Market Liquidity, Active Investment, and Markets for Information*

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Abstract

I study a financial market in which active investors choose among investment strategies that exploit information about different fundamentals. The presence of other active investors generates illiquidity. However, active investors pursuing sufficiently different strategies serve as noise traders for each other, and hence also supply each other with liquidity. The strategies can therefore be substitutes or complements. These liquidity externalities have implications for trade volume, price comovement, liquidity commonalities, herding behavior and the informational role of prices. I also study how these externalities affect markets for information. A monopolistic information vendor deliberately induces investor herding, whereas competition fosters information diversity. Finally, I propose a benign rationale for why some financial institutions both sell information and engage in proprietary trading.

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1 Introduction

Many financial market trades are motivated by the desire to profit from superior information about the value of a traded asset, and a key role of asset prices is to reflect the information contained in these trades. Since there is a plethora of information relevant for the value of an asset, active investment managers specialize in a variety of investment strategies. Such investment "styles" have been documented for mutual funds as well as for hedge funds.¹

Against this background, this paper explores several questions: How do investors that pursue different investment strategies interact in the market? Does the presence of investors that follow one strategy benefit or harm investors that follow another strategy? Which strategies do investors choose? What does the existence of diverse strategies imply for the sale of financial information, and how does a market for information affect the investors' choice?

The common view is that active investors exert negative externalities on each other, as they compete for profits by trading in the same asset (e.g., Grossman and Stiglitz, 1980). However, consider a hedge fund that follows a contrarian investment strategy. By trading "against" the market, the fund may supply liquidity to other investors (much like a market-maker). If it discontinued trading, other investment strategies could suffer from a decline in liquidity – contrary to the view that the withdrawal of some active investors always benefits the remaining active investors.

This paper studies a model in which active investors play such a role in liquidity provision, even though their strategies do not mimic "market-making". The main contribution is to show that investors choose strategies that can deprive each other of, or supply each other with, liquidity. As further discussed, these liquidity externalities can give rise to various financial market phenomena, such as herding behavior, price comovement, or comovement in liquidity. Lastly, the paper shows that the liquidity-providing role of active investors provides a novel ratio-

¹Equity investment strategies can, for example, be distinguished by industry, geography or value vs. growth. Well-known hedge fund strategies include takeover arbitrage, macro-trading or mean-reversion strategies. Chan et al. (2002), Barberis and Shleifer (2003), and Goetzmann and Brown (2003) provide empirical studies of "style" investing.
nale for selling financial information, and further implies a positive externality of information market competition on financial markets.

In the basic model, investors trade in a single asset, the fundamental value of which is driven by two factors. Investors choose, that is, specialize in active trading strategies that are based on one of the two factors. For instance, the factors may represent macroeconomic and company-specific fundamentals, and investment strategies may weight information about these fundamentals differently ("macro-trading" vs. "stock-picking").

Uninformed investors in the model participate in a market with two classes of active investors, each of which possesses a distinct information advantage. During trading, this information asymmetry generates illiquidity, meaning that the investors’ order flow has an impact on the price. While the two investor classes may contribute differently to the illiquidity in the market, all investors are exposed to the same illiquidity. In this sense, the illiquidity represents a source of externality.

Both investor classes contribute to the illiquidity because the uninformed investors want to extract any information contained in the order flow. Since more active investment increases the information contained in the order flow, active investors tend to suffer from each other’s presence. However, active investors can also benefit from each other’s presence: Investors who trade on different fundamentals may submit relatively uncorrelated trades. If so, each investor class represents "noise traders" with respect to the other class. Put differently, investors in one class can camouflage their trades better, the more total trade volume is generated by the other class. In this sense, active investors with different strategies provide each other with liquidity.

These effects bear on the investors’ strategy choice. Each investor weighs the "cost" of (acquiring the relevant information for) a strategy against its "uniqueness" (relative to the strategies chosen by other investors). All else equal, an investor prefers a cheap strategy. But as more investors compete in that strategy, an expensive strategy becomes more appealing. Due to the illiquidity caused by the cheap strategy, however, the expensive strategy might not be chosen even if it were profitable in the absence of the cheap strategy. That is, the cheap strategy may crowd out the expensive strategy. By contrast, a very cheap strategy may
attract so many investors that their trades provide sufficient liquidity for the expensive strategy to become profitable; in which case, the cheap strategy promotes the expensive strategy. Thus, depending on the circumstances, strategies can be substitutes or complements.²

I find that these externalities have interesting implications for various market characteristics and phenomena: (i) sufficiently widespread "conventional" investment strategies may be a prerequisite for more complex investment strategies; (ii) as the information environment of a market improves, total trade volume increases, average trade volume decreases, but new large volume investors emerge (cf. Chordia et al., 2008b); (iii) in emerging markets, stocks followed by more analysts comove more with the market, unless the higher analyst coverage coincides with a higher forecast dispersion (cf. Chan and Hameed, 2006); (iv) the liquidity of larger stocks tends to be more sensitive to, but less driven by, common liquidity shocks than that of smaller stocks (cf. Chordia et al., 2000; Hasbrouck and Seppi, 2001); (v) herding in illiquid markets accompanies expansions in trading activity, whereas herding in liquid markets accompanies contractions in trading activity; and (vi) contrary to common wisdom (cf. Verrecchia, 1982), a decrease in information costs may make prices less informative even though the market becomes more efficient.

Liquidity externalities also provide a new rationale for (expanding) the sale of information. Instead of producing information privately at some cost \( c \), investors often rely on the services of commercial information vendors. Such vendors can offer information at a price \( p \) below \( c \), because information can be duplicated at virtually no cost and fixed costs can be spread over many customers. The question is whether a vendor is willing to lower the price. In a similar framework with only one investment strategy, Admati and Pfleiderer (1988) show that a monopolist would cannibalize on her own revenues if she were to increase the competition

²In most financial market models (e.g., Grossman and Hart, 1980) information choices by different investors are strategic substitutes, with a few exceptions. In Barlevy and Veronesi (2000), due to a non-normal distribution of shocks, initial learning can cause prices to become less informative which in turn increases the value of acquiring more information. In Li and Yang (2008), investors who endogenously acquire information may induce insiders to exit the financial market and invest their wealth in real assets. This makes room for more endogenously informed investors. Neither paper addresses the role of liquidity externalities.
among her customers. Consequently, she sets $p$ equal to $c$, and the financial market is de facto the same as without the information vendor.

This result remains valid in the present setting when the monopolist provides information about both fundamental factors. If, however, the monopolist only provides information about one factor, she voluntarily lowers the price below $c$ and thereby expands her supply of information. This leads to an increase in the number of active investors. Unlike in Admati and Pfleiderer (1988), the vendor is not a "pure" monopolist. Her product competes against information about the other factor, insofar as her customers compete against investors that pursue the other strategy. The monopolist therefore expands supply so as to absorb more of the total demand for information, and to mitigate the negative externality that the other investor class imposes on her customers. In other words, she deliberately crowds out alternative investment strategies.

Thus, the presence of a sole information vendor who provides only a part of all potentially relevant information increases the number of active investors but reduces the diversity of information held by active investors. This serves the interest of the monopolist whose motivation for expanding market participation is to induce investor "herding". As a result, prices may become less informative because of information sales, although the market becomes more efficient.

Competition (which I model as the threat that another vendor enters the market) forces the information vendor to reduce the price, i.e. to expand supply, even more. Because the potential competitor’s product is a perfect substitute, the information vendor is more concerned with deterring entry than with crowding out the alternative investment strategy. Her price reduction leads to a proliferation of the investment strategy based on the information that she provides. The resulting increase in trade volume may in turn provide enough liquidity for alternative investment strategies to become profitable. That is, competition fosters information diversity. This can potentially explain the differences in stock price comovement in developed and undeveloped countries (Morck et al., 2000), as well as the decline in stock price comovement in the US over time (Campbell et al., 2001). It also suggests that information market competition plays an important role in financial market development over and above the direct effect of reducing the price of
The final question addressed in this paper is whether two investors who are endowed with different information can jointly benefit if one of them sells her information to other investors. This is of particular interest when one investor’s information is exclusive, whereas the other investor’s information is also held by other investors. If the latter information is sufficiently widespread, the first investor is indeed willing to compensate the second investor for giving the information away for free. Dispersing the information creates a "herd" of investors who camouflage the first investor’s trades and increase her profits over and above the second investor’s loss. This may explain the co-existence of information sales and proprietary trading within a financial institution: By supplying mundane information to many investors, the institution may improve the liquidity in the market. This in turn may allow it to trade more profitably on information which it does not share with other investors.

The paper belongs to the literature on endogenous information acquisition in financial markets (e.g., Grossman and Stiglitz, 1980; Verrecchia, 1982; Hellwig and Veldkamp, 2008; Van Nieuwerburgh and Veldkamp, 2008a) and to the literature on information sales and financial markets (e.g., Admati and Pfleiderer, 1986, 1988, 1990; Allen, 1990; Garcia and Sangiorgi, 2007; Cespa, 2008). Its main contribution is to examine both topics in a setting where investors choose investment strategies based on different fundamentals, and such investment strategies can be substitutes or complements.

The most closely related papers are Subrahmanyam and Titman (1999), Fishman and Hagerty (1995) and Veldkamp (2006a,b). I extend the model in Subrahmanyam and Titman by allowing investors to choose what type of information to base their trades on, and by introducing a market for information. The distribution of information is therefore completely endogenous in the current paper.

In Fishman and Hagerty, two investors are endowed with identical information, and must decide whether to sell or to trade on their information. Like the monopolistic vendor in my model, they sell information in equilibrium because they want to capture a larger share of the overall trading profits. However, because of the assumed information structure, the framework cannot address the central themes
of this paper, such as information diversity, the crowding out of information, and
the provision of liquidity among informed investors.

Veldkamp introduces different types of information. She focuses on the fact
that the (production and) sale of information involves high fixed but low marginal
costs. This implies that, in a competitive market, information in higher demand
is supplied at a lower price. Thus, agents benefit from purchasing the same in-
formation and may therefore pursue the same investment strategies. While this
herding behavior is common to our models, the mechanisms are different. In my
model, herding results from negative externalities in the asset market whereby
different investment strategies crowd each other out even if the number of poten-
tial investors is infinite. This leads to alternative predictions, notably about the
effect of information market competition on information diversity.

The remainder of the paper is organized as follows. Section 2 outlines the
model and derives the equilibrium in the absence of a market for information.
Section 3 explores different implications of the equilibrium in section 2. Sec-
tion 4 introduces a market for information, and examines equilibrium prices in a
monopoly and in a contestable market. It also provides a rationale for why finan-
cial institutions may simultaneously engage in proprietary trading and information
sales. Section 5 concludes the paper.

2 The Model

2.1 Active Investment

A single asset with uncertain liquidation value \( \bar{V} \sim N(0, 2\sigma^2) \) is traded. The
liquidation value is determined by a pair of fundamental factors, \( \Theta = \{A, B\} \).
For simplicity, I assume that the factors are independent and equally important.
That is, \( \bar{V} = \sum_{\theta} \bar{V}_{\theta} \) with \( \bar{V}_{\theta} \sim N(0, \sigma^2) \) for \( \theta \in \Theta \).

Investors belong to one of two classes. Each class is informed about a different
fundamental. The size of class \( \theta \in \Theta \) is \( n_{\theta} \). The \( i \)th investor in class \( \theta \) receives
data about \( V_{\theta} \) and interprets it with some idiosyncratic bias \( \bar{\epsilon}_{i\theta} \sim N(0, \sigma^2) \). Her
information is thus a signal \( \bar{s}_{i\theta} = \bar{V}_{\theta} + \bar{\epsilon}_{i\theta} \). In short, investors are sorted into classes
with different types of expertise, and individual biases induce some heterogeneity within each class.

Noise traders form a third investor category. Their motives for trading are exogenous to the model, and their total demand for (or supply of) the asset is \( \tilde{y} \sim N(0, \sigma_y^2) \). The probability distributions and the class sizes are commonly known. All investors are risk-neutral and there is no discounting.

Trading proceeds as in Kyle (1985). All investors submit quantity orders to a competitive market maker. Order submission is non-cooperative, simultaneous and anonymous. After observing the aggregate order flow, the market-maker sets a uniform price at which she meets the orders. Finally, \( V \) becomes public. Yet, \( V_A \) and \( V_B \) are not observed individually. So, they cannot be traded separately.

It is standard to solve the game for the Bayes-Nash equilibria in linear and symmetric strategies. In such equilibria, each informed investor’s strategy \( x_{i\theta} = \alpha_{i\theta} s_{i\theta} \) is linear in her signal; and the market-maker’s pricing rule \( p = \lambda z \) is linear in the net imbalance of the order flow \( z \equiv \sum_{\Theta} \sum_{1}^{n_{\theta}} x_{\theta}(s_{i\theta}) + y \). Moreover, investors in the same class follow the same strategy, \( \alpha_{i\theta} = \alpha_{\theta} \) for \( \theta \in \Theta \). A strategy profile is thus a triple \((\alpha_A, \alpha_B, \lambda)\).

The "trading intensity" coefficient \( \alpha_{\theta} \) captures how much the order flow of investors in class \( \theta \) varies with their information. The "price impact" coefficient \( \lambda \) in turn gauges how sensitively the price reacts to any variation in the order flow. The inverse \( 1/\lambda \) is a measure of "market liquidity".

**Lemma 1 (Subrahmanyam and Titman, 1999)** There is a unique Bayes-Nash equilibrium of the trading game in linear and symmetric strategies:

\[
\alpha_\theta^* = \frac{\sigma_y}{\lambda^* [(n_\theta + 1)\sigma^2 + 2\sigma_z^2]} \quad \text{and} \quad \lambda^* = \frac{\sigma^2}{\sigma_y \left[ \sum_{\Theta} T(n_\theta) \right]^{\frac{1}{2}}}
\]

where

\[
T(n_\theta) \equiv \frac{n_\theta (\sigma^2 + \sigma_z^2)}{[(n_\theta + 1)\sigma^2 + 2\sigma_z^2]^2}, \quad \theta \in \Theta.
\]

Individual investors in the same class engage in a Cournot-type competition. Because they trade on similar information, they reinforce each other’s impact on the price. To mitigate this cumulative impact, they cut back their individual
orders. If the two classes differ in size, individual investors in the larger class thus trade less intensively \((n_0^A > n_0^B \iff \alpha_0^A < \alpha_0^B)\).

