Live Prices and Stale Quantities: T+1 Accounting and Mutual Fund Mispricing

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In this paper, we examine a little known aspect of mutual fund accounting, whereby funds do not use contemporaneous fund holdings to calculate net asset values. This practice, sanctioned under SEC Rule 2a-4, uses stale portfolio holdings and gives rise to deviations between reported net asset values (NAVs) and returns and the economic values of those quantities. Using both simulations and a new sample of fund transaction data, we establish that distortions in both NAVs and returns are fairly common, and we discuss the implications of this observation for fund practice and regulation.

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Introduction

Over the past few years, much attention has been directed to the problem of stale net asset values in the mutual fund industry. Stale NAVs, if predictable, can allow some shareholders to benefit at the expense of others. The most obvious examples of this opportunism exist in international funds, where the price used in the calculation of a NAV can be more than 12 hours old.[1]

While stale prices may plague a few types of funds' NAVs, there is a far more common and insidious problem that has avoided public notice. Normal mutual fund accounting, sometimes called "Trade Day Plus One" or "T+1" accounting, *always* uses stale portfolio information in the calculation of NAVs. In the simplest terms, in calculating today's NAV, fund pricing services use today's prices applied to yesterday's portfolio, i.e., live prices and stale quantities. Securities bought or sold on date t do not show up in the NAV on date t; rather the calculations are done *as if* no portfolio trading took place during the day.[2] This set of accounting rules, which is used by virtually all U.S. mutual funds,[3] drives a wedge between the reported "Accounting" NAVs used to calculate the prices at which funds are bought and sold and the actual "Economic" NAVs which represent the value of the funds' portfolios.

Accounting NAVs are used by outsiders to calculate reported fund returns. As a result of T+1 accounting, returns can be distorted as well. Any calculations or decisions which rely on the accounting numbers may be flawed. This includes all decisions made by anyone *outside* of a fund company, whether investors, data vendors, academics, or even fund trustees. We believe that these distortions are only problems for outsiders. Given current technology and practices, investment managers can surely tell what securities they did and did not buy or sell during a day, and are therefore able to report portfolio values and returns that rely on the economic value of the portfolio.

[1] There are many papers written on market timing opportunities created by the use of stale prices in NAV calculations. See, for example, Boudoukh, Richardson, Subrahmanyam, and Whitelaw (2002), Chalmers, Edelen, and Kadlec (2001), Goetzmann, Ivkovich, and Rouwenhorst (2001), and Zitzewitz, E. (2003).
[2] This definition of T+1 accounting is distinct from a definition used in the appendix of a paper by Edelen and Warner (2001). There the authors are interested in the one-day lag between an investor's purchase or redemption order and the time at which the fund managers are informed of the order.
[3] We believe that money market funds, closed-end funds, and certain funds that cater to high-frequency traders do not use T+1 pricing.

In this paper, we describe the source of this problem—the set of accounting rules used by U.S. funds—the implications for value transfer among shareholders, and the potential for false inferences about fund returns. We also discuss the unintended and potentially harmful consequences of this accounting rule as it pertains to fund trading activity. In Section 1, we define T+1 accounting more precisely. In Section 2, we analytically discuss when stale portfolio information will have the largest impact on fund NAVs and returns. In Section 3, we discuss our simulation methodology and what it says about the incidence and frequency of the stale portfolio problem. In Section 4, we present evidence from a sample of domestic equity funds, documenting the incidence and frequency of the problem, as well as empirically identifying the determinants of materially misstated NAVs and returns. In Section 5, we discuss the implications of our work, the potential welfare consequences, and possible responses to our findings.

Section 1. Definition of T+1 Accounting

The accounting rules that determine the procedures for the calculation of Net Asset Value are prescribed in Rule 2a-4, Section 2 under the Investment Company Act of 1940, The rule permits—but does not require—funds to record security transactions as of one business day after the trade date for purposes of determining net asset value.[4] **Appendix A** contains the full text of the Rule.

While this rule is well known by fund accountants, it is less well known even among mutual fund "experts." Most incorrectly assume that net asset values are determined by calculating the number of securities at the end of each day, multiplied by the price (or fair value) of that security as of 4 p.m., less fees (which creates the "net" part of NAV.) We have polled a number of fund experts including academics who study the industry, fund directors, and even investment managers, who were all under this false

[4] If they wish, funds may record trades on the trade date. In fact, there seems to be no reason why funds could not switch between trade-date accounting and t+1 accounting, provided they applied a consistent rule and did not switch arbitrarily. This is apparently rare. However, we are aware of one instance in which a fund that normally used t+1 accounting allegedly made an exception and used trade-date accounting for a specific, highly volatile security. We are aware of a second case in which a fund switches to same-day accounting on those days for which its published balance sheets are reported.

impression.[5] For example, we "Googled" "Net Asset Value" and one of the top links defined it this way:

> Calculating mutual fund net asset values is easy. Simply take the current market value of the fund's net assets (securities held by the fund minus any liabilities) and divide by the number of shares outstanding. (http://mutualfunds.about.com/cs/calculators/l/aa082502a.htm, visited 10/9/2005)

A leading broker defined it in this manner:

> To calculate the NAV, managers add up the fund's assets, subtract the liabilities and divide by the number of shares the public owns. (http://www.ameritrade.com/education/html/encyclopedia/tutorial4/t4_s2.html, visited 10/9/2005)

The SEC's website links to Rule 2a-4, but unless one were to follow this link, one would simply read that:

> (I)f an investment company has securities and other assets worth $100 million and has liabilities of $10 million, the investment company's NAV will be $90 million. (http://www.sec.gov/answers/nav.htm, visited 10/9/2005)

Unfortunately, apart from the SEC's link to Rule 2a-4, these simplistic concepts of how NAVs are calculated are naïve or incorrect. From our discussions with industry experts and regulators, T+1 accounting is the norm in the U.S. fund industry for reported NAV calculations (although internal portfolio management systems use live or T accounting.) In our discussions with one fund auditor, he was unaware of any open-end fund using any method apart from T+1 accounting.

We suspect that this practice is a hold-out from earlier days, when fund managers could not reliably collect information about their portfolios by 4 p.m. nor transmit this information in a timely manner to fund administrators who calculate NAVs. Since the prior day's portfolio was known with certainty when the current day's NAV had to be set, it was likely computationally more convenient to use in the NAV calculations to meet the daily deadlines imposed by newspapers for reporting mutual fund data. If intraday volatility was slight, and trading minimal, the types of distortions would be minimal. Furthermore, with neither the investing public, nor even directors, aware of the rule or its implications, there would be little or no pressure to change. In addition, because fund accounting tends to operate as a cost center (and not a profit center), there might be little incentive to spend money to change a practice that others have not opposed.

[5] In addition to Edelen and Warner (2001), a notable exception is Johnson (2002), whose dissertation briefly discusses T+1 accounting.

A hypothetical example will clarify how T+1 accounting rules operate and the potential implications of these rules. Assume that Fund XYZ has zero expenses, so we can ignore the netting of fees which ordinarily is an integral portion of the calculation of *net* asset values.[6] XYZ holds two securities, A and B, and cash. Table 1 shows the holdings of XYZ as of end of day for three days, as well as the closing prices of A and B.

The most jarring aspect of Table 1 is how NAVs are calculated. Rather than multiplying the 4 p.m. share price times the number of shares held at that time (what we call the Economic NAV), the calculation uses the 4 p.m. share price multiplied by the number of shares owned the prior day (the reported or Accounting NAV). If the fund does not buy or sell securities, then these two are the same. However, on any day where the fund buys or sells securities at anything other than the closing price of the security, the NAV as calculated will not equal to the economic value of the portfolio. In effect, the current set of rules enables and encourages funds to use stale portfolio holdings data.

In Table 1, the stale portfolio data "problem" emerges on Wednesday. The fund manager sells all 100 shares of Security A at \$6.05 per share sometime during the day. This intraday price, \$6.05, is an attractive price relative to the closing price the prior day, but falls short of the closing price as of day's end. The Wednesday NAV pretends as if Security A is still held as of 4 p.m. From the accountant's perspective, the NAV is \$1,130 at 4 p.m. However, if the fund's portfolio were liquidated at that time, we would only find \$1,110 in securities. The difference (\$20) represents the number of shares sold times the difference between the closing price of Security A and the intraday price at which it was sold. While many are concerned that the *price*s used in the calculation of the NAV might be stale for certain securities, the *quantities* used to generate the NAV will <u>always</u> be stale if the firm trades any securities.

This simple example illustrates two important implications of T+1 accounting on returns and dilution. First, calculated using the NAV—the normal practice for investors, financial reporting services, and academic studies—the returns for this fund on Wednesday and Thursday are 2.73% and -1.42 % respectively. However, inside the fund, the portfolio manager would see the actual returns earned by the portfolio, which are

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[6] The accrual of fees gives rise to other little-known concerns, in that it is not uncommon to have the netting on Mondays reflect three days of fees or to have substantial "true-ups" near the end of months or quarters, reflecting the differential between previously accrued and actual expenses.

0.91% and 0.36% respectively. The accounting numbers and the economic returns are completely different.

Second, when NAVs fail to reflect economic value and there is any shareholder activity, there will be inadvertent transfers among fund shareholders. For an open-end fund, the NAV is the price at which the fund will buy and sell shares from shareholders. In our example, suppose that at 4 p.m. on Wednesday, shareholders submitted requests to redeem 50% of the shares in Fund XYZ. They would receive cash equal to 50% of $1130 (the NAV) or $565.00. However, in reality, the fund only had $1110 of securities, so the redeeming shareholders end up taking 50.9% of the assets of the fund, leaving non-redeeming shareholders with 49.1% of the original amount or $535. In our example, the passive ongoing investors give $15 to the redeeming shareholders, a reduction in value of 2.72%. As a result of the accounting system, in this instance, redeeming shareholders will dilute the holdings of non-redeeming shareholders.

Our hypothetical example is indicative of what can, and occasionally does, go wrong in the real world. We have been fortunate to obtain a limited amount of data for a sample of equity funds that allow us to calculate the economic NAVs and see the difference between them and reported NAVs, and the respective returns. In one instance, a fund executed a substantial number of trades on stocks that individually had intraday price movements as great as 12%. The reported NAV in this instance was $1.11 above the economic NAV for that day and $0.08 below the next. Reported and actual returns for the two days in question were also remarkably different. The reported return on the day of purchase was -0.73% versus an actual return of -4.74%, while the reported return on the next day was -1.51% versus an actual return of 2.90%. The fund experienced significant inflows on these days, as it came out of incubation, growing from $65mn to $92mn.

In another real example, reported returns for a certain day were 3.66% while actual returns were 5.31%. On this day, the fund bought heavily (17% of fund AUM and tens of millions of dollars) into a market that continued to rise throughout the day, on a day where the S&P rose by more than 5%. In this case, the fund had just received an inflow that increased fund AUM by 50%.

While these real examples are not representative, they show the potentially pernicious effects of current accounting rules. Investors who bought or sold shares on those two days transacted at prices that were different from the portfolios they bought, and anyone who made a decision looking at the short-run returns could have been misled.

Surely, funds indeed buy and sell securities at prices other than the close, and fund shareholders routinely buy and redeem fund shares. However, neither these facts, nor a hypothetical, pathological example nor data from an actual single fund-day provide compelling evidence that T+1 accounting is a problem. In the remainder of the paper, we provide comparative statics on the potential distortions, provide evidence on their likelihood and size from simulations, and utilize a sample of daily fund data to demonstrate the frequency and severity of the problem of materially misstated NAVs and returns arising from stale portfolio information. In brief, we find that it is not rare for NAVs and returns to be slightly "off" due to T+1 Accounting, and we discuss the implications of this finding.

Section 2: Preconditions for Distortions

The distortions due to T+1 accounting are a function of the trading activity of a firm and the difference between the closing price of a security and the price at which it was bought and sold during the day. Ignoring accrued expenses, the difference between the accounting NAV and the economic NAV can be expressed as

(1) $$\Delta NAV = \sum_{i \in t-1} q_{i,t-1} p_{i,t} - \sum_{i \in t} q_{i,t} p_{i,t} = \sum_{i \in t} (q_{i,t-1} - q_{i,t}) p_{i,t}$$

where q represents the portfolio holdings and p their prices as of 4 p.m. today. The first term after the first equals sign represents the reported NAV, which is calculated using yesterday's portfolio holdings, and the second term represents the economic NAV, which is calculated using today's portfolio holdings. Alternatively, the distortion is the difference in the portfolio weights times the closing prices.

If the fund neither buys nor sells securities, the changes in the portfolio weights is zero and hence ΔNAV = 0. However, if it buys or sells securities, there are typically at least two trades: a liquidation of one security and a purchase of another, where cash might be one of the two. The first precondition to this distortion being large is the presence of meaningful portfolio turnover, i.e., the purchase and sale of securities. As an empirical matter, portfolio turnover is indeed meaningful for mutual funds. Table 2 reports fund turnover data from Morningstar. The median turnover for domestic equity funds is 194% per annum. The 75th percentile turnover measure is 371% per annum. It is important to remember that turnover measures the minimum of the fund's buys and sells, so it is at most half of the amount of trading activity. Furthermore, a fund that was growing (or shrinking) would have positive net buys (or sells) not accounted for in the turnover measure which is the *minimum* of the buys and sells.

Apart from the amount of trading, there is a second condition under which the distortion could be large, relating to the size of intraday price movements relative to closing prices. One can rewrite (1) by separating the assets which are bought and sold within a day.

(2) $$\Delta NAV = \sum_{i \in sold} q_{i,t-1}\left(p_{i,sold} - p_{i,t}\right) + \sum_{i \in bought} q_{i,t-1}\left(p_{i,t} - p_{i,bought}\right)$$

The first term represents the distortions due to assets sold (for cash) and the second term represents distortions due to assets purchased. For this analysis any purchases or sales of "cash" generate no distortions as we ignore interest.

- If the fund sells an asset it had on its books for cash, the NAV will represent the value *as if it had not been sold* today whereas the economic value would reflect the value of cash realized by the sale. ΔNAV will represent the difference between the selling price and the sold security's closing price, multiplied by the number of shares sold. This intraday opportunity cost of having sold the asset is captured by the first term.
- Suppose that a firm had cash on its books yesterday and invests the cash today in a new asset. Ignoring interest, the reported NAV will reflect the value of the cash on the books the prior day (represented by the amount it paid, or $q_{i,t-1}p_{i,bought}$) and the economic value is merely its closing market value. The intraday appreciation or depreciation of the newly purchased assets is captured by the second term.
- Finally, suppose that the fund sells an equal value of one asset and buys another. The NAV will be calculated as if the asset exchange had not occurred and ΔNAV

will reflect the intraday difference in the appreciation/depreciation of the two assets since the exchange.

Thus, the size of the distortion is a function of the extent of trading and intraday price movements between trade time and market close. If a fund were to buy and sell securities only at their closing price each day—or if market prices were flat during the day—these distortions would be eliminated.

While we do not have information on when funds trade, prior research establishes that a substantial amount of trading takes place at times other than the market close. Trading volumes tend to follow a U-shaped pattern, with substantial trading taking place both early and late in the day, with the morning peak generally being higher than the afternoon peak (Jain and Joh 1988, Foster and Viswanathan 1993). Furthermore, investment managers set up costly trading desks to execute their transactions, presumably to do better than merely buying or selling at the market closing price.

These trading desks face substantial intraday volatility in prices. Intraday price movements can be quite large, as suggested in Table 3. Because NAV distortions are due to deviations between intraday prices and market close, we examine high-close and low-close deviations on a daily basis as compared with the daily return. We examine the distributions of these quantities over a ten year period (1995 through October 2005) using the Standard and Poors 500 Index. Since indices are portfolios of securities, their variance is substantially less than for individual stocks, so this provides a somewhat conservative picture of the intraday volatility. During this period, the average (arithmetic) daily return of the index was -3 bp. This return simply represents the percentage change from close to close. In absolute value, the average return is 81 bp per day. This is the same order of magnitude as the average absolute difference between the day's high and the close (65 bp) or the day's low and the close (74 bp.) All three have similar standard deviations (74-77 bp). The amount of intraday variation in prices is as substantial as is the daily return. One feature of intraday volatility which we do not model, but which could affect our results, is that realized volatilities for individual stocks move over time, have long memories, and may be correlated to one another, which could affect the time patterns of the distortions we study (Andersen et al. 2001).

