Firm's Cash Holdings and the Cross–Section of Equity Returns *

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First draft: October 2007 This version: 9 August 2008

Preliminary and incomplete: please do not circulate

Abstract

This paper documents a new empirical finding: an investing strategy that is long in stocks of firms with a high cash-to-assets ratio (High Cash portfolio) and short in stocks of firms with a low cash-to-assets ratio (Low Cash portfolio) produces an average excess return of 42 basis points per month. The Fama-French three factors are not able to explain such a difference in average returns, while a *cash factor (HCMLC)* does. I propose a structural model of the firm's investing and savings decisions that rationalizes the empirical evidence relating corporate cash holdings to the average excess equity returns. I amend the real option model of the firm of Berk, Green, and Naik [1999], to allow for a non-trivial capital structure decision. In my setup firms can finance investment by means equity or retained earnings. Equity issuance involves pecuniary costs such as bankers and lawyers' fees. Corporate savings allow the firm to avoid costly external financing, but yield a return which is lower than shareholders would be able to obtain. Because of risky cash flows, the model generates an additional precautionary savings motive – absent in a risk neutral environment like the one of Riddick and Whited [2008] – that is the key ingredient to explain the positive relationship between corporate cash holdings and average equity returns found in the data.

Keywords: Equity Returns, Precautionary Savings, Growth Options

JEL Classification Numbers : G12 G32 D92

^{*}I am very grateful to Gian Luca Clementi, Douglas Gale, Sydney Ludvigson, and Tom Sargent for their guidance and encouragement. I also thank Francisco Barillas, Alberto Bisin, Jonathan Halkett, Matthias Kredler, Guido Ruta, Gianluca Violante, Neng Wang, and the participants at the NYU Financial Economics Workshop, the NYU Macro Student Lunch Seminar, and the New York FED brown bag seminar

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1 Introduction

The relationship between corporate cash holdings and equity returns has been historically overlooked. Starting with Bhandari [1988], empirical and theoretical studies of the relationship between firms' capital structure and the cross–section of equity returns have focused on the debt–to–equity ratio as the only important explanatory variable.¹ In this paper, I document a positive correlation between the cash–to–assets ratio and the average excess returns and I develop a model to rationalize such a finding.

In the first part of the paper, I perform cross-sectional estimations à la Fama and French [1992] and show that corporate cash holdings and firms' equity returns are positive correlated once I control for the *size* and *value* effects. I also investigate whether a strategy that is long in stocks of firms with a high cash-to-assets ratio (High Cash portfolio) and short in stocks of firms with a low cash-to-assets ratio (Low Cash portfolio) produces an excess return. Following the approach suggested by George and Hwang [2008], I find that the High Cash portfolio earns on average an excess return of 42 basis points per month.

This excess return cannot be explained by the Fama–French three factors. In addition, these factors are not able to explain the excess returns earned by High Cash firms in 75 portfolios generated by a conditional sorting on size, book–to–market and cash holdings. On the other hand, a *cash factor* (*HCMLC*), constructed following the procedure used by George and Hwang [2008] to generate their *leverage factor*, successfully explain the differences in returns, suggesting that *HCMLC* captures a source of risk which is different from the one(s) proxied by the Fama–French factors.

In the second part of the paper, I propose a model of financing and investment decisions of the firm to give a structural interpretation to the positive relationship between a firm's optimal cash policy and its expected equity returns. I amend the real option model of Berk et al. [1999], to allow for a non-trivial capital structure decision. In my setup firms can finance investment by means of retained earnings or equity. Equity issuance involves pecuniary costs such as bankers' and lawyers' fees. Corporate savings allow the firm to avoid costly equity financing, but yield a return that is lower than what shareholders would be able to obtain.

At one extreme, consider a firm that only issues debt. Gomes and Schmid [2008] show that, each time a growth option is exercised, the firm becomes less risky and more levered. Using this argument, they successfully rationalize the negative relationship between book leverage and average excess returns. At the other extreme, I consider a firm that can only issue equity, but it has the option to retain earnings. Since both financing policies are costly, a natural trade–off arises between the choices of distributing dividends

¹See Gomes and Schmid [2008] and references therein.

in the current period and to retain cash avoiding costly external financing in the future. This trade-off determines the firm's optimal saving policy. Besides, I show that when cash flows are correlated with an aggregate shock, riskier firms (e.g. firms with cash flows highly correlated with the aggregate shock) save more *ceteris paribus*. This additional precautionary savings motive – absent in a risk neutral environment like the one of Riddick and Whited [2008] – is the key ingredient that allows the model to generate a positive correlation between expected equity returns and firms' cash holdings.

I convey the main intuition through a simple three-period model. This model, however, cannot be used to simulate a cross-section of firms that I can use to replicate the empirical analysis performed with the data. For this reason, I develop an infinite horizon model. Given its large scale, I study its properties by means of numerical analysis. I perform a calibration exercise and show that the model is able to generate the positive relationship between corporate savings and average equity returns found in the data.

The trade-off between costly external financing and costly cash holdings has already been studied by Kim et al. [1998].² They describe the optimal cash policy of a firm in a three-period model with risk neutral investors and constant risk-free interest rate. Their framework is able to explain many empirical regularities among which the negative correlation of cash holdings with book-to-market and firm size and the positive correlation of cash holdings with the firm's growth options. A the three-period environment, however, is not suitable for simulating a cross-section of firms to replicate, at a theoretical level, the cross sectional regressions performed in empirical studies.

On the other hand, the infinite horizon nature of dynamic trade–off models allows the researcher to use them to replicate the empirical analysis performed with the data. This class of models examines optimal investment decisions together with a non–trivial capital structure choice thus providing a viable theoretical tool to address the problem of endogeneity that afflicts static regressions.

One example can be found in Riddick and Whited [2008].³ They propose an infinite horizon model in which the trade–off between costly equity financing and costly accumulation of cash determines the firm's optimal investment and financing choices and generates a negative firm's propensity to save out of cash flows: a result that contradicts the one in Almeida et al. [2004].

²Other models that provide a theory of optimal corporate savings choice are Almeida et al. [2004] and Acharya et al. [2007] Both of these models share with the work of Kim et al. [1998] the three–periods structure and the risk–neutral environment, but not the trade–off between costly external financing and costly accumulation of cash.

³Other recent examples are Hennessy and Whited [2005] and Tserlukevich [2008]. Both models successfully rationalize the inverse relationship between profitability and leverage, the mean reverting nature of leverage and the dependence between leverage and past profitability of the firm. For a recent extended survey on trade-off and pecking-order theories of capital structure see Frank and Goyal [2007].

My model differs in two dimensions. First and most important, I assume an exogenous stochastic discount factor and risky cash flows so that my model can generate time varying expected returns and an additional precautionary saving motive. Second, Riddick and Whited [2008] use a neoclassical production technology while I use a real option framework with irreversible investment.

Other two examples are the models of Li [2007] and Gomes and Schmid [2008]. They share with my paper the focus on the relationship between firms' investment and financing decisions and the cross section of equity returns.

Motivated by the empirical work of Gompers et al. [2003], Li [2007] builds a dynamic model to evaluate the effect of corporate governance on the cross-section of equity returns. In her set-up, a manager can costly divert resources in a way that induces overinvestment in booms and a slow disinvestment during recessions. Because of this behavior, firms with a better governance earn higher excess returns during booms and lower excess returns during recessions if compared to firms with poor governance.

In Gomes and Schmid [2008], a firm can finance investment using equity or debt. Equity issuance is costly, corporate debt is risky but interest payments are tax-deductible. Because of the tax advantage of debt, an increase in the capital stock implies debt issuance. Hence, a firm that invests becomes less risky – a smaller fraction of its value is determined by risky growth options – and more levered. This mechanism generates a realistic negative relationship between a firm's book leverage and equity returns. Like in Li [2007], the presence of risky corporate debt also allows them to study the effect of investing and financing policies on credit spreads.

Both Li [2007] and Gomes and Schmid [2008] do not allow the firm to retain earnings. In their models cash can only be distributed as dividends to the firm's shareholders or invested in the current period to increase the installed capital. This feature prevents them to analyze the effect of optimal cash holding policies on the cross-section of equity returns.

The outline of the paper is as follows. Section 2 contains the empirical analysis. In Section 3, I propose a simple financing problem in a three–period framework that highlights how a precautionary savings motive can generate the positive correlation found in the data. In Section 4, I characterize the relevant state variables for the infinite horizon model. Section 5 describes the calibration procedure, the simulated optimal financing policies and the results of the simulated cross–sectional regressions. The last section concludes the paper.

2 Cash holdings and the Cross–Section of Equity Returns: Facts

2.1 Data Description

The accounting data are from Compustat Annual. I exclude utilities (SIC codes between 4900 and 4949) and financial companies (SIC codes between 6000 and 6999) because these sectors are subject to heavy regulation.

I construct the book–to–market ratio following the procedure suggested by Fama and French [1993]. ⁴ Companies with a negative book–to–market ratio are excluded from the sample.

The book value of leverage at the end of year t is defined as long term debt (item 9 in Compustat) plus current liabilities (item 34) at the end of year t divided by firm's total assets (item 6) at the end of year t. The cash-to-assets ratio is defined as the value of corporate cash holdings (item 1) over the value of the firm's assets (item 6) net of corporate cash holdings.

Stock prices and quantities are form CRSP. I only consider ordinary common shares (share codes 10 and 11 in CRSP) and I exclude observations relative to suspended, halted, or non–listed shares. I also require that a stock has reported returns for at least 12 months prior to portfolio formation. The monthly risk–free interest rate and the observations relative to the Fama–French factors are taken from Kenneth French's website. ⁵

2.2 Fama–Macbeth Regressions

I start by running cross-sectional regressions in the spirit of Fama and French [1992].⁶ This exercise allows me to check if corporate cash holdings and average stock returns are correlated. Starting in June 1967 and ending in June of 2007, I regress the realized monthly excess equity returns from July of year t to June of year t + 1 on market capitalization evaluated at June of year t (Size), on book-to-market (BM), and on cash-to-assets (CH). The last two variables are evaluated using the data available for the fiscal year ending in year t - 1. In Figure 1, I plot the time-series of the cross-sectional

⁴ "We define book common equity, BE, as the COMPUSTAT book value of stockholders' equity, plus balance sheet deferred taxes and investment tax credit (if available), minus the book value of preferred stock. Depending on availability, we use the redemption, liquidation, or par value (in that order) to estimate the value of preferred stock. Book-to-market equity, BE/ME, is then book common equity for the fiscal year ending in calendar year t - 1, divided by market equity at the end of December of t - 1."(Fama and French [1993], pag. 8)

⁵ http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html ⁶In particular, see their Table 3.

Table 1: Cross–Sectional Correlations

log Size	1.00		
log Book-to-Market	-0.28**	1.00	
$\log Cash-to-Assets$	-0.03*	-0.22**	1.00

Each year from 1967 to 2006, I calculate the cross-sectional correlations among market capitalization evaluated at June of year t, on book-to-market and on cash-to-assets. The last two variables are evaluated using the data for the fiscal year ending in year t - 1. The numbers reported in the Table are the time series averages of the cross-sectional correlations. * indicates a significance level of 5%. ** indicates a significance level of 1%. T-statistics are evaluated using the Newey-West methodology.

correlations among Size, CH, and BM. The corresponding time-series averages are reported in Table 1. Book-to-market is negative correlated to firms size and cash holdings. The correlation between size and cash holdings is negative and significant, but smaller.

Table 2 reports the results of the cross–sectional analysis. The coefficient for each explanatory variable is the time series average of the cross-sectional estimates and the corresponding t–statistic is the time series average divided by its time series standard errors.

Column I confirms the well established findings in Fama and French [1992]. The negative and significant relationship between average returns and the market size (*size premium*) and the positive relationship between average returns and book-to-market (*value premium*). In column II, I use cash-to-assets as an explanatory variable. The regression coefficient is positive, but not significantly different from zero: equity returns and corporate cash holdings are (unconditionally) not correlated. The coefficient on cash-to-assets remains positive and becomes significant only when book-to-market is included as a regressor. In the latter case, both coefficients almost double their value. The last column shows the results when both size and book-to-market are included. Again, the coefficient on cash holdings is positive and significant.

The cross-sectional analysis tells us that cash holdings alone have no predictive power for average stock returns. When I control for the size and value effects, firms with higher cash holdings relative to assets have higher realized returns.

2.3 Portfolio Analysis

I investigate whether a strategy that is long in stocks of firms with a high level of cash holdings and short in stocks of firms with a low level of cash holdings yields an excess return.

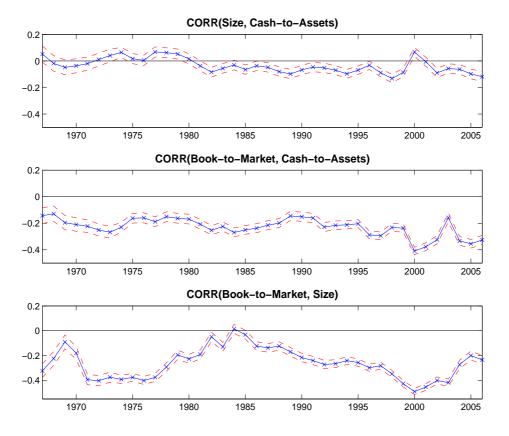


Figure 1: Time Series of Cross–Sectional Correlations

Figure 1 reports the time series of the cross–sectional correlations among market capitalization evaluated at June of year t, book–to–market and cash–to–assets. The last two variables are evaluated using the data available for the fiscal year ending in year t - 1. The dashed lines indicate 95% conficence intervals. The sample period is from 1967 to 2006.