Investors from different classes do not compete in the above sense, since their trades are uncorrelated ex ante. They nonetheless affect each other. Since the market maker expects informed orders from either class, both classes contribute to the illiquidity in the market [via the subfunctions \(T(\cdot)\)]. Market liquidity is thus a channel for interclass externalities. These externalities arise because the investors, even if they possess unrelated information, must trade in the same market.

As in single-class models, the relationship between the number of informed traders and market liquidity is ambiguous. The aggregate order flow of a larger class conveys more (precise) information, since idiosyncratic biases tend to offset each other. At the same time, the intensified competition leads to a larger, more volatile order flow. The two effects have countervailing consequences for market liquidity. Because the information effect gradually vanishes, \(T(\cdot)\) is strictly quasi-concave and has an interior maximum in \(\mathbb{R}^+\).

**Corollary 1** \(\lambda^*\) is unimodal in \(n_0 \geq 0\) for all \(\theta \in \Theta\).

This non-monotonicity will play a central role in the subsequent analysis.\(^3\) It implies that the externality that a given investor class imposes on other investors can increase or decrease in the size of the class. It also implies that increases in \(n_A\) or \(n_B\) can have opposite effects on market liquidity. This has interesting consequences for the formation of investor classes, which we explore in the next section.

Liquidity externalities are not unique to the above market microstructure. For instance, they also arise when investors can submit price-quantity schedules or limit orders. The assumption that the information received by the two investor classes is independent is also not crucial. What matters is that investors in the same class compete more intensively, and that investors as a class affect liquidity in a non-monotonic way. The independence assumption merely accentuates these effects by separating the liquidity externality from the competition effect.

\(^3\)A similar non-monotonicity arises when investors are risk-averse (Subrahmanyam, 1991).
2.2 Investment Specialization

Instead of being endowed with information, investors must now actively acquire information. Formally, the model is extended to include stage 0, in which investors choose to remain uninformed or to conduct a fundamental analysis of the asset. For simplicity, the main analysis assumes that each investor specializes in the analysis of one fundamental factor, and that a truthful exchange of private signals among investors is not enforceable. These assumptions are discussed in more detail in section 2.2.

Fundamental analysis is costly. To produce information about a factor $\theta \in \Theta$, an investor must incur some fixed cost $c_\theta > 0$ to gather and interpret data. For instance, one can think of $c_A$ as the cost of macroeconomic analysis and $c_B$ as the cost of company-specific analysis. Accordingly, investors can be seen as either "macro-traders" or "stock-pickers."

Investors choose their specialization to maximize expected profit

$$\pi_\theta (n_\theta, n_{\theta'}) = \rho_\theta (n_\theta, n_{\theta'}) - c_\theta$$

where $\rho_\theta (n_\theta, n_{\theta'})$ denotes investor $\theta i$’s expected trading profit (gross of information costs). A pure-strategy subgame perfect equilibrium is defined by a pair of investor classes $(n_A, n_B)$ in conjunction with Lemma 1 such that (i) no active investor prefers to switch class or to be uninformed, and (ii) no uninformed investor prefers to become informed. I assume an infinite population of investors who can become informed. Normalizing their outside option to 0, this implies that the expected profit of any investor in equilibrium (if one exists) must be 0. That is, $\pi_\theta (n_\theta, n_{\theta'}) = 0$ for all $\theta \in \Theta$ in equilibrium.\(^4\)

**Payoff externalities**

Because investors in the same class compete with each other, their (expected) trading profits decrease in the size of their own class. This competition effect is illustrated by the downward sloping curve in figure 1. The effect of class size on trading profits across investor classes is less straightforward. Due to the aforemen-

\(^4\)To simplify matters, I ignore integer problems and treat $n_\theta$ as a continuous variable.
tioned non-monotonic liquidity externality, this effect is ambiguous as illustrated by the U-shaped curve in figure 1.

The intuition behind the U-shape is as follows. If the growth of an investor class primarily makes its order flow more informative, the market becomes less liquid. This in turn induces investors in the other class to trade less intensively, and they consequently experience a drop in trading profits. If, by contrast, the order flow primarily becomes more volatile, the market gains liquidity and investors in the other class trade more intensively. Put differently, because the fundamental factors are uncorrelated, different investor classes represent noise traders to each other. More volatile trading by one class ceteris paribus provides (better) camouflage for the trades of the other class.\textsuperscript{5}

\textbf{Crowding out}

I restrict attention to cases where each trading strategy is \textit{per se} profitable, i.e., $\pi_\theta(1, 0) > 0$ for all $\theta \in \Theta$. Without loss of generality, let $c_A \leq c_B$. Thus, in terms of the previous example, a sole investor prefers macroeconomic information over

\textsuperscript{5}In fact, one can show that if the trading intensity $\alpha_\theta$ \textit{were} fixed, the market would become perfectly liquid ($1/\lambda \to \infty$) as a class grows without bound ($n_\theta \to \infty$) – even when exogenous noise trade is negligible ($\sigma_y^2 \geq 0$).
company-specific information, and either of these over no information. In figure 1, her preferences are captured by the diverging but positive intercepts.

A sole trader opts for macro-trading. This not only reduces the profitability of macro-trading but also the profitability of stock-picking (for other investors), as reflected by the decline in both curves. A second active investor then faces the following trade-off. While macroeconomic information is cheaper, macro-trading is more competitive. The second investor also prefers macro-trading only if the cost difference \( c_B - c_A \) exceeds the difference in trading profits \( \rho_B(1,1) - \rho_A(2,0) \).

In this fashion, every investor weighs the "cost" of a trading strategy against its "uniqueness".

As macro-trading expands, macro-traders compete each others' profits away, whereas stock-picking eventually becomes attractive. That is, at some point, the "marginal" investor either prefers to stay out of the market or to become a stock-picker. In figure 1, stock-picking is chosen only if \( \pi_A(n_A,0) \) and \( \pi_B(1,n_A) \) intersect before \( \pi_A(n_A,0) \) hits zero.

This intuition explains the following equilibrium properties. (Mathematical proofs are relegated to the Appendix.)

**Lemma 2** Let \( \mathcal{R}_A \equiv \{ n_A : \pi_A(n_A,0) \leq 0 \} \) and \( \mathcal{R}_B \equiv \{ n_A : \pi_B(1,n_A) \leq 0 \} \). In equilibrium, \( (n_A^*,n_B^*) = (n_A^0,0) \) if and only if \( \mathcal{R}_B \neq \emptyset \) and \( \min \mathcal{R}_A \in \mathcal{R}_B \). Otherwise, there exists a unique equilibrium \( (n_A^*,n_B^*) \) with \( n_A^* \geq n_B^* > 0 \).

The cheaper trading strategy is always more prevalent. Less obvious is that under certain circumstances the other strategy, despite generating profits for a sole trader, is not pursued in equilibrium. This occurs precisely when the illiquidity created by macro-trading renders stock-picking unprofitable, even though the underlying information is unrelated.

**Substitutes vs. complements**

Lemma 2 states that stock-picking is "crowded out" in equilibrium whenever the (unique) root of \( \pi_A(n_A,0) \) falls into a region \( \mathcal{R}_B \) where \( \pi_B(1,n_A) \) is negative. Now suppose that \( \mathcal{R}_B \) is non-empty. Because the root of \( \pi_A(n_A,0) \) can be shifted by varying \( c_A \), it follows that there exists a cost range \( [\underline{c}_A, \bar{c}_A] \) such that crowding
out occurs whenever $c_A \in [\underline{c}_A, \bar{c}_A]$. Intuitively, one can choose the cost of macro-trading such that stock-picking becomes unprofitable. Striking is also the impact of changing the cost of macro-trading outside of this range.

**Proposition 1** A reduction in the cost of $A$-trading decreases the prevalence of $B$-trading above some threshold $\bar{c}_A$ but increases it below some threshold $\underline{c}_A$.

This result is striking because it implies that macro-trading is a strategic substitute for stock-picking when $c_A \geq \bar{c}_A$ but a strategic complement for stock-picking when $c_A \leq \underline{c}_A$ (figure 2). When it is difficult to obtain macroeconomic information, the volume of macro-trading is small, which induces illiquidity in the market. This discourages trading on even more inaccessible company-specific information. By contrast, when macroeconomic information is easily accessible, the massive macro-trading volume camouflages stock-picking, which thereby makes the latter a more profitable trading strategy.

The subsequent analysis focuses on the case where $R_B \neq \emptyset$. This is not as restrictive as it seems, since the analysis can be extended to more than two trading strategies. The assumption $R_B \neq \emptyset$ simply states that some strategies are crowded out under certain cost schedules.
Robustness

This section discusses several issues related to the robustness of the above information equilibrium. They are not crucial for understanding the main analysis which continues in Section 3 with a discussion of the economic implications of Proposition 1.

Cognition  In the above analysis, it is assumed that investors are boundedly rational in the sense that they can base their trading strategy on one fundamental factor only. Such "limits in processing (receiving, storing, retrieving, transmitting) information" (Williamson, 1981, p.553) which induce economic agents to optimally neglect information are commonly referred to as rational inattention. Rational inattention has been documented in financial markets by e.g., Huberman (2001), Huberman and Regev (2001), Massa and Simonov (2005) and Hong et al. (2007).⁶

Notwithstanding, suppose instead that investors can process information about both factors without additional difficulties. That is, they can produce a signal

$$s_{ABi} = \sum \phi (V_i + \epsilon_i)$$

at cost $$c_{AB} = c_A + c_B$$. Though there may now exist other equilibria, it is straightforward to see that pure specialization (as in Lemma 2) remains an equilibrium in this setting. In such an equilibrium, no uninformed investor finds it worthwhile to enter with any type of information. By the same token, no active investor finds it worthwhile to incur the extra cost of acquiring additional information. Moreover, it seems reasonable that the marginal cost of processing information is increasing, so that $$c_{AB} > c_A + c_B$$. In this case, generalism becomes more expensive, and pure specialization becomes a more likely equilibrium outcome.

Communication  The analysis also assumes that investors cannot credibly communicate with each other. Since neither the individual factor realizations $$V_i$$ nor the individual error terms $$\epsilon_i$$ are revealed, a misreported signal is never detected. Consequently, investors cannot commit to share information, as they would shirk effort or communicate false information to trade privately.

The impossibility of communication is not as restrictive as it may seem at first glance. In a similar setting, Colla and Mele (2004) show that information sharing, because it dilutes each investor’s "monopoly" power, arises only if the initial correlation between the signals is sufficiently high. In the present model, information about different fundamental factors is uncorrelated, so that communication across investor classes is unattractive. Thus, the equilibrium in Lemma 2 is robust to communication.

Moreover, communication becomes less attractive when it is costly. Like information acquisition, successful communication typically requires effort (De- watripon and Tirole, 2005). The receiver must exert effort to understand the sender’s message, and the sender must exert effort to make her message intelligible to the receiver. Clearly, such communication costs favor equilibria in which boundedly rational investors specialize in different trading strategies.

**Information** One could also consider a setting where investors can mix information about different factors, while choosing the precision of each type of information. Because of the competition effect, two investors would prefer to be as different as possible, and therefore specialize in distinct factors. With more investors, the incentives to avoid competition induce investors to choose different combinations of information about both factors. Nonetheless, liquidity externalities continue to exist, and each investor’s decision criterion remains to weigh the "cost" of a particular type of information against its "uniqueness". Thus, changes in the cost of one type of information, and the corresponding changes in the demand for this information, should continue to exert positive or negative externalities on the demand for the other type of information.

### 3 Access to Information

In this section, I will employ Proposition 1 to derive implications for various market characteristics, and to shed light on a number of empirically observed phenomena. The model is developed further in section 4, where the cost of information is endogenized.
Participants in financial markets not only experience information shocks but also shocks to the *access* to information. By Proposition 1, even if a shock affects only one investor class, liquidity externalities propagate the shock to other investor classes, and potentially across assets. Spillover effects of this kind induce commonalities in prices, liquidity and trading activity.

In the model, investors’ signals reflect the arrival of new information, whereas changes in the access to information are best viewed as changes in the cost of acquiring information \((c_g)\). In practice, such changes can be both permanent (e.g., the advent of new information technologies) or temporary (e.g., time variation in the media coverage of economic events).

For a single asset, it is immaterial whether the cost changes affect, say, macro-economic information or asset-specific information. What matters is whether the affected investment strategy acts as a substitute for, or as a complement to, the other investment strategy. In the case of multiple assets, however, it is important whether the changes primarily affect investment strategies based on common factors or those based on asset-specific factors. In this case, the discussion below primarily focuses on common factors.

**Investment diversity**

When one investment strategy becomes continuously cheaper, the market moves from the substitute region via the crowding out region to the complement region (figure 2). Consequently, the number of active investors increases, but the number of actively used investment strategies first decreases and then increases.\(^7\)

**Implication 1** *When a subset of information becomes increasingly accessible, active investment becomes more popular while the employed investment strategies first become less and then more diverse.*

In the substitute region, different investment strategies compete for liquidity in the market. As a result, expansions of one investment strategy come at the

\(^7\)By contrast, in competitive rational expectations models with infinitely many investors who can acquire information, a decrease in the cost of one type of information typically implies that (more) investors acquire more of every type of information (Figlewski, 1982).
expense of the other strategy, and active investors become less heterogeneous. By contrast, in the complement region, the expanding investment strategy supplies liquidity to the market, thereby encouraging investors to enter with new investment strategies. In fact, when both strategies are widely used, expansions become mutually reinforcing.

Implication 1 suggests that more complex investment strategies, such as those of hedge funds, may require a sufficient level of "conventional" informed trading to provide sufficient liquidity. This is consistent with the common view that improved market liquidity attracts more informed trading (Chordia et al., 2008a). What stands out in this setting is that the incentives to acquire information feed on liquidity provided by informed trading, as opposed to noise trading, by other investor classes.

The expansion in diversity can be significant. Consider, for example, a three factor-model, $\Theta = \{A, B, C\}$, in which strategies based on $B$ and $C$ are equally costly. It is straightforward to see that, if strategy $A$ crowds out strategy $B$ in the two-factor model, both strategies $B$ and $C$ are crowded out in the three-factor model. Once strategy $A$ becomes sufficiently widespread, both other strategies become equally viable, and the number of employed investment strategies may jump from one to three. New investment strategies emerge even faster when the information environment improves generally, i.e., when $c_A$, $c_B$ and $c_C$ decrease. In that case, strategies $B$ and $C$ are not only made (more) attractive by the increased liquidity provided by $A$-trading, but also by the general decrease in the cost of active investment.