To the extent that stale portfolio reporting affects NAVs, it will naturally affect reported returns as well. Ignoring distributions, reported returns are merely:

$$Ret_{i,t,m} = \frac{NAV_{i,t,m} - NAV_{i,t-1,m}}{NAV_{i,t-1,m}} - 1 \tag{3}$$

where *i* references the fund, *t* the date and *m* the method of calculating the NAV (reported or economic). From the formula, we can see that a distortion in one day's NAV will affect two days' returns, which are capturing changes in NAVs.

Importantly, the return problem is independent of the level of aggregation of returns (daily, monthly, etc.) because a deviation in the daily return on the final day of a period distorts the reported return for the entire period.[7]

Section 3: Simulating the Distortions

Whether NAVs and returns are distorted, and by how much, is an empirical question. To gauge these distortions, we use two approaches. First, we simulate the likelihood and size of the distortions as a function of various parameters. Second, we obtain access to daily trading records for 26 domestic equity funds for a multi-year period, and we calculate the distortions directly. In this section, we discuss the simulation results.

3.A. Simulation overview

To simulate the extent of the potential distortions, we must model the daily trading activity of a fund and intraday returns. We model intraday returns in a simple fashion, following extant literature. Modeling portfolio manager and trading desk activities is a much more novel undertaking. In simplest terms, one needs to capture the circumstances under which a trading desk submits a buy or sell order for a security. Trades could be motivated by a portfolio manager's desire to rebalance her portfolio or to

[7] The return distortions that we report in this paper are all daily returns. However, we have checked that the distortions survive in more aggregated returns: in a table available on request, we re-run several of our analyses using monthly returns, and we find that distortions occur with frequency roughly equal to the frequency with which they occur in daily returns, and that when distortions occur, their magnitudes in monthly returns are about the same size as, or slightly larger than, their magnitudes in daily returns.

respond to fund shareholder purchases and redemptions.[8] Below, we describe our simulation design, including the key parameters and major results. Appendix B (available upon request) provides detailed descriptions of the methodologies and calculations that we outline below.

Figure 1 summarizes the key inputs and outputs of our simulation. The "exogenous" variables are shareholder flows and intraday security prices.[9] Trading decisions are the outcome of certain "policies" which determine the amount and timing of security sales.

Our hypothetical fund learns the prior day's shareholder sales and redemptions, or net shareholder flow, before the market opens. We assume that the manager can only permit her cash balance to fluctuate within some band, and "corrective" trades take place when the fund would exceed these limits. In addition, "fundamental" orders reflect the manager trading to reflect her anticipation of future returns, and we model them in the form of a hazard rate for each security drawn from a mean zero normal distribution.

Trade orders flow to a trading desk where they are executed. We model the trade order as being executed at specific times during the day, as being executed at an average price, as being executed at a "best" or "worst" price, or as being executed at a random time drawn from the empirical distribution of Jain and Joh (1988). We track the various outcomes, in particular the new portfolio, the accounting and economic NAVs (rounded to the nearest $0.01), and the comparable returns at the end of each day. Our model simulates 1,000 days of trading at a hypothetical mutual fund. We key in on the extent to which the accounting NAV is "materially" different from the economic NAV. We understand that in the fund industry if a NAV is misstated by $0.01 as a result of an error,

[8] To our knowledge, ours is the only simulation in the literature that follows a hypothetical mutual fund through the sequential details of an entire trading day, though a number of authors use simulations in their study of mutual funds. For example, Kothari and Warner (2001) generate returns of hypothetical mutual funds by assembling random groups of stocks from the CRSP database; Coles, Daniel, and Nordari (2004) mimic mutual fund returns by generating data with parameters estimated from actual fund returns; Johnson (2004) estimates several costs associated with liquidity provision in order to study funds' transaction costs; and Jones (1980) estimates equity demand equations for mutual funds, as well as for households, insurers, and governments, and then runs simulations to gauge the impact of inflation shocks on equity prices.

[9] Over long periods, shareholder flows are related to performance, which in turn is related to trading activity, but this relationship is considerably stronger for shorter horizons; see Sirri and Tufano (1998). We assume that any single day's flows can be considered as unrelated to the prior day's performance. Similarly, there is evidence that mutual fund trading can affect security returns (Coval and Stafford (2005) among others), but we assume away any price impact factors for simplicity.

best practice is to evaluate whether the fund lost money, and if so, to make fund shareholders whole. We have defined a NAV difference greater than one penny as material, but report other thresholds as well. With respect to returns, we report distortions at various levels, as we are not aware of any standard for materiality.

3.B. Simulation Parameters

Our model has several important input parameters, described below. We describe each parameter and explain our thinking behind the base levels we use for each. Because this model has some highly stylized parameters, we test a range of parameter values.

Intraday stock returns. We model intraday returns on the securities that the fund owns or might buy as discrete-time mean-zero lognormal processes, and we leave the standard deviation of this process as an input parameter. We make the simplifying assumption that returns for each stock are i.i.d. and that returns are uncorrelated across stocks, in essence, stripping out market co-movements.[10] We then use typical values for the annualized standard deviation of returns on stocks to calibrate the volatility of individual stocks. For example, Campbell, Lettau, Malkiel, and Xu (2001) study firm-level volatility for daily U.S. stock returns, and they find that annualized standard deviations of daily returns are roughly 45% or 50% per year. Our simulations use values in the range 10% to 50%, with our base case being 30%.

Fund Flows. Each trading day, we model fund flows, i.e., the net of shareholder purchases and redemptions, as drawn from a mean-zero normal distribution. We think of a fund flow realization as a single shock that arrives in the morning before markets open, and that is an aggregate of the net fund flow that occurred during the prior trading day [11] The standard deviation of the fund flow distribution is an input parameter, and for most simulations we set this value to 1% of fund assets. A standard deviation of daily fund flow equal to 1% of fund assets corresponds to an annual standard deviation of roughly 16% of fund assets. This annual value seems consistent with the empirical observations

[10] In a table available on request, we incorporate a market factor and assign CAPM betas to stocks that are held by the fund to test whether betas are misstated.

[11] For a careful discussion of fund flow and the lag between the time at which an investor places an order and the time at which a fund manager receives information about the order, see the appendix of Edelen and Warner (2001).

of Boudoukh, Richardson, Stenton, and Whitelaw (2004, Table 1) when extreme positive flows (their rightwardly skewed 95% tail) are ignored. Our use of a mean-zero distribution is consistent with the observation by Goetzmann, Massa, and Rouwenhorst (1999) that the average daily net fund flows for the funds in their broad sample is 0.018% of fund assets.

Fund Portfolio. The more stocks a fund holds and trades, the more likely that trading at prices other than the close closing prices might cancel out. Most of our simulations assume that the fund owns fifty or one hundred stocks. These values are in line with averages for a broad sample of funds reported by Wermers (1999, Table 1), who finds that the average number of stocks per mutual fund ranges from about fifty stocks per fund in the 1970s to about ninety stocks per fund in the 1990s.

Cash holding rule. Funds typically hold some cash in their portfolio either as a portfolio decision, but more likely, as a buffer for sales and redemptions. The level of cash held by a fund is often determined as a matter of policy. For most simulations, we assume that a fund manager will be forced to buy securities if cash exceeds 15% of assets. In every case we set the lower bound of cash holdings to 0% of assets, i.e., the fund cannot borrow. Trades that correct cash imbalances reset funds' balances to 7.5% of assets. These parameters are consistent with the mutual fund cash balances that researchers have observed empirically. For example, Falkenstein (1996, Table 1) finds that in 1991 and 1992 the average cash holdings for his sample of equity mutual funds was just less than 9% of assets.

Fundamental Trading Policies. Next we have two parameters that together determine the fund's turnover. These parameters condense the complicated decisions about trading down to two random variables. One parameter sets the daily probability that the fund will elect to trade a given stock. Then, conditional on a trade occurring, we draw the size of the trade from a mean-zero normal distribution, where the second parameter sets the standard deviation of this distribution. We allow these two parameters to vary, but most of our simulations use a 5% daily hazard of trading each stock and a standard deviation of trade size equal to 20% of the fund's position in the stock. Together these values give an average annual turnover for the fund of about 90%, close to Morningstar's reported mean of 94%, but lower than Morningstar's reported median of

194% (Panel B of Table 2),[12] and higher than Wermer's (2000, Table 6) average of 59% from his study of U.S. equity mutual funds (which includes passively-managed index funds), and higher than the asset-weighted average turnover rate reported by ICI[13] for the last twenty years (range: 50% to 82%).

Trade Execution. We assume that the portfolio manager determines how much of a certain security to buy or sell in a day, but a trading desk is charged with executing the trade timing.[14] Trade execution is measured by the time of day at which the fund exercises its trades. We test strategies which trade at various points in the day (e.g., 10 a.m), at the open, at the close, at the average price for the day, at the "best" price for the day, at the "worst" price of the day, and drawn from the empirical distribution of trading activity characterized by of Jain and Joh (1988). Most frequently we model the fund as executing trades at 10 a.m. This assumption is consistent with the well known pattern that trading on major exchanges has peaks in the morning and afternoon (Jain and Joh 1988, Foster and Viswanathan 1993), with the morning peak generally being higher than the afternoon peak.

3.C. Simulation Results

To measure the possible range of distortions, we have examined a large number of potential parameter combinations, and tables showing these results are available upon request. In the interest of efficiency, we present the relevant comparative statics for NAV distortions in Figures 2 and 3 and for return distortions in Figures 4 and 5. Figure 6 plots "tornado diagrams" to show the relative effect of changes in the various variables. Table B1 in Appendix B provides the parameter values for each of the Tables. Our simulation results are best understood through equation (2) above. The quantity of shares traded and the difference between the transaction and closing prices drive distortions in NAVs and returns. Distortions are greater when more securities are traded, and when they are traded at prices other than the close.

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[12] Actual turnover reflects fundamental trading as well as trading relating to shareholder purchase and redemption activity. Our base parameter are set such that total fund turnover will be within reasonable range of the Morningstar mean and median.

[13] See the 2005 Investment Company Factbook, Section II, http://www.ici.org/factbook/05_fb_sec2.html (visited 23 Feb 2006).

[14] We acknowledge, but our model does not consider, that the portfolio manager and trading desk may jointly determine trade execution and that orders may be spread over multiple days.

Frequency of NAV Distortions. As a starting point, using our base parameters, as discussed above, the probability that a unrounded NAV will be off by a half-penny or more, so that reported NAV is off by a penny or more, is 8%.[15] Conditional on realizing a half-penny distortion in unrounded NAV, the average distortion is about a penny. Returns are also misstated when funds use T+1 accounting. Under our base parameters, there is a 61% likelihood that daily reported returns and economic returns will differ by at least 0.5 basis points; conditional on experiencing at least a 0.5 bps distortion, the average distortion is 1-3 bps.[16] More extreme, under our base calibration, there is a 0.5% chance that the daily return is distorted by 10 bps. (Recall that, in our model, there is no drift of security prices nor are we modeling any alpha generation, so the expected fund return is zero.). Of course, these values are very sensitive to changes in the parameter values, and this section discusses the relationships between the parameter values and distortions.

Greater volatility means a higher likelihood of substantial drift between transaction and close, and earlier trading leaves more time for intraday prices to drift. Panels A and B of Figure 2 show the likelihoods of realizing a rounded one penny (or unrounded half penny) deviation in NAV due to these two factors. In each figure, the base parameter case is identified by a black dot. As our intuition suggests, the likelihood of realizing economically important values of ΔNAV rises monotonically with the volatility of securities held by the funds (Fig 2, Panel A) and falls monotonically with the time of day at which trades occur, reaching zero if the fund trades at market close. (Fig 2, Panel B). In the base parameter case, a fund holding 50 stocks employing a "mid-morning" (10 a.m.) execution for stocks with annualized volatilities of 30% has an 8% likelihood of reporting a NAV which is a penny or more different from the economic NAV. This probability rises to 16% if the fund is trading stocks with volatilities of 50%. Funds that trade at the average prices during the day have only a 3% chance of experiencing a penny distortion (Fig 2, Panel B). Funds with either excellent traders (i.e.,

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[15] Given rounding practice, all $0.005 unrounded differences would translate into a different rounded NAV; but even smaller unrounded amounts could change the reported NAV, say if the original unrounded NAV were $20.0045.

[16] Reporting average values of the magnitudes of distortions masks any skewness that might be present in the distributions. In tables available on request, we find little evidence of skewness, either positive or negative, except for "best" and "worst" trading strategies as we define them.

enjoying the best execution for the day) or terrible traders (i.e., who manage to trade at the worst price of the day) have more likely distortions. For our hypothetical fund of 50 stocks, each with annual return standard deviation of 30%, Panel B of Figure 2 shows that these super-traders would have their NAVs distorted 11% of the time. This probability rises to 20% if the super-traders are trading stocks with annualized return standard deviations of 50%.[17] Perversely, as funds improve trading execution, the NAV distortion problem is exacerbated.

As discussed earlier, NAV distortions are smaller for funds with larger portfolios; Panel A of Figure 2 plots distortion probabilities for a fund of one hundred stocks (dashed line) and a fund of fifty stocks (solid line). Consistently, the fund with fewer stocks realizes NAV distortions more frequently. These observations explain perhaps why we would expect the NAV distortion problem to be far more pronounced among actively-managed funds than index funds, as the latter would tend to hold more securities and to trade to produce a NAV that reflects the closing value of the index.

NAV distortions reflect trading intensity. We model "fundamental" trading as a hazard rate that a stock would be traded on any day, and a distribution of trade size, conditional on there being some trading. Panel C of Figure 2 shows the simulation results for these parameters. The solid line traces probabilities of NAV distortions for a fund that trades each stock with a daily hazard of 5%, while the dashed line traces probabilities of NAV distortions for a fund that trades each stock with a daily hazard of 2% NAV distortions increase both with the hazard rate of trading securities (the solid line is everywhere above the dashed line) as well as with the size of trades. While the results are sensitive to the starting conditions, trade size seems to have more influence than the likelihood of trades. For example, our 8% likelihood of a rounded penny distortion only falls to 3% when the likelihood of trading falls from 5% to 2%, but falls to 1% when trade size is halved to 10%.

Trading induced by shareholder sales and redemptions that exceed policy levels of cash holdings increases NAV distortions, as shown in Figure 2, Panel D. More extreme flows with tighter cash constraints drive trading and increase distortions.

[17] This result is not plotted in Figure 2. We report detailed results for many different combinations of input parameters in a series of tables available on request.

However, for the wide range of parameter values that we used, this factor had less of an impact on the likelihood of distortions, when compared with the impact of either the standard deviations of intraday volatility or of the size of fundamental trades. Panel D of Figure 1 plots the daily probability of realizing a half-cent deviation in NAV for a range of upper bounds on cash holdings for two funds, one that has a standard deviation of daily fund flow equal to 2% of assets (solid line) and one that has a standard deviation of daily fund flow equal to 1% of assets (dashed line). Tighter cash holding policy increases distortions, and more so for funds facing larger flows (the solid line).

Magnitude of NAV Distortions. So far we have discussed the probability of realizing a half-cent *or more* deviation in NAV, and we have mentioned that the *average* size of these deviations, when they occur, is roughly a penny. Figure 3 takes the analysis one step further and displays the *distributions* of these deviations, conditional on the realized deviation being greater than a half penny. In Figure 3, each panel uses dark shading to denote realizations of NAV deviations that are less than the median realization; medium shading denotes realizations that are greater than the median realization but less than the distribution's 95% critical value; and the light shading denotes realizations that are greater than the 95% critical value but less than the 99% critical value. In each panel, the base parameter case is outlined with a black border.

Panel A of Figure 3 shows that probability of large distortions rises as the standard deviation of the fund's securities rises. When the fund holds stocks with annualized standard deviations of 50%, the 5% tail of the NAV deviation distribution is above $0.02, indicating that there is more than a 5% chance of seeing an unrounded NAV deviation of 2¢ or more, conditional on the deviation being more than a half cent. Using the 16% probability that the NAV deviation will be greater than a half cent (Fig 2 Panel A), the unconditional probability of realizing an unrounded 2¢ deviation in NAV is about 1%.

Panel B of Figure 3 shows that the probability of large distortions falls as trades occur closer to market close. It also shows that the super-traders who always trade at the best price for the day have roughly a 5% chance of realizing a rounded 3¢ deviation in NAV, conditional on the deviation being greater than a half cent.

Panels C, D, E, and F of Figure 3 report distributions for, respectively, trade size, probability of trading, policy bounds on cash holdings, and fund flows. These variables have smaller influences on the distribution of NAV deviations.