	(I)	(II)	(III)	(IV)	(V)
log Size	-0.16 (-3.05)		-0.19 (-3.91)		-0.15 (-2.93)
log Book-to-market	0.27 (3.38)			0.46 (7. <i>02</i>)	$0.31 \\ (4.47)$
log Cash-to-Assets		0.05 (1.27)	0.04 (1.17)	0.10 (3.14)	0.08 (2.70)

Table 2: Fama–MacBeth Regressions

Staring in June 1967 and ending in June 2006, I regress the realized monthly returns from July of year t to June of year t + 1 on market capitalization in June of year t (Size) and on the relevant accounting variables from the latest fiscal year ending in year t - 1: book-to-market (BM) and cash-to-assets (CH). The cross sectional regression is:

$$R_{it} - R_t^f = \alpha_t + b_{1,t} \log(Size_{i,t-1}) + b_{2,t} \log(BM_{i,t-1}) + b_{3,t} \log(CH_{i,t-1}) + \varepsilon_{i,t}$$

Each coefficient is the time series average of the cross-sectional estimates and the corresponding t-statistics are evaluated dividing the time series averages by their time series standard errors. Data are truncated at the top and bottom 1%.

Following George and Hwang [2008], I form portfolios at a monthly frequency and these portfolios are held for T months. The overall return of the investing strategy at time t is given by the contributions of the single portfolios formed at time t - j, j = 1, ..., T. In order to isolate the contribution of the portfolio formed in month t - j, I run the following cross-sectional regression:

$$R_{it} = \alpha_{jt} + b_{0,jt}R_{i,t-1} + b_{1,jt}\log(Size_{i,t-1}) + b_{2,jt}\log(BM_{i,t-1})$$

$$+ b_{3,jt}Lose_{i,t-j} + b_{4,jt}Winne_{i,t-j} + b_{5,jt}HC_{i,t-j} + b_{6,jt}LC_{i,t-j} + b_{7,jt}HL_{i,t-j} + b_{8,jt}LL_{i,t-j} + \varepsilon_{ijt}$$

$$j = 1...T.$$
(2.1)

The dependent variable is the return to stock i in month t. The independent variables can be separated in two categories. The first one is made up of variables that are known to affect returns. These are the market capitalization of the firm in the previous month $(Size_{i,t-1})$ and the book-to-market $(BM_{i,t-1})$. ⁷ I also include the previous month return $(R_{i,t-1})$ to control for bid-ask bounce. All the control variables are expressed in deviation from their cross-sectional mean.

The second category is made up of dummies related to portfolio strategies. The first two, *Winner* and *Loser*, are included to control for momentum and are constructed following George and Hwang [2004].⁸ The third and the fourth dummies, $HC_{i,t-j}$ and

⁷The book–to–market value is the most recent value to date t which has been reported at least 6 months before portfolio formation. This convention is observed for all the accounting variables.

⁸Let $P_{i,t-j}$ the price of stock *i* at time t-j and $H_{i,t-j}$ the highest price of stock *i* during the period from t-j-T to t-j, I define as Winner (Loser) at time t-j all the stocks in the top (bottom) 20%

Winners	1.00					
Losers	-0.24	1.00				
HL	-0.02	0.08	1.00	$\begin{array}{c} 1.00\\ 0.41 \end{array}$		
LL	0.00	0.01	-0.25	1.00		
HC	-0.01	0.04	-0.16	0.41	1.00	
LC	0.00	0.00	0.15	-0.19	-0.25	1.00

Table 3: Correlations among Portfolios

Each month between January 1967 and December 2006, I calculate the correlations among the six portfolios Winner, Loser, HL, LL, HC, LC. The number reported in the table are the time series averages of the cross-sectional correlations.

 $LC_{i,t-j}$, indicate portfolio strategies formed on cash holdings. $HC_{i,t-j}$ takes value 1 if stock *i* was in the top 20% of the cash-to-assets distribution at time t - j and zero otherwise. $LC_{i,t-j}$ takes value 1 if stock *i* was in the bottom 20% of the cash-to-assets at time t - j and zero otherwise. A similar interpretation holds for $LL_{i,t-j}$ (Low Leverage portfolio) and $HL_{i,t-j}$ (High Leverage portfolio). I add the last two dummies to compare my results with the ones in George and Hwang [2008]. Table 3 reports the correlations among the six portfolios described above. It is worth noting that the correlation coefficient between HC and LL is about 0.4: firms that have high cash holdings relative to the value of assets also tend to have low leverage.

The overall contribution of HC firms to the total return at time t is given by a simple average over all the $b_{5,jt}$, namely $b_{5,t} = \frac{1}{T} \sum_{j=1}^{T} b_{5,jt}$. The average intercept α_t can be interpreted as the excess return of a portfolio that each month hedges the effect on stock returns of all the other independent variables. As a consequence, $\alpha_t + b_{5,t}$ is the return of a strategy that takes each month a long position on the High Cash firms. Finally, $b_{5,t} - b_{6,t}$ is the excess return of a strategy long in the High Cash firms and short in the Low Cash firms.⁹

In Table 4, the regression coefficients are the time series averages of the monthly contribution from January 1967 to December 2006 and the corresponding t–statistics are evaluated dividing the time series average by their time series standard errors.

In all the regressions, the coefficients on the control variables have the expected sign and are all significant. The coefficients on the portfolios formed on cash-to-assets have signs that agree with the results presented in the previous section: High Cash firms earn a higher average return – 42 basis points per month – than Low Cash firms after controlling for size and book-to-market. In regression (b), I replicate the analysis in George and Hwang [2008] by looking at portfolios that consider low leverage firms versus high

of the $\frac{P_{i,t-j}}{H_{i,t-j}}$ distribution.

⁹For a detailed discussion of the parameters' interpretations as returns see chapter 9 in Fama [1976]

			Raw	Returns		
		(a)		(b)		(c)
Intercept	1.05	(4.20)	1.10	(4.35)	1.09	(4.31)
Controls						
R_{t-1}	-0.06	(-17.26)	-0.06	(-17.15)	-0.07	(-17.40)
Size	-0.17	(-3.79)	-0.18	(-3.97)	-0.17	(-3.87)
book-to-market	0.41	(5.89)	0.38	(5.21)	0.40	(5.79)
Portfolios						
Loser	-0.39	(-2.40)	-0.35	(-2.09)	-0.37	(-2.30)
Winner	0.67	(9.88)	0.68	(9.64)	0.67	(9.91)
High Savings (HC)	0.30	(3.23)			0.25	(2.74)
Low Savings (LC)	-0.12	(-2.34)			-0.09	(-1.82)
High Leverage (HL)			-0.26	(-4.13)	-0.23	(-3.95)
Low Leverage (LL)			0.15	(2.87)	0.04	(0.91)

Table 4: Portfolio Analysis: Raw Returns

Each month between January 1967 and December 2006 I estimate a cross-sectional regression where R_{it} is the return in month t to stock i and $R_{i,t-1}$ is the previous month return.

$$\begin{aligned} R_{it} &= \alpha_{jt} + b_{0,jt}R_{i,t-1} + b_{1,jt}\log(Size_{i,t-1}) + b_{2,jt}\log(BM_{i,t-1}) + b_{3,jt}Lose_{i,t-j} + b_{4,jt}Winner_{i,t-j} \\ &+ b_{5,jt}HC_{i,t-j} + b_{6,jt}LC_{i,t-j} + b_{7,jt}HL_{i,t-j} + b_{8,jt}LL_{i,t-j} + \varepsilon_{ijt} \qquad j = 1...T \end{aligned}$$

 $Size_{i,t-1}$ is the market capitalization of the firm in the previous month and $BM_{i,t-1}$ is the book-to-market. $R_{i,t-1}$, $Size_{i,t-1}$ and $BM_{i,t-1}$ are taken in deviation from the correspondent cross-sectional mean. Winner and Loser are dummy variables that control for momentum. Let $P_{i,t-j}$ the price of stock i at time t - j and $H_{i,t-j}$ the highest price of stock i during the period from t - j - T to t - j, Winner (Loser) is equal to 1 at time t - j if the stock is in the top (bottom) 20% of the $\frac{P_{i,t-j}}{H_{i,t-j}}$ distribution. $HC_{i,t-j}$ takes value 1 if stock i was in the top 20% of the savings distribution at time t - j and zero otherwise. $LC_{i,t-j}$ takes value 1 if stock i was in the bottom 20% of the savings distribution time t - j and zero otherwise. A similar interpretation holds for $LL_{i,t-j}$ (low leverage) and $HL_{i,t-j}$ (high leverage). The reported coefficients are the time series averages of the cross-sectional averages taken over the j = 1, ..., 6 holding periods. The corresponding t-statistics are evaluated dividing the time series average by their time series standard errors as suggested in Fama and MacBeth

HCMLC	1.00				
LLMHL	0.77^{**}	1.00			
MKT	0.21^{*}	-0.03	1.00		
SMB	0.31**	0.03	0.31^{**}	1.00	
HML	-0.53**	-0.42**	-0.43**	-0.29*	1.00

Table 5: Correlations among Factors

The table reports the correlations among the five factors HCMLC, HLMLL, MKT, SMB and HML during the period that starts in June 1967 and ends in December 2006. * indicates a significance level of 5%. ** indicates a significance level of 1%. T-statistics are evaluated using the Newey–West methodology.

leverage firms. I also find that firms with lower leverage earn a higher average return thus confirming their findings. In the last regression, both the portfolios formed on cash holdings and the portfolios formed on leverage are included. The positive relationship between cash holdings and equity returns survives even if I control for the leverage portfolios. In this last case, a strategy long in the High Cash firms and short in the Low Cash firms still yields a considerable excess return of 34 b.p. per month.

In what follows, I investigate if the difference in returns can be explained by the Fama–French three factors model. I construct a *cash factor* (*HCMLC*) which is equal to the excess return of a strategy long in the High Cash firms and short in the in the Low Cash firms, namely the difference each month between the coefficients $b_{5,t}$ and $b_{6,t}$ in regression (a) of Table 4. Following George and Hwang [2008], I also construct a *leverage factor* (*LLMHL*) by taking the excess return of a strategy long in the Low Leverge firms and short in the High Leverage firms.

Table 5 reports the correlations among the *cash factor*, the *leverage factor* and the Fama–French three factors. The correlation between the *cash factor* and the *leverage factor* is almost 80% and this is evidence that the two factors might proxy for the same underlying source of risk. The *cash factor* is positively correlated with the market factor (MKT) and the size factor (SMB) and negatively correlated with the value factor (HML). The *leverage factor* has a significant (and negative) correlation with HML only.

In Table 6, I regress the *cash* and *leverage* factors on the Fama–French three factors. Both R–square are small and the intercepts are positive and significant – the risk–adjusted excess returns of a strategy long in high cash firms and short in low cash firms is 62 basis points per month. These results are evidence that the cash and leverage factors proxy for a common source of risk that is not related to the one(s) proxied by the Fama–French factors.

Finally, I evaluate the explanatory power of the HCMLC factor in 75 portfolios generated by a conditional sorting on size, book-to-market and cash-to-assets. In June of

	constant	MKT	SMB	HML	R-squared
HCMLC	0.61 (3.02)	-0.04 (-0.83)	0.16 (1.76)	-0.47 (-4.84)	0.31
LLMHL	$0.66 \\ (4.36)$	-0.12 (-3.35)	-0.03 (-0.60)	-0.39 (-5.70)	0.23

Table 6: HCMLC, LLMHL, and the Fama–French Factors

The table reports the coefficients, the t–statistics and the R–square of a linear regression of the cash and leverage factors on the 3 Fama–French factors. The sample period is from June 1967 to December 2006

year t, stocks are sorted in three size categories (small, medium and large). Whitin each category, stocks are sorted in book–to–market quintiles and within each book–to–market quintile stocks are further sorted in cash holdings quintiles.

The Fama–French three factors are not able to explain the excess returns of high cash versus low cash firms (Table 7). The excess returns of high cash firms over low cash ones (HC-LC) are always positive and significant in all but two cases. In Table 8 the HML factor is replaced by HCMLC and the differences in excess returns (HC-LC) are significant in only two cases. On the other hand, the exclusion of the HML factor produces significant spreads in the excess return of high book–to–market versus low book–to–market firms (HB-LB). When I augment the Fama–French three factors with the cash factor (Table 9), I improve in explaining both the excess returns of high cash versus low cash firms and high book versus low book firms.¹⁰

Table 10 provides further evidence about the explanatory power of the *cash factor*. The *HML* factor adds no explanatory power to the CAPM in a two–pass cross–sectional regression. The values of the GLS R–square are very close and not statistically different form each other.¹¹ When the *HCMLC* factor is added to the CAPM, the value of the GLS R-square doubles. The same happens when *HCMLC* is added to the Fama–French three factors model. The differences in the R-square are also statistically significant at the 1% level.

