Public ($b = 0.35$, $y_d = 0.14$)	0.3	9.29	20.82	30.11	30.9	75

$\gamma = 2$ and for the public firm, respectively.

In the first experiment, we suppose that a public firm issues the same amount of debt $b = 0.85$ and defaults at the same threshold $y_d = 0.47$ as the entrepreneurial firm with $\gamma = 2$. Given these values of coupon b and default threshold y_d, we can calculate the implied market value of equity $E_0 = E(y_0; b, y_d)$ and the market value of the firm $V_0 = V^*(y_0; b, y_d)$. The market leverage is given by the ratio between the market value of debt F_0 and the market value of the firm V_0. Since $E(y; b, y_d) > G(y; b, y_d)$, the imputed market leverage overstates the value of equity for the entrepreneur by ignoring the idiosyncratic risk premium ($E_0 = 11.10$ and $G_0 = 3.77$), thus leading to a leverage ratio 59.8%, substantially lower than the firm's private leverage $L_0 = 81.4\%$. The large difference between the private and market leverage ratios highlights the economic significance of taking idiosyncratic risks into account in order to correctly compute the value of equity.

In the second experiment, we highlight the impact of the entrepreneur's *endogenous* default decision on the leverage ratio. We consider a public firm that has the same technology/environment parameters as the entrepreneurial firm. Moreover, the public firm has the same debt coupon b on the outstanding perpetual debt as the entrepreneurial firm does ($b = 0.85$). The key difference is that the default threshold y_d is endogenously determined.

The default threshold y_d for the public firm is 0.35, lower than the threshold $y_d = 0.47$ for the entrepreneurial firm. Intuitively, facing the same coupon b, the entrepreneurial firm defaults earlier than the public firm because the entrepreneur discounts the future cash flows more heavily due to the non-diversifiable idiosyncratic risk. The implied shorter distance-to-default for the entrepreneurial firm translates into higher 10-year default probability (22% for the entrepreneurial firm versus 10% for the public firm) and higher credit spread (213 basis points for the entrepreneurial firm versus 178 basis points for the public firm). Despite the significant difference in the default thresholds, the leverage ratio for this public firm is close to that of the public firm with the same default threshold as the private firm.

The preceding two experiments helps attribute the differences in leverage ratio between the entrepreneurial firm and the public firm to three effects: the discount effect, the default threshold effect, and the diversification effect. First, with the same coupon and default threshold, the entrepreneur's subjective discount factor lowers the private value of equity and raises the private leverage substantially. Second, again with the same coupon, the entrepreneurial firm defaults earlier than the public firm, which reduce the value of debt and lower the leverage ratio, but the effect is small. Third, the need for diversification makes the entrepreneur issue more debt than the public firm, which substantially raises the leverage ratio of the entrepreneurial firm. While the numerical results are parameter specific, the analysis provides support for our intuition that the entrepreneur's need for diversification and subjective valuation discount for bearing non-diversifiable idiosyncratic risks are key determinants of the private leverage for an entrepreneurial firm.

5 External debt/equity financing, default and cash-out options

We now turn to a richer and more realistic setting where the entrepreneur can diversify idiosyncratic risks by selling both outside debt and equity at time 0. The entrepreneur avoids the downside risk by defaulting if the firm's stochastic revenue falls sufficiently low. In addition, when the firm does well enough, the entrepreneur may want to capitalize on the upside by selling the firm (issuing outside equity) to diversified investors.

In addition to the baseline parameter values from Section 4, we set the effective capital gains tax rate from selling the business $\tau_g = 0.10$, reflecting the tax deferral advantage of the tax timing option. We set the initial investment cost for the project $I = 10$, which is about 1/3 of the market value of project cash-flows. We choose the cash-out cost $K = 27$ to generate a 10-year cash-out probability of 20% (with $\gamma = 2$), consistent with the success rates of venture capital firms in the data (e.g, Hall and Woodward (2008)).

We first study the effects of the cash-out option. Figure 2 plots the private value of equity $G(y)$ and its first derivative $G'(y)$ for an entrepreneur with risk aversion $\gamma = 1$ when he has the option

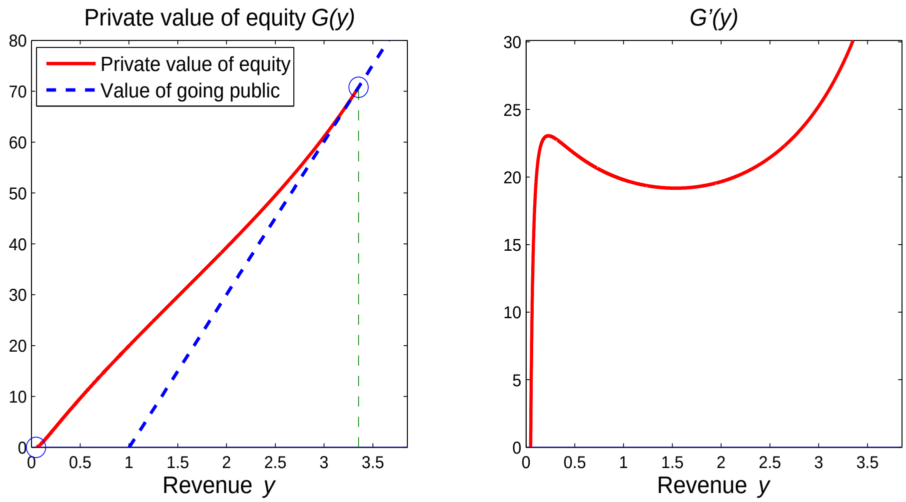

Figure 2: **Private value of equity $G(y)$ as functions of revenue y: the case of debt and equity financing.** We plot the results with the following parameters: $\gamma = 1$, $r = \delta = 0.03$, $\eta = 0.4$, $\mu = 0.04$, $\omega = 0.1$, $\varepsilon = 0.2$, $\alpha = 0.6$, $\tau_e = 0$, $\tau_m = 11.29\%$, $\tau_g = 10\%$, $I = 10$, and $K = 27$.

to cash out but cannot sell external equity. The function $G(y)$ smoothly touches the horizontal axis on the left and the dash line denoting the value of cashing out on the right. The two tangent points give the default and cash-out thresholds, respectively. For sufficiently low values of revenue y, the private value of equity $G(y)$ is increasing and convex because the default option is deep in the money, generating convexity. For sufficiently high values of y, $G(y)$ is also increasing and convex because the cash-out option is deep in the money. For revenue y in the intermediate range, neither default nor cash-out option is deep in the money. In this range, the precautionary saving motive may be large enough to induce concavity. As shown in the right panel of Figure 1, $G'(y)$ first increases for low values of y, then decreases for intermediate values of y, and finally increases for high values of y.

