Do the pecking-order's predictions follow from its premises?*

Christopher A. Hennessy Dmitry Livdan Bruno Miranda

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Abstract

We examine the effect of asymmetric information on the evolution of corporate investment and financing using an empirically testable dynamic structural model. We do so by embedding a privately informed manager in a neoclassical investment framework with costly default. When private information is bad, the manager overinvests (relative to first-best), has negative leverage, and uses dividend cuts or equity flotations to fill any financing gaps. When private information is good, debt provides a positive signal. However, in many states the good type issues equity despite having access to default-free debt. In addition, the good type may overinvest. These predictions contradict the pecking-order folk wisdom. The effects in our model are absent from static models which fail to account for the fact that asymmetric information raises the value of future cash inflows from investment and the shadow cost of cash outflows associated with debt service. Consistent with empirical observation, the simulated firm exhibits mean-reverting leverage and a negative leverage-lagged cash flow relationship. When we mimic traditional reduced-form regressions, coefficient values are inconsistent with what is commonly imputed to pecking-order theory. In addition, "market-timing" proxies are insignificant despite the fact that managers want to exploit mispricing. These results call into question standard reduced-form regressions used to test the significance of informational asymmetry for investment and financing behavior.

1 Introduction

As argued by Myers (1984), the two leading theories of corporate financial behavior are the trade-off and pecking-order theories. The former posits that firms choose their financial mix by equating tax benefits of debt with bankruptcy costs at the margin. The latter posits that asymmetric information causes firms to follow a simple rule of thumb for financing: internal funds should be used first, followed by debt, followed

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by external equity. Given the prominence of these theories, it comes as little surprise that a *voluminous* empirical literature has emerged comparing their ability to explain the stylized facts. For examples, see Shyam-Sunder and Myers (1999) and Fama and French (2002).

In this paper, we ask two fundamental questions related to the validity and meaning of empirical tests of the pecking-order hypotheses. In an economy with asymmetric information, will firms act in the way that Myers and Majluf (1984) and Myers (1984) predict? Relatedly, will firms optimizing under asymmetric information generate regression coefficients that are consistent with those predicted by advocates of the pecking-order?

In order to address these questions, we develop a dynamic structural model embedding a privately informed manager in a neoclassical investment framework with costly default. The model is rich in that it endogenizes investment, debt, savings, equity flotations, dividends, and share repurchases. In addition to providing clear empirical predictions, our model also fills a conspicuous void in the literature. There is a long line of dynamic structural models of the trade-off theory. However, we are unaware of any dynamic structural models that capture the type of ex ante informational asymmetries emphasized by Myers and Majluf (1984), Leland and Pyle (1977) and Ross (1977). This state of affairs has not gone without notice. For example, after presenting his variant of dynamic trade-off theory, Ross (2005) argues that "The introduction of the issues raised by the presence of asymmetric information in the determination of the capital structure and the integration of these issues into the intertemporal neoclassical model are a major challenge."

Some background will be useful in framing the results of our analysis. Empirical tests of the pecking-order have focused on the following predictions, based upon the arguments contained in the seminal paper by Myers and Majluf (1984): (P1) debt is always preferred to equity; (P2) firms use debt in order to fill any financing gaps; (P3) firms only issue equity when the costs of distress become acute; (P4) dividends are sticky and do not adjust in order to fill financing gaps; (P5) asymmetric information induces firms to invest less than they would in an economy with symmetric information; (P6) leverage is declining in lagged profits; and (P7) firms hoard cash in order to avoid distortions associated with asymmetric information.

We begin with a standard neoclassical investment framework where the firm is endowed with a stochastic concave profit function. Each period, the firm is run by a single-period manager who observes a profit shock before outside investors. Following Myers and Majluf (1984), the manager works in the interest of risk-neutral insider-shareholders who do not buy or sell shares in the event of equity flotations or share repurchases. Under these assumptions, equity flotations represent a negative signal and share repurchases represent a positive signal, since they respectively serve to decrease and increase the percentage ownership of insiders. The manager is free to save and can borrow using a standard single-period debt contract. Financing needs are endogenous as the manager also chooses dividends and investment in the interest of insiders.

Although the manager works for a single-period, he is forward-looking and recognizes that his decisions affect net worth and equity value in the subsequent period. Thus, the endogenous evolution of net worth and the exogenous evolution of the firm's "type" represent the underlying source of dynamics. In the model, there is an infinite sequence of signaling games played between a privately informed manager and uninformed investors. We characterize the least-cost separating (perfect Bayesian) equilibria for each level of net worth. The equilibria of the games are properly capitalized into the equity value function by exploiting recursive features of the model.

Our main finding is that firms will exhibit violations of pecking-order predictions P1-P5, whereas P6 and P7 do emerge as equilibrium phenomena in our asymmetric information economy. In equilibrium, when private information is bad, the manager overinvests, has negative leverage, and uses dividend reductions or new equity issues in order to fill any financing gap. When private information is good, debt provides a positive signal. However, in many states the good type issues equity despite having access to default-free debt. In addition, the good type may overinvest.

As one would expect, the regression coefficients generated by a simulated panel of firms are inconsistent with those commonly imputed to pecking-order theory. In particular, in our simulated panel, where each type is assumed to be equally probable, debt issuance only fills 44% of the financing gap, with the remainder of the gap filled by endogenous reductions in dividends or flotations of new equity. This contradicts predictions P1-P4. The simulated model does support P6 and P7 in that firms often hoard cash and exhibit a negative leverage-lagged cash flow relationship. Also consistent with empirical observation, the simulated firms exhibit mean-reverting leverage ratios behavior often attributed to trade-off theory cum transactions costs.

Our model is also related to the market-timing story of Baker and Wurgler (2002). In particular, the manager in our model has market-timing preferences in that he would like to issue overvalued securities and repurchase undervalued equity in order to transfer gains to insider-shareholders. Using simulated data, we run regressions similar to those reported by Baker and Wurgler. In simulated data, the coefficient on their market-timing variable (external finance weighted average q) is insignificant. In a rational securities market, managers with market-timing preferences do not generate significant market-timing coefficients.

It is worthwhile to contrast the results of our model with those emerging from static models and to pinpoint the cause of differences. A key insight provided in this paper is that static models of financing under asymmetric information are likely to overstate the case for debt and to overstate the reduction in investment because they effectively assume that the market imperfection *vanishes* once the first (and only) period is over. This procedure effectively imposes the restriction that the shadow value of a dollar of future internal funds (net worth) is simply a dollar. However, in a world with asymmetric information, managers recognize that future internal funds are worth more than a dollar if they allow the firm to avoid adverse selection costs. This consideration encourages firms to invest more than the full-information first-best, since current investment generates future internal funds. The same consideration encourages saving and discourages debt.

Effectively, asymmetric information introduces concavity (pseudo-risk-aversion) into the value function of the manager, despite the fact that he is risk-neutral. Intuitively, internal funds are especially valuable in low net worth states when the firm is concerned that the flotation of large blocks of securities will send a negative signal. Considerations of efficiency demand that the investor insure the pseudo-risk-averse manager by lowering the firm's debt commitment. However, this *endogenous* risk-aversion also causes the investor to view debt finance as a positive signal. Thus, there is a trade-off between efficient risk-sharing and information revelation.

The other point emphasized by our model is that the literature tends to focus too heavily on predictions regarding firm behavior when private information is good. If one is taking a theory to the data, it must be recognized that private information can be bad as well as good. With this in mind, we note that in the least-cost separating equilibrium, there is no sense in imposing costs on the low type. Therefore, when private information is bad, the firm will be allowed to issue equity on fair terms in order to build up its cash buffer-stock. In addition to precautionary motives, this fact explains why the simulated firms in our model issue less debt than predicted by the pecking-order.