The empirical analysis shows that firms with a high level of cash relative to the book value of assets earn an excess return after controlling for the *size* and *value* effect. The Fama–French three factors are not able to explain such a difference in average returns,

¹⁰The Gibbons, Ross and Shanken F–statistics imply a rejection of the hypothesis that all the pricing errors are jointly equal zero for all the proposed factor models.

¹¹I focus on GLS R-square because it is a measure of the distance of the factors from the minimum– variance frontier (e.g. Lewellen, Nagel, and Shanken [2008]. Kan, Robotti, and Shanken [2008] provide a method on how to derive the asymptotic distribution of the sample R-square.

while a *cash* factor (*HCMLC*) does. This is evidence that *HCMLC* captures a source of risk which is different from the ones proxied by MKT, SMB and HML. In the rest of the paper, I propose a model of the firm with endogenous investing and financing decisions that successfully rationalizes the positive relationship between corporate savings and average equity returns.

Table 7: Time Series Regression I

 $R_i - R^f = \alpha_i + m_i M K T + s_i S M B + h_i H M L + \varepsilon_i$

	α						t(lpha)							adjusted R^2					
	Cash-to-Assets Low 2 3 4 High HC-L0								Cash-	to–Asset	s			Cas	h–to–A	ssets			
	Low	2	3	4	High	HC-LC	Low	2	3	4	High	HC-LC	Low	2	3	4	High		
Low	-0.49	-0.36	-0.31	-0.04	-0.05	0.44	-2.90	-1.81	-1.57	-0.21	-0.20	1.67	0.75	0.78	0.74	0.79	0.77		
2	-0.32	-0.33	-0.19	0.06	0.49	0.81	-1.86	-2.44	-1.35	0.38	2.04	2.84	0.79	0.80	0.85	0.82	0.78		
3	-0.16	-0.11	-0.01	0.18	0.69	0.85	-1.09	-0.71	-0.11	1.46	2.82	3.05	0.80	0.82	0.84	0.84	0.78		
4	-0.03	-0.04	0.14	0.27	0.56	0.59	-0.24	-0.25	1.09	2.35	3.12	2.54	0.82	0.82	0.84	0.85	0.79		
High	0.23	0.17	0.43	0.35	0.67	0.44	1.45	1.17	3.59	2.53	4.19	2.03	0.78	0.79	0.78	0.80	0.78		
HB-LB	0.72	0.53	0.73	0.38	0.72		4.30	2.49	3.69	1.80	3.24								

Small Size

Medium Size

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				α			$\mathrm{t}(lpha)$						adjusted R^2					
	Cash-to-Assets Low 2 3 4 High HC-LC								Cash-	to–Asset	s			Cas	h-to-A	ssets		
	Low	2	3	4	High	HC-LC	Low	2	3	4	High	HC-LC	Low	2	3	4	High	
Low	-0.54	-0.70	-0.26	-0.19	0.02	0.56	-3.03	-3.56	-1.62	-1.24	0.09	2.21	0.77	0.79	0.83	0.83	0.78	
$\frac{2}{3}$	-0.33 -0.34	-0.32 -0.23	-0.19 -0.00	-0.06 -0.15	$0.22 \\ 0.25$	$\begin{array}{c} 0.55 \\ 0.59 \end{array}$	-1.85 -2.41	$-1.55 \\ -1.66$	-1.35 -0.00	-0.41 -1.49	$1.34 \\ 1.67$	$2.12 \\ 2.97$	$0.77 \\ 0.82$	$\begin{array}{c} 0.78 \\ 0.78 \end{array}$	$\begin{array}{c} 0.84 \\ 0.82 \end{array}$	$\begin{array}{c} 0.82 \\ 0.87 \end{array}$	$0.82 \\ 0.81$	
4 High	-0.24 -0.31	-0.30 -0.21	$0.05 \\ -0.11$	-0.03 0.06	$\begin{array}{c} 0.20 \\ 0.04 \end{array}$	$\begin{array}{c} 0.44 \\ 0.35 \end{array}$	-1.86 -2.47	$-2.56 \\ -1.42$	0.39 -0.69	$-0.26 \\ 0.56$	$\begin{array}{c} 1.68 \\ 0.27 \end{array}$	$2.63 \\ 2.14$	$\begin{array}{c} 0.80\\ 0.81 \end{array}$	$\begin{array}{c} 0.81 \\ 0.81 \end{array}$	$\begin{array}{c} 0.83 \\ 0.82 \end{array}$	$\begin{array}{c} 0.80\\ 0.84 \end{array}$	$\begin{array}{c} 0.81 \\ 0.80 \end{array}$	
HB-LB	0.24	0.50	0.16	0.25	0.02		1.32	2.69	0.95	1.51	0.12							

Large Size

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				α			${ m t}(lpha)$							adjusted R^2				
	Cash-to-Assets Low 2 3 4 High HC-LC								Cash-	to–Asset	s			Cas	h-to-A	ssets		
	Low	2	3	4	High	HC-LC	Low	2	3	4	High	HC-LC	Low	2	3	4	High	
Low	-0.20	-0.14	0.13	0.36	0.32	0.52	-1.27	-0.87	0.99	1.72	1.67	2.32	0.75	0.78	0.76	0.75	0.73	
2	-0.23	-0.13	0.07	0.08	0.58	0.80	-1.52	-1.04	0.44	0.68	2.78	3.16	0.76	0.79	0.77	0.79	0.73	
3	-0.40	-0.18	-0.19	0.09	0.34	0.74	-3.10	-1.39	-1.53	0.89	1.85	3.45	0.76	0.79	0.78	0.80	0.77	
4	-0.25	-0.22	-0.27	0.03	0.36	0.61	-1.90	-1.41	-1.75	0.21	2.11	3.16	0.76	0.75	0.78	0.78	0.73	
High	-0.18	-0.20	-0.23	0.06	0.01	0.18	-1.43	-1.52	-1.72	0.43	0.04	1.02	0.78	0.76	0.78	0.75	0.72	
HB-LB	0.03	-0.06	-0.36	-0.30	-0.31		0.16	-0.35	-2.24	-1.34	-1.73							

In June of year t, stocks are sorted in three size categories (small, medium and large). Within each size category, stocks are sorted in book-to-market quintiles and within each book-to-market quintile stock are further sorted in cash holdings quintiles. This Table reports the intercepts (α), with the corresponding Newey-West t-statistics (t(α)) together with the adjusted R^2 of time series regressions over the 75 portfolios using MKT, SMB and HML. HC-LC is the difference in the intercepts of the firms in the high cash holdings quintile with the firms in the low cash holdings quintile. HB-LB is the difference in the intercepts of the firms in the high book-to-market quintile with the firms in the low book-to-market quintile. The sample period is from June 1967 to December 2006

Table 8: Time Series Regression II

 $R_i - R^f = \alpha_i + m_i M K T + s_i S M B + c_i H C M L C + \varepsilon_i$

	α						$\mathrm{t}(lpha)$							adjusted R^2					
	Cash-to-Assets Low 2 3 4 High HC-LC								Cash-	to–Asset	s			Cas	h–to–A	ssets			
	Low	2	3	4	High	HC-LC	Low	2	3	4	High	HC-LC	Low	2	3	4	High		
Low	-0.29	-0.34	-0.53	-0.37	-0.60	-0.31	-1.73	-1.71	-2.67	-2.06	-2.81	-1.71	0.75	0.78	0.75	0.81	0.82		
2	0.20	0.00	0.01	0.02	0.11	-0.09	1.31	0.02	0.08	0.16	0.68	-0.68	0.79	0.80	0.84	0.83	0.83		
3	0.39	0.38	0.47	0.34	0.39	-0.00	3.14	2.52	3.13	2.43	2.93	-0.01	0.81	0.81	0.82	0.83	0.85		
4	0.55	0.54	0.66	0.60	0.48	-0.07	5.03	3.51	4.54	4.03	3.68	-0.55	0.81	0.80	0.81	0.82	0.82		
High	0.95	0.80	1.05	0.85	0.82	-0.13	6.25	5.19	6.83	5.55	5.66	-1.07	0.75	0.75	0.74	0.75	0.77		
HB-LB	1.24	1.14	1.59	1.23	1.42		6.72	5.14	7.76	5.55	6.04								

Small Size

Medium Size

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			α			$\mathrm{t}(lpha)$						adjusted R^2					
Cash-to-Assets Low 2 3 4 High HC-LC								Cash-	to–Asset	s			Cas	h–to–A	ssets		
Low	2	3	4	High	HC-LC	Low	2	3	4	High	HC-LC	Low	2	3	4	High	
-0.48	-0.89	-0.73	-0.89	-0.94	-0.46	-2.68	-5.17	-3.85	-5.09	-4.42	-1.92	0.79	0.78	0.82	0.83	0.80	
-0.04	-0.10	-0.11	-0.26	-0.33	-0.29	-0.20	-0.76	-0.69	-2.07	-2.16	-1.33	0.80	0.79	0.84	0.82	0.85	
0.08	0.21	0.35	0.02	-0.04	-0.12	0.57	1.24	2.06	0.15	-0.25	-0.74	0.82	0.80	0.82	0.86	0.84	
0.36	0.27	0.47	0.30	0.20	-0.16	2.02	1.60	3.65	1.70	1.49	-1.01	0.79	0.78	0.82	0.79	0.82	
$\begin{array}{c} 0.36 \\ 0.83 \end{array}$	$0.44 \\ 1.33$	$0.59 \\ 1.33$	$\begin{array}{c} 0.53 \\ 1.42 \end{array}$	$\begin{array}{c} 0.22 \\ 1.16 \end{array}$	-0.13	$2.10 \\ 4.04$	$2.35 \\ 5.53$	$\begin{array}{c} 3.41 \\ 6.07 \end{array}$	$3.13 \\ 5.96$	$1.11 \\ 4.54$	-0.90	0.75	0.76	0.77	0.80	0.78	
	-0.48 -0.04 0.08 0.36 0.36	$\begin{array}{cccc} -0.48 & -0.89 \\ -0.04 & -0.10 \\ 0.08 & 0.21 \\ 0.36 & 0.27 \\ 0.36 & 0.44 \end{array}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Cash-to-Asset Low 2 3 4 -0.48 -0.89 -0.73 -0.89 -0.04 -0.10 -0.11 -0.26 0.08 0.21 0.35 0.02 0.36 0.27 0.47 0.30 0.36 0.44 0.59 0.53	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	

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	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$						${ m t}(lpha)$					adjusted R ² Cash-to-Assets					
								Cash-to-Assets									
	Low	2	3	4	High	HC-LC	Low	2	3	4	High	HC-LC	Low	2	3	4	High
Low	-0.19	-0.35	-0.37	-0.57	-0.60	-0.41	-1.31	-2.19	-2.16	-2.39	-3.10	-1.97	0.76	0.76	0.72	0.74	0.74
2	-0.07	-0.01	-0.07	-0.27	-0.14	-0.07	-0.39	-0.09	-0.56	-2.40	-1.00	-0.35	0.78	0.80	0.76	0.78	0.78
3	-0.04	0.13	-0.02	-0.02	-0.10	-0.06	-0.28	0.88	-0.17	-0.19	-0.75	-0.35	0.79	0.79	0.78	0.80	0.80
4	0.17	0.25	0.24	0.23	0.22	0.05	1.08	1.68	1.53	1.51	1.44	0.32	0.75	0.75	0.77	0.77	0.77
High	0.36	0.35	0.39	0.47	0.17	-0.19	2.21	2.38	1.96	3.15	0.93	-1.35	0.74	0.72	0.72	0.71	0.70
HB-LB	0.55	0.70	0.76	1.04	0.77		2.93	3.42	2.95	3.89	2.78						

In June of year t, stocks are sorted in three size categories (small, medium and large). Within each size category, stocks are sorted in book-to-market quintiles and within each book-to-market quintile stock are further sorted in cash holdings quintiles. This Table reports the intercepts (α), with the corresponding Newey-West t-statistics (t(α)) together with the adjusted R^2 of time series regressions over the 75 portfolios using MKT, SMB and HCMLC. HC-LC is the difference in the intercepts of the firms in the high cash holdings quintile with the firms in the low cash holdings quintile. HB-LB is the difference in the intercepts of the firms in the high book-to-market quintile with the firms in the low book-to-market quintile. The sample period is from June 1967 to December 2006

Table 9: Time Series Regression III

 $R_i - R^f = \alpha_i + m_i MKT + s_i SMB + h_i HML + c_i HCMLC + \varepsilon_i$

	$\begin{array}{cccccccccccccccccccccccccccccccccccc$						$\mathrm{t}(lpha)$					adjusted R^2					
								Cash-to-Assets					Cash-to-Assets				
	Low	2	3	4	High	HC-LC	Low	2	3	4	High	HC-LC	Low	2	3	4	High
Low	-0.34	-0.49	-0.50	-0.37	-0.61	-0.26	-1.85	-2.34	-2.41	-1.93	-2.66	-1.43	0.75	0.78	0.75	0.81	0.82
2	-0.07	-0.22	-0.20	-0.24	-0.06	0.00	-0.47	-1.76	-1.30	-1.77	-0.40	0.03	0.80	0.81	0.85	0.84	0.83
3	0.12	0.07	0.11	0.00	0.09	-0.03	1.22	0.52	1.02	0.01	0.71	-0.23	0.83	0.83	0.84	0.85	0.86
4	0.19	0.13	0.26	0.22	0.11	-0.09	2.21	1.04	2.47	1.85	0.83	-0.70	0.83	0.83	0.84	0.85	0.84
High	0.47	0.31	0.55	0.37	0.33	-0.14	3.88	2.50	4.71	2.78	2.49	-1.14	0.80	0.79	0.78	0.80	0.81
HB-LB	0.82	0.80	1.05	0.74	0.94		5.05	3.94	5.85	3.82	4.32						