Trading volume

When information becomes more accessible, total trade volume tends to expand because of an increase in the number of active investors. At the same time, the average trade volume tends to decrease because more investors compete in the same strategy. However, the evolution in individual trade volumes is not uniform due to the liquidity externality. If the cost of macroeconomic information falls, the macro-trading volume follows both of the mentioned patterns. But this is not true for stock-picking. In the substitute region, the total stock-picking volume
decreases while the individual stock-picking volume increases; until stock-picking disappears altogether in the crowding out region. When entering the complement region, some stock-pickers reappear with individually high trading volumes. At this point, further expansions in macro-trading induce a "standard" pattern, i.e., an increase in total – but a decrease in individual – stock-picking volume.

**Implication 2** *When a subset of information becomes increasingly accessible, total trade volume tends to increase, average trade volume tends to decrease, and new large volume investment strategies tend to emerge.*

Chordia et al. (2008b) document that the total trade volume at the NYSE has steadily grown over the past decade(s), and that the growth has been largely driven by institutional investors. At the same time, smaller orders have formed an increasing fraction of the trades, although institutional investors remain active in large orders. They also report that the increase in trade volume has coincided with an increase in the production of private information. These trends are broadly consistent with Implication 2. Improvements in the information environment may have led to more (competitive) active investment, with order sizes decreasing for conventional investment strategies and large trade volumes remaining significant for newly emerging, more complex investment strategies.

**Price comovement**

The degree to which different stock prices comove is often taken as a (inverse) measure of the amount of company-specific information that is impounded into stock prices (Roll, 1988). It has been documented that the level of comovement is lower in more developed economies (Morck et al., 2000) and has decreased in the US over the 20th century (Campbell et al., 2001). Some evidence suggests that such patterns may be related to differences in the information environment (Fox et al., 2003; Bushman et al., 2004; Hameed et al., 2005).

Suppose that several assets share (some) common fundamentals, and that information about common fundamentals is more accessible. In that case, asset prices comove more in moderate information environments. To give a simple
example, consider two separately traded stocks, \( S = \{1, 2\} \), and three fundamental factors, \( \Theta = \{A, B, C\} \). The liquidation values are given by

\[
\begin{align*}
\hat{V}_1 &= \hat{V}_A + \hat{V}_B \\
\hat{V}_2 &= \hat{V}_A + \hat{V}_C
\end{align*}
\]

where \( A \) is a macroeconomic factor, and \( B \) and \( C \) are stock-specific factors.

**Implication 3** When macroeconomic information becomes increasingly accessible, price comovement first rises and then falls.

For high levels of \( c_A \), active investors are scarce but pursue diverse strategies (either \( A \), \( B \) or \( C \)). However, when \( c_A \) falls to intermediate levels, macro-trading and the number of active investors expand, while stock-specific investment strategies are crowded out. As a result, the prices of the two stocks comove more. For low levels of \( c_A \), stock-picking becomes attractive again, so that prices increasingly incorporate stock-specific information (again).

In a sample of emerging markets, Chan and Hameed (2006) find comovement to be higher for stocks that are followed by more analysts. The relationship is weaker, however, when a higher number of analysts coincides with a higher forecast dispersion. Implication 3 is consistent with this observation: in illiquid markets, an increase in information acquisition goes together with a decrease in information diversity. However, when a stock is sufficiently liquid, the increase in analyst following may increase diversity, causing the stock to comove less with the market.

For instance, suppose that a more idiosyncratic stock is added to the above model, \( \hat{V}_3 = \hat{V}_D + \hat{V}_E \). If information about the common factor \( A \) is more accessible, this third asset would not only covary less with the market, but it would also attract fewer active investors than the other assets. If information about \( A \) becomes so widespread that investors increasingly acquire information about \( B \) and \( C \), the increase in the number of active investors in stocks 1 and 2 entails an increase in information diversity. As a result, the positive cross-sectional relation between the number of active investors in a stock and the stock’s comovement
with the market becomes weaker.\textsuperscript{8}

Implication 3 can potentially explain why price comovement is different in developed and undeveloped countries, and why it has decreased in the US over time. The observed cross-country variation may indicate that information is more widely available in developed countries than in developing countries. The decline in price comovement in the US may have been driven by improvements in information technology and the development of competitive (business and financial) information markets. Section 4.1 discusses how the level of price comovement can be related to the degree of competition in information markets.

Veldkamp (2006b) also explains price comovement by means of investors’ information choices. In her theory, comovement is the result of complementarities in the investors’ information choices. Due to economies of scale in markets for information, information in higher demand is supplied at a lower price. As a result, investors benefit from acquiring the same information. By contrast, comovement in the present model results from negative externalities in the asset market: it arises in illiquid markets as investment strategies based on common information crowd out investment strategies based on more asset-specific information.

**Liquidity commonality**

It is straightforward to see that shocks to \( c_A \) also induce common changes in the liquidity of the two stocks. That is, the liquidity of each stock comoves with measures of market-wide liquidity, as documented in several papers (Chordia et al., 2000; Hasbrouck and Seppi, 2001; Huberman and Halka, 2001). For instance, a decrease in \( c_A \) reduces (improves) liquidity in the two stocks if both are in the substitute (complement) region.

**Implication 4** Variation in the access to macroeconomic information induces covariation in the liquidity of different assets.

\textsuperscript{8}It should be pointed out that the dispersion of analyst recommendations need not necessarily be a sign of information diversity but can also indicate less precise information. If so, more dispersion indicates less, not more, information (Jin and Myers, 2006). The finding by Chan and Hameed pertains, however, not to the level of dispersion but to its interaction with the number of analysts. Thus, my interpretation presumes that an increase in dispersion indicates an increase in information if it is, at the same time, associated with an increase in analyst coverage.
Illiquidity in this model arises from asymmetric information across investors (including the market-maker) about the value of the stocks. When the availability of macroeconomic information changes, so does the information asymmetry between active investors and less informed market participants. The liquidity provision of uninformed investors (i.e., the market-maker in this model) changes in response, and it changes similarly for both stocks.

If there is heterogeneity with respect to other determinants of liquidity, the sensitivity of liquidity to changes in $c_A$ may differ across stocks in terms of both strength and direction. For instance, suppose that stock 1 is larger and therefore attracts a larger amount of noise trading (cf. Holmström and Tirole, 1993). As a result, stock 1 attracts more macro-traders than stock 2, so that there may be situations in which stock 1 is in the complement region whereas stock 2 is in the substitute. In that case, the stock liquidities move in opposite directions when $c_A$ changes marginally. Such differential responses can mask the importance of common determinants when estimating average liquidity responses across stocks.

Alternatively, suppose that stock 2 is in the crowding out region, whereas stock 1 is so large that macro-trading and stock-picking in that stock are mutually complementary. In stock 2, a marginal increase in $c_A$ has an ambiguous effect on liquidity. By contrast, in stock 1, it reduces macro-trading and thereby liquidity. This in turn reduces stock-picking, which then further reduces liquidity. This negative feedback loop can cause liquidity to spiral downward (until it reaches a new, lower equilibrium level). Despite having higher levels of liquidity, larger stocks may therefore be more sensitive to common variation in liquidity (Chordia et al., 2000). At the same time, they attract more diverse strategies, so that their liquidity may exhibit more idiosyncratic variation. That is, common factors may explain little (or less) of the liquidity variation in large stocks (cf. Hasbrouck and Seppi, 2001).

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9This assumes that the interpretation of macro-economic information is a cognitive task which is specific for each stock. That is, traders must not only specialize in a subset of information but also in a subset of assets (cf. Van Nieuwerburgh and Veldkamp, 2008b).
Herding

There are episodes in financial markets when the market-wide information environment experiences a shock. For instance, macroeconomic information may suddenly become easier or harder to obtain (lower or higher $c_A$). In the present framework, such changes may induce herding behavior (i.e. correlated trading) both within and across stocks. Moreover, the nature of events that trigger herding may depend on the general information environment in which the market operates.

Implication 5 Relatively illiquid markets are prone to herding frenzies, whereas relatively liquid markets are prone to herding panics.

Consider the two-stock example with information about $B$ and $C$ equally accessible. Consider a surge in the supply of macroeconomic information which moves the market from the substitute region into the crowding out region. Both the number of active investors and trading volume rise while investment strategies become more homogeneous, in what resembles a herding frenzy. By contrast, starting from the complement region, such herding occurs only if there is a decrease in the supply of macroeconomic information which moves the market into the crowding out region. In that case, the number of active investors and trading volume fall while investment strategies become more homogeneous, in what resembles a herding panic. In either case, the frenzy or the panic, the correlation in investment strategies increases not only across investors in the same asset but also across different assets.

Informational role of prices

A key role of prices is to aggregate dispersed information (Hayek, 1945). The literature offers three concepts to describe how well prices convey information. Market efficiency refers to the degree to which prices reveal the information held by investors (Fama, 1970). Price informativeness measures how much of the uncertainty about the asset value is reduced when the price is observed. Allocative
efficiency reflects the extent to which price information helps decision-makers to allocate resources efficiently (Tobin, 1982).\textsuperscript{10}

In a single-factor framework, $\Theta = \{A\}$, the three information measures typically evolve in the same direction. If the number of active investors increases, both market efficiency and price informativeness improve, and so does allocative efficiency to the extent that decision-makers benefit from more information about the asset. In a multi-factor model, this need no longer be the case.

**Implication 6** When a subset of information becomes more accessible, market efficiency increases but price informativeness need not increase.

Since market efficiency measures the information contained in prices relative to that possessed by active investors, it can be proxied by the total loss of uninformed (noise) traders. When information becomes cheaper, the number of active investors and active trading volume increase. Consequently, more of the privately held information is revealed by the order flow, and the market-maker can set the price closer to the active investors’ (average) forecast. As a result, the uninformed lose less when trading, i.e., the market becomes more efficient.

In contrast, price informativeness is related to the total, rather than the proportion of existing, information impounded into prices (Chen et al., 2007). This is best measured by the residual uncertainty about the asset value, i.e. the conditional variance $\text{Var}(\tilde{V} | P) = \sigma^2 \left( 1 - \rho^2_{\tilde{V};z} \right)$ where (as shown in the Appendix)

$$\rho^2_{\tilde{V};z} = \frac{1}{2} \sum_{\theta=A,B} \frac{n_{\theta} \sigma^2}{(n_{\theta} + 1) \sigma^2 + 2 \sigma^2_\varepsilon}. \quad (3)$$

Since residual uncertainty decreases in $\rho^2_{\til{V};z}$, I use the latter as a measure of price informativeness. The following comparative statics provide some intuition: $\partial \rho^2_{\til{V};z} / \partial n_{\theta} > 0$ and $\partial^2 \rho^2_{\til{V};z} / \partial n_{\theta}^2 < 0$. That is, price informativeness increases with

\textsuperscript{10}Dow and Gorton (1997) make a similar distinction between market efficiency (which they call price efficiency) and allocative efficiency (which they call economic efficiency), and show that price efficiency does not necessarily entail allocative efficiency. I further distinguish price informativeness (which in their framework coincides with price efficiency) and argue that none of the three measures necessarily implies the others. In fact, in the present framework, the different measures may conflict with each other.
the prevalence of either type of information. However, the marginal increase is decreasing in the prevalence of a given type of information (because the average error asymptotically converges to 0). For a fixed trader population, this implies that the price is the most informative under a balanced information structure \((n_A = n_B)\). All else equal, skewing the information distribution hence reduces price informativeness.\(^{11}\)

This makes it clear why price informativeness need not necessarily increase in the substitute region, where the expansion of strategy \(A\) comes at the expense of strategy \(B\). Intuitively, while the number of active investors increases, they acquire less diverse information. As a result, the market price – while reflecting more of the investors’ acquired information – may reflect less total information. It seems counterintuitive that cheaper information can lead to less informative prices. However, the result manifests the trade-off between the amount of active investment and the diversity of active investment in a setting with different types of information.

Finally, it should be noted that allocative efficiency need not improve even if market efficiency and price informativeness do. In a multi-factor setting, some types of information may be more relevant for allocative decisions than others.\(^{12}\) For instance, Holmström and Tirole (1993) argue that stock-based compensation schemes can enhance managerial incentives because active investors collect information about managerial effort. Suppose that \(B\) relates to managerial effort, whereas \(A\) relates to macro-events outside of managerial control. A stock-based compensation scheme is less effective if strategy \(B\) is crowded out by strategy \(A\) – irrespective of the effect on price informativeness. In fact, the macroeconomic information reflected in the stock price represents "luck" and confounds the role of the price as a signal about "effort".

\(^{11}\)The optimality of the balanced structure is particular to the assumption that both types of information are equally important.

\(^{12}\)According to Dow and Gorton (1997), stock prices play both a retrospective role in evaluating past actions and a prospective role in reflecting the value of investment opportunities. Several papers formalize the idea that managers themselves may extract information from stock prices to improve their investment decisions, for example, whether to continue, expand or modify current firm strategy (e.g., Subrahmanyam and Titman, 1999, 2001; Dow et al., 2006; Goldstein and Gümbel, 2006). Evidence confirms a feedback from stock prices to corporate investment (Wurgler, 2000; Baker et al., 2003; Durnev et al., 2004; Chen et al., 2007).
4 Markets for Information

The preceding analysis has focused on exogenous changes in the access to information. Such changes may pertain to the transparency of economic policy, corporate disclosure rules or the quality of accounting standards. Moreover, one can consider the endogenous supply of information e.g., by commercial suppliers. In this vein, section 4.1 examines the pricing incentives of news vendors who provide a particular investor class with data for their investment decisions. Section 4.2 considers two investors, each with a distinct expertise, who can agree to sell part of their information.

4.1 A Market for Financial News

The model is extended as follows. In stage $-1$, news vendors offer investors subscriptions for $A$-data. In stage 0, each investor decides whether to remain uninformed, to purchase a subscription, or to produce data privately at cost $c_A$. A vendor who sells a subscription communicates in stage 1 the promised data to the subscriber. The marginal cost of communicating data is negligible. News vendors add neither bias nor noise to the data. The analysis focuses on the provision of news about factor $A$, taking the cost of information about $B$ as given. I discuss this assumption at the end of this section.