At this point we provide a summary of closely related papers. Our model is most closely related to that of Lucas and McDonald (1990) who analyze optimal investment timing in a dynamic economy where the manager receives information one-step-ahead of the market. Our model is more general in that it analyzes optimal financial structure, while Lucas and McDonald assume the firm is constrained to finance with external equity. Gomes (2001), Cooley and Quadrini (2001) and Hennessy and Whited (2006) present dynamic models of financing and investment with reduced-form costs of external equity. Although reduced-form costs of external equity have been motivated by appealing to asymmetric information, our model shows that the two approaches are not isomorphic. For example, in our model the firm can costlessly raise equity on fair terms when it realizes a low profit shock. When the profit shock is high, the firm signals its type by issuing less equity, issuing more debt, and investing more. Such cross-sectional predictions do not emerge from models with reduced-form costs of equity.

In the interest of clarity, we note that a number of recent papers have made progress in characterizing optimal contracts in dynamic settings when there is asymmetric information ex post regarding true cash flows. For examples, see DeMarzo and Fishman (2003) and Biais et al. (2006). Our model differs fundamentally from these papers in that the timing of informational asymmetry is different. Our model is in the spirit of Myers and Majluf (1984) in that the manager enjoys superior information ex ante, while cash flow is observable ex post. The alternative models are in the spirit of Bolton and Scharfstein (1990), where the manager privately observes end-of-period cash flows.

The pecking-order hypotheses have been formally assessed in a number of single-period models. The interested reader is referred to Tirole (2006) for a comprehensive survey of this literature. Noe (1988) shows that if the manager has no uncertainty regarding terminal cash flow, then debt is the unique source of external funds in a pooling equilibrium that satisfies standard refinements. Nachman and Noe (1994) identify necessary and sufficient conditions for debt to be the unique source of financing in a pooling equilibrium. A limitation of the Nachman and Noe framework is that it prohibits share repurchases and holds fixed the scale of investment. This is not without loss of generality. For example, Constantinides and Grundy (1990) and Miranda (2006) consider settings with variable project scale and show that share repurchases allow the firm to costlessly overcome adverse selection provided default is costless. DeMarzo and Duffie (1999) derive sufficient conditions for debt to be an optimal separating contract.

Our model is in the spirit of the signaling literature in corporate finance, which was pioneered by Ross (1977) and Leland and Pyle (1977). Although the source of deadweight loss is different, the least-cost separating equilibrium in our model has a clear analog with that constructed in the single-period model of Ambarish, John and Williams (1987). In our model, investment scale and debt are used as signals. Their model uses investment scale and taxable dividends as signals. In our model, overinvestment serves as a positive signal since asymmetric information concerns the profitability of investment, rather than internal resources (net worth). This contrasts with the model of Miller and Rock (1985) where firms with a high private observation of net worth signal this fact by underinvesting. Ambarish, John and Williams (1987) were the first to show that the signal content of real investment hinges upon whether the informational asymmetry concerns net worth or growth options.

The remainder of this paper is organized as follows. Section 2 presents assumptions on technology and describes the structure of the signaling game. Section 3 characterizes the least-cost separating equilibrium. In section 4, we use simulated model data to evaluate the empirical implications of informational asymmetry.

2 Economic Environment

2.1 Technology and Timing

Time is discrete and the firm's horizon is infinite. There is a risk-free asset paying a constant rate of interest $r > 0$. All agents are risk-neutral and share a common discount factor $\beta \equiv (1+r)^{-1}$. Capital (k) decays exponentially at rate $\delta \in [0, 1]$. The following two assumptions describe the production technology and timing of information revelation.

Assumption 1. Operating profits are $\theta\varepsilon k^{\alpha}$ where $\alpha \in (0, 1)$. The shock θ takes values in the set $\{\theta_L, \theta_H\}$ with $0 \leq \theta_L < \theta_H$ and follows a first-order Markov process with π_{ij} denoting the probability of θ_i conditional on lagged type θ_j. The shock θ is persistent with $1 > \pi_{HH} \geq \pi_{HL} > 0$. The idiosyncratic shock ε is independently and identically distributed in the set of positive real numbers $\Re_+$. The density function $f : [\underline{\varepsilon}, \infty) \rightarrow [0, 1]$ is continuous and differentiable.

Assumption 2. At the start of each period, the current θ is privately observed by the manager. Financing and investment is then determined. At the end of the period, ε is revealed simultaneously to the manager and the investor. Realized output is observable, allowing the investor to infer θ at the end of the period.

Under the stated timing assumptions, the manager is "one-step-ahead" of the market. This approximates the environment facing managers. Managers know they observe some information about the firm prior to the market, but they also understand that such information will eventually become public. The model captures this economic reality in a tractable manner. The stated timing assumptions are similar to those adopted by Lucas and McDonald (1990), who also impute to the manager one-step-ahead knowledge.

The firm can raise external funds by borrowing or issuing additional shares of equity. The borrowing technology consists of a one-period debt contract, analogous to those featured in the models of Cooley and Quadrini (2001) and Hennessy and Whited (2006). The face value of debt for the type i firm is denoted b_i. After observing this period's idiosyncratic shock ε, *but prior to the revelation of next period's* θ, the manager decides whether to default. If the firm delivers the promised debt payment, shareholders retain ownership. In the event of default, the now symmetrically-informed lender (per Assumption 2) renegotiates with the firm. The lender has full ex post bargaining power and extracts all bilateral surplus by demanding a payment that leaves the firm indifferent between continuing or not. This set of assumptions allows us to derive endogenous default thresholds, analogous to those obtained from smooth-pasting conditions in continuous-time models. Although the manager makes his default decision prior to observing next period's θ, the persistence of θ implies that a manager with type θ_H exhibits a greater willingness to pay the firm's debt.

The variable w denotes *realized net worth,* which is the sum of capital net of depreciation plus operating profits less debt service.

$$w \equiv (1-\delta)k + \theta\varepsilon k^{\alpha} - b. \tag{1}$$

There are two state variables in the model: the firm's lagged type (exogenous) and its *revised net worth* ($\widetilde{w}$) which is endogenous. Revised net worth is equal to realized net worth if the firm does not default. If the firm defaults, the lender extracts a payment equal to the maximum amount consistent with limited liability. This leaves a defaulting firm with a type-contingent minimal level of net worth. The set of possible values of revised net worth is denoted $\widetilde{W}_j$, where j indexes the manager's type at the time the debt contract was signed.

Suppose that we are at the start of period t, but that the manager has not yet observed θ for period t. The total market value of shareholder's equity, conditional upon having drawn type j in the *prior* period $(t-1)$ is denoted $v_j : \widetilde{W}_j \rightarrow \Re_+$. The default threshold (for net worth) for a firm that is of type j at the time of loan inception is denoted w_j^d. The limited liability (endogenous default) condition can be stated as

$$v_j(w_j^d) = 0. \tag{2}$$

It is easily verified that v_j is strictly increasing in our model. Thus, equation (2) determines a unique default threshold for each borrower type. Further, the option value inherent in the firm implies that $w_j^d < 0$ for $j \in \{L, H\}$. Intuitively, the ability to exploit future positive NPV projects ensures that firms are willing to continue even when net worth is negative. Below, we will speak of -w_j^d as representing the "going-concern" value of the firm.