Panel A of Table 3 provides the capital structure information of an entrepreneurial firm with cash-out option. Since external equity is not allowed in this case, we fix $\psi = 1$. When the market is complete, the firm's cash-out option is essentially an option to adjust the firm's capital structure. In this case, given our calibrated fixed cost K, the 10-year cash-out probability is essentially zero and hence this option value is close to zero for the public firm. Therefore, we expect that the bulk of the cash-out option value for entrepreneurial firms comes from the diversification benefits, not from the option value of readjusting leverage.

When $\gamma = 1$, the presence of the cash-out option lowers the initial coupon to $b = 0.55$ from

Table 3: CAPITAL STRUCTURE OF ENTREPRENEURIAL FIRMS: EXTERNAL EQUITY AND CASH-OUT OPTION

This table reports the results for the setting where the entrepreneur has access to both public debt and equity financing, and cash-out option to exit from his project. The parameters are: $r = \delta = 0.03$, $\eta = 0.4$, $\mu = 0.04$, $\omega = 0.1$, $\varepsilon = 0.2$, $\alpha = 0.6$, $I = 10$, $K = 27$, $\tau_e = \tau_m = 11.29\%$, $\tau_g = 10\%$, and $y_0 = 1$.

	ownership	public debt	public equity	private equity	private firm	private leverage (%)	10-yr default prob (%)	10-yr cash-out prob (%)
	ψ	F_0	$(1-\psi)E_0$	ψG_0	S_0	L_0	$p_d(10)$	$p_u(10)$
				A. Cash-out option				
$\gamma \to 0$	1.00	9.29	0	20.83	30.12	30.9	0.3	0.0
$\gamma = 1$	1.00	12.45	0	9.57	22.02	56.5	4.2	12.3
$\gamma = 2$	1.00	13.68	0	6.24	19.92	68.7	10.1	23.3
				B. External equity and cash-out option				
$\gamma \to 0$	1.00	15.23	0.00	30.07	45.30	33.6	0.4	0.0
$\gamma = 1$	0.69	16.00	8.50	8.26	32.76	48.8	3.8	11.3
$\gamma = 2$	0.65	15.93	9.50	5.67	31.10	51.2	6.0	15.4

$b = 0.68$ for the firm which only has the default option. The private leverage ratio L_0 at issuance is 56.5%, with a credit spread at 138 basis points, compared to the private leverage ratio $L_0 = 67.9\%$ and credit spread 159 basis points when the firm only has the default option. The 10-year default probability is close to zero, but the 10-year cash-out probability is 12.3%, which is economically significant (recall that the 10-year cash-out probability for a public firm is zero). For a higher risk aversion level $\gamma = 2$, the private leverage ratio is 68.7%, smaller than 81.4% for the setting without the cash-out option. Given the opportunity to sell his business to public investors, the entrepreneur substitutes away from risky debt and relies more on the future potential of cashing out to diversify his idiosyncratic risks.

Besides public debt and cash-out option, external equity, e.g. venture capital, is another channel for an entrepreneur to diversify away the idiosyncratic risks. In fact, if it is costless to issue external equity, an risk-averse entrepreneur will want to sell the entire firm to the VC right away. We motivate the costs of external equity through the agency problems of Jensen and Meckling (1976). The more external equity is issued at $t = 0$, the less incentive the entrepreneur will have to exert effort ex post, which we capture in reduced form by linking the expected revenue growth rate to the entrepreneur ownership. More specifically, we model the growth rate μ as a quadratic function of the entrepreneur ownership ψ, $\mu(\psi) = -0.02\psi^2 + 0.04\psi + 0.03$, with $\psi \in [0, 1]$. The functional form is chosen such that the maximum expected growth rate is 5%, when the entrepreneur owns the entire firm ($\psi = 1$), while the lowest growth rate is 3%, when the entire firm is sold ($\psi = 0$). The parameters for $\mu(\psi)$ are chosen to keep the agency costs of external equity modest so as to highlight the substitution effect of external equity.

The results are reported in Panel B of Table 3. If the entrepreneur is risk-neutral, he will clearly prefer to keep 100% ownership. In this case, all the equity in the firm are privately held, the private leverage is 33.6%, and the 10-year probability of default and cash-out are both close to 0. An entrepreneur with $\gamma = 1$ lowers his ownership drops to 69%, which reduces the growth rate to 4.8% (only a 0.2% drop). However, the coupon rises from 0.55 to 0.66, and private leverage rises from 33.6% to 48.8%. The increase in demand for debt due to diversification is economically sizeable, especially considering that the increase is partially offset by the reduced tax benefit of debt due to lower expected growth rates. The 10-year default and cash-out probability both rise, to 3.8% and 11.3% respectively. When $\gamma = 2$, the ownership drops to 65%, while the coupon further rises 0.68, and leverage to 51.2%. The 10-year cash-out probability also rises to 15.4%.

These results demonstrate that entrepreneurial firms have sizeable demand on public debt and cash-out option for diversification purpose even when the agency costs of external equity are small.

6 Idiosyncratic risk, leverage, and risk premia

In this section, we show how idiosyncratic volatility affects leverage and risk premia. Figure 3 shows its effect on leverage. As is well known in the complete markets model, an increase in (idiosyncratic) volatility ϵ raises default risk, hence the market leverage ratio and the coupon rate for the public firm decrease with idiosyncratic volatility. By contrast, facing incomplete markets, risk-averse entrepreneurs want to take on more debt to diversify their idiosyncratic risks when the idiosyncratic volatility is higher. For sufficiently low risk aversion and ϵ, the standard effect of high default risk reducing debt capacity still dominates, which explains the initial drop in coupon and leverage for the case $\gamma = 0.5$. However, for $\gamma = 1$, both coupon and leverage become monotonically increasing in ϵ. This result implies that the private leverage ratio for entrepreneurial firms increases with idiosyncratic volatility even for mild risk aversion.

We next study the impact of idiosyncratic volatility on the risk premium that an entrepreneur

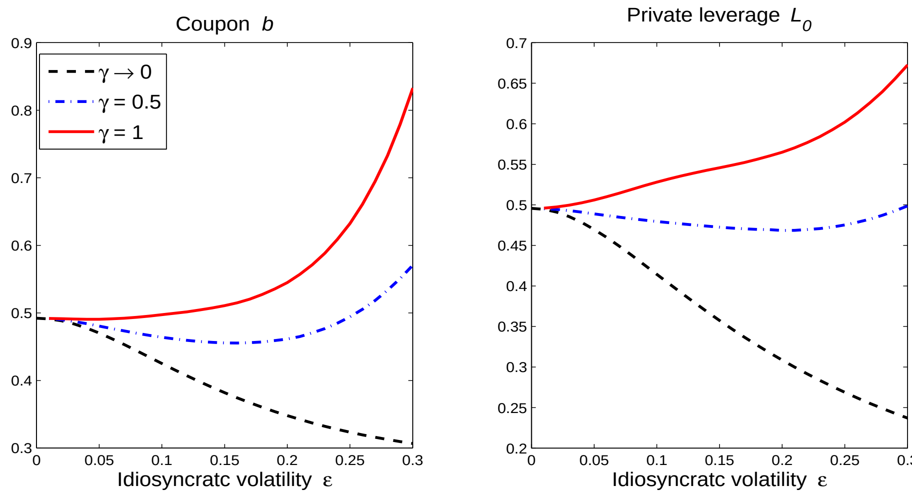

Figure 3: **Comparative statics – optimal coupon and private leverage with respect to idiosyncratic volatilities ϵ: the case of debt and equity financing.** The two panels plot the optimal coupon b and the corresponding optimal private leverage L_0 at $y_0 = 1$. In each case, we plot the results for two levels of risk aversion ($\gamma = 1, 2$) alongside the benchmark complete-market solution ($\gamma \to 0$). The remaining parameters are: $r = \delta = 0.03$, $\eta = 0.4$, $\mu = 0.04$, $\omega = 0.1$, $\alpha = 0.6$, $\tau_e = \tau_m = 11.29\%$, $\tau_g = 10\%$, $I = 10$, and $K = 27$.