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				α			$\mathrm{t}(lpha)$					adjusted R^2					
	Cash-to-Assets							Cash-to-Assets					Cash-to-Assets				
	Low	2	3	4	High	HC-LC	Low	2	3	4	High	HC-LC	Low	2	3	4	High
Low	-0.25	-0.58	-0.41	-0.50	-0.49	-0.24	-1.64	-3.60	-2.34	-3.32	-2.53	-1.04	0.79	0.80	0.84	0.85	0.82
2	-0.03	-0.09	-0.14	-0.21	-0.18	-0.15	-0.17	-0.70	-0.94	-1.48	-1.28	-0.73	0.80	0.79	0.84	0.82	0.86
3	-0.15	0.03	0.16	-0.18	-0.09	0.06	-1.36	0.31	1.07	-1.63	-0.65	0.42	0.83	0.81	0.83	0.87	0.84
4	0.02	-0.12	0.20	0.02	-0.04	-0.05	0.14	-1.23	2.26	0.15	-0.29	-0.34	0.82	0.82	0.84	0.80	0.83
High HB-LB	$-0.16 \\ 0.09$	$-0.04 \\ 0.54$	$\begin{array}{c} 0.07 \\ 0.49 \end{array}$	$\begin{array}{c} 0.10 \\ 0.60 \end{array}$	$-0.16 \\ 0.34$	0.01	$-1.36 \\ 0.49$	-0.28 2.94	$0.59 \\ 3.12$	$0.86 \\ 3.45$	$-0.92 \\ 1.71$	0.06	0.82	0.82	0.83	0.84	0.81

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				α					t	$\epsilon(\alpha)$			adjusted R^2				
	Cash-to-Assets							Cash-to-Assets					Cash-to-Assets				
	Low	2	3	4	High	HC-LC	Low	2	3	4	High	HC-LC	Low	2	3	4	High
Low	0.02	0.03	0.11	-0.05	-0.11	-0.13	0.13	0.22	0.65	-0.30	-0.59	-0.66	0.77	0.79	0.76	0.79	0.77
2	0.01	0.03	0.14	-0.06	0.11	0.11	0.03	0.25	1.02	-0.59	0.74	0.50	0.78	0.80	0.77	0.79	0.79
3	-0.16	-0.01	-0.11	0.03	-0.00	0.15	-1.42	-0.09	-0.99	0.27	-0.03	0.94	0.79	0.80	0.78	0.80	0.80
4	-0.09	-0.01	-0.08	0.02	-0.03	0.05	-0.67	-0.08	-0.59	0.17	-0.20	0.32	0.77	0.77	0.80	0.78	0.78
High	-0.03	-0.04	-0.10	0.11	-0.32	-0.29	-0.25	-0.32	-0.71	0.82	-1.95	-2.05	0.79	0.77	0.79	0.75	0.75
HB-LB	-0.05	-0.07	-0.21	0.16	-0.21		-0.28	-0.42	-1.22	0.83	-1.18						

In June of year t, stocks are sorted in three size categories (small, medium and large). Within each size category, stocks are sorted in book-to-market quintiles and within each book-to-market quintile stock are further sorted in cash holdings quintiles. This Table reports the intercepts (α), with the corresponding Newey-West t-statistics (t(α)) together with the adjusted R^2 of time series regressions over the 75 portfolios using *MKT*, *SMB*, *HML* and *HCMLC*. *HC-LC* is the difference in the intercepts of the firms in the high cash holdings quintile with the firms in the low cash holdings quintile. *HB-LB* is the difference in the intercepts of the firms in the high book-to-market quintile with the firms in the low cash holdings quintile. *HB-LB* is the difference in the intercepts of the firms in the high book-to-market quintile with the firms in the low book-to-market quintile. The sample period is from June 1967 to December 2006

Intercept	MKT	SMB	HML	HCMLC	OLS \mathbb{R}^2	GLS \mathbb{R}^2
1.61	-0.76				0.14	0.08
(2.54)	(-1.29)				(0.19)	(0.05)
1.24	-0.44		0.26		0.15	0.10
(0.83)	(-0.35)		(0.86)		(0.05)	(0.05)
3.38	-2.31			0.34	0.49	0.16
(4.23)	(-3.02)			(1.50)	(0.07)	(0.07)
2.49	-1.77	0.17	0.29		0.61	0.17
(3.32)	(-2.71)	(0.78)	(1.45)		(0.07)	(0.07)
3.25	-2.40	0.03		0.23	0.76	0.19
(7.16)	(-5.47)	(0.13)		(1.33)	(0.10)	(0.07)
2.22	1 50	0.00	0.00	0.07	0.07	
2.22	-1.50	0.00	0.69	0.37	0.87	0.30
(5.16)	(-3.61)	(0.01)	(3.71)	(2.02)	(0.06)	(0.08)

Table 10: Cross–sectional regressions

This Table reports the coefficients, t-statistics (in parenthesis), OLS R^2 , GLS R^2 , T^2 statistic tests with corresponding p-values (in parenthesis) from cross-sectional regressions of average excess returns on factor loadings. The average excess returns are from the 75 portfolios sorted on size, book-to-market and cash holdings. The sample period is from June 1967 to December 2006. The cross-sectional regression takes the following form:

$$E_T(R_t^i - R_t^F) = \lambda_0 + \widehat{\beta}_i \lambda_1 + \nu_i \qquad i = 1...N = 75$$

where $E_T(R_t^i - R_t^F)$ is a $(N \times 1)$ vector of time series average of the excess returns over the N portfolios, λ_0 is the $(N \times 1)$ zero-beta rate vector, λ_1 is the $(K \times 1)$ vector of risk premia over the $(N \times K)$ matrix of factors $\hat{\beta}_i$ estimated in the first step time series regression.

3 A Simple Model

I develop a model in the spirit of Kim, Mauer, and Sherman [1998]. I depart from their risk neutral set–up by adding a stochastic discount factor and risky cash flows.

A firm that expects to have an investment opportunity in the near future needs to decide whether to hoard cash, earning a return lower than the opportunity cost of capital, or distribute dividends in the current period thus increasing the expected investment cost. This trade-off determines the firm's optimal saving policy in the current period.

The assumption that cash flows are correlated with the aggregate risk introduces a precautionary saving motive that induces riskier firm to save more. The precautionary savings motive – absent in a risk neutral environment – is the key ingredient to generate a positive correlation between expected equity returns and a firm's cash holdings.

3.1 Set-up

Consider a three-period model, $t = \{0, 1, 2\}$. At time t = 0, a firm is endowed with initial cash holdings equal to C_0 and an asset (*risky asset*) that produces a random cash flow in period 1 only.

At time 1, after the realization of the risky asset's cash flow, the firm receives an investment opportunity with probability π , $\pi \in [0, 1]$. The opportunity consists in the option of installing an asset (safe asset) that produces a deterministic cash flow, C_2 , at time 2. I assume that C_2 is not pledgeable at time t = 1.

If the firm installs the *safe asset*, then it pays a fixed (sunk) cost I = 1. If the firm does not have enough internal resources to pay for the fixed cost, then it can issue equity. I assume a stochastic cash flow together with a deterministic investment cost to generate a liquidity shock and a consequent need for external financing at time t = 1.¹²

The percentage cost of issuing equity is λ . The firm can also transfer cash from one period to the other at the internal gross rate $\hat{R} < R$, where R is the risk-free gross interest rate. An internal accumulation rate less than the risk-free interest rate can be justified by the fact that the firm pays corporate taxes on interest earned on savings. Moreover, this assumption prevents an unbounded accumulation of cash internally to the firm. The trade-off that the firm faces is distributing dividends today or retaining cash in order to avoid costly external financing tomorrow.

The timing of the model is reported in Figure 2.

¹²I could have used a stochastic investment cost as an alternative way to get a liquidity shock at time 1 like for example in Holmstrom and Tirole [1998] and Holmstrom and Tirole [2000].

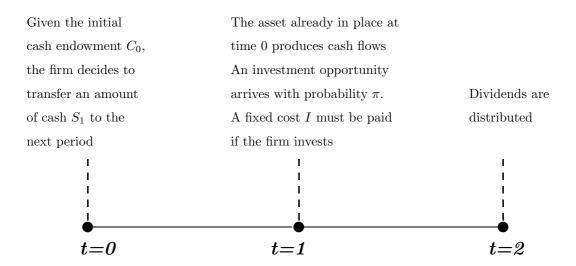


Figure 2: Timing of the Three–Period Model

3.2 Pricing Kernel and Production

To perform assets' valuation, I construct a stochastic discount factor (SDF) adopting the convenient parametrization of Berk, Green, and Naik [1999]. A cash flow produced at time t = 1 is discounted using

$$M_1 = e^{m_1} = e^{-r - \frac{1}{2}\sigma_z^2 - \sigma_z \varepsilon_{z,1}}, \tag{3.1}$$

where $\varepsilon_{z,1} \sim N(0,1)$ is the aggregate shock at time t = 1.¹³ The formulation in equation (3.1) implies that the conditional mean of the SDF, $E_0[M_1]$, is equal to the inverse of the gross risk-free interest rate, e^{-r} .

The pay-off produced by the *risky asset* at time 1 is equal to e^{x_1} , where x_1 is equal to:

$$x_1 = \mu - \frac{1}{2}\sigma_x^2 + \sigma_x \varepsilon_{x,1}. \tag{3.2}$$

The *idiosyncratic shock*, $\varepsilon_{x,1} \sim N(0,1)$, is correlated with the error term of the pricing kernel. It is this last assumption that makes risky the cash flows produced by the asset in place at time 0.

$$M_{t+1} = \beta \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} \qquad \Rightarrow \qquad \log(M_{t+1}) = -\log(R) - \gamma (\log(c_{t+1}) - \log(c_t))$$

¹³Assume that in the background there is a consumer with CRRA preferences, log–normal consumption growth $-\log(\frac{c_{t+1}}{c_t}) \sim N(\mu_c, \sigma_c^2)$ – and discount factor β equal to 1/R. The utility function implies

Because of the log-normality of consumption growth, the logarithm of the pricing kernel is the sum of the (negative) risk-free interest rate plus a normally distributed error term. Setting $-\gamma(\log(c_{t+1}) - \log(c_t))$ equal to $-\frac{1}{2}\sigma_z^2 - \sigma_z\varepsilon_{z,1}$ allows me to recover equation (3.1). For a similar interpretation see Zhang [2005].

I assume that $COV(\varepsilon_{z,1}, \varepsilon_{x,1}) = \sigma_{x,z}$ and, as a consequence, $COV(x_1, m_1) = -\sigma_x \sigma_z \sigma_{x,z}$. As in Berk, Green, and Naik [1999], the *systematic risk* of a project's cash flow, β_{xm} , is equal to $\sigma_x \sigma_z \sigma_{x,z}$.

The value at time zero of a cash flow that will be realized at time 1 is given by the certainty equivalent discounted at the (gross) risk-free interest rate:

$$E_0[e^{m_1}e^{x_1}] = E_0[e^{-r - \frac{1}{2}\sigma_z^2 - \sigma_z \varepsilon_{z,1} + \mu - \frac{1}{2}\sigma_x^2 + \sigma_x \varepsilon_{x,1}}] = e^{-r}e^{-\beta_{xm}}.$$

As β_{xm} increases, the firm's cash flows become more correlated with the *aggregate shock*, hence less valuable.

3.3 Firm's Problem

At time 0, the firm has to decide how much of the initial cash endowment C_0 to distribute as dividends (D_0) and how much to retain (S_1) . Given that the return on internal savings is lower than the risk-free rate, S_1 will always be less than C_0 .

To simplify the problem, I assume that the time 1 present discounted value of the safe project's cash flow, $\frac{C_2}{R}$, is greater than the investment cost when the safe project is entirely equity financed, $1 + \lambda$. This condition is sufficient to ensure that the firm always invests at time 1 if there is an investment opportunity.

Conditional on investing at time 1, the firm issues equity only if corporate savings, $\widehat{R}S_1$, plus the cash flow from the *risky asset*, e^{x_1} , are not enough to pay for the cost of investment. In this case, the dividend at time 1, D_1 , is negative and the firm pays λD_1 in issuance costs. The last period dividend is the cash flow produced by the *safe asset*, $D_2 = C_2$. If the firm does not invest at time 1, all the internal resources are distributed to shareholders and the time 2 dividend is zero.