In the special case where the shock θ is i.i.d., the analysis simplifies since there is no need to keep track of the lagged type as a state variable. In particular,

$$\theta \text{ i.i.d.} \Rightarrow v_H = v_L \equiv v \Rightarrow w_H^d = w_L^d \equiv w^d. \tag{3}$$

In our model, the true θ will be revealed to the investor each period in a separating equilibrium. However, to establish separation, it will be necessary to consider the payoffs to type-i that receives the allocation of type-j where j does not necessarily equal i. Consider first incentives to default on the debt obligation. Let ε_{ij}^d denote the critical value of ε such that type-i would default given that it has received the capital stock and debt obligation of type-j. Now recall that the firm always has the option to default at the end of the period and walk out of the renegotiation with net worth w_i^d. Therefore, the manager will find it optimal to default for any realization of ε such that realized net worth falls below w_i^d. It follows that

$$(1-\delta)k_j + \theta_i \varepsilon_{ij}^d k_j^\alpha - b_j \equiv w_i^d \Rightarrow \varepsilon_{ij}^d \equiv \frac{b_j - (1-\delta)k_j + w_i^d}{\theta_i k_j^\alpha}. \tag{4}$$

Of course, $\varepsilon_{ij}^d \leq \underline{\varepsilon}$ implies there is zero probability of default for type-i that has received the allocation of type-j.

Myers (1984) states that "the modified pecking order story recognizes both asymmetric information and costs of financial distress." The existence of default costs is supported by the empirical studies of Weiss (1990) and Andrade and Kaplan (1998), for example. To account for default costs, we assume that a fraction (ϕ) of going-concern value is spent on legal fees in the event of post-default renegotiations between the firm and the lender.

We now compute the value of lender recoveries in the event of default. Recall that in the event of default the lender demands a revised payment from the firm, call it $b_i^r \neq b_i$, that leaves the firm with revised net worth equal to w_i^d. Therefore, we compute b_i^r using

$$(1-\delta)k_i + \theta_i \varepsilon k_i^\alpha - b_i^r = w_i^d \Rightarrow b_i^r = (1-\delta)k_i + \theta_i \varepsilon k_i^\alpha - w_i^d. \tag{5}$$

Recall that $-w_i^d > 0$ represents the going-concern value of the firm. Thus, equation (5) tell us that the lender seizes the firm's physical assets, all operating profits, and the going-concern value. This leaves equity with a continuation value of zero, consistent with the absolute priority rule. The lender's net recovery in the event of default is computed as:

$$b_i^r + \phi w_i^d = (1-\delta)k_i + \theta_i \varepsilon k_i^\alpha - (1-\phi) w_i^d. \tag{6}$$

In the model, the firm is also allowed to save ($b < 0$). Myers and Majluf (1984) and Myers (1984) argue that firms should attempt to maintain financial slack in order to reduce deadweight losses associated with obtaining external funds under asymmetric information. Shyam-Sunder and Myers (1999) argue that "tax or other costs of holding excess funds" serve as a counterweight to precautionary saving incentives. To account for such costs in a tractable manner, we assume the firm incurs an up-front cost of $\gamma b^2/2$ on deposits into a corporate savings account that offers a gross yield r. This specification of costs of internal funds can be viewed as representing "influence costs." For example, firms with excess cash holdings may be vulnerable to higher compensation demands by employees or suppliers. We opt for this specification rather than tax costs in order to distinguish our model from models based on standard trade-off theory. Below, the variable χ is an indicator function for $b < 0$.

Assumption 3 summarizes the firm's borrowing/savings technology.

Assumption 3. Default is endogenous. In the event of default, the lender has all ex post bargaining power. The deadweight default cost for a firm of type-i is a fraction (ϕ) of going-concern value ($-w_i^d$). The gross yield on corporate saving is r, with the firm incurring a cost of $\gamma b^2/2$ up-front on saving deposits.

In the interest of brevity, let Ω^{ij} denote the expected discounted end-of-period value of shareholders' equity for a type-i that has taken the type-j allocation. We have:

$$\Omega^{ij} \equiv \beta \int_{\varepsilon_{ij}^d}^{\infty} v_i[(1-\delta)k_j + \theta_i \varepsilon k_j^{\alpha} - b_j] f(\varepsilon) d\varepsilon. \tag{7}$$

Also, let ρ_i denote the market price of the debt issued by type i in equilibrium.

$$\rho_i \equiv \beta \left[b_i \int_{\varepsilon_{ii}^d}^{\infty} f(\varepsilon) d\varepsilon + \int_{\underline{\varepsilon}}^{\varepsilon_{ii}^d} [(1-\delta)k_i + \theta_i \varepsilon k_i^{\alpha} - (1-\phi) w_i^d] f(\varepsilon) d\varepsilon \right]. \tag{8}$$

2.2 Managerial Objectives

Each period, the firm is run in the interest of a set of single-period insider-shareholders. For simplicity, we label the insider-shareholders as "the manager." As described in the next assumption, managerial objectives are identical to those assumed by Constantinides and Grundy (1990).

Assumption 4. Each period, the firm is run by a risk-neutral insider-manager who holds a fraction of outstanding shares. The manager does not buy additional shares in the event of an equity flotation and does not tender shares in the event of a share repurchase.

The assumption of risk-neutrality rules out risk exposure as a source of credible signaling, in contrast to the model of Leland and Pyle (1977), for example. The manager holds m shares of stock. The total current number of shares outstanding at the start of the period, inclusive of manager shares, is $c > m$. The number of new shares issued is denoted n, with $n < 0$ implying that the firm is conducting a share repurchase. We define the variable s as $s \equiv n/(c+n)$. Shares are issued and repurchased ex dividend.

Our manager-based modeling framework is used solely for descriptive clarity. More generally, the same equilibrium would be obtained in an economy where managers maximize the value of the claims held by a set of dominant "current shareholders" (e.g. institutional investors) that will neither buy nor sell shares during the current period. It is worth noting that Myers and Majluf (1984) also adopt the assumption that the manager works in the interest of "passive" shareholders who do not alter their stakes even as the firm alters the number of shares outstanding.[1]

Dividends are denoted d and are constrained to be nonnegative. Absent such a constraint, the firm could avoid the costs associated with informational asymmetries by having shareholders inject their own funds directly. Under the stated assumptions, the manager receives a fraction m/c of total dividends and holds an equity stake of $m/(c+n)$ at the end of the period. Therefore, if the manager draws θ_i at the start of the period, he will choose policies to maximize

$$\left(\frac{m}{c}\right)d + \left(\frac{m}{c+n}\right)\beta \int_{\varepsilon^d}^{\infty} v_i[(1-\delta)k + \theta_i \varepsilon k^{\alpha} - b] f(\varepsilon) d\varepsilon. \tag{9}$$

It will be convenient to think of the manager as choosing between alternative "allocations." An allocation is simply a vector $a \equiv (b, d, k, s)$. Suppose now that the manager draws θ_i at the start of the period. Using the definition of s, the objective function for the privately informed manager (9) simplifies, with the type-i manager choosing the allocation that maximizes[2]

$$d + (1-s)\beta \int_{\varepsilon^d}^{\infty} v_i[(1-\delta)k + \theta_i \varepsilon k^{\alpha} - b] f(\varepsilon) d\varepsilon. \tag{10}$$

At this point it is worth noting that the vector a contains all variables relevant to the informed manager's payoff function. Of course, investor beliefs will affect the set of feasible allocations.

Assume the type-i manager reveals the firm's true type by choosing a separating allocation (b_i, d_i, k_i, s_i). If the firm issues new shares ($n > 0$), the equity flotation is worth

$$s_i \left[\beta \int_{\varepsilon_{ii}^d}^{\infty} v_i[(1-\delta)k_i + \theta_i \varepsilon k_i^{\alpha} - b_i] f(\varepsilon) d\varepsilon\right]. \tag{11}$$

It is perhaps less obvious to see that the expression in (11) also represents the cash outflow from a share repurchase, in which case n and s are both negative. To demonstrate this claim, consider a simple example. Note first that the bracketed term in (11) is the expected discounted value of shareholder's equity at the end of the period. Suppose this amount is equal to 100. Suppose also that $c = 50$ and $n = -10$. After the share repurchase, each remaining share will be worth $100/(50-10) = 2.5$. Therefore, in order to induce ten shareholders to tender, the firm must pay $25 (= 10 * 2.5)$. Now note that $s_i * 100 = -(10/40) * 100 = -25$. By construction, each outsider shareholder is indifferent between tendering or not at the margin.