Before turning to the main analysis, I establish that a direct, unrestricted sale with a uniform price is optimal in this model.\footnote{In an indirect sale, the seller sets up an investment fund, and investors can participate in the seller’s knowledge by purchasing fund shares. I do not consider pricing schemes, where the subscription fee is contingent on realized trading profits. In the context of direct sales, this question has not been addressed in the literature. This may be because, in practice, the subscriber’s use of the data, including any ensuing profit, is difficult to monitor or to verify.}

Lemma 3 A direct sale of unlimited subscriptions at a single price is optimal.

Intuitively, consider any price-quantity schedule posted by a monopolist. Since investors are symmetric within each investor class, the highest subscription price paid in equilibrium must equal the expected trading profit of a subscriber. At
all lower prices, subscribers pay less than their reservation price, and the monopolist would earn more by raising the price. Binding quantity restrictions are therefore suboptimal from the vendor’s point of view, while slack restrictions are unnecessary.

Indirect sales can serve as a means to control the number of active investors, and hence to curb competition (Admati and Pfleiderer, 1990). Here, this benefit does not arise because investors can resort to alternative sources of information. Consider an investor who starts a mutual fund. Her optimal investment strategy is that of a single investor. As others will find it worthwhile to enter stage 2 with self-collected A-data, the fund’s expected profit will be $\rho_A(n^*_A, n^*_B) - c_A$. If the investor instead sells data at $p_A = c_A$, she extracts the expected trading profits of all A-investors and earns $n^*_A \rho_A(n^*_A, n^*_B) - c_A$.

While information markets are characterized by low marginal costs, the large-scale and timely dissemination of information often imposes high fixed costs (e.g., maintaining a communication or distribution network). Due to these fixed costs, media industries are often concentrated.\footnote{\textsuperscript{14}Before the merger between Reuters and Thompson in 2007, Bloomberg, Reuters and Thompson accounted for a combined market share of about two-thirds of the financial information services industry. Notwithstanding, compared to other countries, the US financial services industry is arguably one of the most competitive.} In many countries, entry regulations and political capture impose further restrictions on competition (cf. Besley and Prat, 2006). Against this background, a situation in which a news vendor has (some) monopoly power is by no means implausible.

**News monopoly induces predatory pricing**

Let $p_A \in [0, c_A]$ denote the news price. The number of subscriptions, $n^*_A(p_A)$, is endogenously determined as a function of the price. The monopolist chooses $p_A$ to maximize her total profit

$$\Pi(p_A) = \begin{cases} n^*_A(p_A)p_A - c_A & \text{if } p_A \leq c_A \\ 0 & \text{if } p_A > c_A \end{cases}. \quad (4)$$
For $p_A \leq c_A$, free entry of active investors ensures that subscriptions are sold until the subscribers’ expected profits are driven to zero. As a result, $A$-investors’ expected trading profits are fully extracted by the monopolist. Hence,

$$\Pi(p_A) = n_A^*(p_A)\rho(n_A^*(p_A), n_B^*(p_A)) - c_A.$$

This expression highlights not only that $p_A$ jointly determines $n_A$ and $n_B$ but also that maximizing the monopolist’s profit is tantamount to maximizing the total trading profits from strategy $A$.

In a setting with only one type of information, Admati and Pfeiderer (1988) show that a (risk-neutral) monopolist has no incentive to increase supply, i.e., to lower the price. Intuitively, since she is selling information to investors who compete over trading profits, new subscribers’ profits come at the expense of the profit of existing subscribers. In fact, due to the intensified competition, any revenue gain from the new subscribers is always smaller than the revenue loss on the existing subscribers. Hence, expanding the investor base through lower prices cannibalizes on the monopolist’s own profit. So, she chooses $p_A = c_A$.

By contrast, in a setting with more than one type of information, the monopolist may voluntarily expand supply.

**Proposition 2** The news monopolist sets $p_A$ such that $\min R_A = \min R_B$.

Unlike in Admati and Pfeiderer (1988), the news vendor is not a pure monopolist as her product "competes" against information about $B$. This affects her pricing incentives for two reasons. First, the presence of $B$-investors reduces market liquidity and thereby the trading profits of the monopolist’s subscribers. Second, the resources which $B$-investors expend to acquire information do not translate into revenues for the monopolist. By lowering her price, the monopolist can crowd out $B$-investors and attract more subscriptions. Put differently, she can sway $B$-investors to become subscribers. In fact, she reduces the price just enough to render the strategy $B$ unattractive (figure 2).

Relative to the outcome in the absence of a news market, the monopolist increases the trading volume and the number of active investors, which improves market efficiency. At the same time, she reduces the diversity of active investment
strategies, homogenizing expectations and potentially decreasing price informativeness (cf. Implication 6). Thus, in the present model, the impact of information sales on financial market quality is not necessarily benign. Indeed, in the absence of direct competitors, a news vendor increases supply so as to reduce investors’ incentives to seek alternative sources of information.

An anecdote about Reuters in its early days illustrates the above rationale. In the 1850s, Reuters controlled telegraph lines as well as the right to circulate news received from ships of the Austrian Lloyd’s. To develop its business, Reuters offered the main London newspapers subscriptions to its international news dispatches at £30 per month. This was significantly less than a newspaper’s cost of running its own correspondent network. At this price, Reuters had to attract a critical number of daily newspapers to make the service profitable.

The Times initially resented any dependence on Reuters and preferred its own correspondents. However, the value of its own network deteriorated, as rival newspapers gained access to foreign news: "Good though its own network was, it needed to know each evening what telegrams from Reuter were likely to appear in the columns of its competitors next morning, even though it did not necessarily want to print the telegrams itself." Eventually, The Times took out a Reuters subscription. By 1861, Reuters had become indispensable, supplying almost all

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15 The historical account is taken from Read (1999, p.24f).
London newspapers with identical foreign dispatches. Importantly, these short dispatches differed in nature from the reports of overseas correspondents: "The Times kept a correspondent at the front, the famous W. H. Russell, who wrote long mailed dispatches. These made a great impression by revealing military shortcomings, but they were not intended to give the latest news."

The aim of Reuters’ pricing strategy was to replace the newspapers’ own networks. While this gave each newspaper cheaper access to foreign news, it caused foreign news coverage to become more homogenous. In fact, the telegraph dispatches may at the time have crowded out the more complex information typically provided by overseas correspondents.

**News competition promotes information diversity**

I model competition in news markets, which are characterized by high fixed and entry costs, as a contested monopoly in which the incumbent news vendor is threatened by the entry of a competitor. To emphasize the impact of competition on the diversity of investment strategies, consider an economy with two stocks, $\mathcal{M} = \{1, 2\}$, and three fundamental factors, $\Theta = \{A, B, C\}$. The assets’ liquidation values are given by

\[
\begin{align*}
\tilde{V}_1 &= \tilde{V}_A + \tilde{V}_B \\
\tilde{V}_2 &= \tilde{V}_A + \tilde{V}_C
\end{align*}
\]

The factors are i.i.d. and their variances are given by $\sigma^2$. While $A$ is a macroeconomic factor, the others are stock-specific. For simplicity, let $c_B = c_C$.

Liquidity traders invest in the market portfolio so that (a change in) liquidity demand affects each asset in the same way and has a variance of $\sigma_y^2$ per asset. Thus, their trades induce market-wide movements (De Long et al., 1990; Morck et al., 2000), which simplifies the analysis but is not crucial for the results. There is a different market-maker for each stock. Trade occurs simultaneously in all stocks, and market-makers observe only their own order flow.

As before, I focus on the pricing incentives of a news vendor who sells information about $A$. To model contestability, I assume that the incumbent incurs a
non-negligible fixed cost $K_I \in [K_L, K_H]$ related to the news operation. Unlike $c_A$, this cost is private information, though its distribution is common knowledge. To enter the market, the rival must incur an up-front (sunk) cost $S$. For simplicity, I assume that the rival’s operative costs are $K_E < K_L$ and commonly known.\textsuperscript{16}

The timing of the entry game (which takes place in stage $-2$) is as follows. First, the incumbent precommits to a price $p^I_A$, which she can lower but not raise in stage $-1$. This should be interpreted as offering a subscription contract with a fixed duration at a fixed price. Second, the rival observes $p^I_A$ and then decides whether or not to enter the market. Third, if the rival enters, the two vendors engage in price competition (in stage $-1$). If the rival does not enter, the incumbent can set any price in $[0, p^I_A]$ at which to provide the information. I solve the game for Pareto-dominant Perfect Bayesian Equilibria.

Since the incumbent is bound to lose against the rival if the latter enters, the incumbent either surrenders the market or preempts entry. The rival enters if she expects post-entry profits to be at least as large as her entry cost $S$. If she enters, the equilibrium price will equal the incumbent’s break-even price, which implies a revenue of $E \left( K_I \mid p^I_A \right) + c_A$. Hence, the rival enters only if

$$E \left( K_I \mid p^I_A \right) + c_A \geq S + K_E + c_A.$$ 

To preempt entry, the incumbent must therefore choose $p^I_A$ to signal that her operative costs do not exceed $K_I^+$, as defined by

$$K_I^+ = S + K_E.$$

\textbf{Lemma 4} If $E \left( K_I \right) \leq S + K_E$, the news market is uncontested, and the incumbent sets the monopoly price. Otherwise, the market is contested, and the incumbent deters entry if $K_I \leq K_I^+$ and surrenders the market if $K_I > K_I^+$, where $K_I^+ < K_H$. The price in a contested market increases in $S$ and $K_E$, but is lower than the monopoly price.

\textsuperscript{16}Introducing uncertainty or private information about $S$ or $K_E$, or allowing for $K_E \geq K_L$, makes the extension more realistic, but the mechanics and basic intuition behind the results remain the same. Note also that this setup includes Bertrand competition as a special case ($S = 0$ and $K_E = K_I$.)
The equilibrium is intuitive. If entry is not worthwhile when facing an incum- 
\[ K_I \leq K_I^* \] 

bent of average efficiency, entry is deterred without further ado. Hence, the 
\[ K_I > K_I^* \]

incumbent need not be concerned with signalling high efficiency and is de facto a 
monopolist. However, if average efficiency is too low to deter entry, a sufficiently 
\[ S_1, S_2 \]

efficient incumbent wants to signal a higher efficiency, because a higher efficiency 
implies a lower post-entry profit for the rival. This can be achieved by committing 
to lower prices which a less efficient incumbent could not afford to mimic. That 
is, a sufficiently efficient incumbent \( (K_I \leq K_I^*) \) reduces the price and successfully 
defends the market. The deterrence price and the cut-off type \( K_I^* \) increase in \( S \) 
\[ K_E \]

and \( K_E \). Intuitively, when entry is cheaper and the rival is more efficient, the 
incumbent must reduce the price more to deter entry, which only a better incum- 
bent can afford. If the incumbent is too inefficient \( (K_I > K_I^*) \), she cannot credibly 
deter entry and eventually loses the market to the rival.

A reasonable interpretation of \( S \) and \( K_E \) is that these costs capture regulatory 
barriers to entry into information markets and the progress in information technol- 
\[ S_1, S_2 \]

ogy, respectively. For instance, the entry cost \( S \) might consist of two components, 
technological expenditures \( (S_1) \) and the cost of overcoming regulatory “red tape” 
\[ S_2 \]

\( (S_1 K_E)^{-1} \) can then be interpreted as a measure of technological efficiency, 
and \( S_2^{-1} \) as a measure of the openness of the market. With this interpretation, 
Lemma 4 can be invoked to explain cross-country variation in price comovement 
as driven by variation in \( (S_1 K_E)^{-1} \) and \( S_2^{-1} \).

**Proposition 3** More efficient information technology and less entry regulation 
 promote competition in news markets, which in turn promotes diversity among 
active investors and reduces price comovement.

Rival news vendors supplying information about \( A \) compete in perfect substi- 
tutes, while information about \( B \) or \( C \) represents an imperfect substitute. This 
creates a competition "hierarchy" in which the news vendors first and foremost 
compete with each other, and their impact on strategies \( B \) and \( C \) plays a sec- 
\[ 1 \]

ondary role for their pricing incentives. The degree of competition increases in

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\[ 17 \] Djankov et al. (2002) document that regulatory entry barriers tend to be higher in countries 
show that the media sector in such countries is often concentrated and government-controlled.
the threat of entry, and hence in \((S_1K_E)^{-1}\) and \(S_2^{-1}\). Importantly, the competition among news vendors has a positive externality on the demand for alternative information: it expands the supply of news, which in turn improves liquidity and thereby the return to the other investment strategies (figure 4). Intuitively, the larger trade volume based on strategy A camouflages – and hence promotes – alternative trades. In fact, in the limit if \(p_A \to 0\), news become quasi-public and hence no longer exert a negative externality on strategies B and C. As a result, prices incorporate more stock-specific information, and comove less.

The main insight is that competition among news vendors is not only beneficial because it disseminates news more effectively but also because it encourages investors to tap new, alternative sources of information. Thus, competition policy in news markets may have a significant impact on the diversity of active investment strategies and thereby on the quality of financial markets.

**Robustness**

The above analysis takes the cost of information about \(B\) as given. This can reflect the notion that not every type of information can be traded in a market. It may be impossible to credibly communicate certain types of information (Allen, 1990; Michaely and Womack, 1999), or doing so may simply be too complex or costly. Such problems can make direct sale of information infeasible or unprofitable. The
assumption that $c_B$ is fixed is hence reasonable if certain types of investment strategies, to a greater extent than others, require expertise that is difficult to pass on.\footnote{For instance, Goetzmann et al. (2004) find that movie scripts that are less certifiable by hard information are sold at a discount. That is, the scriptwriter incurs a higher cost of selling 'soft' information. (If possible, she might prefer to instead produce the movie herself.) As regards speculative information, Roll (1988, p.564), in his well-known study on stock price comovement, concludes that his results seem “to imply that the financial press misses a great deal of relevant information generated privately.”}

The assumption is also less restrictive than it seems. Even if we introduce a $B$-vendor into the monopoly setting, predatory pricing by the $A$-vendor remains an equilibrium (though it is no longer unique). For example, $p_A$ may be set such that $\min R_A = \max R_B$, and $n_A$ is just small enough for the market not to reach the complement region. At this point, a single investor deems $B$-trading unprofitable. Selling information about $B$ to more than one investor might be profitable, if crowding out $A$-investors improves the market liquidity for $B$-investors. However, at $\min R_A = \max R_B$, the $A$-strategy is a complement to the $B$-strategy but not vice versa. Thus, expanding the supply of $B$-information and thereby crowding out $A$-investors reduces market liquidity and hence the $B$-vendor’s revenues. That is, the $B$-vendor cannot fare better than a single $B$-investor, and therefore stays out of the market. In fact, if the $A$-vendor is a Stackelberg leader, she chooses $p_A$ accordingly to deter entry by a $B$-vendor.