Likelihood and Magnitude of Return Distortions. In Figures 4 and 5 we perform similar analyses for deviations in returns. Return deviations display many of the same patterns as NAV deviations. However, there are two features of returns that make the incidence of return deviations subtly different from the incidence of NAV deviations. First, NAV is an absolute measure, so that deviations of a given size are more likely to occur for funds with higher NAVs than with lower NAVs, simply because the deviation is smaller relative to a larger NAV. In contrast, returns are relative measures, so they do not exhibit this property. Second, returns are measured from one day to the next, so they are calculated using two successive NAVs. Return distortions will be realized only on days for which successive NAVs are distorted by different relative amounts. There would be no distortion in the return between two successive days on which NAVs were distorted by the same relative amount.

In spite of these differences, return deviations generally follow the same patterns as NAV deviations. Figure 4 Panel A shows that the hazard rate of realizing a half-basis-point deviation in return rises as the volatility of the fund's securities rises. When the fund's securities have annual standard deviations of 30% (base parameter case, identified by a black dot), the fund realizes half-basis-point deviations in daily returns about 61% of the time; this hazard rate rises to 71% when the securities' annualized standard deviations rise to 50%. The probabilities that we report in Figure 4 are for daily returns, but they are robust to changes in horizon: in tables available on request, we compute deviations in monthly returns, and we find that half-basis-point deviations occur about 60% of the time in monthly returns as well.

Panel B shows that return deviations occur with lower probability the later in the day the firm trades. As in Figure 2, deviations are more likely to occur when traders are extremely good (best-price traders) or extremely bad (worst-price traders) than they are when the trading desk trades at an averaged price, whether the average is a simple average or an average that is weighted according to the Jain-Joh (1988) empirical distribution. Panel C shows that the probability of realizing a deviation rises with trading volume, whether the increase in volume is related to the hazard rate of trading (sold vs. dashed line) or the size of the trade conditional on the trade occurring (horizontal axis). Finally, Panel D shows that tighter cash bounds and higher fund flows increase the probability of realizing a half-basis-point deviation in returns.

Figure 5 displays distributions of deviations in returns, conditional on the deviation being greater than a half basis point. Patterns here are similar to those in Table 3. Larger return distortions occur as the standard deviation of underlying securities increases (Panel A), as trades occur earlier in the day (Panel B), and as the size of trades increases (Panels C and D). Policy bounds on cash holdings (Panel E) and fund flow (Panel F) have little effect on the conditional distribution of deviations in returns.

Summary of Simulation Results. Figure 6 displays "tornado" diagrams for seven important input parameters. In both panels, a vertical black line marks the daily probability of realizing a half-cent deviation in NAV (Panel A) or a half-basis-point deviation in return (Panel B) for the base case described in detail in section 3.B. above. Horizontal gray bars denote the range of deviation probabilities that obtain as each parameter moves through extreme high and low values. In both panels, variables that influence the quantity of trading (probability of trade and trade size), the volatility of the fund's securities (return standard deviation), and the time between a trade and market close (trade strategy) have the most dramatic effect on the hazard rates of realizing deviations in NAV and returns.

Finally, in tables not reported here but available on request, we check the extent to which estimates of T+1 accounting affects fund betas, return standard deviations, and Sharpe ratios in both daily and monthly data. These deviations are extremely small—deviations in beta, e.g., have orders of magnitude of 0.001—and consequently we do not expect T+1 accounting to impact funds' risk measurements materially.

In sum, our simulation exercise demonstrates that NAV distortions can occur, and that in certain circumstances they can occur frequently and be economically meaningful. Furthermore, daily returns are distorted as well. Distortions are most likely to occur and are most likely to be large when a fund's securities are highly volatile, when the fund trades early in the day, and when the fund makes many and sizeable trades, whether for its own strategic reasons or in order to manage purchases and redemptions. Under our simulation, when distortions occur, they *average* a penny or two for NAV distortions, or a few basis points for return distortions.

Section 4: A Direct Empirical Investigation of the Distortions

Our simulation results are driven by our educated guesses about parameters in our model. More direct tests would involve calculating the distortions using daily fund data, in particular, daily trading data. We were fortunate to obtain data from a number of equity funds for a multi-year period. In particular, we have data for a selection of domestic equity funds, including a range of market caps and strategies. For each fund, we have gross (and net) shareholder flows, full daily portfolio information, and a record of all portfolio trading activity (including some time-of-trade information). To streamline our empirical analysis, we limit our sample to the A-class shares of these domestic equity funds on the basis that the factors that determine distortions to a fund's NAV do not vary by class. Due to the sensitive nature of this data, we have agreed not to reveal the names of the funds, the complexes involved or the exact dates we studied, although we were granted access to all of this information. These funds were not selected in a systematic way in order to prejudice our results, for we believe that until we did these calculations, they had not been done by the funds involved. Any distortions in NAVs or returns we calculate are the result of the funds fully complying with Rule 2a-4, not failing to comply with all stated rules and regulations.

We believe that these funds are representative of many equity funds. Their stated annual turnover varies from a low of 24% to a high of 169%, with a median of 92%, consistent with the Morningstar figures in Table 2 and consistent with our base level turnover parameter in the simulations. The funds have smaller NAVs and larger portfolios than those we simulated, so we would predict that they would have fewer distortions than our base case simulation. Table 4 provides descriptive information for the A-class shares in our sample, including the market cap and strategy of each fund, as well as the mean daily level of the following variables during the time period studied: net assets under management, shares outstanding, NAV, shareholder flows, dollar trading volume, and number of securities held.

To calculate economic NAVs, we follow the methodology outlined in Section 3. Specifically, we first calculate how much fund assets would have changed had T accounting been in place, as defined in Equation (2). We then recalculate the economic NAV by adjusting the fund's net assets on day T by this amount. In addition, we recalculate the daily fund return as defined in Equation (3). Because public sources that report fund performance rely on published penny-rounded NAVs, we recalculate daily returns using penny-rounded economic NAVs.[18] We subsequently compare these returns to comparably-calculated daily returns based on accounting NAVs.

Panel A of Table 5 reports the incidence of differences in the published (i.e., penny-rounded) accounting and economic NAVs, or the incidence of shareholder fund transactions that would have taken place at a different price had the economic NAV been used in place of the accounting NAV. For about 8.5% of fund-day observations, the economic NAV would have differed by a penny or more from the accounting NAV. Panel B of Table 5 reports the incidence of differences in daily returns using published accounting versus economic NAVs. For approximately 14.5% of fund-day observations, the daily returns would have differed by a basis point or more. Note that this number is roughly double the percent of days on which the accounting and economic NAVs differed because a distorted NAV today will cause both today and tomorrow's daily return to be different.

Tables 6 through 8 show the extent of distortions both in NAVs and daily returns along various observable dimensions, including fund and market characteristics. Table 6 shows the distortion on a by-fund basis, revealing that the 8.5% average reported in Table 5 masks considerable variation by fund. For nine funds of our sample of 26, the published accounting and economic NAVs differ by more than a penny on at least 10 percent of days. For one fund, the figure is nearly 1 day out of 5. While our simulation results test whether returns are off by 0.5 basis points, with actual data we see that the likelihood that they are off by twenty-times that level, i.e., 10 bps on a single day, occurs over 5% of the time for five funds out of the 26. For two funds, 10 bp distortions occur

[18] A different penny-rounded economic NAV can result from small adjustments to the unrounded accounting NAV.

12.9 and 26.6% of the days respectively. These averages are taken over a period of five years, and do not represent a short-run phenomenon.

Table 7 disaggregates the data by time consecutive quarters. (At the request of the funds, we do not report the years represented.) In the early part of the sample (over two years long), the published accounting and economic NAVs differ by more than a penny on at least 10 percent of days, while by the end of the sample the frequency is only slightly more than three percent. Throughout, the incidence of 10 bp daily return distortions were more stable over time, occurring in 2 to 4% of all days throughout the five year period.

Table 8 examines the extent to which economic and accounting NAVs differ by the value of a fund's NAV. As expected, funds with larger NAVs more frequently show penny or more distortions in NAVs. For fund NAVs greater than $40, the frequency is one day out of five. The reverse pattern holds for return distortions. Funds with smaller NAVs are more likely to experience more days with returns misstated by over 10 bps, with funds with NAVs under $10 experiencing return distortions on about 7-8% of days.

Tables 5 through 8 also report the size of the distortions, conditional on some distortion being observed. While NAVs and returns may not be distorted "on average," when they are, these distortions can be quite large. Maximum fund-level distortions in NAV (Table 6) range from $0.01 to $1.10. Maximum return distortions (Table 6) range from 8 bps to 443 bps on a single day. Surely, the extreme values represent pathologies, but nonetheless they do occur.

We acknowledge that our actual data is limited to 26 funds over five years, whereas our simulation results are more generalizable—if we can have confidence in the model and our parameterization of it. To test the former, we ran simulations in which we matched, as closely as possible, various moments of the twenty-six funds reported in Tables 4 and 6. Table 9, columns 2-12, report empirical and simulated moments, and columns 13-16 report the simulated and actual frequency of NAV and return distortions.. Our model produces distortions of the same order of magnitude as the actual data, but consistently, the empirical frequency of observed NAV and return distortions is equal to or greater than the simulated distortion frequency.[19] (The simulation underestimates NAV distortions in 24 of 26 cases, and return distortions in 23 of 26 cases.) Given actual simulation parameters, our model seems to slightly underestimate the probabilities of distortions. For the average fund in a sample calibrated to our model, we would predict a 4.7% likelihood of having a unrounded half-penny NAV distortion, whereas in the observed data, the likelihood of a rounded penny distortion is 8.9%. Similarly, using our model, we would predict the average fund would have a 1.5% likelihood of a 10 bp return distortion, whereas in the realized data, this likelihood is 3.4%. In part, the NAV differences could be driven by the incidence of distortions less than $0.005 affecting rounded NAVs in the empirical data. Whatever the source of error in our simulation model, it does not overestimate the actual level of distortions considerably.

Tables 10 and 11 analyze the determinants of the relevant distortions. Table 10 shows a set of regressions examining the distortions against several observable fund and market characteristics, including type of fund (i.e. small, medium or large cap), value of the fund's NAV, year, daily return on the S&P 500, and the number of securities held by the fund. Some of the models add non-public information to which we had access, such as portfolio trading intensity and shareholder flow intensity (both measured as a percent of fund-class assets). Panel A regresses the absolute difference in the published accounting and economic NAVs against these factors, while Panel B regresses the signed difference against them. All factors except for shareholder flow intensity appear to be significant in determining the size of the absolute change, while only the year appears to be significant in determining the size of the signed change.

The empirical results in Table 10 are comparable to the comparative statics that we report in our simulation results. Deviations are larger when the market is moving strongly (i.e., realized volatility is higher) as well as when there is more trading activity by the fund. Distortions were far more common in the earlier period than later. Funds with larger NAVs were more likely to experience a penny deviation. The number of

[19] We suspect that much of the difference between the empirical and simulated values is attributable to the simulation's assumption that the fund holds all its securities in the same amounts. If a fund that reports holding many securities in fact invests more capital in some than others, the effective (for our purposes) number of securities it holds will be much lower, and potential distortions much higher than the number of securities in its portfolio would suggest. However, we suspect that most funds follow IRS guidelines that require 50% of fund assets be composed of stocks that each account for less than 5% of fund assets , so there are checks on this factor.

different securities held by the fund has a negative and significant effect in the regression that uses absolute values but not the regression with signed values. As expected, trading intensity covaries positively with NAV distortions, but is only significant in the absolute value regression.

Table 10 also can shed light on which party to a transaction—the party initiating an order or the party accommodating the order—tends to benefit from discrepancies in NAVs and which party tends to be hurt. The two columns on the far right of Table 10b present coefficients for inflow and outflow: inflow coefficients are negative (significant at the 5% level in the case of the model with AR1 correction), and outflow coefficients are positive (significant at the 10% level in the case of the model with AR1 correction). In these regressions, the left-hand-side variable is the difference *Economic NAV – Accounting NAV*, so that a negative regression coefficient is associated with accounting NAV being greater than economic NAV, and a positive regression coefficient is associated with accounting NAV being less than economic NAV. The inflow coefficient is negative, implying that buyers of fund shares tend to buy on days when accounting NAVs are too high; the outflow coefficient is positive, implying that sellers of fund shares tend to sell on days when accounting NAVs are too low. Though small in magnitude and marginally significant, these patterns suggest that the "moving" party, i.e., the party initiating the transaction, tends to bear the cost of distorted NAVs rather than the counterparty, who might be a buy-and-hold investor. In essence, on average given the pattern of shareholder activity, the distortions seem to create an unintended "transaction cost" borne by transacting shareholders.

Table 11 presents an analysis examining the determinants of distortions in daily published returns. Panel A investigates the absolute change in the daily return, while Panel B examines the signed change. In Panel A, a story emerges that is consistent with Table 10: nearly all of the fund and market characteristics appear to be significant in determining the absolute change in the daily returns. Returns are more likely to be distorted when funds have smaller NAVs, when investing in small or medium cap stocks, when markets are moving strongly, and when the fund holds fewer securities and is trading more actively. As evidenced in Panel B, though, these factors are of limited value in significantly determining signed changes in daily returns.

The results in Tables 10 and 11 suggest that while the presence of distortions may be systematic, predicting their sign—and hence being able to take advantage of them with public data—is more complicated. The R^2 values for the various specifications tell the key tale: Using only public information, our simple empirical models can explain almost none of the variation in NAV or return distortions. However, with non-public information, one can begin to explain a meaningful fraction of the variation in distortions. Unlike the opportunism afforded by using stale prices, which are observable, the scope for opportunism here may be more limited, except for those with information on the fund's trading activities.

In sum, our empirical investigation demonstrates that NAV distortions are common and occasionally large in size. Some observable fund characteristics, i.e., its market cap and NAV appear to be significant in determining differences between accounting and economic NAVs, as well as daily returns. The same holds true for the time period in question, daily market return and the intensity of portfolio trading activity.

Section 5. Interpretation and Implications

Our first goal in writing this paper was to alert otherwise-informed mutual fund experts about the use of stale portfolio information in the construction of net asset values. The simulations and empirical evidence tell a consistent story. The use of stale portfolio information, sanctioned under Rule 2a-4, routinely gives rise to small deviations between accounting and economic NAVs and returns. Our second goal is to frame the discussion about what—if anything—is to be done about the distortions we observe. Financial economists rarely applaud situations when prices and returns are not faithfully reported, and regulators and consumers do not normally condone practices in which parties transact at incorrect prices. Yet, changing industry practice is costly. In business and regulation, it is prudent to consider the costs and benefits of changing long-established practice. In this final section, we seek to establish the agenda for evaluating whether and how to change T+1 accounting rules.

To make the discussion concrete, we propose three generic solutions for debate: (1) Do Nothing, (2) Heightened Disclosure, and (3) Mandate T Accounting. Under "Do Nothing," one would leave the current regulatory system and industry practice as it is.

"Heightened Disclosure" would require funds to periodically disclose: (a) which method (T or T+1) they used, (b) whether they adopted any other methods on any particular day; and (c) any material daily distortions to NAVs and returns as a result of the fund's failure to adopt T accounting. "Mandate T Accounting" would force all fund companies to adopt a same-day pricing system, where the NAV calculation was required to use portfolio values as of 4 p.m. in calculating a fund's NAV.[20]

The costs of the status quo—or benefits from a Change to T Accounting. To choose among these alternatives, one needs to evaluate their costs and benefits. We can identify the classes of costs and benefits, which is an important first step. To measure these costs and benefits would require expertise from industry and regulators. To begin, it is useful to enumerate the costs of the policy of "Do Nothing." In effect, the costs of maintaining the status quo are the basis for any change. We identify costs of doing nothing: shareholder dilution, use of incorrect data in decision making, suboptimal modification of fund trading strategies, possibly incorrect inferences about window dressing, potentials for malfeasance, lack of harmonization of fund financial statements and reported NAVs, reduction of confidence in open-end funds, and perverse incentives for trading.

Value Transfers due to Incorrect NAVs. If NAVs are incorrect, then anyone who transacts (whether buying or redeeming shares), as well as all of the non-transacting shareholders who become their unwitting counterparties, endure potential dilution and anti-dilution. These distortions are zero-sum, with some parties gaining at others' expense.

The size of these value transfers is likely to be large. For the year ending December 2005, the Investment Company Institute reports that that were $1,033.3 billion of new gross sales of stock mutual funds, $883.8 billion of gross redemptions, $178.6 billion of exchanges into stock funds, and $192.3 billion of exchanges out of stock funds.[21] All told there are $2,288 billion in transactions. Assuming an average Net Asset

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[20] One would need to establish exception processes for funds that did not know if a particular trade was executed as of 4 p.m.

[21] http://www.ici.org/stats/mf/trends_12_05.html#TopOfPage, visited February 23, 2006.