[1] See page 189 of Myers and Majluf (1984) for a discussion of this assumption.
[2] We have dropped a multiplicative term (m/c) since it is irrelevant for incentive compatibility.

2.3 The Signaling Game

Although our model is dynamic, in that there is an infinite sequence of state-contingent signaling games, each period's insider-manager is properly viewed as playing a one-shot signaling game. In each signaling game, the manager moves first, offering an allocation a to the investor. The investor then updates his beliefs and either accepts or rejects the allocation. If the investor accepts the offer, the allocation is implemented. If the offer is rejected, the firm is then constrained to finance with internal funds, with $b = s = 0$.

Each manager is forward-looking by construction. In particular, the value functions (v_L, v_H) entering the manager's payoff function (10) will be constructed to correctly capitalize the outcome of all signaling games played by future managers. The equilibrium concept in each round's signaling game is perfect Bayesian equilibrium (PBE). A PBE imposes the following requirements: the manager makes an optimal offer given the investor's beliefs; the investor's beliefs must be obtained through Bayesian updating when possible; and the investor accepts an offer only if his expected profits are weakly positive given his beliefs. We focus attention on the least-cost separating PBE in pure strategies. In a separating PBE, the investor perfectly identifies the firm's type upon observing the manager's offer. The focus on separating equilibria is motivated by the empirical focus of this paper. One of the main appeals of the asymmetric information paradigm in corporate finance is its ability to explain announcement effects associated with security issuances.[3] A model focused on pooling equilibria cannot explain these announcement effects, but one focused on separating equilibria can.[4]

We begin by defining a broad set of technologically feasible allocations $\mathcal{A}$:

$$\mathcal{A} \equiv \{(b, d, k, s) : d \geq 0,\ k \geq 0,\ s \leq 1\}.$$

To derive the least-cost separating equilibrium we begin by solving Program L.[5]

$$\text{PROGRAM L:} \quad \max_{a \in \mathcal{A}} \quad d_L + (1 - s_L)\beta \int_{\varepsilon^d_{LL}}^{\infty} v_L[(1-\delta)k_L + \theta_L \varepsilon k_L^{\alpha} - b_L] f(\varepsilon) d\varepsilon$$

subject to the following budget constraint

$$BC_L : d_L + k_L + \chi\gamma b_L^2/2 - \widetilde{w} \leq$$

$$\beta \left[s_L \int_{\varepsilon^d_{LL}}^{\infty} v_L[(1-\delta)k_L + \theta_L \varepsilon k_L^{\alpha} - b_L] f(\varepsilon) d\varepsilon + b \int_{\varepsilon^d_{LL}}^{\infty} f(\varepsilon) d\varepsilon + \int_{\underline{\varepsilon}}^{\varepsilon^d_{LL}} [(1-\delta)k_L + \theta_L \varepsilon k_L^{\alpha} - (1-\phi) w_L^d] f(\varepsilon) d\varepsilon \right].$$

[3] For example, Asquith and Mullins (1986) document a negative relation between the size of equity issues and share prices. Vermaelen (1981) documents a positive reaction to share repurchases.

[4] Alternatively, one can view our model as solving for the Rothschild-Stiglitz-Wilson allocation discussed by Maskin and Tirole (1992) or the low-information-intensity-optimum of Tirole (2006). Our separating equilibria are the unique PBE of the Tirole security issuance game provided that the probability of the low type is sufficiently high.

[5] This program is solved for net worth levels sufficiently high such that the low type can satisfy BC_L. The low type gets a zero payoff if net worth is sufficiently low such that BC_L cannot be met.

Letting the solution to Program L be denoted $a_L^* \equiv (b_L^*, d_L^*, k_L^*, s_L^*)$, we next solve Program H.

$$\text{PROGRAM H:} \quad \max_{a \in \mathcal{A}} \quad d_H + (1 - s_H)\beta \int_{\varepsilon_{HH}^d}^{\infty} v_H[(1-\delta)k_H + \theta_H \varepsilon k_H^{\alpha} - b_H] f(\varepsilon) d\varepsilon$$

subject to the budget constraint

$$BC_H : d_H + k_H + \chi\gamma b_H^2/2 - \widetilde{w} \leq$$

$$\beta \left[s_H \int_{\varepsilon_{HH}^d}^{\infty} v_H[(1-\delta)k_H + \theta_H \varepsilon k_H^{\alpha} - b_H] f(\varepsilon) d\varepsilon + b \int_{\varepsilon_{HH}^d}^{\infty} f(\varepsilon) d\varepsilon + \int_{\underline{\varepsilon}}^{\varepsilon_{HH}^d} [(1-\delta)k_H + \theta_H \varepsilon k_H^{\alpha} - (1-\phi) w_H^d] f(\varepsilon) d\varepsilon \right]$$

and a no-mimic constraint

$$\begin{aligned} NM_{LH} \quad : \quad & d_L^* + (1 - s_L^*)\beta \int_{\varepsilon_{LL}^d}^{\infty} v_L[(1-\delta)k_L^* + \theta_L \varepsilon (k_L^*)^{\alpha} - b_L^*] f(\varepsilon) d\varepsilon \geq \\ & d_H + (1 - s_H)\beta \int_{\varepsilon_{LH}^d}^{\infty} v_L[(1-\delta)k_H + \theta_L \varepsilon k_H^{\alpha} - b_H] f(\varepsilon) d\varepsilon. \end{aligned}$$

Notice that in solving Program L we did not impose a no-mimic constraint. Lemma 1 shows this is without loss of generality.[6]

Lemma 1. *Assume that a_H^* and a_L^* solve Program H and Program L respectively, and that $s_L^* \geq 0$. Then*

$$d_H^* + (1-s_H^*)\beta \int_{\varepsilon_{HH}^d}^{\infty} v_H[(1-\delta)k_H^* + \theta_H \varepsilon (k_H^*)^{\alpha} - b_H^*] f(\varepsilon) d\varepsilon \geq d_L^* + (1-s_L^*)\beta \int_{\varepsilon_{HL}^d}^{\infty} v_H[(1-\delta)k_L^* + \theta_H \varepsilon (k_L^*)^{\alpha} - b_L^*] f(\varepsilon) d\varepsilon.$$

Proof. See Appendix A.

Let $\widehat{\theta}(a)$ denote the firm type inferred by the investor conditional upon receiving an arbitrary offer a. A PBE in which the firm receives the type-contingent allocations (a_L^*, a_H^*) can be supported by the following investor beliefs.

$$\begin{aligned} \widehat{\theta}(a_H^*) &= \theta_H, \\ \widehat{\theta}(a_L^*) &= \theta_L, \\ \widehat{\theta}(a) &\in \arg \min_{\theta \in \{\theta_L, \theta_H\}} \quad s \int_{\varepsilon_{\theta\theta}^d}^{\infty} v_\theta[(1-\delta)k + \theta \varepsilon k^{\alpha} - b] f(\varepsilon) d\varepsilon + \\ & \quad b \int_{\varepsilon_{\theta\theta}^d}^{\infty} f(\varepsilon) d\varepsilon + \int_{\underline{\varepsilon}}^{\varepsilon_{\theta\theta}^d} [(1-\delta)k + \theta \varepsilon k^{\alpha} - (1-\phi) w_\theta^d] f(\varepsilon) d\varepsilon, \\ \forall a &\notin \{a_L^*, a_H^*\}. \end{aligned} \tag{12}$$

On the equilibrium path, the beliefs in (12) are consistent with Bayes' rule. Off the equilibrium path, the investor imposes "worst-case" beliefs in the sense of Brennan and Kraus (1987). In particular, the investor

[6] In solving Program L, we may confine attention to $s_L \geq 0$ without loss of generality. To see this, note that a share repurchase by the low type can be replaced with a dividend without affecting the value of the objective function.

attaches the lowest possible valuation to any package of securities not issued in equilibrium. If the firm were to issue shares and/or debt, a worst-case belief imputes type θ_L. As another example, if the firm were to repurchase shares (and issue no debt), then a worst-case belief imputes type θ_H.