### 4.2 Proprietary Trading and Newsletters

Some financial institutions engage in proprietary trading activities and, at the same time, provide information to other investors. At first glance, it seems puzzling that they distribute information and trade actively, in particular if sharing information creates more competitors. In the present model, such behavior can be rational when the institution owns different types of information, some of which is also known to others while some is (more) exclusive.

For simplicity, consider two investors (1 and 2) endowed with information about $A$ and $B$, respectively. Information about $A$ is also held by $n_A - 1$ other investors, whereas information about $B$ is exclusive to investor 2. Suppose that...
investor 2 can make a take-it-or-leave-it (cash) offer $P$ to investor 1 such that, if accepted, investor 1 has to give away her information to (infinitely) many other investors. The question is whether there exists a price $P$ such that both investors can benefit from such an agreement.

**Proposition 4** There exists a non-empty interval $N_A$ such that, when $n_A \in N_A$, the two investors would benefit from the following agreement: investor 2 engages in a $B$-strategy and pays $P > 0$ to investor 1, who in return shares her data with infinitely many other investors.

The intuition behind this result is straightforward. The presence of $n_A - 1$ other investors pursuing the $A$-strategy reduces the expected profit of both investor 1 and investor 2. While giving away information about $A$ for free eliminates investor 1’s profit, it also eliminates any negative externality that $A$-investors exert on investor 2’s profit $[T(\infty) \to T(0)]$. As long as $\pi_B(1, 0) > \pi_B(1, n_A) + \pi_A(n_A, 1)$, there are gains from trade that the two investors can share. Intuitively, by flooding the market with information about $A$, they create a "herd" of $A$-investors among which investor 2 can "hide".

5 Conclusion

This paper studies a financial market in which fundamentals are driven by several factors, and active investors choose which factor to base their trading strategies on. The central question is how active investors pursuing different strategies interact when trading in the same market; and how this interaction affects their choice of investment strategy when entering the market. Contrary to common wisdom, a class of active investors in this setting may benefit from the presence of another class of active investors, as long as they pursue different strategies. On the one hand, any class of active investors aggravates the information asymmetry faced by uninformed investors, and hence reduces market liquidity. On the other hand, if the investor classes make (relatively) uncorrelated trades, they serve as "noise" traders for each other. Consequently, their investment strategies can be
complements in the sense that the expansion of one investor class induces an expansion in the other investor class.

Such liquidity externalities have interesting implications for variations in trade volume, price comovement, liquidity commonalities, herding behavior and the informational role of prices. For instance, the framework suggests that (i) sufficiently widespread "conventional" investment strategies may be a prerequisite for more complex investment strategies, such as those of hedge funds; (ii) as the information environment of a market improves, total trade volume increases, average trade volume decreases, but new large volume investors emerge; (iii) in emerging markets, stocks followed by more analysts comove more with the market, unless the higher analyst coverage coincides with a higher forecast dispersion; (iv) the liquidity of larger stocks tends to be more sensitive to, but less driven by, common liquidity shocks than that of smaller stocks; (v) herding in illiquid markets accompanies expansions in trading activity, whereas herding in liquid markets accompanies contractions in trading activity; and (vi) a decrease in information costs may make prices less informative even though the market becomes more efficient.

The paper also examines the impact of information markets in this context. In view of the above liquidity externalities, what is the optimal supply of information from the perspective of a commercial information supplier? I show that a monopolist, who provides information to a particular investor class, wants to expand supply just enough to crowd out other investment strategies. Her motivation to skew the distribution of information among active investors is to absorb more of the demand for information, and to mitigate the negative externality that other investment strategies impose on her customers. This predatory pricing strategy leads to an increase in the number of active investors but a decrease in the diversity of information held by active investors. Accordingly, prices may become less informative although the market becomes more efficient.

In contrast, competition among news vendors expands supply further. Consequently, news-based investment strategies proliferate, and trade volume grows, supplying liquidity that makes alternative investment strategies more profitable. This provides a novel rationale for why information market competition plays
an important role in financial market development. By making financial markets more liquid, it facilitates proprietary investment strategies and fosters information diversity.

Finally, the framework can explain why a financial institution may engage in proprietary trading while also selling information to other investors. By supplying more investors with information to pursue a common investment strategy, the institution may be able to improve the liquidity in the market. This in turn may allow it to trade more profitably on exclusive information which it does not share with other investors.

Appendix

Proof of Proposition 2

The main proof makes use of the following auxiliary result.

Lemma 5 For \(n_0, n_{\theta'} \geq 0\) and \(n \geq n_0\),

(a) \(\pi_{\theta}(n_{\theta}, \cdot)\) decreases in \(n_{\theta}\),

(b) \(\pi_{\theta} (\cdot, n_{\theta'})\) has a unique minimum in \(\mathbb{R}^{+}\),

(c) \(\pi_{\theta} (n_0, n - n_{\theta})\) decreases in \(n_{\theta}\).

Proof of Lemma 5. Given that \(\alpha_{\theta}\) is a constant, I need only consider the behavior of \(\rho_{\theta}(n_{\theta}, n_{\theta'})\) or, more precisely,

\[
\rho_{\theta}(n_{\theta}, n_{\theta'}) = \mathbb{E}[(\hat{V} - \tilde{p})\hat{s}_{i_0}] = \mathbb{E}[(\hat{V} - \lambda\hat{\delta})\alpha_{\theta}\hat{s}_{i_0}]
\]

\[
= \alpha_{\theta}\mathbb{E}(\hat{V}\hat{s}_{i_0}) - \lambda\mathbb{E}\left[\sum_{l=1}^{n_{\theta}} \alpha_{\theta}\hat{s}_{i_0} + \sum_{l=1}^{n_{\theta'}} \alpha_{\theta'}\hat{s}_{i_0'} + \tilde{y}\right] \alpha_{\theta}\hat{s}_{i_0}
\]

\[
= \alpha_{\theta} \left[ \mathbb{E}(\hat{V}\hat{s}_{i_0}) + \mathbb{E}(\hat{V}_{i_0}\hat{s}_{i_0}) \right] - \lambda \sum_{i=1}^{n_{\theta}} \alpha_{\theta}^{2}\mathbb{E}(\hat{s}_{i_0}\hat{s}_{i_0}) + \sum_{l=1}^{n_{\theta'}} \alpha_{\theta'}\alpha_{\theta}\mathbb{E}(\hat{s}_{i_0}\hat{s}_{i_0}) + \alpha_{\theta}\mathbb{E}(\tilde{y}\hat{s}_{i_0}) \right].
\]

Since \(\mathbb{E}(\hat{V}\hat{s}_{i_0}) = \sigma^2\), \(\mathbb{E}(\hat{V}_{i_0}\hat{s}_{i_0}) = 0\), \(\mathbb{E}(\hat{s}_{i_0}\hat{s}_{i_0}) = \sigma^2 + \sigma_{\varepsilon}^2\), \(\mathbb{E}(\hat{s}_{i_0}\hat{s}_{i_0}) = \sigma^2\), \(\mathbb{E}(\hat{s}_{i_0}\hat{s}_{i_0'}) = 0\), and \(\mathbb{E}(\tilde{y}\hat{s}_{i_0}) = 0\) for all \(i\) and all \(i', l \neq i\), this becomes

\[
\rho_{\theta}(n_{\theta}, n_{\theta'}) = \alpha_{\theta}\sigma^2(1 - \lambda\alpha_{\theta}) = \alpha_{\theta}\sigma^2 \left[ \frac{\sigma^2 + \sigma_{\varepsilon}^2}{n_{\theta} + 1} \right]
\]

where the last equality follows from Lemma 1. Substituting for \(\alpha_{\theta}\) and \(\lambda\) in (6) gives

\[
\rho_{\theta}(n_{\theta}, n_{\theta'}) = \left[ \frac{\sigma_{\theta}\sigma^2 (\sigma^2 + 2\sigma_{\varepsilon}^2)}{[\sigma^2 (n_{\theta} + 1) + 2\sigma_{\varepsilon}^2]^2} \right] \left[ \frac{n_{\theta} (\sigma^2 + \sigma_{\varepsilon}^2)}{[(n_{\theta} + 1)\sigma^2 + 2\sigma_{\varepsilon}^2]^2 + T_{\theta'}} \right]^{-1/2}.
\]

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Now define $A \equiv [\rho_0(n_\theta, n_{\theta'})]^{-2}$. This gives

$$A = \frac{n_\theta (\sigma^2 + 2\sigma_0^2) [\sigma^2 (n_\theta + 1) + 2\sigma_0^2]^2}{\sigma_0^2 \sigma^4 (\sigma^2 + 2\sigma_0^2)^2} + \frac{[\sigma^2 (n_\theta + 1) + 2\sigma_0^2]^4}{\sigma_0^2 \sigma^4 (\sigma^2 + 2\sigma_0^2)^2} T_{\theta'}.$$

$A$ strictly increases in $n_\theta$, which proves (a). Recall that $T_{\theta'}$ has a unique maximum. For constant $n_\theta$, this maximum coincides with a unique minimum of $A$, which proves (b). The proof of (c) proceeds in steps (i)-(iii).

(i) First define the functions

$$\lambda^n (n_\theta) \equiv \frac{\sigma^2}{\sigma_y} \left[ \frac{n_\theta (\sigma^2 + 2\sigma_0^2)}{[(n_\theta + 1)\sigma^2 + 2\sigma_0^2]^2} + \frac{(n - n_\theta)(\sigma^2 + 2\sigma_0^2)}{[(n - n_\theta + 1)\sigma^2 + 2\sigma_0^2]^2} \right]^{1/2}$$

and

$$\alpha^n_\theta (n_\theta) \equiv \frac{\sigma^2 [\lambda^n (n_\theta)]^{-1}}{\sigma^2 (n_\theta + 1) + 2\sigma_0^2}.$$

Note $\lambda^n (n_\theta)$ and hence $\alpha^n_\theta (n_\theta)$ are continuously differentiable for $n_\theta \in [0, n]$. Using the first equality in (6), I write

$$\rho_0(n_\theta, n - n_\theta) \equiv \alpha^n_\theta (n_\theta) = \alpha^n_\theta (n_\theta) \sigma^2 \left[ 1 - n_\theta \alpha^n_\theta (n_\theta) \lambda^n (n_\theta) \right].$$

Since this is an algebraic combination of continuously differentiable functions for $n_\theta \in [0, n]$, it is also continuously differentiable for $n_\theta \in [0, n]$.

(ii) Consider (6) again. Note that the term in parentheses decreases in $n_\theta$. Moreover, by Lemma 1, $n_\theta < n_{\theta'} \Rightarrow \alpha_\theta > \alpha_{\theta'}$. Hence, $n_\theta < n_{\theta'} \Rightarrow \rho^n_\theta(n_\theta) > \rho^n_{\theta'}(n_{\theta'})$ (*). Simple inspection of the formulae in Lemma 1 further shows that, for a fixed population, a trader from group $\theta$ with $n_\theta = x$ faces exactly the same decision problem as would a trader from group $\theta'$ with $n_{\theta'} = x$. That is, $\rho^n_\theta (x) = \rho^n_{\theta'} (x)$. Together with equivalence (*), this implies that

$$\rho^n_\theta (n_\theta) > \rho^n_\theta (n - n_\theta) \quad \text{for} \quad n_\theta \in (0, n/2),$$

which in turn implies that $\rho^n_\theta (n_\theta)$ is decreasing over some range in $[0, n]$.

(iii) Now suppose that $\rho^n_\theta (n_\theta)$ is also increasing over some range in $[0, n]$. Since $\rho^n_\theta (n_\theta)$ is continuously differentiable, this requires the existence of some $x^* \in (0, n)$ such that $\frac{\partial \rho^n_\theta}{\partial n_\theta} (x^*) = 0$. Using Lemma 1 and $n_{\theta'} = n - n_\theta$ to eliminate $\alpha_\theta$, $\lambda$ and $n_{\theta'}$ from $\rho^n_\theta(n_\theta)$, one can verify (e.g., in Maple) that $\frac{\partial \rho^n_\theta}{\partial n_\theta} (x^*) = 0$ has no real solution. By contradiction, $\rho^n_\theta (n_\theta)$ is strictly decreasing in $[0, n]$. This proves part (c).

**Preliminary.** I show that $\pi_A (n_A, 0) = \pi_B (1, n_A)$ has a unique solution. That they cross at least once follows from parts (a) and (b) of Lemma 5. To show that they cross at most once, it suffices to show that $\rho_A (n_A, 0) = \rho_B (1, n_A)$ has a unique solution. After substituting for $\alpha_\theta$ and $\lambda$ in (6), the respective functions are given by

$$\rho_A(n_A, 0) = \frac{\sigma_0 \sigma^2 (\sigma^2 + 2\sigma_0^2)}{((n_A + 1)\sigma^2 + 2\sigma_0^2)^{3/2} \sqrt{n_A (\sigma^2 + \sigma_0^2)}}$$

and

$$\rho_B(1, n_A) = \frac{\sigma_0^2 \sigma^2 (\sigma^2 + 2\sigma_0^2) ((n_A + 1)\sigma^2 + 2\sigma_0^2)^{-1}}{2 (\sigma^2 + \sigma_0^2)^{3/2} \sqrt{((n_A + 1)\sigma^2 + 2\sigma_0^2)^2 + 4n_A (\sigma^2 + \sigma_0^2)^2}}.$$

Equating these expressions and rearranging yields

$$1 = \frac{((n_A + 1)\sigma^2 + 2\sigma_0^2)^2 \sqrt{n_A}}{2 (\sigma^2 + \sigma_0^2)^{3/2} \sqrt{((n_A + 1)\sigma^2 + 2\sigma_0^2)^2 + 4n_A (\sigma^2 + \sigma_0^2)^2}}.$$

On both sides, I square and further rearrange to get

$$4 (\sigma^2 + \sigma_0^2)^2 \left( \frac{1}{((n_A + 1)\sigma^2 + 2\sigma_0^2)^2 n_A} + \frac{1}{((n_A + 1)\sigma^2 + 2\sigma_0^2)^2} \right) = 1.$$
The left-hand side goes to infinity for \( n_A \to 0 \) and strictly decreases in \( n_A \). Thus, \( \pi_A (n_A, 0) = \pi_B (1, n_A) \) has exactly one solution, to the left of which \( \pi_A (n_A, 0) > \pi_B (1, n_A) \) and to the right of which \( \pi_A (n_A, 0) < \pi_B (1, n_A) \).