Value of $20,[22] this represents 114.4 billion fund share transactions. As a thought experiment, suppose that 10% of the time, fund shares were sold at a NAV that was $0.01 off. Some investors would gain, others would lose. Then for 11.4 billion transactions, some $114 million per year would be transferred among shareholders due only to distortions in NAVs.

This calculation is understated because it excludes any distortion due to bond funds. In addition, the extreme values in Tables 6-8 suggest that in some instances, these distortions can be far more frequent than 10% of the time, and far larger than a penny. However our calculation is overstated because some of the buys and sells represent the same transaction, and thus are double-counted. To take a conservative lower-bound estimate of the distortions, we suppose that all buys and sells were paired, and use the maximum of buys and sells ($1,212 billion) as the basis of our calculation. For equity funds, this would translate into a lower bound of $61 million per year in transfers among equity fund shareholders.

We suspect that most, if not all, of these value transfers were inadvertent, rather than the result of some investors intentionally taking advantage of others. As a matter of policy and business ethics, one might be less bothered by inadvertent transfers than by intentional ones. However, from the perspective of the shareholder who lost in the transaction, there may be little difference between these two.

Incorrect Decision Making. Under T+1 accounting, the prices and returns that investors rely upon are noisy at best. We cannot tell if any decision by a fund investor would have been changed, had the correct information been available. However, a thought experiment may enlighten us about the value of having the correct information. Suppose one were to force investment management professionals to only get the T+1 NAVs and returns that public investors receive. How much would they pay for the "correct" information? We suspect that no investment manager would be content to use 2a-4 style, stale portfolio calculations of daily returns to assess performance. Rather, the information technology officer who insisted that "This is the best we can do" might find

[22] On January 29, 2006, we visited the Morningstar website (www.morningstar.com). Using their "premium screener" tool for mutual funds, we found quartile breaks for NAV for funds in the category "All Domestic Stock." For this category of funds, the median NAV was $20.68. We use a $20 NAV in our calculation above as well as in our simulations in Section 3.

himself looking for a new job. We are hard-pressed to understand why a fund investor would feel differently, once informed. While the deviations are mostly small, a glance at the extremes in Tables 12-14 remind us that one day performance numbers may be misstated by 10s to 100s of basis points in pathological circumstances.

Suboptimal Trading Patterns. We know that some funds are keenly aware of the implications of the stale portfolio rules imposed by rule 2a-4. When shareholder flows are extreme and trading significant, then the types of distortions we document can be material. Were trading desks to ignore these distortions, they could end up creating large wealth transfers among shareholders. For example, if faced with large net redemptions and the resultant need to trade securities to generate cash, trading securities at prices other than the close can cause large discrepancies outlined at the beginning of this paper. Therefore, trading desks sometimes adopt strategies of "hitting the NAV," trading at or near the closing prices. This can be accomplished a variety of ways, but most simply by moving trading activity near to or at the close of the market. A cost of "hitting the NAV" is that the trading desk's sole focus is no longer getting the best price. This could lead to suboptimal execution.[23] We do not know the frequency of this practice nor of the costs. To the extent this trading practice exists, movement to T accounting could cause a shift in the distribution of trades, with fewer trades occurring late in the day.

Incorrect Inferences about Window Dressing. Mutual funds have occasionally been criticized for practicing "window dressing," or trading at the end of the quarter to present a more favorable portfolio to consumers (Haugen and Lakonishok 1988, Lakonishok *et al*. 1991, O'Neal 2001, Ng and Wang 2004, among others). Window dressing is a very temporary phenomenon—dealing with returns over a few days—and we worry that research on window dressing could be drawing incorrect inferences about fund behavior based on distorted T+1 accounting results. Studies that compare fund returns at the end of a period to fund returns on other trading days should be on solid ground insofar as funds are consistent in their accounting practices. If this assumption holds, taking differences should wash out NAV and return distortions that are unrelated to excess cosmetic trading. However, the assumption of consistency in NAV reporting

[23] Of course, in the case of index funds, the intent is to meet, not beat, the tracking index, making end-of-day trading a natural execution policy.

might not be valid. For example, we are aware of at least one fund that reports NAV using T+1 accounting, except on two days per year when it prepares financial reports and calculates NAV using T accounting. The day before the end of the year, the portfolio is valued as $P_{t-1}Q_{t-2}$; the end of the year it is P_tQ_t, and the next day it would be $P_{t+1}Q_t$. Trying to draw inferences about returns at year end would be complicated, at best. We do not know how prevalent this practice is.

As a separate but related example, we suspect that T+1 accounting would tend to attenuate "leaning for the tape" effects (Carhart et al. 2002), thus making them harder to detect, since stocks bought on the last day of a period for the purpose of "pumping" the price would not be reflected in NAV until the first day of the following period.

Reconciliation of financial statements and reported NAVs. For financial reporting, funds are not permitted to use T+1 accounting. Rather for quarterly statements, they report portfolio values that reflect portfolio positions and prices as of the reporting date.[24] A change to T accounting for the calculation of Net Asset Values would lead to a consistent harmonization between the NAVs in financial statements and those reported to investors on a daily basis, and reflect what accountants believe to be appropriate accounting for financial reporting.

Potential Malfeasance. We worry that these distortions could lead to potential opportunities for malfeasance. Anyone who knew (or could suspect) whether a fund had bought or sold securities at prices more attractive than the closing price would be in a position to benefit from this information. A fund insider (such as a portfolio manager or someone privy to the trading activities of the fund) could profitably trade on the basis of the distorted NAVs. This type of activity would almost surely be prohibited under law and regulation as a violation of fiduciary duty, and could be thus addressed. Traders who knew about particular trades executed by a fund might also be able to use this information to profitably trade. Furthermore, while it has not been our intent in this paper to determine algorithms that fund outsiders could employ to take advantage of these distortions, perhaps statistical techniques that might accomplish this goal, once people become aware of the opportunity. We cannot understand why it might be wise to

[24] The AICPA Investment Company Audit Guide has a specific prohibition against using T+1 accounting for financial reporting. (See footnote 15 of Chapter 1.)

leave open these opportunities, which could lead to intentional, rather than inadvertent, value transfers.

Confidence in the Fund Industry. The recognition that NAVs and returns are incorrect—and that the industry is unconcerned with correcting the distortions—can only decrease public confidence in the mutual fund industry. While we cannot measure the welfare consequences of this diminished level of confidence, we doubt it can be optimal. In addition, mutual fund shareholders have demonstrated a propensity to vote with their feet: in the wake of the 2003 timing crisis, fund flows moved against funds caught in the investigation and in favor of other funds. Wellman and Houge (2005) estimate that within six months of the scandal, "assets of investigated funds were reduced by almost 13% relative to the non-investigated funds" (p. 134).

Perverse Incentives for Traders. As we note before, it is ironic that as traders become more proficient, they tend to exacerbate the problem that we describe here. Distortions are larger if traders are able to enjoy substantial intraday returns. Therefore anything we do to encourage traders to "do better" will make the distortions larger.

Costs of Changing from the Status Quo. Against this set of costs of the status quo (or benefits of change) must be balanced the costs of change. We can see four costs associated with change: (1) one time costs to change IT systems to provide portfolio composition information by the time the NAV is set; (2) costs of "exception reporting" when T accounting cannot be used; (3) higher levels of NAV restatements; and (4) costs of disclosure.

One-Time Information Technology Costs. Generally, the information on the securities that a fund buys and sells is captured in a portfolio management or trading system. Portfolio managers, risk managers, trading supervisors and others depend on this system to track investment and trading performance. At some point, information on the fund's portfolio holdings is transmitted to a separate pricing system that establishes the fund NAV. Fundamentally, changing to a T accounting system does not require that any new type of information be transferred; rather only that the information hand-off take place quicker. In industry parlance, this type of quick handoff is facilitated by "straight through processing" (or STP), whereby information is entered promptly and passed between various systems to permit real-time monitoring and reporting. For funds—or outsourced service providers—whose systems have made progress toward greater use of STP, the change to T accounting should have relatively small costs. There is generally a trend toward the use of STP in financial services, and this change is consistent with that trend. Less technology-savvy funds may not be set up to make this change without incurring information technology costs.[25] We do not have the information to estimate the size of these costs, but this information could be gathered from industry experts. Were T accounting not mandated, but encouraged through disclosure, firms who produced more timely information might enjoy a competitive advantage to the extent that investors preferred up to date NAVs and returns.

In the extreme, some firms might not be able to produce NAVs quickly enough and might miss the traditional deadlines set by newspapers for publishing daily NAVs. Industry observers have commented that this would be devastating for these funds. However as a practical matter, to save money, fewer newspapers are publishing fund NAVs in print editions, and when they do, they are publishing data on fewer funds.[26] If antiquated newspaper deadlines are the bottleneck, perhaps funds and newspapers might mutually benefit from a coordinated move to publishing NAVs on-line, which would likely push the deadline somewhat later.

Costs of Exceptions. Even in a system where T accounting were mandated, there would surely be instances in which it was not possible to use T accounting for certain securities on certain dates. A fund may not have timely information on whether a transaction was consummated on a foreign market, a large trade or IPO purchase that needed to be allocated among funds might not be allocated in time, or an operational failure might make it unclear whether a certain transaction took place. Funds—and their boards—would need to establish a set of policies to handle these exceptions. One possible rule would be to state that T accounting could be used, and if a fund did not have

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[25] We were warned by some that the processing power (and the corresponding expense of purchasing the appropriate technology) that is required to update accounting systems quickly may be material. For example, a fund that tracks the Russell 2000 might need to incorporate *thousands* of trades on June 30th of each year when the index re-balances. This would likely be costly. However, on the other hand, failure to incorporate these trades into the NAV under a T+1 system could lead to large distortions on June 30th for these funds.

[26] For example, the Chicago Tribune no longer publishes complete stock and fund prices in its weekday editions. See http://www.editorandpublisher.com/eandp/departments/online/article_display.jsp?vnu_content_id=1001842100

confirmation that a trade had taken place by a certain time, it was acceptable to assume that it had not taken place (i.e., revert to T+1 accounting as an exception for certain securities.) In some sense, fund financial reporting already has a set of principles that define a trading receivable or payable, and these financial accounting rules could be invoked. To the extent that financial statements would classify a trade as a receivable or payable, it should be considered as consummated.

Probability of NAV Restatements. Under T+1 accounting, funds have twenty four hours to ensure that the portfolio information incorporated into a NAV calculation is correct. Funds that use T accounting must process all of their transactions in the roughly two hours between the NYSE close at 4 p.m. and the NASDAQ transmission deadline at 5:55 p.m. Given this tight deadline, the probability may be higher that errors in portfolio composition will occur and that reported NAVs will need to be revised at a later date.

Costs of Disclosure. Under our "disclosure" alternative, funds would not be required to change their method of NAV calculation. However, they would be required to periodically disclose: (a) which method (T or T+1) they used, (b) whether they adopted any other methods on any particular day; (c) any material daily distortions to NAVs and returns as a result of the fund's failure to adopt T accounting. The first two of these disclosures would not require substantial financial costs.

Funds not adopting T accounting could be required to report the distortions due to T+1 accounting, either quarterly or annually. We did these calculations to perform this analysis for this paper; the algorithms are fairly straightforward. In essence, they are akin to performance metrics that use post-trade benchmark to assess trading performance. Implementing this type of ex post performance analysis would involve ongoing costs to collect and process this information. However, if this information were only produced periodically these costs could be reduced. The only funds required to make these disclosures would be ones that choose not to incur the expenses to move to T accounting.

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In enumerating these costs and benefits of changing from T+1 accounting, we seek to be impartial. Initial reactions to this paper from consumer advocates and industry executives tend to focus exclusively on a small set of arguments. Our goal is to call these distortions to attention, and frame the debate so that regulators, fund trustees, fund auditors, consumer advocates, and investment management professionals can carefully examine whether shareholders' interests are best served by a rule that was conceived and justified decades ago. We also hope that it spurs additional research into this new topic.

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Figure 1: Schematic of Simulation Model

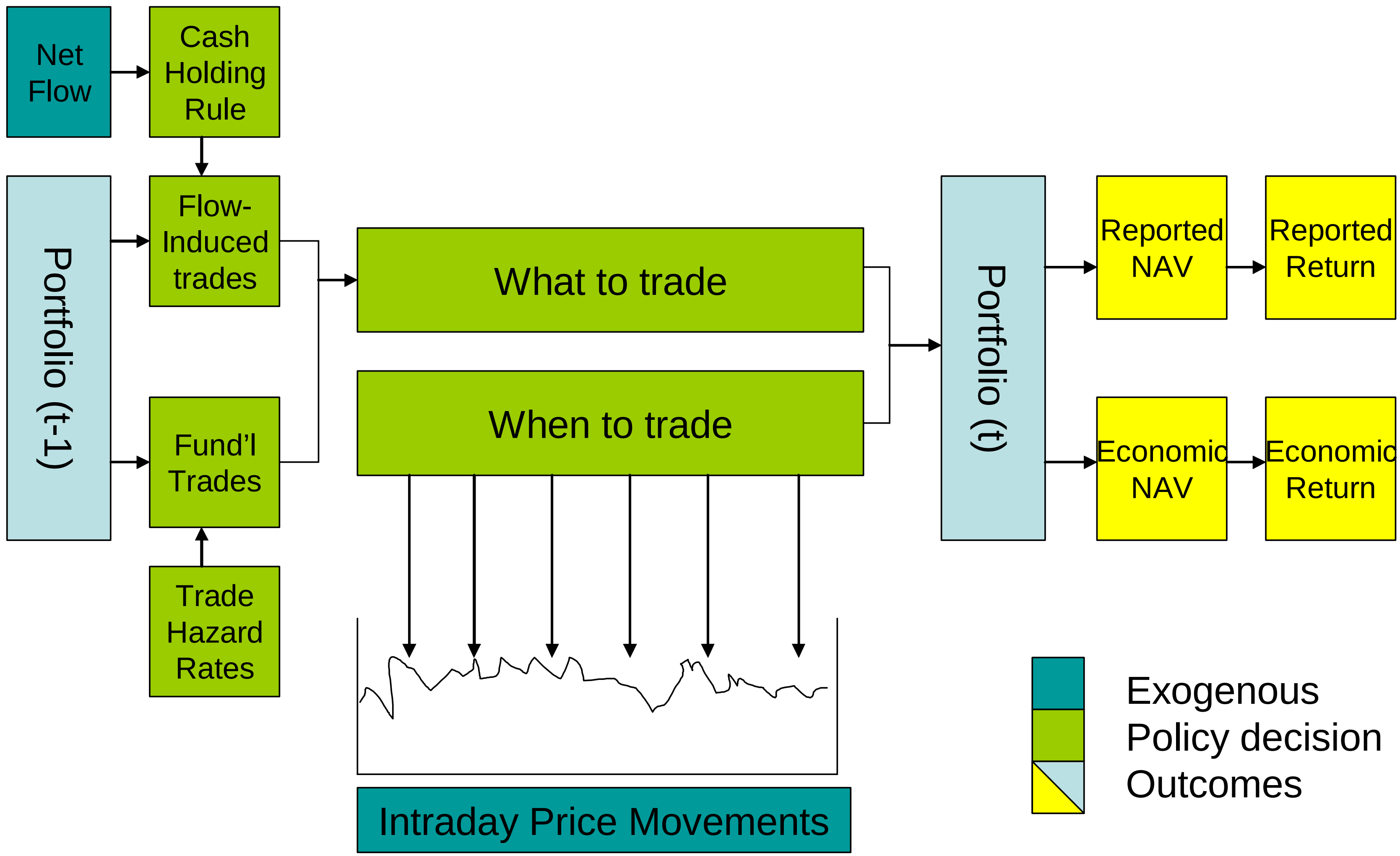


Figure 2. Daily likelihood of realizing unrounded half-penny deviations in NAV. Panels display the daily hazard of a simulated fund realizing a half-cent or more difference between accounting NAV and economic NAV, Pr[|ΔNAV|≥½¢], for various values of important input parameters in a simulation of 1,000 trading days. Parameters are: the annualized standard deviation of returns on securities held by the fund (Panel A); the strategy that the fund uses to execute trade orders (Panel B); the standard deviation of the size of the fund's trades, measured as a fraction of the fund's position in the traded security (Panel C); and the maximum amount of cash, measured as a fraction of total assets, that the fund is permitted to hold (Panel D). Unless otherwise noted, the fund holds fifty stocks, each with an annualized standard deviation of returns of 30%; the fund executes its trades at 10:00am; the daily hazard rate of trading a given stock is 5%; the standard deviation of the size of a trade, conditional on a trade occurring, is 20% of the fund's position in the traded security; and no less than 0% nor more than 15% of the fund's assets may be held in cash. A black dot identifies this base case in each figure. In panel A, the solid line represents a fund that holds fifty stocks, while the dashed line represents a fund that holds one hundred stocks. In Panel B, the trading strategies are always trade at market open ("O"), always trade at 10am, always trade at 11am, always trade at 12pm, always trade at 1pm, always trade at 2pm, always trade at 3pm, always trade at market close ("C"), trade at the average price for the day ("A"), trade at the best price for the day ("B"), trade at the worst price for the day ("W"), and trade at a random time of day according to the empirical distribution of Jain and Joh (1988, "JJ"). In Panel C, the solid line represents a fund that trades each security with daily probability of 5%, while the dashed line represents a fund that trades each security with daily probability of 2%. In Panel D, the solid line represents a fund that has a standard deviation of daily fund flows equal to 2% of assets, while the dashed line represents a fund that has a standard deviation of daily fund flows equal to 1% of assets. In all panels, the vertical axis measures the daily hazard of realizing a half-cent or more difference between accounting NAV and economic NAV, Pr[|ΔNAV|≥½].