We now verify that the solutions to Programs L and H in conjunction with beliefs (12) constitute a PBE. First note that the beliefs are consistent with Bayes' rule on the equilibrium path. Second, note that any offer $a_0 \notin \{a_L^*, a_H^*\}$ that would be acceptable to the investor necessarily satisfies both BC_L and BC_H, because the beliefs minimize the value of the package of securities.

We now verify that each type will choose to make his type-specific offer. Consider first the low type's incentive to offer some allocation $a_0 \notin \{a_L^*, a_H^*\}$ that would be acceptable to the investor given the beliefs in (12). Since a_0 is acceptable, it must satisfy BC_L. Thus, a_0 was in the feasible set for Program L and the low type must prefer a_L^* to a_0. This same argument shows that the low type will not make an offer that is rejected, since such an allocation is equivalent to getting zero outside funding, and zero outside funding is always acceptable to the investor. Therefore, the low type prefers the offer a_L^* to all other allocations other than a_H^*. Finally, NM_{LH} ensures the low type prefers a_L^* to a_H^*.

Consider next the high type's incentive to offer some allocation $a_0 \neq a_H^*$ that would be accepted by the investor given the beliefs in (12). Note that the allocation a_0 was in the feasible set when we solved Program H. To see this, note that BC_H is necessarily satisfied, since the offer is acceptable even with worst-case beliefs. Since a_0 also satisfies BC_L, we know a_0 was in the feasible set for Program L. The optimality of a_L^* in Program L implies that NM_{LH} would be satisfied if the high type were to offer a_0. The optimality of a_H^* for Program H implies the high type prefers a_H^* to a_0. This same argument shows that the high type will not make an offer that is rejected, since such an allocation is equivalent to zero outside funding, and zero outside funding is always acceptable to the investor.

The final step in the construction will be to define the equity value function recursively, using

$$\begin{aligned} v_j(w) \quad \equiv \quad & \pi_{Hj} \left[d_H^* + (1-s_H^*)\beta \int_{\varepsilon_{HH}^d}^{\infty} v_H[(1-\delta)k_H^* + \theta_H \varepsilon (k_H^*)^\alpha - b_H^*] f(\varepsilon) d\varepsilon \right] \\ & +(1-\pi_{Hj}) \left[d_L^* + (1-s_L^*)\beta \int_{\varepsilon_{LL}^d}^{\infty} v_L[(1-\delta)k_L^* + \theta_L \varepsilon (k_L^*)^\alpha - b_L^*] f(\varepsilon) d\varepsilon \right]. \end{aligned} \tag{13}$$

Although above we have dropped w as an argument in the optimal allocation vectors, it is worth stressing at this point that the allocations $(a_L^*, a_H^*)(w)$ are wealth-contingent.

3 Equilibrium

The full-information first-best investment policy solves

$$
\begin{aligned}
k_i^{FB} \quad &\in \quad \arg\max_k \quad \beta \int_{\underline{\varepsilon}}^{\infty} [(1-\delta)k + \theta_i \varepsilon k^{\alpha}] f(\varepsilon) d\varepsilon - k \\
&\Rightarrow \quad k_i^{FB} = \left[\frac{\alpha \theta_i E(\varepsilon)}{r+\delta} \right]^{1/(1-\alpha)}.
\end{aligned} \tag{14}
$$

The following remark provides a description of the full-information economy. This provides a useful benchmark for assessing the relative allocative efficiency of the economy where managers have private information.

Remark. If the profit shock θ_i is public information, default is costly ($\phi > 0$), and corporate saving is costly ($\gamma > 0$), then the firm implements first-best investment (k_i^{FB}) using a combination of equity and default-free debt. There will be no corporate saving ($b_i \geq 0$).

Although they do not emphasize the point, one of the key premises of Myers and Majluf (1984) and Myers (1984) is the existence of costs of default. The critical role of this assumption is illustrated in the following subsection, which shows that the firm can implement first-best investment with costless signaling through debt issuance provided that default is costless. Essentially, when default is costless, the firm relies entirely upon debt to signal good information.

3.1 The Possibility of Costless Separation

Constantinides and Grundy (1990) adopt the assumption that default is costless. In their static model, they show that a firm with variable investment scale can costlessly implement the first-best investment policy even if the set of financing instruments is limited to equity and standard debt. In their model, the issuance of any security is a negative signal. However, equity is more sensitive to the manager's private information than debt. First-best is achieved with the firm issuing debt in excess of the amount needed to fund the investment. The excess funds are then used to finance a share repurchase. Effectively, the negative signal content of the debt flotation is just compensated by the positive signal provided by the share repurchase. Of course, the probability of default is likely to be high under such a separating policy. However, this has no effect on total firm value under their adopted assumption of costless default.

Consider now our dynamic model. Let p_i^{FB} denote the net present value of investment for a firm that implements the first-best investment policy:

$$
p_i^{FB} \equiv \beta[(1-\delta)k_i^{FB} + \theta_i E(\varepsilon)(k_i^{FB})^{\alpha}] - k_i^{FB}. \tag{15}
$$

For a firm that can implement the full-information first-best each period using fairly-priced external financing (with no deadweight losses), the type-contingent going-concern value of the firm (-w_L^{d*}, $-w_H^{d*}$) can be found as the solution to the following system

$$\begin{aligned} -w_H^{d*} &= \pi_{HH}[p_H^{FB} - \beta w_H^{d*}] + (1-\pi_{HH})[p_L^{FB} - \beta w_L^{d*}] \\ -w_L^{d*} &= \pi_{HL}[p_H^{FB} - \beta w_H^{d*}] + (1-\pi_{HL})[p_L^{FB} - \beta w_L^{d*}]. \end{aligned} \tag{16}$$

Assumption 1 guarantees that the solution to the above system satisfies

$$w_H^{d*} \leq w_L^{d*}. \tag{17}$$

Finally, we note that if a firm can implement first-best investment using fairly-priced external financing with zero default cost then equity value is linear in internal resources, with

$$v_i(\widetilde{w}) = v_i^{FB}(\widetilde{w}) \equiv \widetilde{w} - w_i^{d*}.$$

This brings us to Proposition 1, which provides weak conditions on primitives such that the firm implements first-best provided that $\phi = 0$.

Proposition 1. *If the idiosyncratic shock ε is bounded above by $\overline{\varepsilon}$ and there are no costs of default, the firm achieves the full-information first-best valuation v_i^{FB} implementing first-best investment k_i^{FB} for $i \in \{L, H\}$. If $\widetilde{w} < k_H^{FB}$, the low type sets $b_L = 0$ and the high type sets $b_H = (1-\delta)k_H^{FB} + \theta_L \overline{\varepsilon}(k_H^{FB})^\alpha - w_L^{d*}$. If $\widetilde{w} \geq k_H^{FB}$, both firms finance investment using internal funds exclusively.*

Proof. Fix $k_i = k_i^{FB}$ and choose s_i to satisfy BC_i. The proposed set of financing policies clearly solves Programs L and H ignoring the NM_{LH} constraint and we need only verify it is satisfied. Suppose first $\widetilde{w} < k_H^{FB}$. Consider the above policies and set $d_L = \max\{0, \widetilde{w} - k_L^{FB}\}$ and $d_H = 0$. But note that NM_{LH} is slack since

$$d_L + (1 - s_L)\Omega^{LL} \geq (1 - s_H)\Omega^{LH} = 0.$$

Suppose next that $\widetilde{w} \geq k_H^{FB}$. Then NM_{LH} is satisfied with $s_i = b_i = 0$ and $d_i = \max\{0, \widetilde{w} - k_i^{FB}\}$. To see this, note that each type is financing with internal funds in this case so the low type must be better off choosing k_L^{FB}.■

The proof of Proposition 1 is worthy of further discussion since it runs parallel to Proposition 4 from Constantinides and Grundy (1990). Under the policies specified in our proposition, the high type is able to costlessly separate from the low type by raising the face value of its debt sufficiently high such that an imposter low type would default regardless of the realized value of ε. Of course, such a high debt payment increases the likelihood of default by the high type. However, the assumption that $\phi = 0$ ensures this is of no consequence. Of course, this method of separation becomes suspect once one introduces costs of default, which is one of the stated premises of Myers and Majluf (1984) and Myers (1984).