For use below, I define \( \mathcal{R}_B = \{n_A, \pi_A\} \) and \( \min \mathcal{R}_A = n_A^0 \).

**First part:** \( \min \mathcal{R}_A \in \mathcal{R}_B \neq \emptyset \). I start with the sufficient condition, \( n_A \leq n_A^0 \leq \pi_A \Rightarrow (n_A^*, n_B^*) = (n_A^0, 0) \). It is straightforward to verify that the set of inequalities \( \mathcal{R}_A \leq n_A^0 \leq \pi_A \) is equivalent to the condition \( \pi_B (1, n_A^0) < \pi_A (n_A^0, 0) = 0 \). Suppose that this condition holds. I now check different candidate equilibria. (i) Note that \( (n_A^0, 0) \) trivially satisfies the free-entry condition. (ii) Conjecture an equilibrium with \( n_A > n_A^0 \) and \( n_B \geq 0 \). For all \( n_A > n_A^0 \), \( \pi_A (n_A, n_B) < \pi_A (n_A^0, n_B) < \pi_A (n_A^0, 0) = 0 \), that is, \( A \)-traders would incur a loss. Hence this cannot be an equilibrium. (iii) Conjecture an equilibrium with \( n_A < n_A^0 \) and \( n_B \geq 0 \) and distinguish the cases (iiia) \( n_A + n_B = n \leq n_A^0 \) and (iiib) \( n_A + n_B > n_A^0 \).

(iiia) Note that \( \pi_B (n, 0) > \pi_B (1, n - 1) \). Given that \( n < n_A^0 \), this follows from the initial assumption \( \pi_A (n_A^0, 0) > \pi_B (1, n_A^0) \) and that \( \pi_A (n_A, 0) \) and \( \pi_B (1, n_A) \) cross only once. Then, by Lemma 5(c), \( \pi_A (n, 0) < \pi_A (n - 1, 1) < \cdots < \pi_A (1, 1) \) whereas \( \pi_B (1, n - 1) > \pi_B (2, n - 2) > \cdots > \pi_B (n - 1, 1) \). Thus, this cannot be an equilibrium because, for any \( (n_A, n_B) \) such that \( n_A + n_B < n_A^0 \), \( B \)-traders would want to switch to \( A \)-data.

(iiib) Denote \( n_B = n_A^0 - n_A \) so that \( n_B > n_B^0 \). This provided, note that \( \pi_B (n_A^0, 0) = 0 > \pi_B (1, n_A^0 - 1) > \pi_B (n_B, n_A) > \pi_B (n_B, n_A) \). The first two inequalities follow from Lemma 5(c), and the last inequality follows from Lemma 5(a). Together, they imply that this cannot be an equilibrium because \( B \)-traders would expect to make a loss.

Finally, the necessary condition, \( (n_A^*, n_B^*) = (n_A^0, 0) \Rightarrow \pi_B (1, n_A^0) < \pi_A (n_A^0, 0) \), holds because, if the latter inequality was violated, \( A \)-traders would on the margin switch to \( B \)-data, and \( (n_A^0, 0) \) would not be an equilibrium.

**Second part:** \( \min \mathcal{R}_A \notin \mathcal{R}_B \) or \( \mathcal{R}_B = \emptyset \). I must show that, when \( n_A^0 \notin \mathcal{R}_A \), there exists a unique pair \( (n_A^*, n_B^*) \) that satisfies \( n_A^* > n_B^* > 0 \) and \( \pi_B (n_A^*, n_B^*) = 0 \) for \( \theta = A, B \). I proceed in steps (i)-(vi).

(i) Note that neither \((0, 0), (0, n_B)\) nor \((n_A, 0)\) can be an equilibrium. This follows respectively from \( \pi_A (1, 0) > 0 \) for \( \theta = A, B \). \( \pi_A (1, n_B - 1) > \pi_B (n_B, 0) \) (Lemma 5(c) and \( c_A \leq c_B \)), and the proof for the first part of the proposition.

(ii) Note that there exists an information structure where positive profits are equally shared. Consider a point \( n \) where \( \pi_B (1, n) > \pi_A (n, 0) > 0 \), whose existence follows from \( n_A^0 \notin \mathcal{R}_A \). Ignoring integer problems, this implies that it is on the margin profitable for an \( A \)-trader to instead become a \( B \)-trader. In doing so, she marginally lowers the expected profit in the \( B \)-group but marginally raises it in the \( A \)-group (Lemma 5(c)). Still, it might still be profitable for the next marginal \( A \)-trader to switch. However, since \( \pi_B (n - 1, 1) < \pi_A (1, n - 1) \) and the profit functions are continuous, there exists a unique \( x \) where \( \pi_B (n - x, x) = \pi_A (x, n - x) \). Given \( \pi_A (n, 0) > 0 \), profits must be positive for both groups.

(iii) It follows from the monotonicity in Lemma 5(c) that such an indifference point exists for any \( n \) (though not always with positive profits).

(iv) If \( \pi_B (n_B, n_A) = \pi_A (n_A, n_B) > 0 \) – the total population is changed, both \( n_A \) and \( n_B \) have to move in the same direction to maintain the indifference. To see this, note that after substituting for \( \alpha_0 \) and \( \lambda \) in (6), \( \pi_A (n_A, n_B) = \pi_B (n_A, n_B) \) can be written out and rearranged to

\[
\left( \frac{1}{(n_A + 1) \sigma^2 + 2 \sigma^2} - \frac{1}{(n_B + 1) \sigma^2 + 2 \sigma^2} \right) = \frac{c_A - c_B}{\sigma^4 (\sigma^2 + 2 \sigma^2)}
\]

Suppose this holds for a given \( n_A \) and \( n_B \). Now suppose that the change in population lowers \( \lambda \). In order for the equation to still hold, we need that the term in the parentheses to the left becomes smaller. It cannot be that only one group increases (or decreases) because, if so, the group that does not grow in size would end up with positive profits (lower price impact, same or less number of traders). To maintain the equality, both groups have to increase (decrease) when \( \lambda \) falls (rises).

(v) As both groups increase, aggregate expected profits eventually decrease (and even become negative). To see
this, write them as

\[
\begin{align*}
&n_A \frac{\sigma^2_A (\sigma^2 + 2\sigma^2_C)}{((n_A + 1)\sigma^2 + 2\sigma^2_C)^2} \left( \frac{n_A (\sigma^2 + 2\sigma^2_C)}{((n_A + 1)\sigma^2 + 2\sigma^2_C)^2} + \frac{n_B (\sigma^2 + 2\sigma^2_C)}{((n_B + 1)\sigma^2 + 2\sigma^2_C)^2} \right)^{-1/2} - n_A c_A \\
&+ n_B \frac{\sigma^2_B (\sigma^2 + 2\sigma^2_C)}{((n_B + 1)\sigma^2 + 2\sigma^2_C)^2} \left( \frac{n_A (\sigma^2 + 2\sigma^2_C)}{((n_A + 1)\sigma^2 + 2\sigma^2_C)^2} + \frac{n_B (\sigma^2 + 2\sigma^2_C)}{((n_B + 1)\sigma^2 + 2\sigma^2_C)^2} \right)^{-1/2} - n_B c_B = \\
&\sigma^2_y \frac{((n_A + 1)\sigma^2 + 2\sigma^2_C)^2 + n_B (\sigma^2 + 2\sigma^2_C)}{((n_B + 1)\sigma^2 + 2\sigma^2_C)^2} \left( \frac{n_A (\sigma^2 + 2\sigma^2_C)}{((n_A + 1)\sigma^2 + 2\sigma^2_C)^2} + \frac{n_B (\sigma^2 + 2\sigma^2_C)}{((n_B + 1)\sigma^2 + 2\sigma^2_C)^2} \right)^{-1/2} - n_A c_A - n_B c_B
\end{align*}
\]

The first expression, total trading profits, is concave in the sense that, if both \(n_A\) and \(n_B\) increase the marginal trading gain decreases. Moreover, total trading profits converge to zero as \(n_A + n_B \to 0\). By contrast, the information costs increase linearly in \(n_A + n_B\).

(vi) Since the above functions are continuous, the preceding arguments imply that – starting from \(\pi_B(n_A, n_B) = \pi_A(n_A, n_B) > 0\) – there exist a unique population size \(n^* > n_A + n_B\) such that, at the respective indigence point, total trading profits and hence average trading profits are zero. This point identifies the unique equilibrium information structure. ■

**Ad Implication 6**

**Market efficiency.** I need to show that total trading profits are lower under \((n_A, 0)\) than under \((n_A, n_B)\). First, note that \(\pi_A(n_A + n_B, 0) < \pi_A(n_A, n_B) = \pi_B(n_B, n_A)\) where the inequality follows from Lemma 5 (c), and the equality follows from the fact that \((n_A, n_B)\) denotes an equilibrium outcome. These relations imply that

\[
(n_A + n_B) \pi_A(n_A + n_B, 0) < n_A \pi_A(n_A, n_B) + n_B \pi_B(n_B, n_A).
\]

This inequality can be rearranged to

\[
(n_A + n_B) \rho_A(n_A + n_B, 0) - n_A \rho_A(n_A + n_B, 0) - c_A < n_A \rho_A(n_A, n_B) - n_A \rho_A(n_A, n_B) - n_B \rho_B(n_B, n_A) - n_B \rho_B(n_B, n_A) - n_B (c_B - c_A)
\]

which – due to \(c_B > c_A\) – implies

\[
(n_A + n_B) \rho_A(n_A + n_B, 0) < n_A \rho_A(n_A, n_B) + n_B \rho_B(n_B, n_A)
\]

where the left-hand side and the right-hand side represent total trading profits for \((n_A + n_B, 0)\) and \((n_A, n_B)\) respectively. Finally, it is well-known that \(n A \rho_A(n, 0) < (n') \rho_A(n', 0)\) if \(n' > n\) (see, e.g., Admati and Pfleiderer, 1988). Since \(n' > n_A + n_B\), it follows that

\[
n_A \rho_A(n_A, 0) < (n_A + n_B) \rho_A(n_A + n_B, 0) < n_A \rho_A(n_A, n_B) + n_B \rho_B(n_B, n_A)
\]

which proves the proposition.

**Price informativeness.** First, note that \(\text{Var}(\tilde{V} | p) = \text{Var}(\tilde{V} | z)\) as the price is based exclusively on order flow information. This conditional variance is given by \(\text{Var}(\tilde{V} | z) = \text{Var}(\tilde{V}) (1 - \rho^2_{V, z})\) where

\[
\rho^2_{V, z} = \frac{\text{Var}(\tilde{V})}{\text{Var}(\tilde{V})} = \frac{\text{Cov}(\tilde{V}, \tilde{z})}{\text{Var}(\tilde{V})} = \frac{(n_{AA} + n_{AB}) \sigma^2}{2 \sigma^2} = \frac{1}{2} \left( \frac{n_A \sigma^2}{\sigma^2(n_A + 1) + 2\sigma^2} + \frac{n_B \sigma^2}{\sigma^2(n_B + 1) + 2\sigma^2} \right) \lambda
\]

which proves the proposition.
Note that
\[
\frac{\partial \rho_{V,A}^2}{\partial \eta_a} = \frac{\sigma^2 (\sigma^2 + 2 \sigma_y^2)}{\sigma^2 (\sigma^2 + 2 \sigma_y^2) + 1} > 0 \quad \text{and} \quad \frac{\partial \rho_{V,A}^2}{\partial \eta_B} = -\frac{2 \sigma^4 (\sigma^2 + 2 \sigma_y^2)}{\sigma^2 (\sigma^2 + 2 \sigma_y^2)^3} < 0.
\]

I now show that price informativeness can decrease when B-information is crowded out. Define \( I(n_A, n_B) = 2\rho_{V,A}^2 \) as a measure of price informativeness. Suppose that \( n_A^B(c_A) \leq n_A \), and denote the equilibrium information structure by \((n_A, n_B)\). Price informativeness is then given by
\[
I(n_A, n_B) = \sum_{\theta = A, B} n_\theta \sigma^2 \left( \frac{1}{4 \sigma^2 + 2 \sigma_y^2} \right)^{1/2} \left( \frac{n_A (\sigma^2 + \sigma_y^2)}{(n_A + 1) \sigma^2 + 2 \sigma_y^2} + \frac{1}{4 \sigma^2 + 2 \sigma_y^2} \right)^{-1/2} = C_B,
\]
(7)

When the participation constraint of an A-trader is also binding,
\[
I(n_A, n_B) = \frac{n_A \sigma^2}{\sigma^2 (n_A + 1) + 2 \sigma_y^2},
\]
and denote the equilibrium information structure \((n_A, n_B)\) is an equilibrium.

Suppose these conditions hold for \( c_A \), and consider a change in cost to \( c_A' < c_A \). This will increase \( n_A \), and may in turn crowd out the B-investor. When this happens, the equilibrium number of A-investors is determined by
\[
\frac{\sigma^2 \sigma_y (\sigma^2 + 2 \sigma_y^2)}{(n_A + 1) \sigma^2 + 2 \sigma_y^2} \left( \frac{n_A (\sigma^2 + \sigma_y^2)}{(n_A + 1) \sigma^2 + 2 \sigma_y^2} + \frac{1}{4 \sigma^2 + 2 \sigma_y^2} \right)^{-1/2} = C_A',
\]
(8)

I now compute the equilibrium information structure(s) for \( \sigma^2 = 1, \sigma_y = 10, \sigma^2 = 10, c_A = 1/2 \) and \( c_A' = 1/(2.1) \). (I use \( C_B \) as my degree of freedom to ensure that \( n_B = 1 \).) Using condition 8, I first determine the equilibrium number of A-investors for \( c_A \),
\[
\frac{210}{(n_A + 21)^2} \left( \frac{11 n_A}{(n_A + 21)^2} + \frac{1}{44} \right)^{-1/2} = \frac{1}{2^2},
\]
which yields \( n_A = 12.236 \). Similarly, I compute the number of A-investors that enter once the B-investor has been crowded out via condition 9,
\[
\frac{210}{(n_A + 21)^2} = (n_A + 21) \sqrt{n_A},
\]
which yields \( n_A = 14.238 \). The B-investor will indeed drop out because the maximum of \( T(n_A) \) is at \( n = 21 \) in this case. That is, the increase in \( n_A \) occurs in a range where the liquidity externality on B-investors is negative.