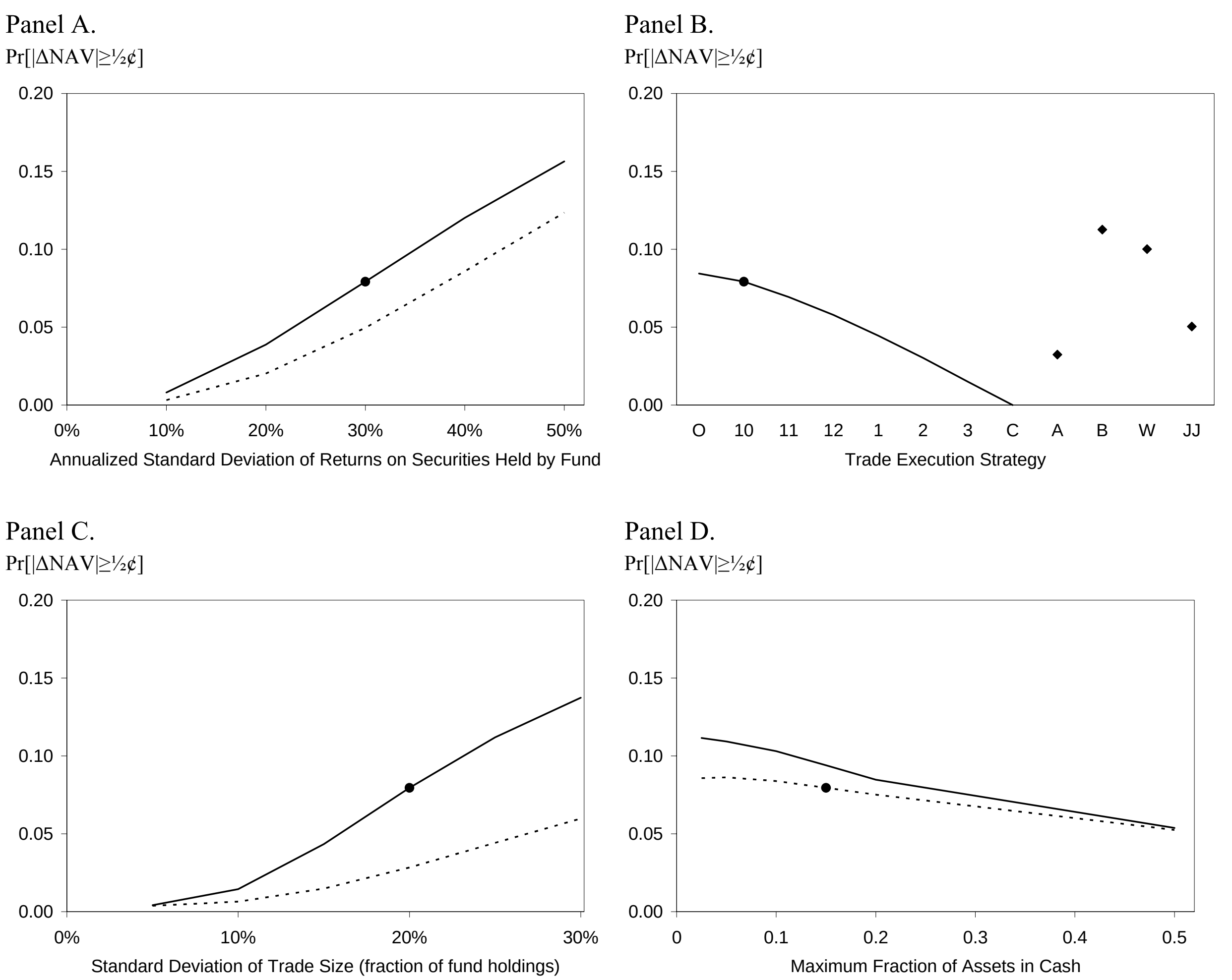


Figure 3. Distributions of deviation in unrounded NAVs when the deviation is greater than ½¢. Simulation and baseline parameters are as described in Figure 2. Each panel fixes all input parameters, except for the parameter on the horizontal axis, and the base case is outlined with a black border. For all panels, the dark shading denotes realizations of |ΔNAV| that are less than the median realization, the medium shading denotes realizations of |ΔNAV| that are greater than the median realization but less than in the distribution's 95% critical value, and the light shading denotes realizations of |ΔNAV| that are greater than the 95% critical value but less than the 99% critical value. All distributions are conditional on |ΔNAV|≥½¢.

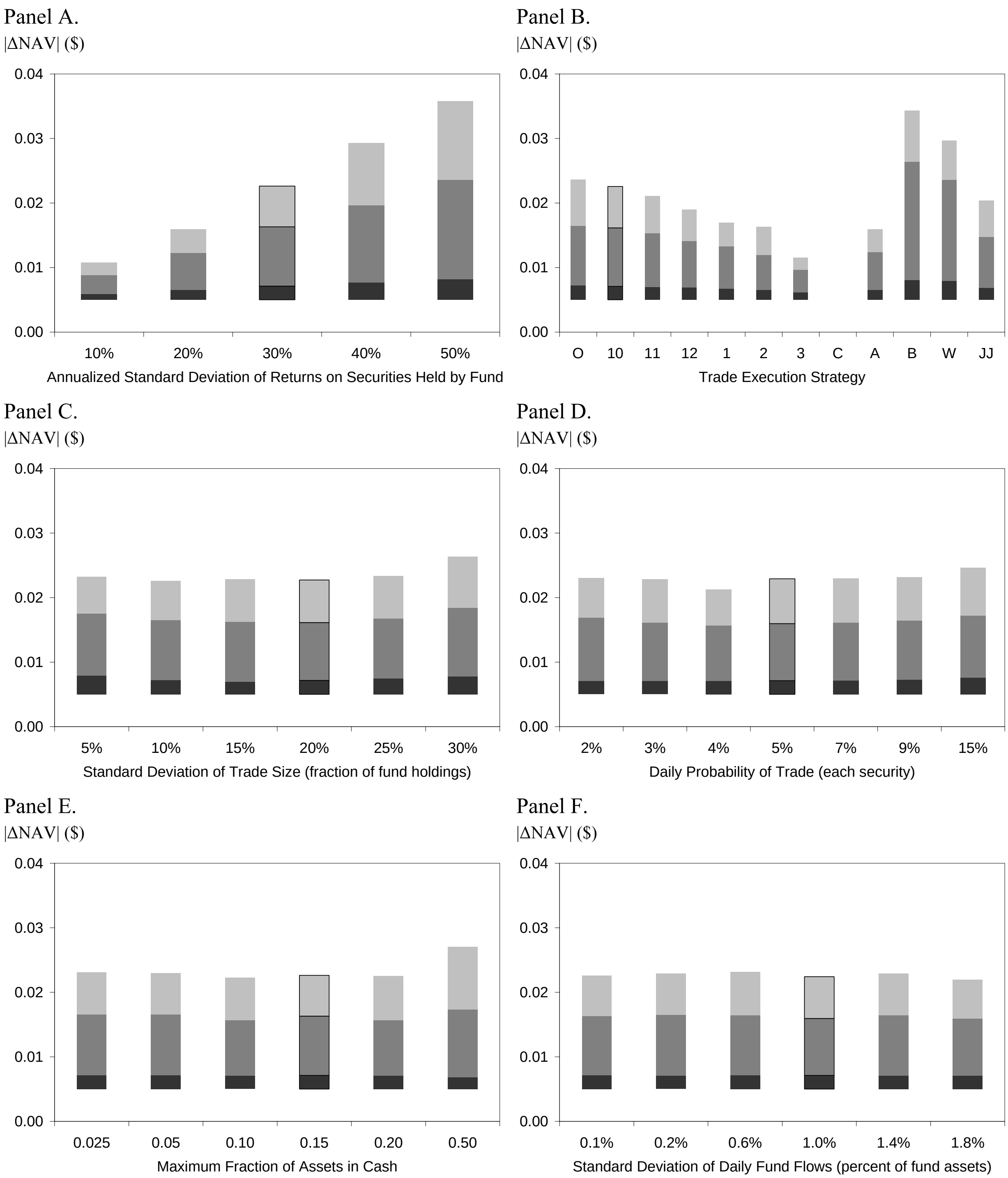


Figure 4. Daily likelihood of realizing half-basis-point deviations in return. Panels display the daily hazard of a simulated fund realizing a half-basis-point or more difference between accounting return and economic return, Pr[|ΔR|≥½bp], for various values of important input parameters in a simulation of 1,000 trading days. Parameters are: the annualized standard deviation of returns on securities held by the fund (Panel A); the strategy that the fund uses to execute trade orders (Panel B); the standard deviation of the size of the fund's trades, measured as a fraction of the fund's position in the traded security (Panel C); and the maximum amount of cash, measured as a fraction of total assets, that the fund is permitted to hold (Panel D). Unless otherwise noted, the fund holds fifty stocks, each with an annualized standard deviation of returns of 30%; the fund executes its trades at 10:00am; the daily hazard rate of trading a given stock is 5%; the standard deviation of the size of a trade, conditional on a trade occurring, is 20% of the fund's position in the traded security; and no less than 0% nor more than 15% of the fund's assets may be held in cash. A black dot identifies this base case in each figure. In panel A, the solid line represents a fund that holds fifty stocks, while the dashed line represents a fund that holds one hundred stocks. In Panel B, the trading strategies are always trade at market open ("O"), always trade at 10am, always trade at 11am, always trade at 12pm, always trade at 1pm, always trade at 2pm, always trade at 3pm, always trade at market close ("C"), trade at the average price for the day ("A"), trade at the best price for the day ("B"), trade at the worst price for the day ("W"), and trade at a random time of day according to the empirical distribution of Jain and Joh (1988, "JJ"). In Panel C, the solid line represents a fund that trades each security with daily probability of 5%, while the dashed line represents a fund that trades each security with daily probability of 2%. In Panel D, the solid line represents a fund that has a standard deviation of daily fund flows equal to 2% of assets, while the dashed line represents a fund that has a standard deviation of daily fund flows equal to 1% of assets. In all panels, the vertical axis measures the daily hazard of realizing a half-cent or more difference between accounting return and economic return, Pr[|ΔR|≥½bp].

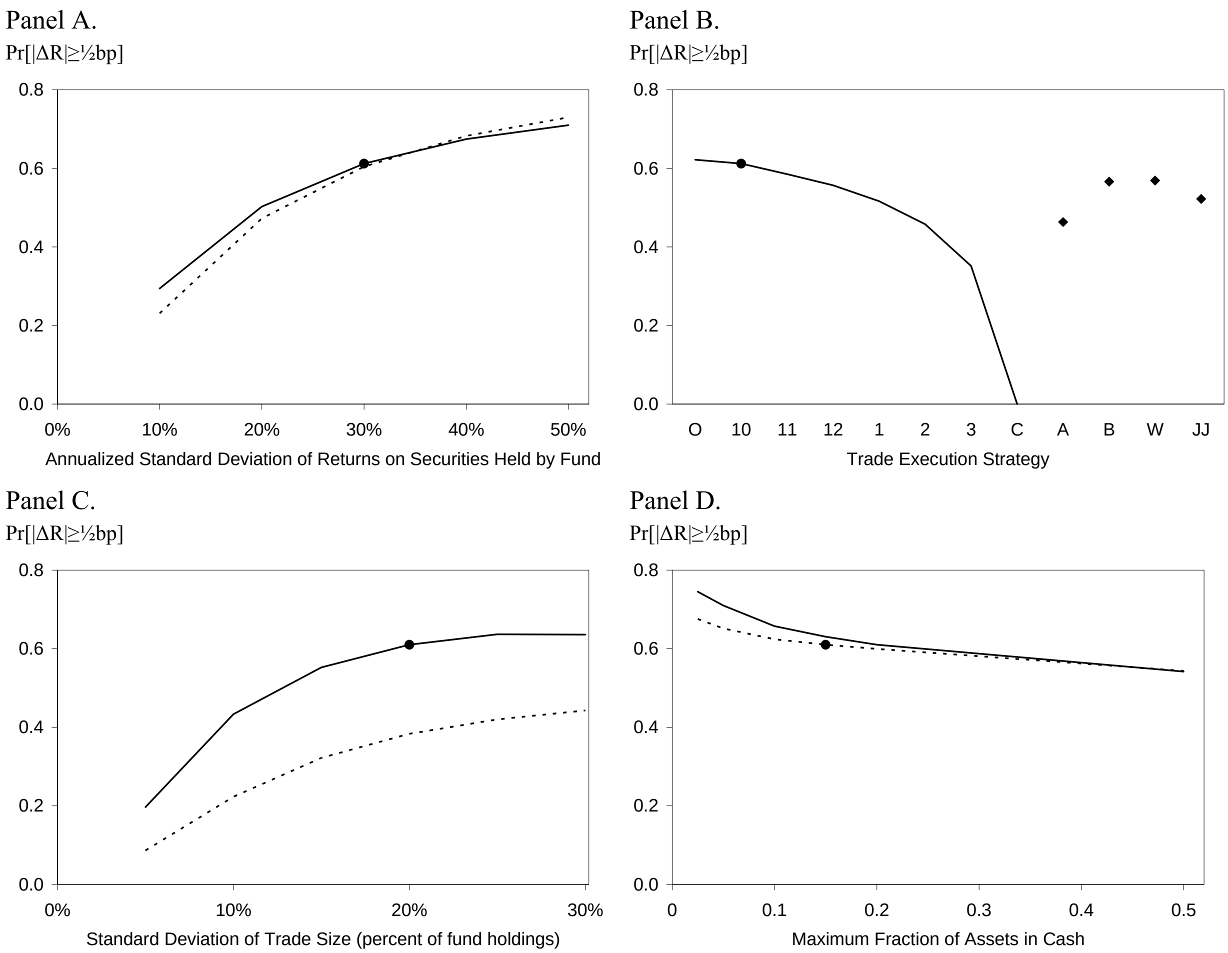
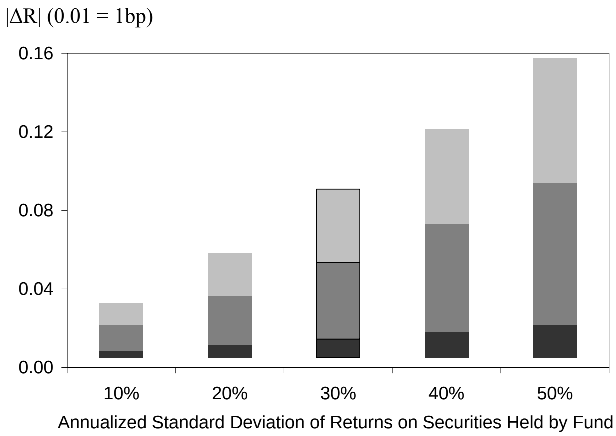
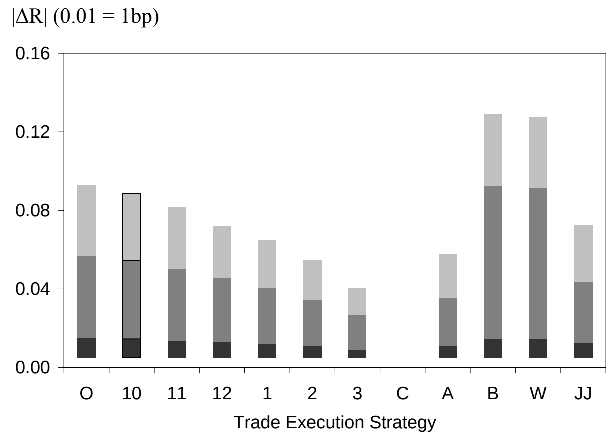


Figure 5. Distributions of deviation in return when the deviation is greater than ½bp. Simulation and baseline parameters are as described in Figure 4. Each panel fixes all input parameters, except for the parameter on the horizontal axis, and the base case is outlined with a black border. For all panels, the dark shading denotes realizations of |ΔR| that are less than the median realization, the medium shading denotes realizations of |ΔR| that are greater than the median realization but less than in the distribution's 95% critical value, and the light shading denotes realizations of |ΔR| that are greater than the 95% critical value but less than the 99% critical value. All distributions are conditional on |ΔR|≥½bp.