The following result shows that $\phi = 0$ is not a necessary condition for costless separation of types in our model, although the proposition does suggest that costless separation with $\phi > 0$ requires fairly strong restrictions on the underlying production technology.

Proposition 2. *If capital has a use-life of one period ($\delta = 1$) and the low type cannot use the capital productively ($\theta_L = 0$), then the firm achieves full-information first-best valuation v_i^{FB} implementing first-best investment k_i^{FB} for $i \in \{L, H\}$ using only external equity to fill any financing gap.*

Proof. Fix $k_i = k_i^{FB}$; $b_i = 0$; $s_L = 0$; $d_L = \widetilde{w}$; $d_H = \max\{0, \widetilde{w} - k_H^{FB}\}$, with s_H determined according to BC_H. Clearly, this set of policies solves Programs L and H ignoring the NM_{LH} constraint. Depending on the firm's net worth, the NM_{LH} constraints are

$$\begin{aligned} \widetilde{w} &\leq k_H^{FB} : \widetilde{w} - w_L^{d*} \geq -(1 - s_H) w_L^{d*} \\ \widetilde{w} &> k_H^{FB} : \widetilde{w} - w_L^{d*} \geq \widetilde{w} - k_H^{FB} - w_L^{d*}. \end{aligned}$$

Both constraints are satisfied.■

As a limiting case, Proposition 2 is of some interest for the analysis that follows. Under the stated assumptions, the low type places a value of zero on any physical capital. Therefore, the low type has no interest in raising funds through an equity issuance, despite the fact that it would seem to benefit from issuing overvalued equity. However, the money raised in the equity issuance is of no value to the low type since the funds go to unproductive capital.

We turn our attention in the next subsection to the nature of equilibrium when the firm cannot achieve the full-information first-best valuation.

3.2 Low Type Policies

In this subsection and the next, it is assumed that $\phi > 0$ which is necessary to preclude the non-distorting equilibrium described in Proposition 1. In order to express the optimality conditions compactly, we define some additional variables. It will be convenient to compute the marginal effect of b on the expected discounted end-of-period value of shareholders' equity for a high-type (low-type) that has taken the high-type allocation. We have

$$\begin{aligned} \Omega_b^{HH} &\equiv -\beta \int_{\varepsilon_{HH}^d}^{\infty} v'_H[(1-\delta)k_H + \theta_H \varepsilon k_H^{\alpha} - b_H] f(\varepsilon) d\varepsilon \qquad (18) \\ \Omega_b^{LH} &\equiv -\beta \int_{\varepsilon_{LH}^d}^{\infty} v'_L[(1-\delta)k_H + \theta_L \varepsilon k_H^{\alpha} - b_H] f(\varepsilon) d\varepsilon. \end{aligned}$$

Similarly, we may compute the marginal effect of k on the expected discounted end-of-period value of shareholders' equity for a high-type (low-type) that has taken the high-type allocation as follows:

$$\begin{aligned} \Omega_k^{HH} &\equiv \beta \int_{\varepsilon_{HH}^d}^{\infty} v_H'[(1-\delta)k_H + \theta_H \varepsilon k_H^{\alpha} - b_H][1-\delta+\alpha\theta_H\varepsilon k_H^{\alpha-1}]f(\varepsilon)d\varepsilon \qquad (19) \\ \Omega_k^{LH} &\equiv \beta \int_{\varepsilon_{LH}^d}^{\infty} v_L'[(1-\delta)k_H + \theta_L \varepsilon k_H^{\alpha} - b_H][1-\delta+\alpha\theta_L\varepsilon k_H^{\alpha-1}]f(\varepsilon)d\varepsilon. \end{aligned}$$

We will also need to introduce the indicator function Φ denoting a firm whose equity will be worth zero if the low type is realized in the *subsequent* period. Of course, equity value hinges upon net worth, so we shall write $\Phi(w)$. Similarly, we shall let μ denote the multiplier on the NM_{LH} constraint and write it as $\mu(w)$ to emphasize that this multiplier is also contingent upon the net worth.

In order to interpret the optimality conditions in this model, it will be useful to have a sense of the magnitude of the shadow value of internal resources. This is the subject of Lemma 2.

Lemma 2. *The shadow value of internal resources on the equilibrium path is*

$$v_L'(w) = 1 + \pi_{HL}\mu(w)(\Omega^{HH} - \Omega^{LH})/\Omega^{HH} \qquad (20)$$

$$v_H'(w) = \pi_{HH}[1 + \mu(w)(\Omega^{HH} - \Omega^{LH})/\Omega^{HH}] + (1-\pi_{HH})[1-\Phi(w)]. \qquad (21)$$

Proof. See Appendix A.

In order to provide some intuition for Lemma 2 we must foreshadow some results that follow. We begin first with a discussion of equation (20), which quantifies the shadow value of internal funds for a firm with lagged type θ_L. On the equilibrium path, a firm with lagged type θ_L will always have strictly positive equity value in the subsequent period regardless of ε and regardless of the observed type. This is because the low type saves ($b_L \leq 0$) implying that it will enter the subsequent period with $w > 0$ regardless of the ε shock. If the realized type in the subsequent period is θ_L, a dollar of internal funds is just worth a dollar, as the firm essentially obtains financing on fair terms. However, if the realized type in the subsequent period is θ_H, a dollar of internal funds will be worth more than a dollar if it allows the firm to avoid costs associated with asymmetric information. These effects are clearly illustrated in (21) as $v_L' > 1$ when the NM_{LH} constraint binds.

Consider next equation (21), which quantifies the shadow value of internal funds for a firm with lagged type θ_H. In contrast to the low type, the high type may take on debt in order to signal its type ($b_H > 0$). If the firm then experiences a sufficiently low draw of ε, end-of-period net worth can be negative. If the next realized type is θ_L, it may be optimal to shut down.[7] This gives rise to a variant of the Myers (1977) debt overhang problem, in that the possibility of shut-down in bad states causes the firm to place a lower value

[7]Technically, this occurs when the low type cannot satisfy BC_L even if $s_L = 1$ and $d_L = 0$. In contrast, on the non-default region, equity must have strictly positive value if θ_H is drawn.

of internal funds, ceteris paribus. However, equation (21) tells us that the shadow value of internal funds will still exceed unity if the precautionary motive swamps the overhang effect.

Lemma 2 is of fundamental importance for our dynamic theory of investment and financing under asymmetric information. In particular, standard one-period models of the firm implicitly force $v' = 1$. This is because a firm that simply vanishes is never forced to confront informational asymmetries after the initial round of financing is obtained. Hence, in static models, the value of a dollar received at the terminal date of the firm is simply a dollar. In contrast, in a forward-looking framework, the manager recognizes that a dollar of internal funds in the future can have precautionary value.