demands. We decompose the entrepreneur's risk premium into two components: the systematic risk premium $\pi^s(y)$ and the idiosyncratic risk premium $\pi^i(y)$. In the appendix, we show that the systematic risk premium $\pi^s(y)$ and the idiosyncratic risk premium $\pi^i(y)$ may be respectively expressed as follows:

$$\pi^s(y) = \eta\omega \frac{G'(y)}{G(y)} y = \eta\omega \frac{d \log G(y)}{d \log y}, \tag{18}$$

$$\pi^i(y) = \frac{\gamma r}{2} \frac{(\epsilon y G'(y))^2}{G(y)}. \tag{19}$$

The systematic risk premium $\pi^s(y)$ defined in (18) takes the same form as in standard asset pricing models. It is the product of (market) Sharpe ratio η, systematic volatility ω, and the elasticity of $G(y)$ with respect to y, where the elasticity captures the impact of optionality on risk premium. Despite this standard interpretation for the systematic risk premium, it is worth pointing out that $\pi^s(y)$ also indirectly reflects the non-diversifiable idiosyncratic risks that the entrepreneur bears, and risk aversion γ indirectly affects $\pi^s(y)$ through its impact on $G(y)$.

Unlike $\pi^s(y)$, the idiosyncratic risk premium $\pi^i(y)$ defined in (19) *directly* depends on risk aversion γ, and $(\epsilon y G'(y))^2$, the conditional (idiosyncratic) variance of the entrepreneur's equity $G(y)$. The conditional (idiosyncratic) variance term reflects the fact that the idiosyncratic risk

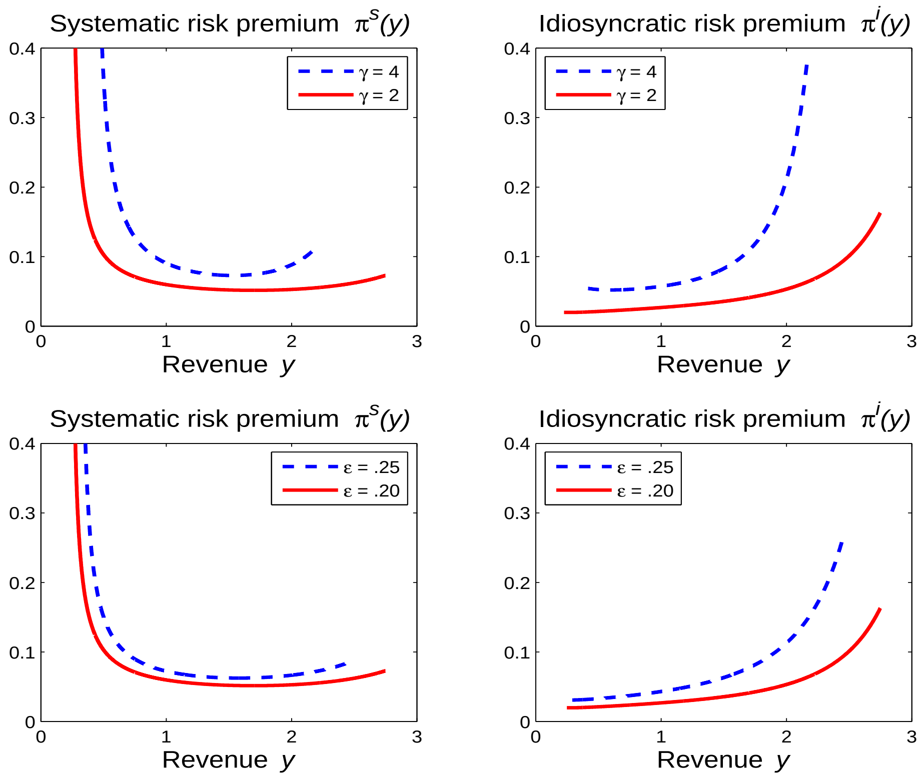

Figure 4: **Systematic and idiosyncratic risk premia as functions of revenue y: the case of debt financing and cash-out optin.** This figure plots the systematic and idiosyncratic risk premia that the entrepreneur demands for holding the inside equity of a private firm. The top panels plot the results for two levels of risk aversion ($\gamma = 2, 4$). The bottom panels plot the results for two levels of idiosyncratic volatility ($\epsilon = 0.20, 0.25$).The remaining parameters are: $\gamma = 2$ (when changing ϵ), $r = \delta = 0.03$, $\eta = 0.4$, $\mu = 0.04$, $\omega = 0.1$, $\epsilon = 0.2$ (when changing γ), $\alpha = 0.6$, $\tau_e = 0$, $\tau_m = 11.29\%$, $\tau_g = 10\%$, $I = 10$, and $K = 27$.

premium $\pi^i(y)$ is determined by the entrepreneur's precautionary saving demand, which depends on the conditional variance of idiosyncratic risks (Caballero (1991) and Wang (2006)).

We examine the behavior of these risk premia at the default and cash-out thresholds in Figure 4. First consider the default threshold. The entrepreneur's equity is a levered position in the firm. When the firm approaches default, the systematic component of the risk premium $\pi^s(y)$ behaves similarly to the standard valuation model. That is, the significant leverage effect around the default boundary implies that the risk premium diverges to infinity when y approaches y_d.

The idiosyncratic risk premium $\pi^i(y)$ behaves quite differently. Using the boundary conditions (12)-(13), we deduce that both idiosyncratic and systematic risk premia are finite at the cash-out threshold. Figure 4 indicates that idiosyncratic risk premium peaks up at the cash-out threshold, while systematic risk premia are much smaller. Both idiosyncratic and systematic risk premium may not be monotonic with respect to the revenue y. Systematic risk premia tend to be small when both default and cash-out options are not deep in the money. Idiosyncratic risk premia tend to be small when the firm is close to default.