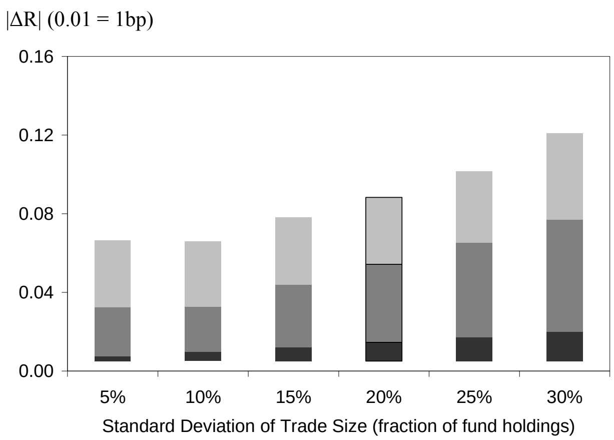
Panel A.



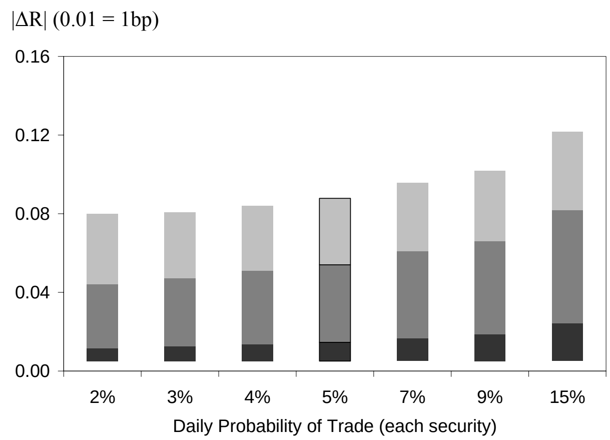
Panel B.



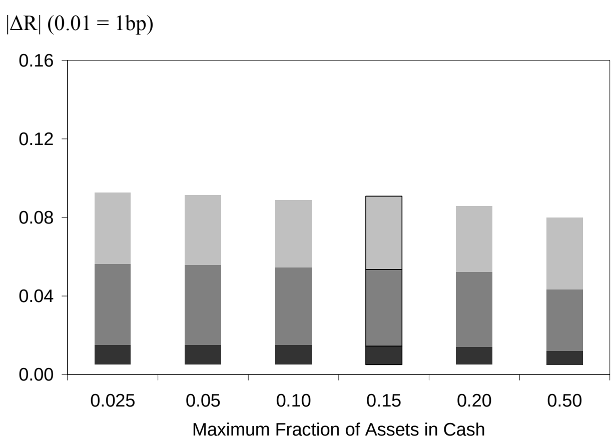
Panel C.



Panel D.



Panel E.



Panel F.

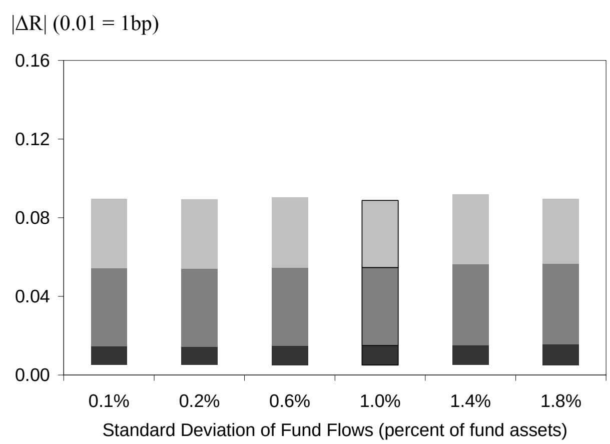
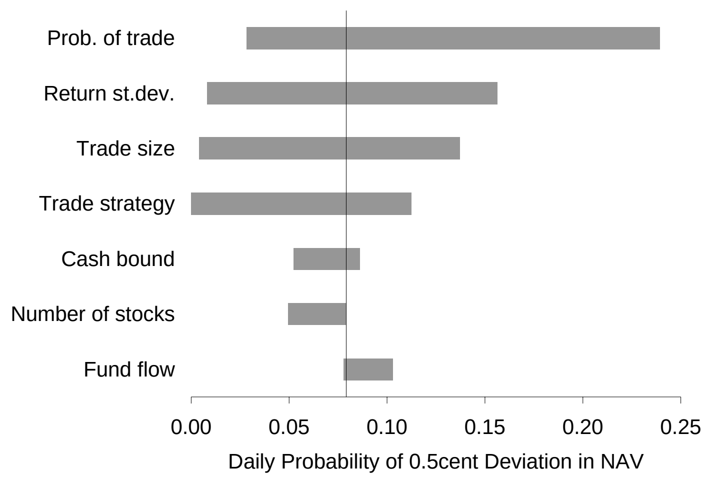


Figure 6. "Tornado" diagrams for daily probabilities of deviation in NAV and return. Panels display ranges of daily probabilities of realizing a half-cent deviation in NAV (Panel A) or a half-basis-point deviation in return (Panel B) for important input parameters. In each panel, a solid vertical line represents the daily probability for a base case: the base case assumes that the fund holds fifty stocks, each with an annualized standard deviation of returns of 30%; that the fund executes its trades at 10:00am; that the daily hazard rate of trading a given stock is 5%; that the standard deviation of the size of a trade, conditional on a trade occurring, is 20% of the fund's position in the traded security; and that no less than 0% nor more than 15% of the fund's assets may be held in cash. In Panel A, the base-case daily probability of realizing a half-cent deviation in NAV is $\Pr[|\Delta NAV|\geq½¢] = 0.08$. In Panel B, the base-case daily probability of realizing a half-basis-point deviation in return is $\Pr[|\Delta R|\geq½bp] = 0.61$. In each panel, gray bars display the range of daily probabilities as bounded by extreme assumptions on each input parameter. In the simulations, the daily probability of trading each security held by the fund ranges from 2% to 15%; the return standard deviation of securities held by the fund ranges from 10% annually to 50% annually; the standard deviation of trade size, conditional on making a trade, ranges from 5% to 30% of the fund's position in the security; the fund's trading strategy ranges from trading at market close to trading at the best price of the day (Panel A) or to trading at market open (Panel B); the upper bound on cash holdings ranges from 50% of fund assets to 2.5% of fund assets; the number of stocks held by the fund ranges from 100 to 50; and the daily standard deviation of fund flow ranges from 0.1% of fund assets to 2% of fund assets.

Panel A.



Panel B.

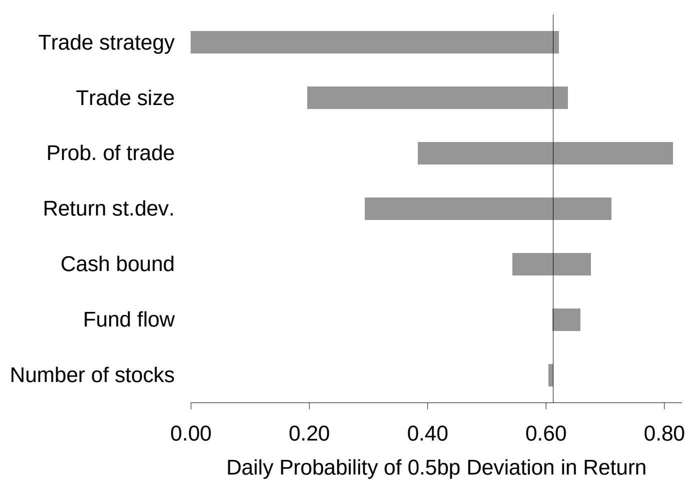


Table 1: Calculation of Net Asset Value, Accounting-based Returns, and Economic returns on Fund XYZ

Note: The hypothetical calculation assumes that the fund holds two securities, has a single fund share, and has no expenses to focus attention to the mechanics of NAV and returns

	Number of Shares held at 4 pm as of:			
	Monday	**Tuesday**	**Wednesday***	**Thursday**
Number of Shares of A	100	100	100	100
Number of Shares of B	100	100	0	0
Value in Cash	0	0	605	605

* During the day on Wednesday, the fund sold its 100 B shares for $6.05 each.

	Closing Prices on 4 pm as of:			
	Monday	**Tuesday**	**Wednesday**	**Thursday**
Share A	$ 5.00	$ 5.00	$ 5.05	$ 5.09
Share B	$ 6.00	$ 6.00	$ 6.25	$ 6.20
Cash	$ 1.00	$ 1.00	$ 1.00	$ 1.00

NAV calculations and published return as of 4 p.m. on

N(t-1)*P(t)		**Tuesday**	**Wednesday**	**Thursday**
Accounting Value of A		$ 500.00	$ 505.00	$ 509.00
Accounting Value of B		$ 600.00	$ 625.00	$ -
Accounting Value of Cash		$ -	$ -	$ 605.00
NAV		$ 1,100.00	$ 1,130.00	$ 1,114.00
Calculated daily return	one day		2.73%	-1.42%
	two day			1.27%

Economic value of the portfolio as of 4 p.m. on

		Tuesday	**Wednesday**	**Thursday**
Value of A holdings (Nt*Pt)		$ 500.00	$ 505.00	$ 509.00
Value of B holdings (Nt*Pt)		$ 600.00	$ -	$ -
Value of Cash holdings (Nt*Pt)		$ -	$ 605.00	$ 605.00
NAV		$ 1,100.00	$ 1,110.00	$ 1,114.00
Calculated daily return	one day		0.91%	0.36%
	two day			1.27%

Table 2: The extent of annual turnover reported by Morningstar for Equity Funds. 2005

The Morningstar website (http://www.morningstar.com, visited 7/6/2005) "premium fund screener" tool provides statistics on mutual fund turnover. Morningstar defines annual turnover as the minimum of a fund's security purchases and sales relative to the fund's AUM. The table below reports Morningstar's quartile breakpoints for turnover for domestic stock funds.
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Turnover percent quartiles reported by Morningstar for Domestic Stock Funds

Quartile	Lower bound	Upper bound
1	0	95
2	95	194
3	194	371
4	371	4,874

Table 3: Distribution of Daily Intraday and Daily Returns, S&P 500 Index, January 3, 1995 - October 11, 2005

	Distribution of Absolute Value of			
	High - Close	Close-Low	Daily Return	Daily Return
Mean	0.0065	0.0074	0.0081	-0.0003
Standard Deviation	0.0074	0.0074	0.0077	0.0112
Minimum	0.0000	0.0000	0.0000	-0.0542
25th Percentile	0.0012	0.0023	0.0026	-0.0063
Median	0.0041	0.0053	0.0060	-0.0006
75% Percentile	0.0092	0.0103	0.0110	0.0056
Maximum	0.0796	0.0803	0.0737	0.0737

Table 4. Summary Statistics for A-Class Funds Used in Empirical Investigation

The following table provides summary statistics of the funds used in our empirical analysis. Our sample includes only domestic equity funds and covers a multi-year time period. We restrict our analyis to A-class share NAVs and daily returns because the factors that determine distortions to these areas do not vary by class. For each fund class, the table below provides market cap and broad investment strategy (specialized or traditional), as well as mean daily levels for the duration of our sample for the following variables: assets under management, shares outstanding, NAV value, number of securities in portfolio, gross shareholder inflows and outflows, and gross and net portfolio trading activity. The table is sorted by decreasing incidence of NAV distortion (Fund 1 = highest , Fund 26 = lowest).

				Mean Daily Level:							
Fund	Market Cap	Fund Strategy	Number of Trading Days in Sample	Assets Under Management ($000)	Shares Outstanding	NAV Size ($)	Number of Securities in Portfolio	Gross Daily Inflows ($000)	Gross Daily Outflows ($000)	Gross Daily Trading Volume ($000)	Net Daily Trading Volume ($000)
1	Large Cap	Specialized	1,183	$ 3,000,000	50,000,000	> 40	100	$ 2,000	$ 2,800	$ 15,900	$ (900)
2	Medium Cap	Traditional	673	500,000	30,000,000	15-20	200	2,000	800	5,300	1,200
3	Large Cap	Traditional	1,253	900,000	40,000,000	15-20	200	500	1,400	11,800	(1,100)
4	Large Cap	Specialized	376	100,000	20,000,000	0-10	100	700	200	2,300	400
5	Large Cap	Traditional	1,256	12,000,000	220,000,000	> 40	200	10,600	12,900	56,700	(3,200)
6	Medium Cap	Specialized	1,253	3,000,000	200,000,000	10-15	200	8,200	7,100	21,400	600
7	Medium Cap	Traditional	1,256	1,000,000	50,000,000	20-25	200	1,600	1,400	8,200	< 100
8	Large Cap	Traditional	1,164	900,000	60,000,000	10-15	100	1,800	1,000	11,100	500
9	Medium Cap	Traditional	1,243	200,000	20,000,000	0-10	100	800	500	2,700	100
10	Large Cap	Traditional	1,118	2,000,000	100,000,000	15-20	100	4,100	2,300	12,100	1,100
11	Medium Cap	Traditional	499	200,000	20,000,000	10-15	100	900	500	2,100	600
12	Large Cap	Traditional	580	200,000	20,000,000	10-15	200	700	400	7,800	< 100
13	Small Cap	Traditional	448	50,000	< 5,000,000	10-15	300	300	200	600	200
14	Large Cap	Traditional	1,235	17,000,000	830,000,000	20-25	300	12,600	16,700	116,700	(3,500)
15	Medium Cap	Traditional	1,246	4,000,000	370,000,000	10-15	100	5,500	5,500	34,100	200
16	Large Cap	Traditional	1,123	500,000	40,000,000	10-15	100	900	900	3,500	< 100
17	Small Cap	Traditional	1,166	300,000	20,000,000	10-15	200	900	600	1,300	200
18	Large Cap	Traditional	1,249	5,000,000	390,000,000	10-15	100	6,900	6,300	37,200	600
19	Large Cap	Traditional	1,227	1,000,000	90,000,000	10-15	600	1,800	1,600	9,200	(100)
20	Large Cap	Traditional	922	800,000	80,000,000	10-15	200	300	800	4,100	(700)
21	Large Cap	Traditional	1,088	1,000,000	90,000,000	10-15	100	600	1,300	7,300	(900)
22	Large Cap	Traditional	1,256	700,000	70,000,000	10-15	2,000	900	1,000	6,200	(700)
23	Large Cap	Traditional	914	600,000	60,000,000	10-15	700	500	600	13,300	(1,600)
24	Large Cap	Traditional	1,243	3,000,000	210,000,000	15-20	1,200	3,200	4,300	57,300	(6,400)
25	Large Cap	Traditional	1,246	18,000,000	1,020,000,000	15-20	200	7,900	18,100	64,200	(8,600)
26	Large Cap	Traditional	1,256	1,000,000	110,000,000	10-15	1,700	1,600	1,500	11,700	(1,600)

Table 5. Incidence of | Δ Published NAV | and | Δ Published Return | by Extent of Distortion
Panel A summarizes the differences between the published (i.e. penny-rounded) accounting and economic NAVs in absolute dollar terms. It provides the distribution of these differences by selected dollar ranges, and for the fund-days within each range shows the mean, median and maximum absolute dollar difference between the unrounded economic and accounting NAVs. Panel B summarizes the differences in one-day absolute returns using published (i.e. penny-rounded) accounting and economic NAVs. Similar to Panel A, it provides the distribution of these differences by selected basis point ranges, and for the fund-days within each range shows the mean, median and maximum absolute basis point difference.