The Lagrangian for Program L is

$$\begin{aligned} L &= d_L + (1 - s_L)\beta \int_{\varepsilon^d_{LL}}^{\infty} v_L[(1-\delta)k_L + \theta_L \varepsilon k_L^{\alpha} - b_L] f(\varepsilon) d\varepsilon + \\ & \quad \lambda_L \{\widetilde{w} - d_L - k_L - \chi\gamma b_L^2/2 + \beta s_L \int_{\varepsilon^d_{LL}}^{\infty} v_L[(1-\delta)k_L + \theta_L \varepsilon k_L^{\alpha} - b_L] f(\varepsilon) d\varepsilon \\ & \quad + \beta b_L \int_{\varepsilon^d_{LL}}^{\infty} f(\varepsilon) d\varepsilon + \beta \int_{\underline{\varepsilon}}^{\varepsilon^d_{LL}} [(1-\delta)k_L + \theta_L \varepsilon k_L^{\alpha} - (1-\phi) w_L^d] f(\varepsilon) d\varepsilon\} \\ & \quad + \eta_L d_L + \psi_L (1 - s_L). \end{aligned}$$

The first-order conditions for d_L and s_L are

$$1 - \lambda_L + \eta_L = 0 \tag{22}$$

$$(\lambda_L - 1)\Omega^{LL} - \psi_L = 0. \tag{23}$$

From equations (22) and (23) it follows that $\eta_L \Omega^{LL} = \psi_L$. This brings up two relevant scenarios. Suppose first that $\psi_L > 0$. It follows that $\eta_L > 0$ and $d_L = 0$. It follows that the low type is getting a payoff of zero since $s_L = 1$. If ε has bounded support, the constraint NM_{LH} is then satisfied by having the high type take on a sufficient amount of debt such that the low type would default in all states. If ε has unbounded support, $\psi_L(w) > 0$ can never occur for w on the continuation region. To see this, note that the proposed equilibrium would entail the low type getting zero. But then the low type would always opt for the high type allocation unless $s_H = 1$ and $d_H = 0$. Of course, this implies both types get a payoff of zero which contradicts being on the continuation region. For the remainder of this subsection we confine attention to the economically interesting case where $\psi_L = 0$.

The first-order condition pinning down b_L is

$$\beta \left[\int_{\varepsilon^d_{LL}}^{\infty} [v_L'((1-\delta)k_L + \theta_L \varepsilon k_L^{\alpha} - b_L) - 1] f(\varepsilon) d\varepsilon \right] = -\chi\gamma b_L + \beta \frac{\partial \varepsilon^d_{LL}}{\partial b_L} f(\varepsilon^d_{LL}) \phi w_L^d. \tag{24}$$

From Lemma 2 it follows that the left side of (24) is positive, and strictly so provided that NM_{LH} will bind with positive probability in the subsequent period. It follows that b_L must be negative. The optimality condition for k_L takes a similar form, with

$$\beta\left[\int_{\underline{\varepsilon}}^{\infty}[v_L'((1-\delta)k_L+\theta_L\varepsilon k_L^{\alpha}-b_L)][1-\delta+\alpha\theta_L\varepsilon k_L^{\alpha-1}]f(\varepsilon)d\varepsilon\right]=1.$$

The next proposition follows directly from the first-order conditions for the debt and capital of the low type.

Proposition 3. *If the manager observes θ_L, the firm overinvests relative to first-best with $k_L > k_L^{FB}$ if there is a positive probability of NM_{LH} binding in the subsequent period. The low type engages in precautionary saving with*

$$b_L = -\frac{\beta}{\gamma}\left[\int_{\underline{\varepsilon}}^{\infty}[v_L'((1-\delta)k_L+\theta_L\varepsilon k_L^{\alpha}-b_L)-1]f(\varepsilon)d\varepsilon\right]. \tag{25}$$

Dividends and equity issuance for the low type are contingent upon net worth with

$$\begin{aligned}\widetilde{w} &< k_L-\beta b_L+\gamma b_L^2/2 \Rightarrow d_L=0 \;\; \textit{and } s_L>0\\ \widetilde{w} &\geq k_L-\beta b_L+\gamma b_L^2/2 \Rightarrow d_L=\widetilde{w}-(k_L-\beta b_L+\gamma b_L^2/2) \;\; \textit{and } s_L=0.\end{aligned}$$

The intuition for the low type policies are as follows. The fact that the low type would benefit from security mispricing causes NM_{LH} to be the key incentive constraint. In order to discourage imitation by the low type, the least-cost separating equilibrium makes the low type as well off as possible. This is a standard feature of signaling models. However, in most signaling models, the optimal policy entails giving the low type the full-information first-best allocation. This does *not* hold in our dynamic model. In our model, the low type is given a "second-best" allocation which accounts for the fact that the shadow value of internal resources exceeds unity. Consequently, the low type overinvests relative to first-best and engages in precautionary saving despite the fact that such saving entails deadweight losses. It is also worth noting that both b_L and k_L are invariant to net worth ($\widetilde{w}$). Effectively, the low type satisfies BC_L by varying dividends and equity issuance only. Note, *this is the exact opposite of the pecking-order prediction that debt (and only debt) is used to achieve budget balance.*

3.3 High Type Policies

The Lagrangian for Program H is[8]

$$
\begin{aligned}
L \;=\;& d_H + (1-s_H)\beta\int_{\varepsilon^d_{HH}}^{\infty} v_H[(1-\delta)k_H + \theta_H\varepsilon k_H^{\alpha} - b_H]f(\varepsilon)d\varepsilon \\
&+\lambda_H\{\widetilde{w} - d_H - k_H - \chi\gamma b_H^2/2 + \beta s_H\int_{\varepsilon^d_{HH}}^{\infty} v_H[(1-\delta)k_H + \theta_H\varepsilon k_H^{\alpha} - b_H]f(\varepsilon)d\varepsilon \\
&+\beta b_H\int_{\varepsilon^d_{HH}}^{\infty} f(\varepsilon)d\varepsilon + \beta\int_{\underline{\varepsilon}}^{\varepsilon^d_{HH}}[(1-\delta)k_H + \theta_H\varepsilon k_H^{\alpha} - (1-\phi)w^d_H]f(\varepsilon)d\varepsilon\} \\
&+\mu\{d_L^* + (1-s_L^*)\beta\int_{\underline{\varepsilon}}^{\infty} v_L[(1-\delta)k_L^* + \theta_L\varepsilon(k_L^*)^{\alpha} - b_L^*]f(\varepsilon)d\varepsilon \\
&-d_H - (1-s_H)\beta\int_{\varepsilon^d_{LH}}^{\infty} v_L[(1-\delta)k_H + \theta_L\varepsilon k_H^{\alpha} - b_H]f(\varepsilon)d\varepsilon\} \\
&+\eta_H d_H + \psi_H(1-s_H).
\end{aligned}
$$

The first-order conditions for d_H and s_H are

$$1 - \lambda_H - \mu + \eta_H = 0 \tag{26}$$

$$(\lambda_H - 1)\Omega^{HH} + \mu\Omega^{LH} - \psi_H = 0 \tag{27}$$

Substituting (26) into (27), we obtain

$$\eta_H\Omega^{HH} = \mu(\Omega^{HH} - \Omega^{LH}) + \psi_H. \tag{28}$$

It is straightforward to establish $\psi_H(w) = 0$ on the continuation region ($w > w_j^d$). To see this, suppose to the contrary that $\psi_H > 0$. It follows from (28) that $\eta_H > 0$ and $d_H = 0$. Thus, the high type gets a payoff of zero. However, the low type would then also receive an allocation with a payoff of zero since the NM_{HL} constraint demands

$$d_H + (1-s_H)\Omega^{HH} \geq d_L + (1-s_L)\Omega^{HL} \geq d_L + (1-s_L)\Omega^{LL}. \tag{29}$$

But this contradicts $v_j(w) > 0$. Without loss of generality we shall treat $\psi_H = 0$ as we solve for optimal policies on the firm's continuation region.