7 Project choice: Asset substitution versus risk sharing

So far we have focused on the entrepreneur's financing decisions assuming that the entrepreneur has made the investment in the project. We now turn to the effect of non-diversifiable risk on the entrepreneur's project choice decision. Jensen and Meckling (1976) point out that there is an important incentive problem associated with debt. They argue that after debt is in place, managers have incentive to take very risky projects due to the convexity feature of equity payoffs. However, there is little empirical evidence in support of risk shifting.[16] We argue that risk aversion and precautionary saving demand substantially mitigate the benefit of asset substitution and potentially overturn the Jensen and Meckling risk shifting argument.

We consider the following project choice problem. Suppose the risk-averse entrepreneur can choose among a continuum of mutually exclusive projects with different idiosyncratic volatilities ϵ in the interval $[\epsilon_{\min}, \epsilon_{\max}]$ *after debt is in place.* Let F_0 be the market value of existing debt with the coupon payment b. The entrepreneur then chooses idiosyncratic volatility $\epsilon^+ \in [\epsilon_{\min}, \epsilon_{\max}]$ to maximize his own utility. As shown in Section 3, the entrepreneur effectively chooses ϵ^+ to maximize his private value of equity $G(y_0)$, taking the debt contract (b, F_0) as given. Let this maximized value be $G^+(y_0)$.

In a rational expectation equilibrium, the lender anticipates the entrepreneur's *ex post* incentive of choosing the level of idiosyncratic volatility ϵ^+ to maximize $G(y_0)$, and prices the initial debt contract accordingly in the competitive capital markets. Therefore, the entrepreneur *ex ante* maximizes the private value of the firm, $S(y_0) = G^+(y_0) + F_0$, taking the competitive market debt pricing into account. This joint investment and financing problem is a fixed point problem.

Figure 5 illustrates the solution of this optimization problem. We set $\epsilon_{\min} = 0.05, \epsilon_{\max} = 0.35$. For simplicity, we focus on the case without external equity ($\psi = 1$). When $\gamma \to 0$, the entrepreneur chooses the highest idiosyncratic volatility project with $\epsilon_{\max} = 0.35$. The optimal coupon payment

[16] See Andrade and Kaplan (1998), Graham and Harvey (2001), Rauh (2008), among others.

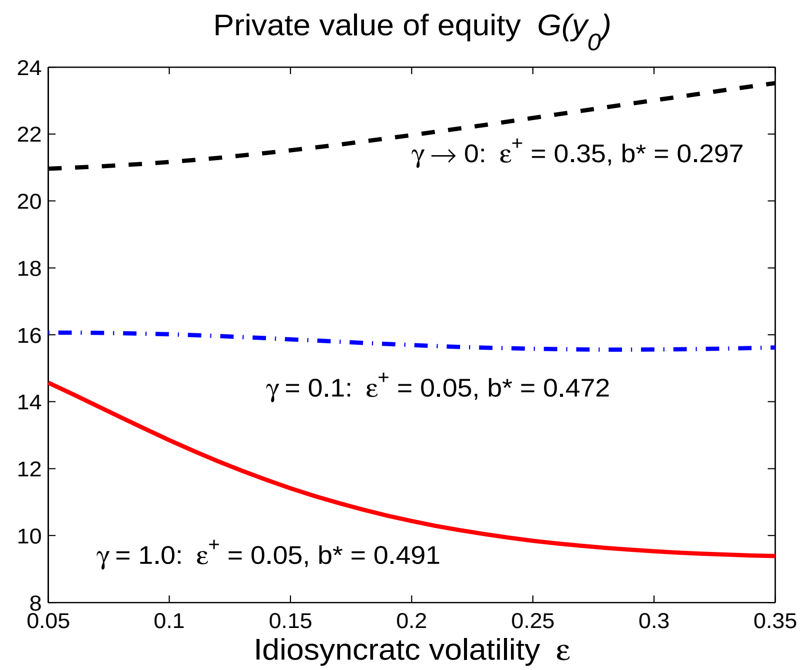

Figure 5: **Private equity value as function of idiosyncratic volatility after optimal debt is in place.** This figure plots the private value of equity for different choices of idiosyncratic volatility ϵ after debt issuance. The coupon is fixed at the optimal value corresponding to given risk aversion. The remaining parameters are: $r = \delta = 0.03$, $\eta = 0.4$, $\mu = 0.04$, $\omega = 0.1$, $\epsilon_L = 0.05$, $\epsilon_L = 0.35$, $\alpha = 0.6$, $\tau_e = \tau_m = 11.29\%$, $\tau_g = 10\%$, $I = 10$, and $K = 27$.

is 0.297. In this case, the entrepreneur effectively faces complete markets. The Jensen and Meckling (1976) argument applies because the market value of equity is convex and the asset substitution problem arises. When the entrepreneur is risk averse, he demands a premium for bearing the non-diversifiable idiosyncratic risks, which tends to lower his private value of equity $G(y_0)$. When this effect dominates, the entrepreneur prefers projects with lower idiosyncratic volatility. For example, for $\gamma = 1.0$, the entrepreneur chooses the optimal project with $\epsilon_{\min} = 0.05$, with the corresponding optimal coupon payment 0.491. Even when the degree of risk aversion is low (e.g., $\gamma = 0.1$, which implies an idiosyncratic risk premium of 2 basis points for $\epsilon = 0.05$, or 20 basis points for $\epsilon = 0.20$), we still find that the risk aversion effect dominates the risk shifting incentive.

From this numerical example, we find that in our model the precautionary saving incentive tends to dominate the asset substitution incentive. Importantly, our argument applies to publicly traded firms as well, provided that managerial compensation is tied to firm performance and managers are not fully diversified. This suggests that the reason that we do not find much empirical evidence supporting asset substitution may be simply due to risk aversion/precautionary saving.

8 Conclusion

Entrepreneurial investment opportunities are often illiquid and non-tradable. Entrepreneurs cannot costlessly diversify away from the significant project-specific risks potentially for reasons such as incentives and informational asymmetry. Therefore, standard law-of-one-price-based valuation/capital structure paradigm in corporate finance cannot be directly applied to entrepreneurial finance. An entrepreneur acts as both a household choosing consumption/saving and portfolio decisions, and a producer making dynamic investment and financing decisions for his business project. The entrepreneur's dual and interdependent roles as both a household and a producer motivate us to develop a neoclassic dynamic finance framework (Ross (2005)) for entrepreneurial finance by building on the non-diversification feature of the entrepreneurial business.

In the baseline setting where the external financing is risky debt, we show that more risk-averse entrepreneurs choose to take on more leverage for diversification benefits. Therefore, the bank charges a higher credit spread and the entrepreneur ends up default earlier ceteris paribus. By essentially the same diversification argument, two otherwise identical firms, the entrepreneurial firm shall be more levered and riskier than the public firm.

External equity and cash-out option provide additional channels for diversification, which reduces the attractiveness of leverage as the diversification instrument. We show that the capital budgeting calculation for the entrepreneur firm goes beyond the CAPM, even if CAPM holds for an otherwise identical public firm. In addition to demand compensation for the systematic risk premium, the entrepreneur also requires compensation for the systematic risk premium, which increases with his risk aversion, his equilibrium inside ownership, and idiosyncratic variance. Finally, we revisit the classic insight that equity is a levered (call) position on the firm and hence the entrepreneur has strong incentives to engage in excessive risk taking (Jensen and Meckling (1976). We show that this asset substitution effect significantly weakens even for entrepreneurs with low to moderate levels of risk aversion.