The problem of the firm can be written as 14

$$V_0 \equiv \max_{S_1 \ge 0} D_0 + E_0[M_1 D_1] + E_0[M_2 D_2]$$
(3.3)

¹⁴In this economy, I also need M_2 , the pricing kernel to evaluate a random pay–off in period 2.

$$M_2 = e^{-2r - \frac{1}{2}\sigma_z^2 - \sigma_z \varepsilon_{z,z}}$$

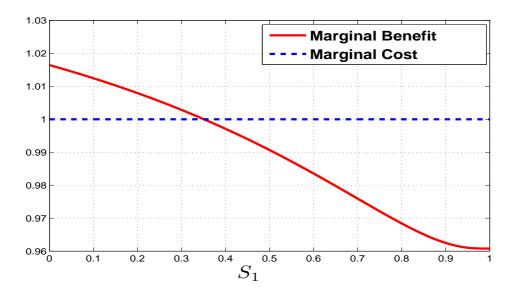


Figure 3: Euler Equation

subject to :

$$D_{0} = C_{0} - \frac{S_{1}}{\hat{R}},$$

$$D_{1} = \begin{cases} (1 + \lambda \Delta_{1})(S_{1} + e^{x_{1}} - 1) & \text{with probability } \pi \\ S_{1} + e^{x_{1}} & \text{with probability } 1 - \pi \end{cases}$$

$$D_{2} = \begin{cases} C_{2} & \text{with probability } \pi \\ 0 & \text{with probability } 1 - \pi \end{cases},$$

where Δ_1 is an indicator function that takes value 1 if the internal resources at time 1 are not enough to pay for the fixed cost of investment $(e^{x_1} + S_1 < 1)$.¹⁵

Assuming an interior solution, the optimal saving policy is such that the firm equates the cost and the benefit of saving an extra unit of cash:

$$1 = \widehat{R}E_0[M_1] + \pi\lambda\widehat{R}E_0[M_1\Delta_1].$$
(3.4)

The marginal cost is simply the forgone dividend at time 0. The marginal benefit is given by the expected dividend that the firm will distribute next period plus the expected reduction in issuance cost if the firm will issue equity. Figure 3 shows that this value is decreasing in the amount of retained cash. Figure 4 reports the firm's optimal retention policy as a function of the cash flows mean, the probability of getting an investment

 $^{^{15}}$ In the appendix, I provide a condition for the existence and the uniqueness of an interior solution for the firm's problem (proposition A.1).

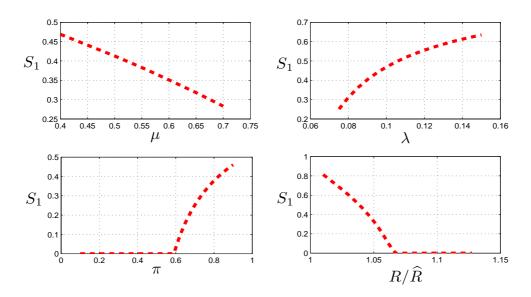


Figure 4: The effects of varying μ , λ , π , and R

opportunity, the cost of external financing, and the risk–free rate. These results are summarized in proposition A.4.

As the mean of cash flows increases, the firm expects to have more liquid resources to finance the investment and this causes a reduction in the marginal benefit of saving, hence the firm optimally lowers the time 0 amount of retained cash.

Without equity issuance cost, the firm does not save given that the return on internal savings is less than the risk-free interest rate. On the other hand, a positive value of λ generates a positive expected financing cost. Hence, an increase in λ produces an increase in the marginal benefit of retaining cash and this, in turn, induces the firm to retain more cash.

The marginal benefit of retaining cash is also increasing in the probability of receiving an investment opportunity because a higher probability of investing next period produces a higher expected financing cost.

The risk-free rate measures the opportunity cost of internal savings. The higher the risk-free rate relative to the internal rate, the lower the marginal benefit of retaining cash for the firm. As the ratio R/\hat{R} increases, it becomes more expensive for the firm to internally accumulate cash and as a consequence the firm reduces the amount of cash transferred to the next period.

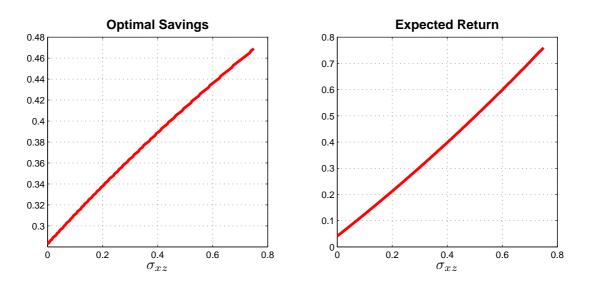


Figure 5: The Effects of a Change in Risk

3.4 Risk, Savings, and Expected Equity Returns

In this section, I explain how the covariance of the *risky asset*'s cash flow with aggregate risk affects the firm's savings decision and expected returns.

Exploiting the properties of the covariance between two random variables, I rewrite the Euler equation in (3.4) as

$$1 = \widehat{R}E_0[M_1] + \pi \lambda \widehat{R} \Big(E_0[M_1] Prob_0(\Delta_1 = 1) + COV[M_1, \Delta_1] \Big).$$

Under risk-neutrality, the covariance term disappears from the Euler Equation and risk plays no role in determining the firm's optimal saving policy. Here, instead, an increase in the covariance term will lower the expected value of the firms' cash flows in those future states in which the firm is more likely to issue equity (namely when the firm decides to invest and the realization of the aggregate shock is low). As a consequence, an increase in riskiness leads to an increase in the time t = 1 expected financing cost and the firm reacts by increasing savings at time 0. This comparative statics is illustrated in the left panel of Figure 5 and formalized in proposition A.2.

The expected return between time 0 and time 1 is the ratio of the time 0 expected future dividends over the time 0 ex-dividend value of the firm:

$$E[R_{0,1}^e] = \frac{E_0[D_1 + E_1(\frac{M_2}{M_1}D_2)]}{E_0[M_1D_1] + E_0[M_2D_2]}.$$
(3.5)

When the cash flows are uncorrelated with the stochastic discount factor the expected equity return is equal to the risk-free interest rate R. On the other hand, when there is

no investment opportunity ($\pi = 0$) or no equity issuance cost ($\lambda = 0$) the optimal policy for the firm is to set $S_1^* = 0$. This will make the expected equity return independent from the saving policy These three cases are of no interest if the objective is the analysis of the relationship between savings and expected equity returns. Hence, risk, a positive expectation of future investment, and costly equity issuance are essential ingredients to answer the main question of this paper.

A change in the firm's systematic risk affects expected returns through two channels. The first channel works through the direct effect of a change in σ_{xz} . An increase in risk will reduce the time 0 ex-dividend value of the firm while the expected future dividends are not affected: expected return will increase. At the same time, a change in σ_{xz} will affect the optimal choice of S_1^* (proposition A.2). Both the numerator and the denominator in equation (3.5) depend positively on the optimal level of firm's savings. This indirect effect moves the time 0 ex-dividend value and the expected future dividends in the same direction so the overall effect on expected equity returns is indeterminate. In the appendix, I provide a sufficient condition under which an increase in σ_{xz} leads to higher expected equity returns (proposition A.3). The right panel of Figure 5 reports the positive relationship between risk and expected equity returns.

In the next section, I extend the three–period model to an infinite horizon set–up so that I can use simulation methods to replicate some of the empirical analysis performed with the data.

4 An Infinite Horizon Model

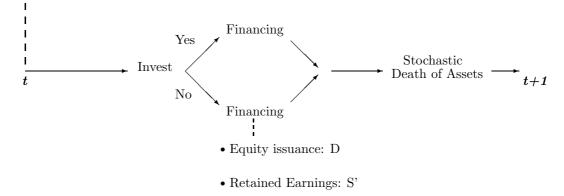
The timing of the model is as follows. A firm starts period t endowed with an amount of internal resources equal to the sum of savings with the cash flows produced by the assets in place. At the beginning of each period, the firm has the option of installing an asset. After the investment decision has been taken, the firm chooses the amount of dividends to distribute/equity to raise and the amount of cash to retain. Assets are subject to stochastic depreciation. The latter happens before the end of the period (Figure 6).¹⁶

I use the model to simulate a panel of firms that differ for their cash flows' correlation with the aggregate shock. I show that, everything else being equal, the riskier firms are the ones that save the most to avoid future costly equity issuance. This precautionary saving motive is the key mechanism to generate a positive correlation between expected equity returns and corporate cash holdings.

¹⁶Berk, Green, and Naik [1999], Carlson, Fisher, and Giammarino [1999], Sundaseran and Wang [2008] and Tserlukevich [2008] are recent example of infinite–horizon, partial–equilibrium models of the firm in a real option framework

Figure 6: Timing of the Infinite Horizon Model

- current risk–free interest rate: r
- Quality of the new investment opportunity: $Q = \{1 \text{ if } h, 0 \text{ if } l\}$
- Cash flows and previous period savings
- Assets in place: $n = n_H + n_L$



4.1 Interest Rate and Pricing Kernel

The pricing kernel is very similar to the one described in Section 3.2. The only difference is that the one period risk–free interest rate follows an autoregressive process:¹⁷

$$r_{t+1} = (1-\rho)\bar{r} + \rho r_t + \sigma_r \varepsilon_{r,t+1}.$$

The unconditional mean of the risk-free interest rate is \bar{r} , the persistence ρ and the conditional variance is σ_r . The shock to the risk-free rate, $\varepsilon_{r,t+1} \sim N(0,1)$, is assumed to be independent and identically distributed.¹⁸

The pricing kernel used at time t to evaluate a pay-off at time t + 1 is

$$M_{t+1} = e^{m_{t+1}} = e^{-r_t - \frac{1}{2}\sigma_z^2 - \sigma_z \varepsilon_{z,t+1}}.$$
(4.1)

The aggregate shock, $\varepsilon_{z,t+1} \sim N(0,1)$, is correlated with the shock to the firm's cash flows. I will describe this correlation in the next section.

The conditional mean of M_{t+1} is equal to the inverse of the gross risk-free interest rate. In addition, the implied Sharpe ratio, the ratio between the conditional standard

¹⁷A time–varying interest rate allows the model to generate time–varying average expected returns.

¹⁸This assumption can be relaxed by allowing the shock to the interest rate to be correlated with the shock to the stochastic discount factor.

deviation and conditional mean of the stochastic discount factor, is constant and equal to $\sqrt{e^{\sigma_z^2} - 1}$. I use this measure to calibrate the value for σ_z .

4.2 Production

Assets differ with respect to their risk. An asset of type h (high risk asset) has a higher correlation with the *aggregate shock* than an asset of type l (low risk asset). The investment opportunity can be of low risk type with probability θ and of high risk type with probability $1 - \theta$, $\theta \in [0, 1]$. If the firm decides to invest, it has to pay a fixed cost equal to I. In what follows, I normalize the cost of investment to 1 to simplify the notation. This can be done without loss of generality.

The pay-off of an asset at time t is equal to $e^{x_{i,t}}$, where $x_{i,t}$ can be written as

$$x_{i,t} = \mu - \frac{1}{2}\sigma_x^2 + \sigma_x \varepsilon_{i,t} \qquad i = h, l.$$
(4.2)

The *idiosyncratic shock* in (4.2), $\varepsilon_{i,t} \sim N(0,1)$, is correlated with the *aggregate shock* in (4.1).

I assume that the variance–covariance matrix among $\varepsilon_{z,t+1}$, $\varepsilon_{h,t+1}$ and $\varepsilon_{l,t+1}$ is equal to:

$$\begin{pmatrix} 1 & \sigma_{h,z} & \sigma_{l,z} \\ \sigma_{h,z} & 1 & \sigma_{h,z}\sigma_{l,z} \\ \sigma_{l,z} & \sigma_{h,z}\sigma_{l,z} & 1 \end{pmatrix},$$

where $\sigma_{i,z}$ is the correlation of $\varepsilon_{i,t+1}$ with the aggregate shock $\varepsilon_{z,t+1}$ and $\sigma_{h,z} > \sigma_{l,z} > 0$. It follows that an individual asset correlation with the pricing kernel is equal to $-\sigma_x \sigma_z \sigma_{i,z}$

Let $\beta_{x_{i,z}} = \sigma_x \sigma_z \sigma_{i,z}$ and assume that a firm has *n* assets in place. The present discounted value of the cash flows that will be produced tomorrow by the *n* assets in place is:

$$\pi E_t \left[e^{m_{t+1}} \sum_{i=1}^n e^{x_{i,t+1}} \right] = \pi e^{-r_t + \mu} \sum_{i=1}^n e^{-\beta_{x_i,z}}.$$
(4.3)

As in Berk et al. [1999], I define the firm systematic risk, $\beta_{x,z}$, to be an average of the individual assets correlation with the pricing kernel so that I can rewrite equation (4.3) as

$$\pi E_t \left[e^{m_{t+1}} \sum_{i=1}^n e^{x_{i,t+1}} \right] = \pi n e^{\mu} e^{-\beta_{x,z}} e^{-r_t}, \qquad (4.4)$$

where $\beta_{x,z}$ is equal to $-e^{\log\left(\sum_{i=1}^{n} \frac{e^{-\beta_{x,i}z}}{n}\right)}$. Equation (4.4) has a natural interpretation: the present discounted value of tomorrow cash flows is the certain equivalent – given by the expected value of the cash flows $(\pi n I e^{\mu})$ multiplied by a risk adjustment $(e^{-\beta_{x,z}})$ – discounted using the risk-free interest rate.