Price informativeness in this example drops from
\[
I(n_A, 1) = \frac{12.236}{(12.236 + 1) + 20} + \frac{1}{(1 + 1) + 20} = 0.41361
\]

Note that
\[
\frac{\partial \rho_{V,A}^2}{\partial \eta_a} = \frac{\sigma^2 (\sigma^2 + 2 \sigma_y^2)}{\sigma^2 (\sigma^2 + 2 \sigma_y^2) + 1} > 0 \quad \text{and} \quad \frac{\partial \rho_{V,A}^2}{\partial \eta_B} = -\frac{2 \sigma^4 (\sigma^2 + 2 \sigma_y^2)}{\sigma^2 (\sigma^2 + 2 \sigma_y^2)^3} < 0.
\]

I now show that price informativeness can decrease when B-information is crowded out. Define \( I(n_A, n_B) = 2\rho_{V,A}^2 \) as a measure of price informativeness. Suppose that \( n_A^B(c_A) \leq n_A \), and denote the equilibrium information structure by \((n_A, n_B)\). Price informativeness is then given by
\[
I(n_A, n_B) = \sum_{\theta = A, B} n_\theta \sigma^2 \left( \frac{1}{4 \sigma^2 + 2 \sigma_y^2} \right)^{1/2} \left( \frac{n_A (\sigma^2 + \sigma_y^2)}{(n_A + 1) \sigma^2 + 2 \sigma_y^2} + \frac{1}{4 \sigma^2 + 2 \sigma_y^2} \right)^{-1/2} = C_B,
\]
(7)

When the participation constraint of an A-trader is also binding,
\[
I(n_A, n_B) = \frac{n_A \sigma^2}{\sigma^2 (n_A + 1) + 2 \sigma_y^2},
\]
and denote the equilibrium information structure \((n_A, n_B)\) is an equilibrium.

Suppose these conditions hold for \( c_A \), and consider a change in cost to \( c_A' < c_A \). This will increase \( n_A \), and may in turn crowd out the B-investor. When this happens, the equilibrium number of A-investors is determined by
\[
\frac{\sigma^2 \sigma_y (\sigma^2 + 2 \sigma_y^2)}{(n_A + 1) \sigma^2 + 2 \sigma_y^2} \left( \frac{n_A (\sigma^2 + \sigma_y^2)}{(n_A + 1) \sigma^2 + 2 \sigma_y^2} + \frac{1}{4 \sigma^2 + 2 \sigma_y^2} \right)^{-1/2} = C_A',
\]
(8)

the structure \((n_A, n_B)\) is an equilibrium.

I now compute the equilibrium information structure(s) for \( \sigma^2 = 1, \sigma_y = 10, \sigma^2 = 10, c_A = 1/2 \) and \( c_A' = 1/(2.1) \). (I use \( C_B \) as my degree of freedom to ensure that \( n_B = 1 \).) Using condition 8, I first determine the equilibrium number of A-investors for \( c_A \),
\[
\frac{210}{(n_A + 21)^2} \left( \frac{11 n_A}{(n_A + 21)^2} + \frac{1}{44} \right)^{-1/2} = \frac{1}{2^2},
\]
which yields \( n_A = 12.236 \). Similarly, I compute the number of A-investors that enter once the B-investor has been crowded out via condition 9,
\[
\frac{210}{(n_A + 21)^2} = (n_A + 21) \sqrt{n_A},
\]
which yields \( n_A = 14.238 \). The B-investor will indeed drop out because the maximum of \( T(n_A) \) is at \( n = 21 \) in this case. That is, the increase in \( n_A \) occurs in a range where the liquidity externality on B-investors is negative.

Price informativeness in this example drops from
\[
I(n_A, 1) = \frac{12.236}{(12.236 + 1) + 20} + \frac{1}{(1 + 1) + 20} = 0.41361
\]
Proof of Lemma 3

First, note that price discrimination without rationing at each price is equivalent to selling unlimited subscriptions at the lowest offered price.

Second, consider price-quantity schedules of the following form: The seller determines a set of prices \( p_A^n > p_A^{n-1} > \cdots > p_A^1 \) and the maximum number of subscriptions \( y_A^n, y_A^{n-1}, \ldots, y_A^1 \) she is willing to sell at each price. Suppose that a quantity restriction is binding in the sense that more traders would like to purchase a subscription at that price, say \( p_A^1 \). There are two possible cases. (i) If it is unprofitable for any additional trader to purchase data at \( p_A^1 \), the information seller fares better by increasing \( y_A^1 \) and hence the number of subscriptions sold at \( p_A^1 \). (ii) If there are traders who purchase data at \( p_A^1 \), the information seller fares better by setting \( y_A^1 = 0 \). To see this, note that all \( A \)-traders make the same trading profit, irrespective of the individual price paid for the data. Thus, if some traders do not incur a loss when buying data at \( p_A^1 \), the \( y_A^1 \) traders that buy data at \( p_A^1 \) make a (total) profit of at least \( A_i y_A^1 (p_A^1 - p_A^{i+1}) \). When \( y_A^1 = 0 \), these traders would be willing to buy subscriptions at \( p_A^1 \). Thus, having a binding quantity restriction is not optimal. But quantity restrictions that are not binding are unnecessary.

Finally, consider the indirect sale of information, e.g., through a fund. In this case, the seller trades on behalf of its subscribers, and a contract prescribes a fixed subscription fee and a profit-sharing rule. If only a fixed fee is paid, the seller’s trading strategy and hence the fund’s expected profit is equivalent to that of a single trader. That is, the seller is better off directly selling the data (to many traders). Now suppose that the sharing rule induces the seller to trade as aggressively as \( n_A \) traders. That is, the fund’s expected profit will be equal to the combined expected profit of \( n_A \) individual traders with exactly the same signal. Competition is more intense for any number of traders with ‘photocopied’ errors than for the same number of traders with ‘personalized’ errors (Admati and Pfleiderer, 1986). This is because, in the first case, the traders commonly know that all of them will submit identical orders, perfectly reinforcing each other’s impact on the price. As a result, selling data to \( n_A \) traders that interpret it differently generates a higher expected profit than trading as aggressively as \( n_A \) traders with the same signal. Since the seller extracts the entire trading profits of her subscribers, she is better of selling her signal directly.

Proof of Proposition 2

Suppose that \( n_A^0(c_A) < \frac{2A}{A} \), and define \( \bar{p}_A \) by \( n_A^0(\bar{p}_A) = \frac{2A}{A} \). Now consider any price \( p_A \in (p_A, c_A) \). For any such price, \( n_A^0(p_A) < \frac{2A}{A} \), both types of data will be acquired in equilibrium, and the monopolist’s gross profit (which is equal to \( A \)-traders’ total trading profits) is given by

\[
\Pi_A^g(n_A, n_B) = n_A \left( \frac{\sigma^2 \gamma (\sigma^2 + 2\sigma^2)}{(n_A + 1) \sigma^2 + 2\sigma^2} \right) \left( \frac{n_A (\sigma^2 + \sigma^2)}{[(n_A + 1) \sigma^2 + 2\sigma^2]^2} + \frac{n_B (\sigma^2 + \sigma^2)}{[(n_B + 1) \sigma^2 + 2\sigma^2]^2} \right)^{-1/2}.
\]

Now suppose that the monopolist lowers the price to \( \bar{p}_A \), thereby crowding out all \( B \)-traders. Her gross profit in this case is given by

\[
\Pi_A^g(\bar{p}_A, 0) = \bar{p}_A \left( \frac{\sigma^2 \gamma (\sigma^2 + 2\sigma^2)}{(n_A + 1) \sigma^2 + 2\sigma^2} \right) \left( \frac{\bar{p}_A (\sigma^2 + \sigma^2)}{[(n_A + 1) \sigma^2 + 2\sigma^2]^2} \right)^{-1/2}.
\]
It is straightforward but tedious to show that \( n_A > n_A \). I therefore omit the proof, which rests on the logic that, unless their number increases in response to a fall in \( p_A \), \( A \)-traders would earn a positive profit (which cannot be an equilibrium).

I now need to show that \( \Pi_A^B(n_A,0) > \Pi_A^B(n_A,n_B) \) or, equivalently, that

\[
\frac{\Pi_A^B(n_A,0)}{\Pi_A^B(n_A,n_B)} = \frac{\sigma^2 \sigma_A^2 \sigma_B^2}{\sigma^2 + \sigma_A^2 + \sigma_B^2} 
\frac{n_B}{n_A} (n_A(n_A+1)\sigma^2 + 2\sigma_B^2) \left( \frac{(n_A+1)\sigma^2 + 2\sigma_B^2}{n_A(n_A+1)\sigma^2 + 2\sigma_B^2} \right) \left( \frac{(n_A+1)\sigma^2 + 2\sigma_B^2}{n_A(n_A+1)\sigma^2 + 2\sigma_B^2} \right)^{-1/2} 
= \frac{\sqrt{n_A(n_A+1)\sigma^2 + 2\sigma_B^2}}{\sigma^2 + \sigma_A^2 + \sigma_B^2} \left( \frac{(n_A+1)\sigma^2 + 2\sigma_B^2}{n_A(n_A+1)\sigma^2 + 2\sigma_B^2} \right) \left( \frac{(n_A+1)\sigma^2 + 2\sigma_B^2}{n_A(n_A+1)\sigma^2 + 2\sigma_B^2} \right)^{1/2}.
\]

is greater than 1. This condition can be written as

\[
1 + \frac{n_B}{n_A} \frac{(n_A+1)\sigma^2 + 2\sigma_B^2}{(n_A+1)\sigma^2 + 2\sigma_B^2} > \frac{n_A}{(n_A+1)\sigma^2 + 2\sigma_B^2}.
\]

The value of the left-hand side is greater than 1. The value of the right-hand side is smaller than 1 if

\[
\frac{n_B}{n_A} \frac{(n_A+1)\sigma^2 + 2\sigma_B^2}{(n_A+1)\sigma^2 + 2\sigma_B^2} > \frac{n_A}{(n_A+1)\sigma^2 + 2\sigma_B^2}.
\]

This is true because, by the definition of \( p_A \), \( T(n_A) \) is increasing in \( n_A \) for \( n_A < n_A \). Thus, the monopolist is better off crowding out \( B \)-information.

**Proof of Lemma 4**

If \( E(K_I) \leq S + K_E \), any uninformative precommitment price preempts entry. That provided, it is clearly a Perfect Bayesian Equilibrium for all incumbent types to choose the monopoly price. It is also straightforward to see that, from the incumbent’s perspective, this is a Pareto-dominant equilibrium.

However, if \( E(K_I) > S + K_E \), a pooling price does not preemt entry. Therefore, some (of the more efficient) incumbent types have an incentive to reveal their type in order to deter the challenger. I first conjecture a Perfect Bayesian equilibrium such that all types below a cut-off type \( K_I \) preempt entry by setting a uniform price \( p_A^* \) and all types above \( K_I \) surrender the market. Incentive-compatibility requires that

\[
n_A(p_A^*) \rho(n_A^*(p_A^*), n_B^*(p_A^*)) - c_A - K_I < 0 \quad \text{for all } K_I > K_I^+.
\]

and that

\[
n_A(p_A^*) \rho(n_A^*(p_A^*), n_B^*(p_A^*)) - c_A - K_I \geq 0 \quad \text{for all } K_I \leq K_I^+.
\]

This trivially implies that \( p_A^* \) must satisfy

\[
n_A(p_A^*) \rho(n_A^*(p_A^*), n_B^*(p_A^*)) - c_A - K_I^+ = 0.
\]

To deter entry, the cut-off value must further satisfy

\[
E(K_I \leq K_I^+) \leq K_I^+.
\]

Since \( E(K_I) > S + K_E \) implies \( K_I^+ < E(K_I) < K_H \) and \( K_I \) is continuously distributed, there exists a unique \( K_I^* \in [K_L,K_H] \) such that all \( K_I < K_I^* \) satisfy this condition. Since \( p_A^* \) increases in the cut-off value \( K_I^* \), the Pareto-dominant Perfect Bayesian equilibrium (from the incumbent’s perspective) is to set the cut-off value as high as possible, that is, to \( K_I^* = K_I^+ \). To establish Pareto-dominance formally, it is easy to verify that all types above \( K_I^* \) earn zero profits in any Perfect Bayesian equilibrium, and that all types below \( K_I^* \) prefer a higher
cut-off value not only because it preempts entry for more incumbent types but also because it increases the precommitment price and hence the profit of any incumbent type. If there is entry, price is set to incumbent’s break-even price.

Lower $S$ or $K_E$ make it less likely that the information market is uncontestable ($E(K_f) \leq S + K_E$). When the market is contestable, lower $S$ or $K_E$ decrease $K_f^*$ and thereby also the equilibrium price. To see this, first note that $K_f^*$ increases in $S$ and $K_E$ (by definition: $K_f^* = S + K_E$), that $K_f^*$ increases in $K_f^*$ (by definition: $E(K_f \leq K_f^*) = K_f^*$), and that the equilibrium price $p_f^*$ increases in $K_f^*$ (by definition: $n_f^*(p_f^*) = E(K_f^* + c_A)$). By implication, $K_f^*$ and $p_f^*$ increase in $S$ and $K_E$.

### 5.1 Proof of Proposition 3

Price comovement can, for example, be measured by the average (absolute) correlation coefficient between individual asset prices and the market index:

$$\hat{\rho}_M = |\mathcal{M}|^{-1} \sum_{a \in \mathcal{M}} |\rho_{aM}|$$

where

$$\rho_{aM} = \frac{\text{Cov}(p_a, p_M)}{\sqrt{\text{Var}(p_a)\text{Var}(p_M)}}$$

and $p = \sum_{a \in \mathcal{M}} p_a$.