Our paper also makes methodological contributions to financial valuation and decision making. For example, this paper builds and extends the standard complete-markets Black-Scholes-Merton option pricing and real options methodology to settings where investment opportunities are illiquid and not marked-to-market, and decision makers are not-diversified. It is straightforward to see that our model also applies to valuation of non-diversified executives' stock option and the impact of executives' decision making on a firm's capital structure and investment decisions.

We do not model the fundamental frictions causing markets to be incomplete and entrepreneurs to be non-diversified. We view endogenous incomplete markets as a complementary perspective, which can have fundamental implications such as promotion of entrepreneurship and contract design. We leave these questions for future research.

Appendices

A Market Valuation and Capital Structure of a Public Firm

Well-diversified owners of a public firm face complete markets. Given the Sharpe ratio η of the market portfolio and the riskfree rate r, there exists a unique stochastic discount factor (SDF) $(\xi_t : t \geq 0)$ satisfying (see Duffie (2001)):

$$d\xi_t = -r\xi_t dt - \eta\xi_t dB_t, \ \xi_0 = 1. \tag{A.1}$$

Using this SDF, we can derive the market value of the unlevered firm, $A(y)$, the market value of equity, $E(y)$, and the market value of debt $D(y)$. The market value of the firm is equal to the sum of equity value and debt value:

$$V(y) = E(y) + D(y). \tag{A.2}$$

By the Girsanov theorem, under the risk-neutral probability measure Q, the standard Brownian motion B^Q satisfies $dB_t^Q = dB_t + \eta dt$ (see Duffie (2001)). We then rewrite the dynamics of the cash flows (4) as follows:

$$dy_t = \nu y_t dt + \omega y_t dB_t^Q + \epsilon y_t dZ_t, \tag{A.3}$$

where ν is the risk-adjusted drift defined by $\nu \equiv \mu - \omega\eta$.

A. 1 Valuation of Public Unlevered Firm

We start with the after-tax unlevered firm value $A(y)$, which satisfies the following differential equation:

$$rA(y) = (1-\tau_m)(y-w) + \nu y A'(y) + \frac{1}{2}\sigma^2 y^2 A''(y). \tag{A.4}$$

This is a second-order ordinary differential equation (ODE). We need two boundary conditions to obtain a solution. One boundary condition describes the behavior of $A(y)$ when $y \to \infty$. This condition must rule out speculative bubbles. To ensure $A(y)$ is finite, we assume $r > \nu$ throughout the paper. The other boundary condition is related to abandonment. As in the standard option exercise models, the firm is abandoned whenever the cash flow process hits a threshold value y_a for the first time. At the threshold y_a, the following value-matching condition is satisfied

$$A(y_a) = 0, \tag{A.5}$$

because we normalize the outside value to zero. For the abandonment threshold y_a to be optimal, the following smooth-pasting condition must also be satisfied:

$$A'(y_a) = 0. \tag{A.6}$$

Solving equation (A.4) and using the no-bubble condition and boundary conditions (A.5)-(A.6), we obtain

$$A(y) = (1-\tau_m)\left[\left(\frac{y}{r-\nu}-\frac{w}{r}\right)-\left(\frac{y_a}{r-\nu}-\frac{w}{r}\right)\left(\frac{y}{y_a}\right)^{\theta_1}\right], \tag{A.7}$$

where the abandonment threshold y_a is given in

$$y_a = \frac{r-\nu}{r}\frac{\theta_1}{\theta_1-1}w, \tag{A.8}$$

where

$$\theta_1 = -\sigma^{-2}\left(\nu-\sigma^2/2\right)-\sqrt{\sigma^{-4}\left(\nu-\sigma^2/2\right)^2+2r\sigma^{-2}} < 0. \tag{A.9}$$

We next turn to the valuation and capital structure decision of a levered firm.

A. 2 Valuation of Public levered Firm

First, consider the market value of equity. Let y_d be the corresponding default threshold. After default, equity is worthless, in that $E(y) = 0$ for $y \leq y_d$. This gives us the value matching condition $E(y_d) = 0$. Before default, equity value $E(y)$ satisfies the following differential equation:

$$rE(y) = (1-\tau_m)(y-w-b)+\nu yE'(y)+\frac{1}{2}\sigma^2y^2E''(y),\ y \geq y_d. \tag{A.10}$$

When $y \to \infty$, $E(y)$ also satisfies a no-bubble condition. Solving this ODE and using the boundary conditions, we obtain

$$E(y;y_d) \quad = \quad (1-\tau_m)\left[\left(\frac{y}{r-\nu}-\frac{w+b}{r}\right)-\left(\frac{y_d}{r-\nu}-\frac{w+b}{r}\right)\left(\frac{y}{y_d}\right)^{\theta_1}\right]. \tag{A.11}$$

Equation (A.11) shows that equity value is equal to the after-tax present value of profit flows minus the present value of the perpetual coupon payments plus an option value to default. The term $(y/y_d)^{\theta_1}$ may be interpreted as the "risk-adjusted" Arrow-Debreu price of a unit of claim contingent on the event of default. Using the smooth-pasting condition,

$$\left.\frac{\partial E(y)}{\partial y}\right|_{y=y_d} = 0, \tag{A.12}$$

we obtain the optimal default threshold given by

$$y_d^* = \frac{r-\nu}{r}\frac{\theta_1}{\theta_1 - 1}\left(b+w\right). \tag{A.13}$$

After debt is in place, there is a conflict between equityholders and debtholders. Equityholders choose the default threshold y_d to maximize equity value $E(y; y_d)$.

The market value of debt before default satisfies the following differential equation:

$$rD\left(y\right) = b + \nu y D'\left(y\right) + \frac{1}{2}\sigma^2 y^2 D''\left(y\right),\ y \geq y_d. \tag{A.14}$$

The value-matching condition is given by:

$$D\left(y_d\right) = \alpha A\left(y_d\right). \tag{A.15}$$

We also imposes a no bubble condition when $y \to \infty$. Solving the valuation equation, we have

$$D\left(y\right) = \frac{b}{r} - \left[\frac{b}{r} - \alpha A\left(y_d\right)\right]\left(\frac{y}{y_d}\right)^{\theta_1}, \tag{A.16}$$

For given coupon rate b and default threshold y_d, using equation (A.2), we may write the market value of the levered firm value $V\left(y; y_d\right)$ as follows:

$$V\left(y; y_d\right) \quad = \quad A\left(y\right) + \frac{\tau_m b}{r}\left[1 - \left(\frac{y}{y_d}\right)^{\theta_1}\right] - \left(1-\alpha\right)A\left(y_d\right)\left(\frac{y}{y_d}\right)^{\theta_1}, \tag{A.17}$$

Equation (A.17) shows that the levered market value of the firm is equal to the after-tax unlevered firm value plus the present value of tax shields minus bankruptcy costs.