Assets currently in place can disappear randomly (stochastic depreciation). I define $Y_{i,j}$ to be an i.i.d. random variable associated with an asset in place j of type i that takes value 0 with probability π and value 1 with probability $1 - \pi$. If $Y_{i,j}$ is equal to zero than the asset will be lost, otherwise it survives to the next period.

4.3 Financing

In each period, the firm has to decide if to invest or not and, conditional on the investment decision, how much dividends to distribute/equity to issue and how much cash to retain. The firm takes these decisions knowing the number of high risk assets $(n_{h,t})$, the number of low risk assets $(n_{l,t})$, the savings accumulated from the previous period (S_t) , the current level of the risk-free interest rate (r_t) and the quality of the new investing opportunity (Q_t) .¹⁹

The sources of funds are the after taxes profits generated at the beginning of time t^{20} plus the beginning of period cash holdings, S_t . The uses of funds are equal to dividends distributions, D_t , plus the (discounted) amount of cash that the firm decides to have at the beginning of the next period, S_{t+1} , plus the fixed cost of investment if the firm decides to install a new asset. Retaining cash is costly because the firm pays the corporate tax, τ , on the interests earned on savings so that the internal accumulation rate is $\hat{R}_t = e^{r_t} - \tau(e^{r_t} - 1) < e^{r_t} = R_t$, where R_t is the gross risk-free interest rate at time t.

Let \mathcal{I}_t be an indicator variable that equals one if the firm invests at time t and zero otherwise. Then the firm's budget constraint can be written as

$$S_t + (1 - \tau) \left(\sum_{j=0}^{n_{l,t}} e^{x_{l,j}} + \sum_{j=0}^{n_{h,t}} e^{x_{h,j}} \right) = D_t + \frac{S_{t+1}}{\widehat{R}_t} + \mathcal{I}_t.$$
(4.5)

If $D_t < 0$, the firm can raise equity by paying a percentage issuance cost equal to λ . I define Δ_t to be an indicator variable that takes value of one if the firm issues equity $(D_t < 0)$ and zero otherwise, so that the return paid by the firm to the shareholders at time t is equal to $(1 + \lambda \Delta_t)D_t$.

The trade-off that the firm faces is given by the choice of distributing dividends today or retain cash in order to avoid costly external financing tomorrow.

 $^{^{19}}Q$ takes a value of one if the new investment is of the low risk type, otherwise Q is equal to zero.

²⁰Let $n_{h,t}$ and $n_{l,t}$ be the beginning of period number of type h and type l assets in place respectively. Then the after cash profits generated by the $(n_{h,t} + n_{l,t})$ assets are equal to $(1 - \tau) \left(\sum_{j=0}^{n_{l,t}} e^{x_{l,j}} + \sum_{j=0}^{n_{h,t}} e^{x_{h,j}} \right)$

4.4 Equity Valuation

The value of equity – equal to the present discounted value of the firm's future dividends – is the solution to^{21}

$$V(n_h, n_l, C, r, Q) \equiv \max_{D, \mathcal{I}, S' \ge 0} (1 + \lambda \Delta) D + E \left[M' V(n'_h, n'_l, C', r', Q') \right]$$
(4.6)

subject to:

$$C = D + \frac{S'}{\widehat{R}} + \mathcal{I}, \qquad (4.7)$$

$$C' = S' + (1 - \tau) \left(\sum_{j=0}^{n'_l} e^{x_{l,j}} + \sum_{j=0}^{n'_h} e^{x_{h,j}} \right),$$
(4.8)

$$n'_{h} = \sum_{j=1}^{n_{h}+Q\mathcal{I}} Y'_{h,j} \qquad n'_{l} = \sum_{k=1}^{n_{l}+(1-Q)\mathcal{I}} Y'_{l,j}, \qquad (4.9)$$

$$Prob(Y'_{i,j} = 1) = \pi \qquad Prob(Y'_{i,j} = 0) = 1 - \pi \qquad i = 1, 2 \qquad \forall \ j, \ k \ .$$

To simplify the notation, I perform a change of state variable introducing a new variable that summarizes the total amount of the beginning of period internal resources available to the firm. I call this variable C and it is defined as the sum of after tax profits plus the amount of cash transfered internally from the previous period. This transformation allows me to rewrite the firm's budget constraint as in (4.7). Equation (4.8) is the law of motion for C.

Equation (4.9) describes the law of motion of the assets in place as a function of the realizations of the i.i.d. random variables $Y_{i,j}$. This equation depends on the realization of Q only if the firm decides to invest in the current period ($\mathcal{I} = 1$).

4.5 Optimal Financing Policy

By the envelope condition, the Euler equation for savings is

$$(1 + \lambda \Delta) \ge \widehat{R}E \Big[M'(1 + \lambda \Delta') \Big]$$

In what follows, I assume an interior solution and I also assume that the firm does not issue equity in the current period – so that $\Delta = 0$. The Euler equation becomes

$$1 = \frac{\widehat{R}}{R} + \frac{\widehat{R}\lambda Prob(\Delta'=1)}{R} + \widehat{R}\lambda COV[M', \Delta'], \qquad (4.10)$$

 $^{^{21}\}mathrm{From}$ now on I will suppress time indexes and I will denote next period values with a prime.

where I have exploited the fact that E[M'] = 1/R, $E[M'\Delta'] = E[M']E[\Delta'] + COV[M', \Delta']$, and $E[\Delta'] = Prob(\Delta' = 1)$.

Equation (4.10) is the analogue of equation (A.5): the firm equates the marginal cost of saving an extra unit of cash – the forgone dividend in the current period – to the marginal benefit – the expected dividend that the firm will distribute next period plus the expected reduction in issuance cost if the firm will need to issue equity.

Having risky assets is not necessary to generate a precautionary saving motive. Without the covariance term, I would get an Euler Equation very similar to the one in Riddick and Whited [2008]. In such a situation, firms with the same number of assets in place (equal size) will choose the same saving policy because the probability of issuing equity next period is the same for all of them.

In this model, risk induces heterogeneity in savings policies controlling for firm's size. When cash flows are correlated with the aggregate shock, riskier firms will expect lower cash flows in those future states when there is investment and the realization of the aggregate shock is low. As a consequence, riskier firms save more to reduce the expected financing cost everything else being equal.

To study how the probability of investing next period affects the optimal retention policy it is sufficient to notice that a firm will issue equity next period only if it decide to invest. As a consequence, the probability of issuing equity next period is just equal to the probability of investing next period multiplied by the probability of issuing equity conditional on investing. Bearing this in mind, the Euler Equation can be rewritten including the probability of investing next period as

$$1 = \frac{\widehat{R}}{R} + \frac{\widehat{R}\lambda Prob(\mathcal{I}'=1)Prob(\Delta'=1|\mathcal{I}'=1)}{R} + \widehat{R}\lambda COV[\tilde{M}',\Delta']$$

If the probability of investing next period is zero, then the firm will never retain cash because the probability of issuing costly equity is zero. On the other hand, the marginal benefit of retaining an extra unit of cash is increasing in the the probability of investing next period, hence the precautionary motive is stronger in times when investing opportunities are likely to arise.

5 Calibration

The model's parameters are separated in the three groups reported in Table 11. The first group includes the parameters that are taken from other studies. Following Riddick and Whited [2008], I set the corporate tax rate τ equal to 0.3 and the survival probability of each installed asset π equal to 0.85. The proportional equity issuance cost is set equal to

		A priori information				
Variable	Value	Description				
Ι	1.00	sunk cost for investment				
λ	0.10	equity issuance cost				
π	0.85	survival probability				
au	0.30	tax on income from interest rates				
	Inter	est rate and pricing kernel				
Variable	Value	Description				
\bar{r}	0.04	unconditional mean of r_{t+1}				
ρ	0.75	persistence of r_{t+1}				
σ_r	0.02	conditional variance of r_{t+1}				
σ_z	0.40	conditional variance of $\log(M_{t+1})$				
		Technology				
Variable	Value	Description				
μ	-0.68	mean of the cash flows distribution				
σ_x	1.15	variance of the cash flows distribution				
β_h	0.45	correlation asset type h				
β_l	0.25	correlation asset type l				
θ	0.35	probability of getting a project of type \boldsymbol{h}				

Table 11: Parameters' Values

0.1. This value is close to the *seven percent* rule found by Chen and Ritter [2000].

The second group contains the four parameters governing the processes for the pricing kernel and interest rate: $\rho, \bar{\mathbf{r}}, \sigma_{\mathbf{r}}, \sigma_{\mathbf{z}}$. I set the first three to match the unconditional mean, the unconditional variance, and the first order autocorrelation of the annual risk-free interest rate over the post war period. The remaining parameter, $\sigma_{\mathbf{z}}$, is chosen to match the value of the Sharpe ratio.

The last group is made up by the parameters that govern the production process: μ , σ_x , β_h , β_l , θ . I set their values to match five unconditional moments: average equity premium, standard deviation of equity premium, average investment-to-capital ratio, average book-to-market ratio, and average savings-to-capital ratio.

I briefly describe the theoretical counterparts of the targeted financial and accounting data. The value of equity is the ex-dividend value of the firm at the end of each period before the death of the assets in place. The one-period equity return at time t is the ratio between the value of the firm at time t and the ex-dividend value of the firm at time t - 1:²²

$$R_{t-1,t} = \frac{V_t}{V_{t-1} - D_{t-1}}.$$
(5.1)

 $^{^{22}}$ This definition of equity return is the same as the one used in Zhang [2005] and Gomes and Schmid [2008]

Table 12: Balance Sheet

Assets	Liabilities
S_{t+1}/\widehat{R}_t $K_t = n_{l,t} + n_{h,t}$	A_t

The accounting variables are evaluated at the end of each. A stylized balance sheet is described in Table 12. Total assets at time t (A_t) are equal to the amount of internal resources that are transferred to the next period (S_{t+1}/\hat{R}_t) plus the book value of capital $(n_{l,t} + n_{h,t})$.

The book-to-market value at time t equals to the ratio of the book value of capital over the ex-dividend value of equity: $BM_t = \frac{K_t}{V_t - D_t}$. The last two variables targeted in the calibration exercise are the investment-to-capital ratio, defined as the cost of investment (I) over the book value of capital (K_t), and the cash-to-capital ratio, defined as the amount of internal resources that are transferred to the next period (S_{t+1}/\hat{R}_t) over the book value of capital (K_t). In Table 13, the calibrated values are compared to their empirical counterparts.

Table 13: Sample Moments

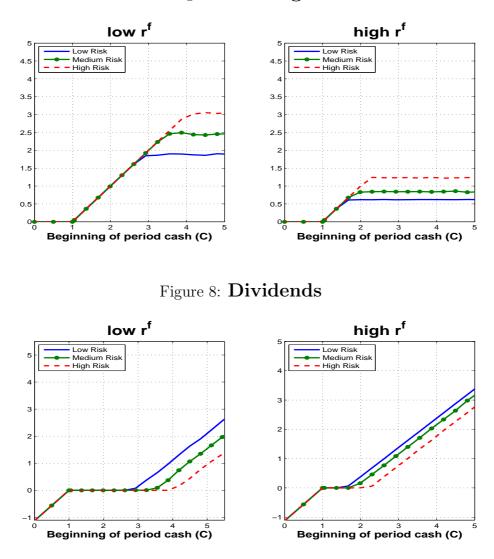
Variable	Description	Data	Model
$E[R^f]$	annual risk–free interest rate	0.018	0.04
$std[R^f]$	std. dev. risk–free interest rate	0.030	0.026
$\rho[R^f]$	autocorrelation risk–free interest rate	0.570	0.75
$\sigma(M)/E(M)$	Sharpe Ratio	0.400	0.431
$E[R-R^f]$	annual equity premium	0.056	0.052
$std[R-R^f]$	std. dev. equity premium	0.168	0.164
K/(V-D)	Book-to-market	0.670	0.557
S/K	Savings-to-capital ratio	0.170	0.169
I/K	Investment-to-capital ratio	0.145	0.162

I take the values for the autocorrelation of the annual risk-free interest rate, the average annual equity premium and the corresponding standard deviation from Canova and De Nicolo' [2003]. The investment-to-capital ratio is from Gomes and Schmid [2008]. The values for the unconditional mean and standard deviation of the risk-free interest rate, the average Sharpe Ratio and the book-to-market ratio are from Zhang [2005]. The empirical counterpart of the savings-to-capital ratio can be found in Opler, Pinkowitz, Stulz, and Williamson [1999].

5.1 Optimal Policies

This section illustrates how the precautionary saving motive affects the firms' optimal retention policies. I consider three firms that have invested in the current period and have six assets in place. The low–risk firm only has low–risk assets installed. The medium–risk firm has three low–risk assets and three high–risk assets in place. Finally, the high–risk firm has only high–risk assets installed.

In the left panel of Figure 7, I depict the optimal retention policy when the





risk-free interest rate is at its lowest level; in the the right panel, I illustrate the optimal retention policy when the risk-free interest rate is at its highest level. Similarly for dividends in Figure 8. In all the figures, quantities are reported as a function of the beginning of period cash holdings C.