<table>
<tr><th colspan="5">Panel A: | Δ Published NAV ($) | by Dollar Ranges</th></tr>
<tr><th></th><th>Percent of</th><th colspan="3">Difference in Unrounded NAV ($)</th></tr>
<tr><th>Range of Distortions</th><th>Fund-Days</th><th>Mean</th><th>Median</th><th>Maximum</th></tr>
<tr><td>| ΔNAV | = $0.00</td><td>91.52%</td><td>0.001</td><td>0.000</td><td>0.010</td></tr>
<tr><td>| ΔNAV | = $0.01</td><td>7.83%</td><td>0.004</td><td>0.002</td><td>0.019</td></tr>
<tr><td>| ΔNAV | = $0.02</td><td>0.42%</td><td>0.018</td><td>0.018</td><td>0.027</td></tr>
<tr><td>$0.03 <= | ΔNAV | <= $0.05</td><td>0.18%</td><td>0.033</td><td>0.030</td><td>0.053</td></tr>
<tr><td>| ΔNAV | > $0.05</td><td>0.05%</td><td>0.177</td><td>0.087</td><td>1.105</td></tr>
<tr><td>| ΔNAV | >= $0.01</td><td>8.48%</td><td></td><td></td><td></td></tr>
</table>

<table>
<tr><th colspan="5">Panel B: | Δ Published Return (bps) | by Basis Point Ranges</th></tr>
<tr><th></th><th>Percent of</th><th colspan="3">Difference in Published One-Day Return (bps)</th></tr>
<tr><th>Range of Distortions</th><th>Fund-Days</th><th>Mean</th><th>Median</th><th>Maximum</th></tr>
<tr><td>| ΔReturn | = 0 bps</td><td>84.79%</td><td>-</td><td>-</td><td>-</td></tr>
<tr><td>0 bps < | ΔReturn | <= 1 bps</td><td>0.73%</td><td>0.16</td><td>0.06</td><td>1.00</td></tr>
<tr><td>1 bps < | ΔReturn | <= 10 bps</td><td>12.00%</td><td>5.35</td><td>5.50</td><td>10.00</td></tr>
<tr><td>10 bps < | ΔReturn | <= 50 bps</td><td>2.44%</td><td>14.55</td><td>12.24</td><td>49.82</td></tr>
<tr><td>50 bps < | ΔReturn | <= 100 bps</td><td>0.02%</td><td>67.18</td><td>65.48</td><td>86.39</td></tr>
<tr><td>| ΔReturn | > 100 bps</td><td>0.02%</td><td>234.72</td><td>156.98</td><td>443.01</td></tr>
<tr><td>| ΔReturn | > 1 bps</td><td>14.48%</td><td></td><td></td><td></td></tr>
</table>

Table 6. Incidence of | ΔNAV | and | ΔReturn | by Fund

Panel A summarizes by fund the mean, median and maximum difference between the unrounded accounting and economic NAVs. It also provides for each fund the percent of days where the published economic NAV would have been different from the published accounting NAV. Panel B summarizes by fund the mean, median and maximum difference between one-day absolute returns using published (i.e. penny-rounded) accounting and economic NAVs. Similar to Panel A, it also provides for each fund the percent of days where the one-day absolute returns were different by 10 basis points or greater.

Panel A: | ΔNAV | by Fund

Fund	Number of Days with Change in Published NAV	Percent of Days with Change in Published NAV	Difference in Unrounded NAV ($) Mean	Median	Maximum
1	221	19.78%	0.002	0.001	0.087
2	90	17.98%	0.004	0.001	1.105
3	214	17.08%	0.002	0.001	0.085
4	195	16.76%	0.003	0.001	0.148
5	108	16.08%	0.002	0.001	0.062
6	145	12.93%	0.001	0.000	0.052
7	134	12.34%	0.001	0.001	0.042
8	152	12.11%	0.001	0.000	0.024
9	143	11.50%	0.001	0.000	0.026
10	120	9.75%	0.001	0.000	0.024
11	120	9.62%	0.001	0.000	0.015
12	55	9.48%	0.002	0.000	0.181
13	104	8.93%	0.001	0.000	0.040
14	89	7.21%	0.001	0.000	0.018
15	86	6.90%	0.001	0.000	0.022
16	81	6.41%	0.001	0.000	0.016
17	27	5.92%	0.000	0.000	0.022
18	58	4.88%	0.001	0.000	0.018
19	54	4.32%	0.000	0.000	0.005
20	31	3.36%	0.000	0.000	0.021
21	39	3.13%	0.000	0.000	0.006
22	36	2.87%	0.000	0.000	0.029
23	25	2.74%	0.000	0.000	0.021
24	30	2.41%	0.000	0.000	0.013
25	30	2.41%	0.000	0.000	0.007
26	27	2.15%	0.000	0.000	0.029

Panel B: | ΔReturn | by Fund

Fund	Number of Days with Difference in One-Day Return Greater than 10 bps	Percent of Days with Difference in One-Day Return Greater than 10 bps	Difference in Published One-Day Return (bps) Mean	Median	Maximum
4	100	26.60%	5.59	0.00	140.56
9	160	12.87%	2.32	0.00	39.58
12	47	8.10%	2.83	0.00	164.83
6	65	5.19%	1.45	0.00	20.75
2	34	5.05%	3.56	0.00	443.01
11	20	4.01%	1.63	0.00	20.09
3	42	3.35%	1.70	0.00	42.11
15	36	2.89%	1.00	0.00	16.64
17	33	2.83%	1.03	0.00	31.85
13	10	2.23%	1.32	0.00	30.03
20	20	2.17%	0.68	0.00	25.87
8	23	1.98%	1.56	0.00	20.44
18	21	1.68%	0.65	0.00	17.22
23	15	1.64%	0.56	0.00	18.43
26	19	1.51%	0.43	0.00	33.33
22	19	1.51%	0.51	0.00	34.08
10	5	0.45%	0.89	0.00	13.56
16	5	0.45%	0.96	0.00	20.01
21	2	0.18%	0.48	0.00	10.87
1	2	0.17%	0.64	0.00	14.17
14	2	0.16%	0.64	0.00	10.60
24	1	0.08%	0.29	0.00	10.91
25	1	0.08%	0.26	0.00	11.79
7	1	0.08%	1.00	0.00	11.55
19	-	0.00%	0.58	0.00	8.31
5	-	0.00%	0.54	0.00	9.70

Table 7. Incidence of | ΔNAV | and | ΔReturn | by Quarter

Panel A summarizes by quarter the mean, median and maximum difference between the unrounded accounting and economic NAVs. It also provides for each quarter the percent of days where the published economic NAV would have been different from the published accounting NAV. Panel B summarizes by quarter the mean, median and maximum difference between one-day absolute returns using published (i.e. rounded to the penny) accounting and economic NAVs. Similar to Panel A, it also provides for each quarter the percent of days where the one-day absolute returns were different by 10 basis points or greater.

Panel A: | ΔNAV | by Quarter

Quarter	Number of Days with Change in Published NAV	Percent of Days with Change in Published NAV	Difference in Unrounded NAV ($) Mean	Median	Maximum
1	117	9.91%	0.001	0.000	0.022
2	172	13.56%	0.001	0.001	0.025
3	104	8.04%	0.001	0.000	0.015
4	144	11.02%	0.001	0.000	0.030
5	266	19.91%	0.004	0.001	1.105
6	199	14.53%	0.002	0.000	0.085
7	204	14.00%	0.002	0.000	0.148
8	232	16.04%	0.002	0.001	0.062
9	194	12.85%	0.002	0.000	0.181
10	104	6.85%	0.001	0.000	0.013
11	101	7.26%	0.001	0.000	0.029
12	59	3.98%	0.001	0.000	0.036
13	59	4.25%	0.001	0.000	0.087
14	81	5.24%	0.001	0.000	0.031
15	86	5.81%	0.001	0.000	0.040
16	45	3.43%	0.000	0.000	0.012
17	46	3.68%	0.000	0.000	0.008
18	41	3.12%	0.000	0.000	0.013
19	40	3.12%	0.000	0.000	0.009
20	36	2.69%	0.000	0.000	0.018

Panel B: | ΔReturn | by Quarter

Quarter	Number of Days with Difference in One-Day Return Greater than 10 bps	Percent of Days with Difference in One-Day Return Greater than 10 bps	Difference in Published One-Day Return (bps) Mean	Median	Maximum
1	25	2.12%	1.07	0.000	39.58
2	43	3.39%	1.33	0.000	31.85
3	23	1.78%	0.85	0.000	24.08
4	31	2.37%	0.98	0.000	23.53
5	57	4.27%	2.49	0.000	443.01
6	44	3.21%	1.45	0.000	42.11
7	44	3.02%	1.69	0.000	140.56
8	59	4.08%	1.74	0.000	43.51
9	68	4.50%	1.96	0.000	164.83
10	28	1.84%	0.92	0.000	45.48
11	29	2.08%	1.07	0.000	34.08
12	19	1.28%	0.62	0.000	21.87
13	32	2.31%	0.62	0.000	16.88
14	44	2.84%	0.83	0.000	38.23
15	54	3.65%	1.05	0.000	46.62
16	17	1.30%	0.58	0.000	19.38
17	16	1.28%	0.57	0.000	20.75
18	18	1.37%	0.49	0.000	19.12
19	25	1.95%	0.47	0.000	16.50
20	7	0.52%	0.31	0.000	14.62

Table 8. Incidence of | ΔNAV | and | ΔReturn | by NAV

Panel A summarizes by accounting NAV the mean, median and maximum difference between the unrounded accounting and economic NAVs. It also provides for each NAV the percent of days where the published economic NAV would have been different from the published accounting NAV. Panel B summarizes by accounting NAV the mean, median and maximum difference between one-day absolute returns using published (i.e. penny-rounded) accounting and economic NAVs. Similar to Panel A, it also provides for each NAV the percent of days where the one-day absolute returns were different by 10 basis points or greater.

Panel A: | ΔNAV | by NAV

NAV ($)	Number of Days with Change in Published NAV	Percent of Days with Change in Published NAV	Difference in Unrounded NAV ($) Mean	Median	Maximum
0-5	12	4.10%	0.001	0.000	0.022
5-10	262	4.91%	0.001	0.000	0.148
10-15	597	5.59%	0.001	0.000	0.181
15-20	443	7.83%	0.001	0.000	0.031
20-25	233	14.62%	0.002	0.001	0.085
25-30	199	21.24%	0.004	0.001	1.105
30-35	94	15.67%	0.002	0.001	0.051
35-40	59	15.49%	0.002	0.001	0.032
>40	431	21.63%	0.002	0.001	0.087

Panel B: | ΔReturn | by NAV

NAV ($)	Number of Days with Difference in One-Day Return Greater than 10 bps	Percent of Days with Difference in One-Day Return Greater than 10 bps	Difference in Published One-Day Return (bps) Mean	Median	Maximum
0-5	21	7.17%	1.87	0.00	45.48
5-10	445	8.33%	1.24	0.00	140.56
10-15	88	0.82%	0.95	0.00	164.83
15-20	77	1.36%	0.89	0.00	20.44
20-25	14	0.88%	1.28	0.00	42.11
25-30	28	2.99%	2.62	0.00	443.01
30-35	6	1.00%	1.08	0.00	18.96
35-40	1	0.26%	0.77	0.00	12.23
>40	3	0.15%	0.70	0.00	14.17

Table 9. Empirical and simulated moments for twenty-six funds. Columns (2)-(7), (9), (11), (13), and (15) present empirical moments for twenty-six fund families, as reported in Tables 4 and 6. Columns (2), (3), (5), (8), (10), (12), (14), and (16) present average moments for 1,000 trials of a 1,000-day trading simulation. Simulations are as described in Figures 2-6. Input parameters are chosen individually for each fund, in order to match the empirical moments as closely as possible. Simulations achieve perfect matches for number of stocks (column 2), average NAV (column 3), and standard deviation of daily fund flow (column 5); for brevity data and simulation results are presented jointly for these variables. For all simulations, mean daily fund flow is zero (cf. column 4). For all simulations but one, mean daily net trading is zero (cf. column 6); exception is fund 12, for which the simulated level is -0.001.

			Daily fund flow, fraction of assets		Daily net trading, fraction of assets			Daily gross trading, fraction of assets							
	# of stocks	Average NAV	Mean	St. Dev.	Mean	Standard deviation		Mean		Standard deviation		Pr[|ΔNAV|] ≥ ½¢		Pr[|ΔR|] ≥ 10bp	
Fund	Data & Sim.	Data & Sim.	Data	Data & Sim.	Data	Data	Sim.	Data	Sim.	Data	Sim.	Data	Sim.	Data	Sim.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
1	100	62	0.000	0.001	0.000	0.004	0.002	0.005	0.007	0.010	0.007	0.20	0.27	0.00	0.01
2	170	17	0.002	0.014	0.002	0.014	0.014	0.011	0.019	0.015	0.013	0.18	0.11	0.05	0.02
3	190	19	–0.001	0.002	–0.001	0.008	0.003	0.013	0.012	0.021	0.011	0.17	0.11	0.03	0.02
4	80	6	0.005	0.008	0.004	0.014	0.010	0.023	0.065	0.027	0.641	0.17	0.10	0.27	0.16
5	220	54	0.000	0.001	0.000	0.002	0.002	0.005	0.001	0.004	0.002	0.16	0.00	0.00	0.00
6	150	15	0.000	0.004	0.000	0.005	0.004	0.007	0.009	0.010	0.007	0.13	0.02	0.05	0.00
7	220	22	0.000	0.002	0.000	0.005	0.003	0.008	0.009	0.009	0.006	0.12	0.04	0.00	0.00
8	80	14	0.001	0.004	0.001	0.006	0.004	0.012	0.012	0.017	0.010	0.12	0.05	0.02	0.01
9	110	9	0.001	0.007	0.001	0.010	0.007	0.014	0.017	0.018	0.012	0.12	0.03	0.13	0.02
10	50	19	0.001	0.003	0.001	0.005	0.003	0.006	0.007	0.008	0.009	0.10	0.03	0.00	0.01
11	110	11	0.002	0.005	0.003	0.012	0.005	0.010	0.014	0.011	0.013	0.10	0.06	0.04	0.03
12	160	11	0.002	0.025	0.000	0.019	0.025	0.039	0.030	0.061	0.017	0.09	0.07	0.08	0.02
13	250	15	0.004	0.016	0.004	0.013	0.016	0.011	0.017	0.017	0.011	0.09	0.02	0.02	0.00
14	270	21	0.000	0.001	0.000	0.004	0.002	0.007	0.007	0.007	0.005	0.07	0.02	0.00	0.00
15	150	11	0.000	0.002	0.000	0.005	0.003	0.009	0.008	0.009	0.007	0.07	0.01	0.03	0.00
16	80	13	0.000	0.003	0.000	0.005	0.003	0.007	0.007	0.008	0.007	0.06	0.01	0.00	0.01
17	220	12	0.001	0.004	0.001	0.005	0.004	0.004	0.005	0.004	0.004	0.06	0.00	0.03	0.00
18	130	13	0.000	0.002	0.000	0.005	0.003	0.007	0.007	0.008	0.006	0.05	0.01	0.02	0.00
19	560	14	0.000	0.003	0.000	0.005	0.003	0.009	0.008	0.009	0.005	0.04	0.00	0.00	0.00
20	2,050	11	0.000	0.003	–0.001	0.009	0.003	0.009	0.013	0.013	0.007	0.04	0.01	0.03	0.00
21	1,690	11	0.000	0.005	–0.002	0.009	0.005	0.012	0.015	0.018	0.008	0.04	0.02	0.04	0.01
22	170	10	–0.001	0.001	–0.001	0.005	0.002	0.005	0.005	0.008	0.005	0.03	0.00	0.02	0.00
23	110	12	–0.001	0.002	–0.001	0.004	0.003	0.007	0.008	0.012	0.008	0.03	0.02	0.00	0.01
24	720	10	0.000	0.005	–0.003	0.013	0.005	0.022	0.026	0.038	0.014	0.03	0.07	0.02	0.04
25	1,160	16	0.000	0.002	–0.002	0.008	0.003	0.019	0.025	0.032	0.014	0.02	0.14	0.00	0.03
26	250	18	–0.001	0.001	0.000	0.002	0.002	0.004	0.002	0.005	0.002	0.02	0.00	0.00	0.00

Table 10A. Regression Analysis of | Δ Published NAV ($) |

Panels A and B show our regression analysis used to identify which factors significantly determine changes in the published NAV, i.e. differences in the penny-rounded accounting and economic NAVs. Both Panels use a random effects model on fund with robust standard errors, with Panel A regressing the absolute change in the published NAV against several observable dimensions of fund and market characteristics and Panel B regressing the signed change in the published NAV against the same dimensions. The observable fund and market characteristics used as control variables include 1) the categorized published accounting NAV using the same buckets as in Table 8; 2) the type of fund; 3) the year; 4) the absolute and signed daily return on the S&P 500 index; 5) the gross and net intensity of trading, calculated as the dollar volume of trading as a percent of fund class assets; 6) gross shareholder flow intensity, calculated as the gross flows as a percent of fund class assets; and 7) the number of securities held in the fund.