While y_d^* is chosen to maximize $E(y)$, coupon b is chosen to maximize *ex ante* firm value $V(y)$. Substituting (A.13) into (A.17) and using the following first-order condition:

$$\frac{\partial V\left(y_0\right)}{\partial b} = 0, \tag{A.18}$$

we obtain the optimal coupon rate b^* as a function of y_0. We also verify that the second order condition is satisfied.

Now consider the special case without operating cost (i.e. $w = 0$). We have an explicit expression for the optimal coupon:

$$b^* = y_0 \frac{r}{r-\nu}\frac{\theta_1 - 1}{\theta_1}\left(1 - \theta_1 - \frac{\left(1-\alpha\right)\left(1-\tau_m\right)\theta_1}{\tau_m}\right)^{1/\theta_1}. \tag{A.19}$$

Substituting (A.13) and (A.19) into (A.17), we obtain the following expression for $V^*(y)$, firm value when debt coupon is optimally chosen:

$$V^*(y) = \left[1 - \tau_m + \tau_m \left(1 - \theta_1 - \frac{(1-\alpha)(1-\tau_m)\theta_1}{\tau_m}\right)^{1/\theta_1}\right] \frac{y}{r-\nu}. \tag{A.20}$$

Note that this firm value formula only applies at the moment of debt issuance and will equal to firm value when the entrepreneur cashes out.

B Proof of Theorem 1

First, consider the entrepreneur's optimality after he exits. The entrepreneur solves the standard complete-markets consumption/portfolio choice problem (Merton (1971)). His wealth follows from the following dynamics

$$dx_t = \left(r\left(x_t - \phi_t\right) - c_t\right) dt + \phi_t \left(\mu_p dt + \sigma_p dB_t\right). \tag{B.1}$$

The entrepreneur's value function $J^e(x)$ is given by the following explicit form:

$$J^e(x) = -\frac{1}{\gamma r} \exp\left[-\gamma r \left(x + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2}\right)\right]. \tag{B.2}$$

The consumption and portfolio rules are given by

$$\overline{c}(x) = r\left(x + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2}\right), \tag{B.3}$$

$$\overline{\phi}(x) = \frac{\eta}{\gamma r \sigma_p}. \tag{B.4}$$

Next, turn to the entrepreneur's decision making while he runs the private firm. The entrepreneur receives his non-tradeable business income, pays operating costs, services the debt and pays taxes until he exits via either default or cash-out. Before exiting from his business, the entrepreneur's financial wealth evolves as follows:

$$dx_t = \left(r\left(x_t - \phi_t\right) + \psi\left(1 - \tau_e\right)(y - b - w) - c_t\right) dt + \phi_t \left(\mu_p dt + \sigma_p dB_t\right), \tag{B.5}$$

for $t < \min(T_d, T_u)$, where T_d is the default time and T_u is the cash-out time. The entrepreneur receives $(1 - \tau_e)(y_t - w - b)$ from his business in flow terms after servicing the debt and paying taxes.

Using the principle of optimality, we claim that the value function $J^s(x,y)$ satisfies the following HJB equation:

$$\begin{aligned}\delta J^s(x,y) &= \max_{c,\phi} u(c) + \big(rx + \phi\left(\mu_p - r\right) - c + \psi\left(1-\tau_e\right)(y-b-w)\big) J_x^s(x,y) \\ &\quad + \mu y J_y^s(x,y) + \frac{(\sigma_p \phi)^2}{2} J_{xx}^s(x,y) + \frac{\sigma^2 y^2}{2} J_{yy}^s(x,y) + \phi \sigma_p \omega y J_{xy}^s(x,y)\,. \quad \text{(B.6)}\end{aligned}$$

The first-order conditions (FOC) for consumption c and portfolio allocation ϕ are as follows:

$$u'(c) = J_x^s(x,y)\,,\ \ \phi = \frac{-J_x^s(x,y)}{J_{xx}^s(x,y)}\left(\frac{\mu_p - r}{\sigma_p^2}\right) + \frac{-J_{xy}^s(x,y)}{J_{xx}^s(x,y)}\frac{\omega y}{\sigma_p}, \quad \text{(B.7)}$$

Conjecture that the value function $J^s(x,y)$ is exponential in wealth and is given by equation (B.8), where $G(y)$ is a function to be determined.

Let $J^s(x,y)$ denote the entrepreneur's value function. We conjecture that $J^s(x,y)$ takes the following explicit exponential form:

$$J^s(x,y) = -\frac{1}{\gamma r}\exp\left[-\gamma r\left(x + \psi G(y) + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2}\right)\right], \quad \text{(B.8)}$$

where $G(y)$ is given in Theorem 1. As shown in Miao and Wang (2007), $G(y)$ is the entrepreneur's certainty equivalent wealth per unit of the entrepreneur's inside equity of the firm. We will refer to $G(y)$ as the entrepreneur's private value of equity.

Under the conjectured value function (B.8), we show that the optimal consumption rule and the portfolio rule are given by (7) and (8), respectively. Substituting these expressions back into the HJB equation (B.6) gives the differential equation (9) for $G(y)$.

We now turn to boundary conditions. First, consider the lower default boundary. At the instant of default, the entrepreneur walks away from his firm's liability and the lender liquidates the firm's asset. The entrepreneur's financial wealth x does not change immediately after default, in that $x_{T_d} = x_{T_d-}$. In addition, the entrepreneur's value function should remain unchanged at the moment of default. That is, the following value-matching condition holds along the default boundary as in standard option exercising problems:

$$J^s(x,y) = J^e(x). \quad \text{(B.9)}$$

The above equation implicitly defines the lower default boundary for cash flow y as a function of wealth x: $y = y_d(x)$. Note that in general, the default boundary depends on the entrepreneur's wealth level. Because the default boundary is optimally chosen, the following smooth-pasting conditions at $y = y_d(x)$ must be satisfied:[17]

$$\frac{\partial J^s(x,y)}{\partial x} = \frac{\partial J^e(x)}{\partial x}, \tag{B.10}$$

$$\frac{\partial J^s(x,y)}{\partial y} = \frac{\partial J^e(x)}{\partial y}. \tag{B.11}$$

The first smooth-pasting condition (B.10) states that the marginal change in cash flow y has the same marginal effect on the entrepreneur's value functions just before and immediately after defaulting on the firm's liability. Similarly, the second smooth-pasting (B.11) states that the marginal effect of wealth x must be the same on the entrepreneur's value functions just before and immediately after defaulting on the firm's liability. Unlike the complete-markets endogenous default models (Leland (1994)), the entrepreneur's financial wealth x enters as an additional state variable, which gives rise to the second smooth-pasting condition.