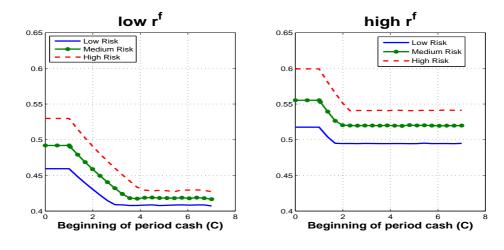
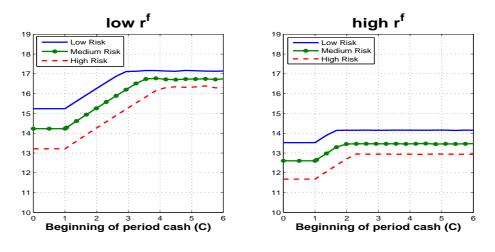


Figure 9: Book-to-Market





Equity is only issued when firms do not have enough internal resources to finance the cost of investment (C < 1). Firms retain cash if they are able to fully finance investment with internal resources $(C \ge 1)$ and they only distribute dividends when they are able to save the unconstrained optimal level of cash. Notice that the high–risk firm starts to distribute dividends at a higher level of C. Controlling for number of installed assets, riskier firms save more. The intuition for such a result is quite simple. Given that the *aggregate shock* is i.i.d., all firms have the same expected cash flows. The high–risk firm, however, will have a lower cash flows with respect to a low–risk firm conditional on a low realization of the *aggregate shock*, namely exactly in the state in which the probability of external financing is the highest. Hence, the high–risk firm, having a higher expected financing cost, saves more everything else being equal.

All firms save more when the interest rate is low. This is not surprising because the

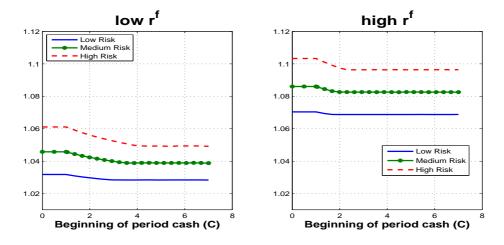


Figure 11: Expected Equity Returns

calibrated values are such that a firm will invest in both types of assets when the riskfree interest rate is at its lowest level and it will only invest in the low-risk assets when the risk-free interest rate is at its highest level. Such a property generates a realistic pro-cyclical investment rate and a counter-cyclical book-to-market ratio. Because of the pro-cyclicality of investment, firms save more when the risk-free interest rate is low.

Figures 9 and 10 report the book-to-market and the ex-dividend value of equity, respectively. The book-to-market is flat for values of C less than the cost of investment, it is decreasing in C when firms save and do not distribute dividends and it is again flat when firms distribute dividends. This behavior is entirely determined by the ex-dividend value of equity given that the book value of capital is constant.

Two firms that only differ in C can have different book-to-market values. This happens when two firms (e.g. low-risk firms) do not distribute dividends but are retaining a positive amount of cash. Given that the two firms have identical future investment opportunities, the difference in book-to-market is an indirect measure of their different expected financing cost. Put differently, a higher book-to-market value signals a higher exposure to *financing risk*.

Expected equity returns are depicted in Figure 11. By construction, the high–risk firm has a higher expected equity return than the low–risk firm; the high–risk firm also retains more cash. The model is able to generate a positive relationship between expected equity returns and corporate cash holdings.

5.2 Empirical Implications

I simulate 500 panels of length 800 each containing 2000 firms. For each one of them, realized excess equity returns at time t are regressed on the natural logarithm of the ex-dividend value of the firm at time t - 1, on the natural logarithm of book-to-market at time t - 1 and on the cash-to-capital at time t - 1.²³ Then, I evaluate the time series average of the cross-sectional estimates and the corresponding t-statistic dividing the time series average by its time series standard errors.

Table 14 compares the regression coefficients derived by averaging the results over

	-	1	2		و	3
	model	data	model	data	model	data
log Size	-0.02 (-8.03)	-0.16 (-3.05)			-0.05 (-9.33)	-0.15 (-2.93)
log Book-to-market	0.14 (11.06)	0.27 (3.38)			0.28 (10.45)	0.31 (4.47)
Cash-to-Capital			-0.06 (-9.95)	0.05 (1.27)	0.10 (7.80)	0.08 (2.70)

Table 14: Simulated Fama–MacBeth Regressions

In each period t, realized excess equity returns at time t are regressed on the natural logarithm of the value of the firm at time t - 1, on the natural logarithm of book–to–market at time t - 1 and on the ratio of corporate savings over book value of installed assets at time t - 1. The cross sectional regression is:

$$R_{it} - R_{t-1}^f = \alpha_{t-1} + b_{1,t-1} \log \left(V_{t-1} - D_{t-1} \right) + b_{2,t-1} \log \left(BM_{t-1} \right) + b_{3,t-1} \frac{S_t / \hat{R}_{t-1}}{K_{t-1}} + \varepsilon_{i,t}$$

Each reported coefficient is the time series average of the cross-sectional estimates and the corresponding t-statistic is evaluated dividing the time series average by its time series standard errors. The results are generated by the simulation of 500 panels of length 800 each containing 2000 firms. The empirical counterparts are from Section 2.2.

the 500 simulations with their empirical counterparts from Table 2.

The first regression replicates at an annual frequency the first equation in Table 2 . The model is qualitatively able to replicate the *size* and *value* effects found by Fama and French [1992]. In the second regression, I only use corporate savings as an explanatory variable. In the data the regression coefficient is positive, but not significantly different

²³I do not take the natural logarithm of the ratio of cash-to-capital to keep observations relative to firms with zero savings. When I only consider firms with positive savings and I use the natural logarithm the results will be qualitatively the same.

from zero: equity returns and corporate savings are not correlated. On the other hand, the model generates a negative relationship between equity returns and corporate savings when corporate savings is the only independent variable: the unconditional correlation is negative. In the last equation, I regress equity returns on corporate savings controlling for size and book-to-market. In the data, the regression coefficients on size and bookto-market remain significant and their values do not change much while the coefficient on corporate savings, still positive, becomes significant. Something very similar happens in the simulated data, the only difference is that the coefficient on corporate savings changes its sign.

Why are savings important in explaining the cross-section of equity returns? In the model, riskier firms are the ones that save the most to reduce expected financing costs controlling for the number of assets in place. In addition, riskier firms are also the one that bear the higher expected equity return and this creates a positive correlation between the latter and corporate savings. It is very difficult, if not impossible, to observe in the data the composition of assets in place in order to precisely determine a firm's riskiness. Corporate savings are just a proxy for assets' composition – hence for risk – and for this reason they are relevant in explaining the differences in the cross-section of equity returns.

6 Conclusion

In this paper, I document a new empirical finding: corporate cash holdings and firms' equity returns are positive correlated once I control for the *size* and *value* effects.

I also show that the Fama–French three factors do not to explain the excess return earned by High–Cash over Low–Cash firms, while a *cash* factor (HCMLC) does. This is evidence that HCMLC captures a source of risk which is different from the ones proxied by MKT, SMB and HML.

To rationalize the empirical findings, I propose a model in which firms face a tradeoff between the choice of distributing dividends in the current period and retaining cash to avoid costly external financing in the future. Ex-ante, all firms have the same expected cash flows, but high-risk firms, whose cash flow which is more correlated with the aggregate shock, will have relatively lower cash flows exactly in those states in which the probability of external financing is the highest. Hence, *ceteris paribus*, high-risk firms have a higher expected financing cost and they optimally decide to retain more cash. In turn, this implies a positive relationship between expected equity returns and corporate cash holdings. Ideally, a structural corporate finance model would include all three main sources of financing available to a firm: internal funds, external equity, and corporate debt. Writing down such a model is not difficult, but its solution is computationally intense. My future research will focus on the extension of the model presented in this paper to include risky corporate debt.

A Proofs of Propositions

A.1 Existence and Uniqueness of the Optimal Retention Policy

Proposition A.1 A unique interior solution to the firm's problem exists if

$$1 + \pi \lambda \Phi_2|_{S_1=0} > \frac{R}{\widehat{R}}$$

where $\Phi_2 = \Phi(\zeta)$, $\Phi(\cdot)$ is the CDF of a standard normal random variable and

$$\zeta = \frac{\log(1 - S_1) - \mu + .5\sigma_x^2 + \beta_{xm}}{\sigma_x}$$

Proof: Rewrite the firm's problem as

$$\begin{aligned} \max_{S_1 \ge 0} C_0 - S_1 + E_0 \Big[M_1 \Big((1 - \pi) (e^{X_1} + S_1) \Big) \Big] + E_0 \Big[M_1 \Big(\pi (1 + \lambda \Delta_1) (e^{X_1} + S_1 - 1) \Big) \Big] + E_0 \Big[M_2 \Big(\pi C_2) \Big) \Big]. \end{aligned}$$

Let $\kappa = \log (1 - S_1)$, then $E_0 \Big[M_1 \Big(\pi (1 + \lambda \Delta_1) (e^{X_1} + S_1 - 1) \Big) \Big]$ can be rewritten as
 $\pi E \Big[M_1 \big((1 + \lambda) (e^{X_1} + S_1 - 1) \big) \Big| X_1 < \kappa \Big] \Phi \Big(\frac{\kappa - \mu + 0.5\sigma_x}{\sigma_x} \Big)$
 $+ E \Big[M_1 \big(e^{X_1} + S_1 - 1 \big) \Big| X_1 \ge \kappa \Big] \Big(1 - \Phi \Big(\frac{\kappa - \mu + 0.5\sigma_x}{\sigma_x} \Big) \Big)$

The above expression can be further simplified using the following two results.

Result A.1 Let X and Y be two correlated normal random variables. X has mean μ_x and variance σ_x , Y has mean μ_y and variance σ_y . Let ρ be the their correlation coefficient. Then

$$E\left[e^{Y}|X \leq \bar{x}\right] = e^{\mu_{y} + \frac{\sigma_{y}^{2}}{2}} \left(\frac{\Phi\left(\frac{\bar{x}-\mu_{x}}{\sigma_{x}} - \rho\sigma_{y}\right)}{\Phi\left(\frac{\bar{x}-\mu_{x}}{\sigma_{x}}\right)}\right),\tag{A.1}$$

where Φ is the cumulative distribution function of a standard normal variable.

Result A.2 Let X and Y be two correlated normal random variables. X has mean μ_x and variance σ_x , Y has mean μ_y and variance σ_y . Let σ_{xy} be the their covariance. Then:

$$E[e^{X}e^{Y}|X \ge \bar{x}] = e^{\mu_{y} + \mu_{x} + \frac{\sigma_{y}^{2} + \sigma_{x}^{2} + 2\sigma_{xy}}{2}} \left(\frac{1 - \Phi(\frac{\bar{x} - \mu_{x} - \sigma_{x}^{2} - \sigma_{xy}}{\sigma_{x}})}{1 - \Phi(\frac{\bar{x} - \mu_{x}}{\sigma_{x}})}\right);$$
(A.2)

where Φ is the cumulative distribution function of a standard normal variable.

These two results can be derived using any standard statistics textbook (e.g. Casella and Berger [2002]).

Using the results in lemma A.1 and A.2, $E_0 \Big[M_1 \Big(\pi (1 + \lambda \Delta_1) (e^{X_1} + S_1 - 1) \Big) \Big]$ simplifies to

$$\frac{\pi}{R} \Big((1 + \Phi_1 \lambda) e^{\mu + \beta_{X,M}} + (S_1 - 1)(1 + \Phi_2 \lambda) \Big),$$

where $\Phi_1 = \Phi(\zeta - \sigma_x)$, $\Phi_2 = \Phi(\zeta)$, $\zeta = \frac{\kappa - \mu + .5\sigma_x^2 + \beta_{xm}}{\sigma_x}$ and $\Phi(\cdot)$ is the CDF of a standard normal random variable.

The first order condition with respect to S_1 is

$$\frac{1}{\widehat{R}} + \phi = \frac{1 - \pi}{R} + \frac{\pi}{R} \left(e^{\mu + \beta_{X,M}} \lambda \Phi'(\zeta - \sigma_x) \frac{1}{\sigma_x(S_1 - 1)} + (1 + \Phi_2 \lambda) + (S_1 - 1)\lambda \Phi'(\zeta) \frac{1}{\sigma_x(S_1 - 1)} \right).$$

Now I can exploit the fact that $\Phi'(\zeta - \sigma_x) = \Phi'(\zeta)e^{-0.5(\sigma_x^2 + \sigma_x\zeta)}$ and get the following first order condition

$$\frac{1}{\widehat{R}} + \phi = \frac{1}{R} + \frac{\pi\lambda}{R} \Phi_2.$$

 Φ_2 is decreasing in S_1 and goes to 0 as S_1 approaches 1. As a consequence, Φ_2 reaches its maximum value when S_1 is equal to zero. The firm will save a positive amount if and only if $\frac{\pi\lambda}{R}\Phi_2|_{S_1=0} > \frac{1}{\hat{R}} - \frac{1}{R}$, which is equivalent to require $\pi\lambda\Phi_2|_{S_1=0} > \frac{R}{\hat{R}} - 1$. Since Φ_2 is decreasing in S_1 and by assumption $\frac{R}{\hat{R}} > 1$, a unique interior solution exists.

A.2 Optimal Retention Policy and Risk

Proposition A.2 The optimal retention policy is increasing in the firm's riskiness.