The average correlation coefficient indicates how much of the variation of a single price is explained by market-wide variations and is typically a good approximation of the $R^2$ in a regression.

Each $A$-trader trades in both assets. Let $\alpha_{Aa}$ denote $A$-traders’ trading intensity when trading in asset $a$. By definition of the market makers’ pricing functions,

$$p_1 = \lambda_1 \left( \sum_{i=1}^{n_A} x_{1A}(\bar{s}_{1A}) + \sum_{i=1}^{n_B} x_{B}(\bar{s}_{1B}) + \bar{y} \right) \quad \text{and} \quad p_2 = \lambda_2 \left( \sum_{i=1}^{n_A} x_{2A}(\bar{s}_{2A}) + \sum_{i=1}^{n_C} x_{C}(\bar{s}_{2C}) + \bar{y} \right)$$

which implies the following ‘market index’

$$p_M = \lambda_1 \left( \sum_{i=1}^{n_A} x_{1A}(\bar{s}_{1A}) + \sum_{i=1}^{n_B} x_{B}(\bar{s}_{1B}) + \bar{y} \right) + \lambda_2 \left( \sum_{i=1}^{n_A} x_{2A}(\bar{s}_{2A}) + \sum_{i=1}^{n_C} x_{C}(\bar{s}_{2C}) + \bar{y} \right) .$$

Price variances are given by

$$\text{Var}(p_1) = (\lambda_1)^2 \text{Var} \left( \sum_{i=1}^{n_A} x_{1A}(\bar{s}_{1A}) + \sum_{i=1}^{n_B} x_{B}(\bar{s}_{1B}) + \bar{y} \right) = (\lambda_1)^2 \text{Var} \left( \sum_{i=1}^{n_A} \alpha_{1A}\bar{s}_{1A} + \sum_{i=1}^{n_B} \alpha_{B}\bar{s}_{1B} + \bar{y} \right)$$

$$= (\lambda_1)^2 \left[ \text{Var} \left( \sum_{i=1}^{n_A} \alpha_{1A} (\bar{v}_{1} + \bar{\epsilon}_i) \right) + \text{Var} \left( \bar{\alpha}_{B} \sum_{i=1}^{n_B} (\bar{v}_{B} + \bar{\epsilon}_i) \right) + \sigma_y^2 \right]$$

$$= (\lambda_1)^2 \left[ (n_A\alpha_{1A})^2 \sigma_A^2 + (\alpha_{1A})^2 n_A \sigma_A^2 + (n_B\alpha_{B})^2 \sigma_B^2 + (\alpha_{B})^2 n_B \sigma_B^2 + \sigma_y^2 \right]$$

and, analogously,

$$\text{Var}(p_2) = (\lambda_2)^2 \left[ (n_A\alpha_{2A})^2 \sigma_A^2 + (\alpha_{2A})^2 n_A \sigma_A^2 + (n_C\alpha_{C})^2 \sigma_C^2 + (\alpha_{C})^2 n_C \sigma_C^2 + \sigma_y^2 \right] .$$
The variance of the market index is

\[
\text{Var}(p_M) = \text{Var} \left( \sum_{i=1}^{n_A} x_{1A} \tilde{s}_{iA} + \lambda_2 \sum_{i=1}^{n_B} x_{2A} \tilde{s}_{iA} + (\lambda_1 + \lambda_2) \bar{y} + \lambda_1 \sum_{i=1}^{n_B} x_{B} \tilde{s}_{iB} + \lambda_2 \sum_{i=1}^{n_C} x_{C} \tilde{s}_{iC} \right)
\]

\[
= \text{Var} \left( \sum_{i=1}^{n_A} x_{1A} \tilde{s}_{iA} + \lambda_2 \sum_{i=1}^{n_A} x_{2A} \tilde{s}_{iA} \right) + \text{Var} [ (\lambda_1 + \lambda_2) \bar{y} ]
\]

\[
+ (\lambda_1)^2 \left[ (n_B \sigma_B^2 \sigma^2 + (\sigma_B^2) n_B \sigma_B^2 \sigma^2 + (\alpha_B^2)^2 n_B \sigma_B^2 \sigma^2 \right]
\]

\[
= (\lambda_1 \sigma_{1A}^2 + \lambda_2 \sigma_{2A}^2)^2 \left[ (n_A \sigma_A^2 \sigma^2 + n_A \sigma_A^2 \sigma^2 + (n_A \sigma_A^2)^2 \sigma^2 + (n_A \sigma_A^2)^2 \sigma^2 + (\alpha_A)^2 \sigma^2 + \sigma_A^2 \sigma^2 \right]
\]

The covariances between individual asset prices and the market index are given by

\[
\text{Cov}(p_{1,M}) = \text{Cov} \left( \sum_{i=1}^{n_A} x_{1A} \tilde{s}_{iA} + \sum_{i=1}^{n_B} x_{B} \tilde{s}_{iB} + \bar{y} \right), \lambda_1 \left( \sum_{i=1}^{n_A} x_{1A} \tilde{s}_{iA} + \sum_{i=1}^{n_B} x_{B} \tilde{s}_{iB} + \bar{y} \right) + \lambda_2 \left( \sum_{i=1}^{n_A} x_{2A} \tilde{s}_{iA} + \bar{y} \right)
\]

\[
= \text{Cov} \left( \lambda_1 \sum_{i=1}^{n_A} x_{1A} \tilde{s}_{iA} + \lambda_1 \sum_{i=1}^{n_B} x_{B} \tilde{s}_{iB} + \lambda_1 \tilde{y}, (\lambda_1 \sigma_{1A} + \lambda_2 \sigma_{2A}) \sum_{i=1}^{n_A} \tilde{s}_{iA} + \lambda_1 \sum_{i=1}^{n_B} x_{B} \tilde{s}_{iB} + (\lambda_1 + \lambda_2) \bar{y} \right)
\]

\[
= \text{Cov} \left( \lambda_1 \sum_{i=1}^{n_A} o_{1A} \tilde{s}_{iA} + \lambda_1 \sum_{i=1}^{n_B} x_{B} \tilde{s}_{iB} + \lambda_1 \tilde{y}, (\lambda_1 \sigma_{1A} + \lambda_2 \sigma_{2A}) \sum_{i=1}^{n_A} \tilde{s}_{iA} \right) +
\]

\[
\text{Cov} \left( \lambda_1 \sum_{i=1}^{n_A} x_{1A} \tilde{s}_{iA} + \lambda_1 \sum_{i=1}^{n_B} x_{B} \tilde{s}_{iB} + \lambda_1 \tilde{y}, \lambda_1 \sum_{i=1}^{n_B} x_{B} \tilde{s}_{iB} \right) +
\]

\[
\text{Cov} \left( \lambda_1 \sum_{i=1}^{n_A} x_{1A} \tilde{s}_{iA} + \lambda_1 \sum_{i=1}^{n_B} x_{B} \tilde{s}_{iB} + \lambda_1 \tilde{y}, (\lambda_1 + \lambda_2) \bar{y} \right)
\]

\[
= (\lambda_1 \sigma_{1A}) (\lambda_1 \sigma_{1A} + \lambda_2 \sigma_{2A}) \left[ (n_A \sigma_A^2 \sigma^2 + n_A \sigma_A^2 \sigma^2 + (n_A \sigma_A^2)^2 \sigma^2 + (n_A \sigma_A^2)^2 \sigma^2 + (\alpha_A)^2 \sigma^2 + \sigma_A^2 \sigma^2 \right]
\]

and, similarly,

\[
\text{Cov}(p_{2,M}) = (\lambda_2 \sigma_{2A}) (\lambda_1 \sigma_{1A} + \lambda_2 \sigma_{2A}) \left[ (n_A \sigma_A^2 \sigma^2 + n_A \sigma_A^2 \sigma^2 + (n_A \sigma_A^2)^2 \sigma^2 + (n_A \sigma_A^2)^2 \sigma^2 + (\alpha_A)^2 \sigma^2 + \sigma_A^2 \sigma^2 \right]
\]

The correlation coefficients are thus

\[
\rho_{1,M} = \frac{1}{\sqrt{(\lambda_1)^2 \left[ (n_A \sigma_A^2 \sigma^2 + (\alpha_A)^2 \sigma_A^2 \sigma^2 + (n_B \sigma_B^2 \sigma^2 + (\alpha_B)^2 \sigma_B^2 \sigma^2 + \sigma_A^2 \sigma^2 + (\sigma_B)^2 \sigma_B^2 \sigma^2 + \sigma_A^2 \sigma^2 \right]
\]

\[
\frac{(\lambda_1 \sigma_{1A}) (\lambda_1 \sigma_{1A} + \lambda_2 \sigma_{2A}) \left[ (n_A \sigma_A^2 \sigma^2 + n_A \sigma_A^2 \sigma^2 + (n_A \sigma_A^2)^2 \sigma^2 + (n_A \sigma_A^2)^2 \sigma^2 + (\alpha_A)^2 \sigma^2 + \sigma_A^2 \sigma^2 \right] + (\lambda_1 \sigma_{1A}) \left[ (n_B \sigma_B^2 \sigma^2 + (\alpha_B)^2 \sigma_B^2 \sigma^2 \right] + (\lambda_1 \sigma_{1A}) \left[ (n_C \sigma_C^2 \sigma^2 + (\alpha_C)^2 \sigma_C^2 \sigma^2 \right]}
\]

\[
\sqrt{(\lambda_1 \sigma_{1A}) (\lambda_1 \sigma_{1A} + \lambda_2 \sigma_{2A}) \left[ (n_A \sigma_A^2 \sigma^2 + n_A \sigma_A^2 \sigma^2 + (n_A \sigma_A^2)^2 \sigma^2 + (n_A \sigma_A^2)^2 \sigma^2 + (\alpha_A)^2 \sigma^2 + \sigma_A^2 \sigma^2 \right] + (\lambda_1 \sigma_{1A}) \left[ (n_B \sigma_B^2 \sigma^2 + (\alpha_B)^2 \sigma_B^2 \sigma^2 \right] + (\lambda_1 \sigma_{1A}) \left[ (n_C \sigma_C^2 \sigma^2 + (\alpha_C)^2 \sigma_C^2 \sigma^2 \right]}
\]

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Simple inspection shows that these correlation coefficients must be smaller than 1 as long as \((n_B, n_C) \neq (0,0)\). This is intuitive. For instance, if \(n_C > 0\), the price of asset 1 is independent of the \(C\)-factor, whereas the market index is not.

By contrast, consider the case where the monopolist engages in predatory selling and crowds out information about the idiosyncratic signals \((n_B = n_C = 0)\). In this case, \(A\)-traders face no rival trader group in either asset market. Using \(\alpha_{1A} = \alpha_{2A} = \alpha_A\), the correlation coefficient then becomes

\[
\rho_{1M} = \frac{(\lambda_1 \alpha_A) (\lambda_1 \alpha_A + \lambda_2 \alpha_A) \left( n_A \right)^2 \sigma^2 + n_A \sigma^2_y + (\lambda_1) \left( \lambda_1 + \lambda_2 \right) \sigma^2_y}{\sqrt{\left( \lambda_1 \alpha_A \right)^2 \left( n_A \right)^2 \sigma^2 + n_A \sigma^2_y + \lambda_1 \left( \lambda_1 + \lambda_2 \right) \sigma^2_y} \sqrt{\left( \lambda_1 \alpha_A \right)^2 \left( n_A \right)^2 \sigma^2 + \lambda_1 \left( \lambda_1 + \lambda_2 \right) \sigma^2_y} \sqrt{\left( \lambda_1 \alpha_A \right)^2 \left( n_A \right)^2 \sigma^2 + n_A \sigma^2_y + \lambda_1 \left( \lambda_1 + \lambda_2 \right) \sigma^2_y}}
\]

That, \(n_A = n_B > 0\) is more likely to occur for lower \(S_1\), \(K_E\) or \(S_2\) follows from Lemma 4 and the related discussion in the text.

**Proof of Proposition 4**

It suffices to show that there are aggregate gains from trade that the two investors can achieve by entering into the contract. Without the contract, their joint expected trading profit is given by the sum

\[
\rho_B(1, n_A) + \rho_A(n_A, 1) = \frac{\sigma_y \sigma^2 \left( \sigma^2 + 2 \sigma^2_y \right)}{(2 \sigma^2 + 2 \sigma^2_y)^2} \left[ T(1) + T(n_A) \right]^{-1/2} + \frac{\sigma_y \sigma^2 \left( \sigma^2 + 2 \sigma^2_y \right)}{(2 \sigma^2 + 2 \sigma^2_y)^2} \left[ T(1) + T(n_A) \right]^{-1/2}
\]

If investor 1 gives away her information for free, then there will be infinitely many \(A\)-investors and \(T(\infty) \approx 0\). In that case, their joint trading profit is equal to investor 2’s profit as a sole active investor:

\[
\rho_B(1, \infty) = \frac{\sigma_y \sigma^2 \left( \sigma^2 + 2 \sigma^2_y \right)}{(2 \sigma^2 + 2 \sigma^2_y)^2} \left[ T(1) + 0 \right]^{-1/2}.
\]

I want to know whether, for some initial \(n_A > 1\), the inequality \(\rho_B(1, \infty) > \rho_B(1, n_A) + \rho_A(n_A, 1)\) is
satisfied. Substituting the above expressions into the inequality gives

\[
\frac{\sigma_y \sigma^2 (\sigma^2 + 2\sigma^2)}{(2\sigma^2 + 2\sigma^2)^2} [T(1)]^{-1/2} > \frac{\sigma_y \sigma^2 (\sigma^2 + 2\sigma^2)}{(2\sigma^2 + 2\sigma^2)^2} \left( \frac{1 + T(n_A)}{T(1)} \right)^{1/2} \quad \text{for} \quad n_A > \frac{16 (\sigma^2 + \sigma^2)^2}{(2\sigma^2 + 2\sigma^2 + 2\sigma^2 n_A)^2}
\]

Finally, substituting for \( T(\cdot) \) and rearranging yields

\[
4n_A - 8 > \frac{16 (\sigma^2 + \sigma^2)^2}{(2\sigma^2 + 2\sigma^2 + 2\sigma^2 n_A)^2}
\]

The left-hand side is increasing in \( n_A \), whereas the right-hand side is decreasing in \( n_A \). Hence, this inequality is satisfied for sufficiently high \( n_A \). ■

References


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