Next, consider the upper cash-out boundary. At the instant of cashing out, the entrepreneur sells his firm to well diversified investors and collects firm value $V^*(y)$ given in (A.17). Since the entrepreneur needs to pay the fixed cost K, retire debt at par F_0, buys out existing outside equity, and pay capital gains taxes, his wealth x_{T_u} immediately after cashing out satisfies

$$x_{T_u} = x_{T_u-} + V^*(y_{T_u}) - F_0 - (1-\psi)E_0(y_u) - K - \tau_g\left(\psi V^*(y_{T_u}) - K - (I - (1-\psi)E_0)\right). \tag{B.12}$$

Note that $V^*(y_{T_u}) = E_0(y_u)$. The entrepreneur's value function satisfies the following value-matching condition:

$$J^s(x,y) = J^e(x + \psi V^*(y) - F_0 - K - \tau_g\left(\psi V^*(y) - K - (I - (1-\psi)E_0)\right)). \tag{B.13}$$

This equation implicitly determines the upper cash-out boundary $y_u(x)$. Using the same arguments as those for the lower default boundary, the entrepreneur's optimality implies the following smooth-pasting conditions at $y = y_u(x)$:

$$\frac{\partial J^s(x,y)}{\partial x} = \frac{\partial J^e(x + \psi V^*(y) - F_0 - K - \tau_g\left(\psi V^*(y) - K - (I - (1-\psi)E_0)\right))}{\partial x}, \tag{B.14}$$

$$\frac{\partial J^s(x,y)}{\partial y} = \frac{\partial J^e(x + +\psi V^*(y) - F_0 - K - \tau_g\left(\psi V^*(y) - K - (I - (1-\psi)E_0)\right))}{\partial y}. \tag{B.15}$$

Using the conjectured value function (B.8), we show that the default and cash-out boundaries $y_d(x)$ and $y_u(x)$ are independent of wealth. We thus simply use y_d and y_u to denote the default and

[17] See Krylov (1980), Dumas (1991), and Dixit and Pindyck (1994).

cash-out thresholds, respectively. Using the value matching and smooth pasting conditions (B.9)-(B.11) at y_d, we obtain (10) and (11). Similarly, using the value-matching and smooth-pasting conditions (B.13)-(B.15) at y_u, we have (12) and (13).

Having characterized the entrepreneurial firm's decision problem after time 0 and before exit, we now turn to the firm's time-0 investment and financing decision. Let $\underline{x}$ denote the entrepreneur's endowment of financial wealth. If the entrepreneur chooses to start his business, his time-0 financial wealth x_0 immediately after financing his business is equal to endowment $\underline{x}$ minus his own equity contribution $(I - F_0 - (1-\psi)E_0)$, which is given by the difference between the investment cost I and external financing raised (the sum of debt proceeds F_0 and external equity $(1-\psi)E_0$), in that

$$x_0 = \underline{x} - (I - F_0 - (1-\psi)E_0). \tag{B.16}$$

At time zero, the entrepreneur chooses a coupon rate b to solve the following problem:

$$\max_b J^s\left(\underline{x} + F_0 + (1-\psi)E_0 - I, y_0\right), \tag{B.17}$$

subject to the requirement that outside debt and equity are competitively priced, i.e. $F_0 = F(y_0)$, and $E_0 = E_0(y_0)$. In Appendix C, we provide an explicit formulae for $F(y)$ and $E_0(y)$.

Therefore, the entrepreneur launches the project if his value function from the project is higher than the value function without the project, i.e.,

$$\max_b J^s\left(\underline{x} + F_0 + (1-\psi)E_0 - I, y_0\right) > J^e\left(\underline{x}\right). \tag{B.18}$$

In Section 6, we analyze the model's predictions on risk premia. We rewrite the valuation equation (9) for the entrepreneur's private value of equity $G(y)$ as follows:

$$\pi^s(y) + \pi^i(y) = \frac{(1-\tau_e)(y-b-w)}{G(y)} + \frac{1}{G(y)}\left(\mu y G'(y) + \frac{\sigma^2 y^2}{2} G''(y)\right) - r, \tag{B.19}$$

The first term and the second term on the right side of (B.19) measure the current yield and the expected percentage change in private value of equity $G(y)$, respectively. The sum of these two terms is the total expected return for private value of equity $G(y)$. Subtracting the risk-free rate from the expected return gives the total risk premium for $G(y)$.

At the default boundary y_d, we can show that the idiosyncratic risk premium $\pi^i(y_d)$ is given by

$$\pi^i(y_d) = \lim_{y \to y_d} \frac{\gamma r}{2} \frac{\left(\epsilon y G'(y)\right)^2}{G(y)} = \gamma r (\epsilon y_d)^2 G''(y_d). \tag{B.20}$$

Using $G''(y_d)$ implied by Theorem 1, we may write $\pi^i(y_d) = -2\gamma r(1-\tau_e)(y_d - b - w)\epsilon^2/\sigma^2$, which is always positive (due to the option value of default, i.e., $y_d < b + w$). That is, the quadratic dependence of the idiosyncratic risk premium $\pi^i(y)$ on $G'(y)$ makes its value finite at the default boundary y_d.

C Market Values of the Entrepreneurial Firm's Outside Debt and Equity

When the entrepreneur neither defaults nor cashes out, the market value of his debt $F(y)$ satisfies the following ODE:

$$rF(y) = b + \nu y F'(y) + \frac{1}{2}\sigma^2 y^2 F''(y), \; y_d \le y \le y_u. \tag{C.1}$$

At the default trigger y_d, debt recovers the fraction α of after-tax unlevered firm value, in that $F(y_d) = \alpha A(y_d)$. At the cash-out trigger y_u, debt is retired and recovers its face value, in that $F(y_u) = F_0$. Solving (C.1) subject to these boundary conditions gives

$$F(y) = \frac{b}{r} + \left(F_0 - \frac{b}{r}\right)\overline{q}(y) + \left[\alpha A(y_d) - \frac{b}{r}\right]\underline{q}(y), \tag{C.2}$$

where

$$\overline{q}(y) = \frac{y^{\theta_1} y_d^{\theta_2} - y^{\theta_2} y_d^{\theta_1}}{y_u^{\theta_1} y_d^{\theta_2} - y_u^{\theta_2} y_d^{\theta_1}}, \tag{C.3}$$

$$\underline{q}(y) = \frac{y^{\theta_2} y_u^{\theta_1} - y^{\theta_1} y_u^{\theta_2}}{y_u^{\theta_1} y_d^{\theta_2} - y_u^{\theta_2} y_d^{\theta_1}}. \tag{C.4}$$

Here, θ_1 is given by (A.9) and

$$\theta_2 = -\sigma^{-2}\left(\nu - \sigma^2/2\right) + \sqrt{\sigma^{-4}\left(\nu - \sigma^2/2\right)^2 + 2r\sigma^{-2}} > 1. \tag{C.5}$$