Proof: Let's consider the first order condition when an interior solution exists and let's evaluate the total differential with respect to S_1^* and σ_{xz} :

$$0 = \left(\frac{\Phi'_2}{\sigma_x(S_1^* - 1)}\right) dS_1^* + \left(\frac{\Phi'_2\sigma_z}{\sigma_x}\right) d\sigma_{xz}.$$
 (A.3)

It follows that

$$\frac{dS_1^*}{d\sigma_{x,z}} = -\frac{\left(\frac{\Phi_2'\sigma_z}{\sigma_x}\right)}{\left(\frac{\Phi_2'}{\sigma_x(S_1^*-1)}\right)} = -\sigma_z(S_1^*-1) > 0, \qquad (A.4)$$

since the firm will never choose S_1^* bigger or equal to 1 given that the return on internal savings is less than the risk-free rate.

An alternative proof uses the following Euler equation

$$1 = \widehat{R}E_0[M_1] + \pi \lambda \widehat{R} \Big(E_0[M_1] Prob_0(\Delta_1 = 1) + COV[M_1, \Delta_1] \Big).$$

Let's consider two firms, h and l, with different values of the correlation with the SDF, σ_{hz} and σ_{lz} such that $\sigma_{hz} > \sigma_{lz}$. Notice that $Prob(\Delta_{i,1} = 1)$ is equivalent to $Prob(\sigma_{iz}\varepsilon_z + \sqrt{1 - \sigma_{iz}^2}\varepsilon_i < \frac{\log(1 - S_i^*) - \mu + 0.5\sigma_x^2}{\sigma_x})$, where ε_i and ε_z are two independent standard normally distributed variables. Assume that the two firms choose the same optimal saving policy, then $S_h^* = S_l^* \Rightarrow Prob(\Delta_{l,1} = 1) = Prob(\Delta_{h,1} = 1)$.

The firms' Euler equations imply $COV[M_2, \Delta_{h,1}] = COV[M_2, \Delta_{l,1}]$. But this is not possible given that σ_{hz} and σ_{lz} are different. To show the last claim, rewrite $COV[M_2, \Delta_{i,1}]$ as

$$E[M_2\Delta_{i,1}] - E[M_2]E[\Delta_{i,1}] = Prob(\Delta_{i,1} = 1)\Big(E[M_2 \mid \Delta_{i,1} = 1] - \frac{1}{R}\Big),$$

where I have used the fact that $E[M_2\Delta_{i,1}] = Prob(\Delta_{i,1} = 1)E[M_2 \mid \Delta_{i,1} = 1]$ and $E[\Delta_{i,1}] = Prob(\Delta_{i,1} = 1)$. Given that $Prob(\Delta_{l,1} = 1) = Prob(\Delta_{h,1} = 1)$, then it must be the case that $E[M_2 \mid \Delta_{h,1} = 1] = E[M_2 \mid \Delta_{l,1} = 1]$. Using lemma A.1, it turns out that

$$\frac{E[M_2 \mid D_2^h < 0]}{E[M_2 \mid D_2^l < 0]} = \frac{\Phi\left(\frac{\log(1-S_h^*) - \log(1-\tau) - \mu - \sigma_x^2 + \sigma_x \sigma_z \sigma_{hz}}{\sigma_x}\right)}{\Phi\left(\frac{\log(1-S_l^*) - \log(1-\tau) - \mu - \sigma_x^2 + \sigma_x \sigma_z \sigma_{lz}}{\sigma_x}\right)} = 1.$$

Under the assumption $S_h^* = S_l^*$, the above ratio implies $\sigma_{lz} = \sigma_{hz}$, which contradicts the assumption of $\sigma_{hz} > \sigma_{hz}$.

The next step is to show that $S_h^* < S_l^*$ cannot be a solution when $\sigma_{hz} > \sigma_{hz}$. By contradiction, assume that the opposite is true. Then $S_h^* < S_l^*$ implies $Prob(\Delta_{h,1} = 1) > Prob(\Delta_{l,1} = 1)$ because $Prob(\Delta_{i,1} = 1)$ is decreasing in S_i^* and unaffected by σ_{iz} . Given that the Euler equation must hold for both firms, it must be the case that $COV[M_2, \Delta_{l,1}] > COV[M_2, \Delta_{h,1}]$, which is the same to require $(\log(1 - S_l^*) + \sigma_x \sigma_z \sigma_{lz}) > (\log(1 - S_h^*) + \sigma_x \sigma_z \sigma_{hz})$. The last inequality contradicts the fact that $S_h^* < S_l^*$ and $\sigma_{hz} > \sigma_{hz}$. So the only possibility left is that a riskier firm saves a larger amount of cash.

A.3 Expected Returns and Risk

Proposition A.3 The firm's expected return is increasing in the firm's riskiness if, given the optimal savings policy S_1^* , the following inequality holds:

$$\sigma_x e^{\mu - \beta_{M,X}} \ge \frac{(1 + \pi \lambda \Phi_2)}{(1 + \pi \lambda \Phi_1)} (1 - S_1^*).$$
(A.5)

Proof: To asses how a change in riskiness affects expected equity returns, I take the first

derivative of

$$E[R_{0,1}^e] = \frac{E_0\Big[(1-\pi)(e^{x_1}+S_1^*)+\pi(1+\lambda\Delta_1)(e^{x_1}+S_1^*-1)\Big]+E_0[\frac{M_2}{M_1}\pi C_2]}{E_0\Big[M_1\Big((1-\pi)(e^{x_1}+S_1^*)+\pi(1+\lambda\Delta_1)(e^{x_1}+S_1^*-1)\Big)\Big]+E_0[M_2\pi C_2]} = \frac{f(\sigma_{xz})}{g(\sigma_{xz})}$$

with respect to σ_{xz} . Applying the quotient rule, $\frac{dE[R_{0,1}^e]}{d\sigma_{xz}} = \frac{f_{\sigma_{xz}}g - fg_{\sigma_{xz}}}{g^2}$, where $f_{\sigma_{xz}}$ and $g_{\sigma_{xz}}$ are the derivatives of $f(\sigma_{xz})$ and $g(\sigma_{xz})$ w.r.t. σ_{xz} . The close form expression for the two derivatives are²⁴

$$g_{\sigma_{xz}} = \frac{1-\pi}{R} \left(-\sigma_x \sigma_z e^{\mu - \beta_{M,X}} + \frac{dS_1^*}{d\sigma_{xz}} \right) + \frac{\pi}{R} \left(-\sigma_x \sigma_z e^{\mu - \beta_{M,X}} (1 + \lambda \Phi_1) + \frac{\lambda \Phi_1'}{\sigma_x} e^{\mu - \beta_{M,X}} \left(\frac{dS_1^*/d\sigma_{xz}}{S_1^* - 1} + \sigma_z \right) + (1 + \lambda \Phi_2) \frac{dS_1^*}{d\sigma_{xz}} + (S_1^* - 1) \frac{\lambda \Phi_2'}{\sigma_x} \left(\frac{dS_1^*/d\sigma_{xz}}{S_1^* - 1} + \sigma_z \right) \right) \\ = \frac{1}{R} \left(-\sigma_x \sigma_z e^{\mu - \beta_{M,X}} (1 + \pi \lambda \Phi_1) + (1 + \pi \lambda \Phi_2) \frac{dS_1^*}{d\sigma_{xz}} \right)$$

and

$$f_{\sigma_{xz}} = (1-\pi)\frac{dS_1^*}{d\sigma_{xz}} + \pi \left(\frac{\lambda \Phi_3'}{\sigma_x}e^{\mu} \left(\frac{dS_1^*/d\sigma_{xz}}{S_1^* - 1} + \sigma_z\right)\right) + (1+\lambda\Phi_4)\frac{dS_1^*}{d\sigma_{xz}} + (S_1^* - 1)\left(\frac{\lambda \Phi_4'}{\sigma_x}\frac{dS_1^*/d\sigma_{xz}}{S_1^* - 1} + \sigma_z\right)\right) = [(1+\pi\lambda\Phi_4)\frac{dS_1^*}{d\sigma_{xz}}],$$

where $\Phi_3 = \Phi(\zeta - \frac{\beta x, m}{\sigma_x} - \sigma_x)$ and $\Phi_4 = \Phi(\zeta - \frac{\beta x, m}{\sigma_x})$. Then, it is possible to rewrite $(f_{\sigma_{xz}}g - fg_{\sigma_{xz}})$ as

$$(1+\pi\lambda\Phi_4)\frac{dS_1^*}{d\sigma_{xz}} - \frac{1}{R}\left(-\sigma_x\sigma_z e^{\mu-\beta_{M,X}}(1+\pi\lambda\Phi_1) + (1+\pi\lambda\Phi_2)\frac{dS_1^*}{d\sigma_{xz}}\right)E[R_{1,2}^e],$$

If the above expression is positive then a positive change in σ_{xz} will increase expected returns. This implies:

$$\frac{E[R_{1,2}^e]}{R} > \frac{(1+\pi\lambda\Phi_4)\frac{dS_1^*}{d\sigma_{xz}}}{(1+\pi\lambda\Phi_2)\frac{dS_1^*}{d\sigma_{xz}} - \sigma_x\sigma_z e^{\mu-\beta_{M,X}}(1+\pi\lambda\Phi_1)}$$

Given that in the model $\frac{E[R_{1,2}^e]}{R}$ is always positive, a sufficient condition is to have

$$\sigma_x \sigma_z e^{\mu - \beta_{M,X}} (1 + \pi \lambda \Phi_1) \ge (1 + \pi \lambda \Phi_2) \frac{dS_1^*}{d\sigma_{xz}}.$$
(A.6)

I can further simplify equation (A.6) by using the fact $\frac{dS_1^*}{d\sigma_{xz}} = \sigma_z(1 - S_1^*)$ to get

$$\sigma_x \sigma_z e^{\mu - \beta_{M,X}} (1 + \pi \lambda \Phi_1) \geq (1 + \pi \lambda \Phi_2) \sigma_z (1 - S_1^*) \Rightarrow \sigma_x e^{\mu - \beta_{M,X}} \geq \frac{(1 + \pi \lambda \Phi_2)}{(1 + \pi \lambda \Phi_1)} (1 - S_1^*)$$

²⁴In what follows, I use the fact that $\Phi'_1 = \Phi(\zeta - \sigma_x)' = \Phi(\zeta)' e^{\log(1-S_1^*) - \mu + \beta_{M,X}} = \Phi'_2 e^{\log(1-S_1^*) - \mu + \beta_{M,X}}$ so that the terms $e^{\mu - \beta_{M,X}} \frac{\lambda \Phi'_1}{\sigma_x} big(\frac{dS_1^*/d\sigma_{xz}}{S_1^* - 1} + \sigma_z)$ and $(S_1^* - 1) \frac{\lambda \Phi'_2}{\sigma_x} (\frac{dS_1^*/d\sigma_{xz}}{S_1^* - 1} + \sigma_z)$ cancel each other.

A.4 Optimal Retention Policy: Additional Properties

Proposition A.4 The optimal retention policy is:

- decreasing in the mean of the cash flow process μ ;
- decreasing in the risk-free rate R;
- increasing in the probability of getting an investment opportunity π ;
- increasing in the cost of external financing λ .

Proof: Let's consider the first order condition when an interior solution exists and let's evaluate the total differential with respect to S_1^* and μ :

$$0 = \left(\frac{\Phi'_2}{\sigma_x(S_1^* - 1)}\right) dS_1^* + \left(-\frac{\Phi'_2}{\sigma_x}\right) d\mu \Rightarrow \frac{dS_1^*}{d\mu} = -\frac{\left(-\frac{\Phi'_2}{\sigma_x}\right)}{\left(\frac{\Phi'_2}{\sigma_x(S_1^* - 1)}\right)} = (S_1^* - 1) < 0.$$

The optimal retention policy is decreasing in the mean of the cash flow process μ since the firm will never choose S_1^* bigger or equal to 1 given that the return on internal savings is less than the risk-free rate.

The total differential w.r.t. R and S_1^* implies that the optimal retention policy is decreasing in the risk-free rate R:

$$\frac{1}{\widehat{R}}dR = \lambda \pi \left(\frac{\Phi_2'}{\sigma_x(S_1^*-1)}\right) dS_1^* \Rightarrow \frac{dS_1^*}{dR} = \lambda \pi \frac{\left(\frac{\Phi_2'}{\sigma_x(S_1^*-1)}\right)}{\widehat{R}} < 0.$$

The total differential w.r.t. π and S_1^* implies that the optimal retention policy is increasing in the probability of investing π :

$$0 = \lambda \Phi_2 d\pi + \lambda \pi \left(\frac{\Phi_2'}{\sigma_x(S_1^* - 1)}\right) dS_1^* \Rightarrow \frac{dS_1^*}{d\pi} = -\pi \frac{\left(\frac{\Phi_2'}{\sigma_x(S_1^* - 1)}\right)}{\Phi 2} > 0.$$

The total differential w.r.t. λ and S_1^* implies that the optimal retention policy is increasing in the cost of external financing λ :

$$0 = \pi \Phi_2 d\lambda + \lambda \pi \left(\frac{\Phi_2'}{\sigma_x (S_1^* - 1)} \right) dS_1^* \Rightarrow \frac{dS_1^*}{d\lambda} = \lambda \frac{\left(\frac{\Phi_2'}{\sigma_x (S_1^* - 1)} \right)}{\Phi 2} > 0.$$

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