Panel A: Dependent Variable: | Δ Published NAV ($) |

Random Effects Model on Fund with Robust Standard Errors

	Base Model	With SP500 Index Absolute Return	With SP500 Index Signed Return	With Number of Securities Held	With Gross Trading Intensity	With Net Trading Intensity	With Absolute Index Return and Gross Trading Intensity Interacted	With Flow Intensity	Full Model	Full Model with AR1 Correction
Public Information:										
Categorized Published Accounting NAV ($)	0.00036	0.00038	0.00037	0.00035	0.00043	0.00035	0.00042	0.00037	0.00041	0.00039
	0.00007***	0.00007***	0.00007***	0.00007***	0.00009***	0.00005***	0.00006***	0.00007***	0.00006***	0.00006***
Fund Type (Base = Large Cap):										
Medium Cap	0.00099	0.001	0.00099	0.00091	0.00112	0.00039	0.00115	0.00092	0.00113	0.00115
	0.00049**	0.00049**	0.00050**	0.00051*	0.00055**	0.00016**	0.00054**	0.00044**	0.00057**	0.00034***
Small Cap	0.00031	0.00033	0.00032	0.00026	0.00057	-0.00103	0.00058	0.00014	0.00051	0.00056
	0.00016*	0.00016**	0.00016*	0.00017	0.00024**	0.00073	0.00025**	0.0002	0.00027*	0.00055
SP500 Index Absolute Return (%)		0.00039					-0.00119		-0.00117	-0.00124
		0.00010***					0.001		0.00099	0.00005***
SP500 Index Signed Return (%)			-0.00006							
			0.00006							
Number of Securities Held				-2.25E-07					-1.67E-08	-7.04E-08
				8.51E-8***					1.47E-07	2.58E-07
Non-Public Information:										
Gross Trading Intensity (bps)					0.00001		0.00000		-2.82E-06	-5.06E-06
					0.00001**		0.00001		0.00001	3.17E-7***
Net Trading Intensity (bps)						0.00003				
						0.00002*				
Absolute Index Return (%) * Gross Trading Intensity (bps)							13.36543		13.25223	13.69914
							9.23112		9.14786	0.19925***
Gross Inflows Intensity (bps)								3.50E-06	-1.82E-06	-6.65E-07
								2.87E-06	1.49E-06	1.77E-07
Gross Outflows Intensity (bps)								9.61E-07	2.92E-06	-1.67E-06
								1.03E-06	3.64E-06	1.33E-06
Intercept	-0.0005	-0.00095	-0.0005	-0.00036	-0.00225	-0.00063	-0.00085	-0.00063	-0.00084	-0.00046
	0.00031	0.00041**	0.00032	0.00035	0.00113**	0.00033*	0.00032***	0.00037*	0.00030***	0.00038
Observations	27473	27447	27447	27473	27473	27473	27447	27471	27445	27445
Number of Panels (Funds)	26	26	26	26	26	26	26	26	26	26
Overall R^2	0.01	0.02	0.01	0.01	0.13	0.22	0.25	0.03	0.25	0.24

Notes:
[1] * significant at 10%; ** significant at 5%; *** significant at 1%
[2] All models control for yearly effects.

Table 10B. Regression Analysis of Δ Published NAV ($)

Panels A and B show our regression analysis used to identify which factors significantly determine changes in the published NAV, i.e. differences in the penny-rounded accounting and economic NAVs. Both Panels use a random effects model on fund with robust standard errors, with Panel A regressing the absolute change in the published NAV against several observable dimensions of fund and market characteristics and Panel B regressing the signed change in the published NAV against the same dimensions. The observable fund and market characteristics used as control variables include 1) the categorized published accounting NAV using the same buckets as in Table 8; 2) the type of fund; 3) the year; 4) the absolute and signed daily return on the S&P 500 index; 5) the gross and net intensity of trading, calculated as the dollar volume of trading as a percent of fund class assets; 6) gross shareholder flow intensity, calculated as the gross flows as a percent of fund class assets; and 7) the number of securities held in the fund.

Panel B: Dependent Variable: Δ Published NAV ($)

Random Effects Model on Fund with Robust Standard Errors

	Base Model	**With SP500 Index Absolute Return**	**With SP500 Index Signed Return**	**With Number of Securities Held**	**With Gross Trading Intensity**	**With Net Trading Intensity**	**With Absolute Index Return and Gross Trading Intensity Interacted**	**With Flow Intensity**	**Full Model**	**Full Model with AR1 Correction**
Public Information:										
Categorized Published Accounting NAV ($)	0.00003	0.00003	0.00003	0.00003	-0.00007	0.00003	-0.00005	0.00002	-0.00005	-0.00002
	0.00003	0.00003	0.00003	0.00003	0.00009	0.00003	0.00007	0.00003	0.00007	0.00005
Fund Type (Base = Large Cap):										
Medium Cap	-0.00011	-0.00012	-0.00011	-0.00013	-0.00019	0.00026	-0.00021	-0.00008	-0.00024	-0.00034
	0.00021	0.00021	0.00021	0.00022	0.00025	0.00012**	0.00025	0.00018	0.00027	0.00027
Small Cap	-0.0001	-0.0001	-0.0001	-0.00011	-0.0003	0.001	-0.00029	0.00002	-0.00028	-0.00031
	0.00008	0.00008	0.00008	0.00009	0.00019	0.00072	0.00017*	0.00013	0.00017	0.00044
SP500 Index Absolute Return (%)		-0.00005					0.00116		0.00114	0.00127
		0.0001					0.00109		0.00108	0.00006***
SP500 Index Signed Return (%)			0.00008							
			0.00006							
Number of Securities Held				-2.82E-08					-8.78E-08	-1.03E-07
				4.38E-08					8.20E-08	2.06E-07
Non-Public Information:										
Gross Trading Intensity (bps)					-0.00001		0.00000		3.48E-06	4.87E-06
					0.00001		0.00001		0.00001	3.58E-7***
Net Trading Intensity (bps)						-0.00002				
						0.00002				
Absolute Index Return (%) * Gross Trading Intensity (bps)							-10.25917		-10.17763	-11.50633
							10.09647		10.01863	0.22691***
Gross Inflows Intensity (bps)								-2.56E-06	-1.32E-06	-4.49E-07
								2.64E-06	1.39E-06	1.93E-7**
Gross Outflows Intensity (bps)								2.57E-07	3.17E-06	2.64E-06
								1.16E-06	2.87E-06	1.48E-6*
Intercept	0.00011	0.00015	0.0001	0.00013	0.00135	0.00015	0.00001	0.00019	0.00006	-0.00014
	0.00017	0.00025	0.00017	0.00019	0.00105	0.00018	0.00041	0.00022	0.00038	0.00033
Observations	27473	27447	27447	27473	27473	27473	27447	27471	27445	27445
Number of Panels (Funds)	26	26	26	26	26	26	26	26	26	26
Overall R^2	0.00	0.00	0.00	0.00	0.05	0.15	0.12	0.01	0.12	0.12

Notes:
[1] * significant at 10%; ** significant at 5%; *** significant at 1%
[2] All models control for yearly effects.

Table 11A. Regression Analysis of | Δ Published Return |

Panels A and B show our regression analysis used to identify which factors significantly determine changes in published daily returns, i.e. differences in one-day returns based on penny-rounded accounting and economic NAVs. Both Panels use a random effects model on fund with robust standard errors, with Panel A regressing the absolute change in the published return against several observable dimensions of fund and market characteristics and Panel B regressing the signed change in the published return against the same dimensions. The observable fund and market characteristics used as control variables include 1) the categorized published accounting NAV using the same buckets as in Table 8; 2) the type of fund; 3) the year; 4) the absolute and signed daily return on the S&P 500 index; 5) the gross and net intensity of trading, calculated as the dollar volume of trading as a percent of fund class assets; 6) gross shareholder flow intensity, calculated as the gross flows as a percent of fund class assets; and 7) the number of securities held in the fund.

Panel A: Dependent Variable: | Δ Published Return (bps) |

Random Effects Model on Fund with Robust Standard Errors

	Base Model	With SP500 Index Absolute Return	With SP500 Index Signed Return	With Number of Securities Held	With Gross Trading Intensity	With Net Trading Intensity	With Absolute Index Return and Gross Trading Intensity Interacted	With Flow Intensity	Full Model	Full Model with AR1 Correction
Public Information:										
Categorized Published Accounting NAV ($)	0.03738	0.05457	0.00535	0.02707	0.00173	-0.01204	0.00757	-0.02094	-0.00875	-0.03288
	0.05543	0.05654	0.04954	0.05569	0.04226	0.03587	0.04075	0.0421	0.04166	0.05188
Fund Type (Base = Large Cap):										
Medium Cap	0.81105	0.81826	0.80836	0.67531	0.8811	0.39418	0.89113	0.75323	0.74543	0.78344
	0.56336	0.55577	0.42212*	0.56164	0.51103*	0.31837	0.47620*	0.31226**	0.34224**	0.29364***
Small Cap	0.35142	0.35855	0.35348	0.24907	0.51713	-0.48981	0.5188	0.23166	0.36556	0.41923
	0.33659	0.3311	0.25367	0.3345	0.31817	0.40026	0.30491*	0.21313	0.22316	0.47637
SP500 Index Absolute Return (%)		0.22192					-0.2965		-0.28228	-0.05495
		0.05237***					0.35665		0.35219	0.02722**
SP500 Index Signed Return (%)			0.00846							
			0.03169							
Number of Securities Held				-0.00044					-0.00039	-0.00047
				0.00012***					0.00011***	0.00022**
Non-Public Information:										
Gross Trading Intensity (bps)					0.00863		0.00369		0.00353	0.00258
					0.00264***		0.00315		0.0031	0.00018***
Net Trading Intensity (bps)						0.01801				
						0.00576***				
Absolute Index Return (%) * Gross Trading Intensity (bps)							4164.99995		4066.941	1019.53466
							3334.03055		3304.15039	113.17027***
Gross Inflows Intensity (bps)								0.00268	0.00162	-0.00241
								0.00177	0.00105	0.00011***
Gross Outflows Intensity (bps)								0.00118	-0.00134	-0.00029
								0.001	0.00184	0.00079
Intercept	1.06491	0.78389	1.16792	1.28883	0.17883	1.1034	0.509	1.14977	0.71031	1.10991
	0.30314***	0.31810**	0.26436***	0.33452***	0.47728	0.24206***	0.43999	0.24018***	0.44743	0.32168***
Observations	27447	27447	27447	27447	27447	27447	27447	27445	27445	27445
Number of Panels (Funds)	26	26	26	26	26	26	26	26	26	26
Overall R^2	0.01	0.01	0.01	0.01	0.13	0.21	0.16	0.03	0.17	0.07

Notes:
[1] * significant at 10%; ** significant at 5%; *** significant at 1%
[2] All models control for yearly effects.

Table 11B. Regression Analysis of Δ Published Return

Panels A and B show our regression analysis used to identify which factors significantly determine changes in published daily returns, i.e. differences in one-day returns based on penny-rounded accounting and economic NAVs. Both Panels use a random effects model on fund with robust standard errors, with Panel A regressing the absolute change in the published return against several observable dimensions of fund and market characteristics and Panel B regressing the signed change in the published return against the same dimensions. The observable fund and market characteristics used as control variables include 1) the categorized published accounting NAV using the same buckets as in Table 8; 2) the type of fund; 3) the year; 4) the absolute and signed daily return on the S&P 500 index; 5) the gross and net intensity of trading, calculated as the dollar volume of trading as a percent of fund class assets; 6) gross shareholder flow intensity, calculated as the gross flows as a percent of fund class assets; and 7) the number of securities held in the fund.

Panel B: Dependent Variable: Δ Published Return (bps)

Random Effects Model on Fund with Robust Standard Errors

	Base Model	**With SP500 Index Absolute Return**	**With SP500 Index Signed Return**	**With Number of Securities Held**	**With Gross Trading Intensity**	**With Net Trading Intensity**	**With Absolute Index Return and Gross Trading Intensity Interacted**	**With Flow Intensity**	**Full Model**	**Full Model with AR1 Correction**
Public Information:										
Categorized Published Accounting NAV ($)	0.00145	0.00163	0.00102	0.00162	-0.02836	0.00104	-0.01992	0.00082	-0.02012	-0.02874
	0.01719	0.01735	0.01726	0.01943	0.04114	0.01714	0.03421	0.01865	0.03655	0.02548
Fund Type (Base = Large Cap):										
Medium Cap	0.0016	0.00175	0.00127	0.00239	-0.0209	0.09911	-0.03165	0.00478	-0.04707	-0.07818
	0.12149	0.12167	0.12154	0.1292	0.13755	0.08652	0.13342	0.11165	0.14243	0.11293
Small Cap	0.00356	0.0038	0.00341	0.00424	-0.06128	0.27942	-0.05318	0.01471	-0.07548	-0.11622
	0.09216	0.09218	0.09207	0.09644	0.11633	0.33682	0.11052	0.10000	0.10876	0.1945
SP500 Index Absolute Return (%)		0.00841					0.51972		0.52019	0.67923
		0.05335					0.41057		0.40877	0.04285***
SP500 Index Signed Return (%)			0.05419							
			0.03267*							
Number of Securities Held				0.00000					-0.00001	-0.00002
				0.00004					0.00005	0.00009
Non-Public Information:										
Gross Trading Intensity (bps)					-0.00277		0.0024		0.00237	0.00281
					0.00276		0.00357		0.00355	0.00027***
Net Trading Intensity (bps)						-0.00616				
						0.00747				
Absolute Index Return (%) * Gross Trading Intensity (bps)							-4393.22475		-4405.54567	-5783.10007
							3871.87904		3866.31578	174.91764***
Gross Inflows Intensity (bps)								-0.00031	0.0002	0.00043
								0.00117	0.0007	0.00014***
Gross Outflows Intensity (bps)								0.00027	0.0012	0.00176
								0.00117	0.00147	0.00112
Intercept	-0.0085	-0.01701	-0.01081	-0.01021	0.38796	-0.00007	-0.21338	-0.00369	-0.21997	-0.25151
	0.10678	0.12541	0.10642	0.13008	0.45923	0.11105	0.39621	0.12305	0.40218	0.17671
Observations	27447	27447	27447	27447	27447	27447	27447	27445	27445	27445
Number of Panels (Funds)	26	26	26	26	26	26	26	26	26	26
Overall R^2	0.00	0.00	0.00	0.00	0.01	0.02	0.04	0.00	0.04	0.04

Notes:
[1] * significant at 10%; ** significant at 5%; *** significant at 1%
[2] All models control for yearly effects.

Appendix A
Full Text of Rule 2a-4
'Definition of "current net asset value" for use in computing periodically the current price of redeemable security'

(a) The current net asset value of any redeemable security issued by a registered investment company used in computing periodically the current price for the purpose of distribution, redemption, and repurchase means an amount which reflects calculations, whether or not recorded in the books of account, made substantially in accordance with the following, with estimates used where necessary or appropriate:

(1) Portfolio securities with respect to which market quotations are readily available shall be valued at current market value, and other securities and assets shall be valued at fair value as determined in good faith by the board of directors of the registered company.

(2) Changes in holdings of portfolio securities shall be reflected no later than in the first calculation on the first business day following the trade date. (emphasis added)

(3) Changes in the number of outstanding shares of the registered company resulting from distributions, redemptions, and repurchases shall be reflected no later than in the first calculation on the first business day following such change.

(4) Expenses, including any investment advisory fees, shall be included to date of calculation. Appropriate provision shall be made for Federal income taxes if required. Investment companies which retain realized capital gains designated as a distribution to shareholders shall comply with paragraph (h) of Section 210.6-03 of Regulation S-X.

(5) Dividends receivable shall be included to date of calculation either at ex-dividend dates or record dates, as appropriate.

(6) Interest income and other income shall be included to date of calculation.

(b) The items which would otherwise be required to be reflected by subparagraphs (4) and (6) above need not be so reflected if cumulatively, when netted, they do not amount to as much as one cent per outstanding share.

(c) Notwithstanding the requirements of paragraph (a) above, any interim determination of current net asset value between calculations made as of the close of the New York Stock Exchange on the preceding business day and the current business day may be estimated so as to reflect any change in current net asset value since the closing calculation on the preceding business day.

Source: Rules and Regulations promulgated under the Investment Company Act of 1940, 29 FR 19101, Dec. 30, 1964, as amended at 35 FR 314, Jan. 8, 1970; 47 FR 56844, Dec. 21, 1982 (http://www.law.uc.edu/CCL/InvCoRls/rule2a-4.html)