Equation (C.2) admits an intuitive interpretation. It states that debt value is equal to the present value of coupon payment plus the changes in value when default occurs and when cash-out occurs. Note that $\overline{q}(y_0)$ can be interpreted as the present value of a \$1 if cash-out occurs before default, and $\underline{q}(y_0)$ can be interpreted as the present value of a \$1 if the entrepreneur goes bankrupt before cash-out. Using $F_0 = F(y_0)$, we have that the initial debt issuance is given by

$$F_0 = \frac{b}{r} - \left(\frac{b}{r} - \alpha A(y_d)\right)\frac{\underline{q}(y_0)}{1 - \overline{q}(y_0)}. \tag{C.6}$$

Similarly, for outside equity claim, we have the following valuation equation:

$$rE_0(y) = (1-\tau_e)(y-w-b) + \nu y F'(y) + \frac{1}{2}\sigma^2 y^2 F''(y), \; y_d \le y \le y_u, \tag{C.7}$$

subject to the following boundary conditions:

$$E_0(y_u) = V^*(y_u), \tag{C.8}$$

$$E_0(y_d) = 0. \tag{C.9}$$

Solving the above valuation equation, we have that the value of outside equity $E_0(y)$ is given by

$$\begin{aligned} E_0(y) = \; & (1-\tau_e)\left(\frac{y}{r-\nu} - \frac{w+b}{r}\right) + \left[V^*(y_u) - (1-\tau_e)\left(\frac{y_u}{r-\nu} - \frac{w+b}{r}\right)\right]\overline{q}(y) \\ & -(1-\tau_e)\left(\frac{y_d}{r-\nu} - \frac{w+b}{r}\right)\underline{q}(y). \end{aligned} \tag{C.10}$$

The initial outside equity issuance E_0 is then given by $E_0 = E_0(y_0)$.

D Capital gain taxes and operating leverage

D. 1 Effects of capital gain taxes

In the presence of capital gains taxes with $\tau_g = 0.10$, the benefit from cash-out falls. Table D.1 shows that the 10-year cash-out probability decreases, and the entrepreneur takes on more debt in order to diversify idiosyncratic risks. However, the quantitative effects are small in our numerical example. We may understand the intuition from the value-matching condition (12). At the cash-out threshold y_u, when $\psi = 1$, the entrepreneur obtains less value $(1-\tau_g)V^*(y_u)$, but enjoys tax rebate $\tau_g(K+I)$. Thus, these two effects partially offset each other, making the effect of capital gains taxes small. Clearly, if the cash-out value is sufficiently large relative to the cash-out and investment costs, then the effect of the capital gains tax should be large.

D. 2 Effects of operating leverage

How does operating leverage affect an entrepreneurial firm's financial leverage? Intuitively, operating leverage increases financial distress risk, and thus should limit debt financing. Panel 3 of Table D.2 confirms this intuition for the complete-markets case (the limiting case with $\gamma \to 0$). As the operating cost w increases from 0.2 to 0.4, the 10-year default probability rises from 2.2% to 6.2%, and the firm issues less debt. On the other hand, equity value also decreases because operating

Table D.1: CAPITAL STRUCTURE OF ENTREPRENEURIAL FIRMS: CAPITAL GAIN TAXES

This table reports the results for the setting where the entrepreneur has both default and cash-out options to exit from his project. The parameters are: $r = \delta = 0.03$, $\eta = 0.4$, $\mu = 0.04$, $\omega = 0.1$, $\varepsilon = 0.2$, $\alpha = 0.6$, $I = 10$, and $K = 27$. The initial revenue is $y_0 = 1$. We report results for two business income tax rates ($\tau_e = 0, \tau_m(11.29\%)$), two capital gain tax rates ($\tau_g = 0, 10\%$), and two levels of risk aversion ($\gamma = 1, 2$). The case "$\gamma \to 0$" corresponds to the complete-market model, where the "cash-out" option effectively allows the firm to adjust leverage once.

	coupon b	public debt F_0	private equity G_0	private firm S_0	private leverage (%) L_0	credit spread (bp) CS	10-yr default probability (%) $p_d(10)$	10-yr cash-out probability (%) $p_u(10)$
					$\tau_e = 0,\ \tau_g = 0$			
$\gamma = 1$	0.11	3.20	19.95	23.14	13.8	32	0.0	12.3
$\gamma = 2$	0.42	10.11	10.36	20.47	49.4	115	1.9	24.1
					$\tau_e = \tau_m,\ \tau_g = 0$			
$\gamma = 1$	0.54	12.29	9.92	22.22	55.3	138	3.9	15.5
$\gamma = 2$	0.66	13.57	6.47	20.04	67.7	186	9.8	26.8

costs lower the operating profits. As a result, the effect on financial leverage ratio is ambiguous. In our numerical examples, this ratio increases with operating costs.

Our analysis above shows that risky debt has important diversification benefits for entrepreneurial firms. This effect may dominate the preceding "crowding-out" effect of operating leverage. Table D.2 confirms this intuition. As w increases from 0.2 to 0.4, an entrepreneur with $\gamma = 1$ raises debt with increased coupon payments from 0.59 to 0.62. However, the market value of debt decreases because both the 10-year default probability and the cash-out probability increase with w. The private equity value also decreases with w and this effect dominates the decrease in debt. Thus, the private leverage ratio rises with operating costs. This result also holds true for a more risk-averse entrepreneur with $\gamma = 2$. Note that the more risk-averse entrepreneur relies more on risky debt to diversify risk. As a result, the 10-year default probability increases substantially from 26.9% to 50.6% for $\gamma = 2$. But the 10-year cash-out probability decreases from 23.7% to 22.3%.

Table D.2: THE EFFECTS OF OPERATING LEVERAGE: THE CASE OF DEBT FINANCING AND CASH-OUT OPTION

This table reports the results for the setting where the entrepreneur has both default and cash-out options to exit from his project. The parameters are: $r = \delta = 0.03$, $\eta = 0.4$, $\mu = 0.04$, $\omega = 0.1$, $\varepsilon = 0.2$, $\alpha = 0.6$, $\tau_e = \tau_m$, $\tau_g = 10\%$, $I = 10$, and $K = 27$. The initial revenue is $y_0 = 1$. We report results for two levels of risk aversion ($\gamma = 1, 2$) alongside the complete-market solution ($\gamma \to 0$).

	coupon	public debt	private equity	private firm	private leverage (%)	credit spread (bp)	10-yr default probability (%)	10-yr cash-out probability (%)
	b	F_0	G_0	S_0	L_0	CS	$p_d(10)$	$p_u(10)$
					$\gamma \to 0$			
$w = 0.2$	0.35	8.03	16.73	24.76	32.4	132	2.2	0.0
$w = 0.4$	0.33	6.72	13.40	20.12	33.4	194	6.2	0.0
					$\gamma = 1$			
$w = 0.2$	0.59	10.94	6.34	17.28	63.3	237	14.1	13.1
$w = 0.4$	0.62	9.41	3.98	13.39	70.3	356	28.4	13.5
					$\gamma = 2$			
$w = 0.2$	0.73	11.95	3.57	15.53	77.0	315	26.9	23.7
$w = 0.4$	0.84	10.48	1.57	12.05	86.9	503	50.6	22.3

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