Rearranging (28) we obtain

$$\eta_H = \mu[(\Omega^{HH} - \Omega^{LH})/\Omega^{HH}]. \tag{30}$$

Condition (30) is of fundamental importance. It tells us that whenever NM_{LH} binds, the manager would be better off if the firm could implement a rights-issue in which shareholders are paid a negative dividend. That

[8] Note, Program H is solved only if the low type achieves a strictly positive continuation value in Program L. The preceding subsection discussed the nature of equilibrium for lower net worth levels.

is, when NM_{LH} binds, the manager would be better off if the firm could avoid turning to outside investors for funds. The adverse selection problem is manifest in condition (30), with the term in squared brackets representing the relative difference in true equity values for the two types. From this optimality condition it also follows that the high type will never pay a dividend when NM_{LH} binds since

$$\mu > 0 \Rightarrow \eta_H > 0 \Rightarrow d_H = 0. \tag{31}$$

The optimality condition pinning down b_H is

$$\begin{aligned} &\beta \left[\int_{\varepsilon^d_{HH}}^{\infty} [v'_H((1-\delta)k_H + \theta_H \varepsilon k_H^{\alpha} - b_H) - 1] f(\varepsilon) d\varepsilon - \frac{\partial \varepsilon^d_{HH}}{\partial b_H} f(\varepsilon^d_{HH}) \phi w^d_H \right] + \chi \gamma b_H \\ = \; & \left(\frac{\mu \Omega^{LH}}{\lambda_H} \right) [1 - s_H] \left[\frac{\Omega^{HH}_b}{\Omega^{HH}} - \frac{\Omega^{LH}_b}{\Omega^{LH}} \right]. \end{aligned} \tag{32}$$

We now conjecture, and then verify, that the high type will choose $b_H \leq 0$ if the NM_{LH} constraint is slack at the current level of net worth. Under the conjecture that $b_H \leq 0$, there is zero probability of shut-down regardless of next period's realized type. It follows from Lemma 2 that $v'_H \geq 1$ in this case. From (32) it follows immediately that $\mu = 0 \Rightarrow b_H \leq 0$. Condition (32) leads directly to a key implication of the model, that signaling must be a concern if the firm issues a positive amount of debt. This conclusion is stated as Lemma 3.

Lemma 3. *If the manager observes θ_H and there is a positive probability of NM_{LH} binding in the subsequent period, then a necessary condition for $b_H > 0$ is that NM_{LH} is binding in the current period.*

The optimality condition pinning down k_H is

$$\begin{aligned} &1 - \beta \left[\int_{\varepsilon^d_{HH}}^{\infty} [v'_H((1-\delta)k_H + \theta_H \varepsilon k_H^{\alpha} - b_H)][1 - \delta + \alpha \theta_H \varepsilon k_H^{\alpha-1}] f(\varepsilon) d\varepsilon \right] \\ &-\beta \left[\int_{\underline{\varepsilon}}^{\varepsilon^d_{HH}} [1 - \delta + \alpha \theta_H \varepsilon k_H^{\alpha-1}] f(\varepsilon) d\varepsilon + \frac{\partial \varepsilon^d_{HH}}{\partial k_H} f(\varepsilon^d_{HH}) \phi w^d_H \right] \\ = \; & \left(\frac{\mu \Omega^{LH}}{\lambda_H} \right) [1 - s_H] \left[\frac{\Omega^{HH}_k}{\Omega^{HH}} - \frac{\Omega^{LH}_k}{\Omega^{LH}} \right]. \end{aligned} \tag{33}$$

By way of contrast, the optimality condition in a full-information neoclassical economy is

$$1 - \beta \left[\int_{\underline{\varepsilon}}^{\infty} [1 - \delta + \alpha \theta_H \varepsilon k_H^{\alpha-1}] f(\varepsilon) d\varepsilon \right] = 0.$$

Anticipating the results of our numerical analysis, it is worth noting the appearance of the multiplicative term v'_H in the capital optimality condition (33). If $v'_H > 1$, the desire to avoid adverse selection costs provides an added incentive for capital accumulation. However, the debt overhang effect in our model may also cause $v'_H < 1$, thus discouraging capital accumulation.

From (32) and (33) it also follows that

$$
\begin{aligned}
\mu &= 0 \Rightarrow b_H = b_H^P \text{ and } k_H = k_H^P \\
&\quad \text{where} \\
b_H^P &\equiv -\frac{\beta}{\gamma}\left[\int_{\underline{\varepsilon}}^{\infty}[v_H'((1-\delta)k_H^P + \theta_H\varepsilon(k_H^P)^{\alpha} - b_H^P) - 1]f(\varepsilon)d\varepsilon\right] \\
&\quad \text{and} \\
1 &= \beta\left[\int_{\underline{\varepsilon}}^{\infty}[v_H'((1-\delta)k_H^P + \theta_H\varepsilon(k_H^P)^{\alpha} - b_H^P)][1-\delta+\alpha\theta_H\varepsilon(k_H^P)^{\alpha-1}]f(\varepsilon)d\varepsilon\right].
\end{aligned}
$$

Proposition 4 spells out some important implications of the above optimality conditions.

Proposition 4. *If NM_{LH} is ever binding, then $\exists\ \widetilde{w}_0$ at which NM_{LH} switches from binding to nonbinding. For all $\widetilde{w} > \widetilde{w}_0$, the high type overinvests relative to first-best with $k_H = k_H^P > k_H^{FB}$ and saves with $b_H = b_H^P$. For $\widetilde{w} > \widetilde{w}_0$, dividends and equity issuance are contingent upon net worth with*

$$
\begin{aligned}
\widetilde{w} &< k_H^P - \beta b_H^P + \gamma(b_H^P)^2/2 \Rightarrow d_H = 0 \quad \textit{and } s_H > 0 \\
\widetilde{w} &\geq k_H^P - \beta b_H^P + \gamma(b_H^P)^2/2 \Rightarrow d_H = \widetilde{w} - k_H^P - \beta b_H^P + \gamma(b_H^P)^2/2 \quad \textit{and } s_H = 0.
\end{aligned}
$$

Proof. See Appendix A.

Propositions 3 and 4 show that much of the folk-wisdom regarding the empirical content of asymmetric information is suspect. First, we note that the propositions show that it is possible for asymmetric information to induce *both* types of firms to *overinvest* relative to first-best. Part of the causal mechanism behind the model's prediction of overinvestment is that costs of adverse selection generate a precautionary motive for undertaking policies that generate *future* internal funds. This mechanism is the sole source of the high type's overinvestment incentive when NM_{LH} is slack. Anticipating results in the next subsection, signaling potentially provides an additional motive for overinvestment by the high type. We have already noted that the low type violates the pecking order's prescription to use debt as the prime source of external funds. A second point worthy of note is that *it is possible for even the high type to exhibit blatant violations of the pecking-order prescriptions in terms of financing.* In particular, when the financing gap is small and NM_{LH} is nonbinding, Proposition 4 shows that the high type simultaneously issues equity and saves.

Proposition 4 does not discuss optimal policies when NM_{LH} is binding. Discussion of the economic content of conditions (32) and (33) for $\mu > 0$ will be delayed until the next subsection, which links the optimality conditions to single-crossing conditions common to the signaling and mechanism design literatures.

3.4 Single-Crossing Conditions

In this subsection we consider lower net worth states such that NM_{LH} is binding, so that separation enters explicitly into the decision-making process of the high type. In the previous subsection it was established that $d_H = 0$ when NM_{LH} binds. Let us now consider the normalized payoff $u(b, k, s; \theta_i)$ to a type-i manager that takes some arbitrary allocation (b, k, s) such that $d = 0$

$$u(b, k, s; \theta_i) \equiv (1 - s)\beta \int_{\varepsilon^d}^{\infty} v_i[(1 - \delta)k + \theta_i \varepsilon k^{\alpha} - b]f(\varepsilon)d\varepsilon. \tag{34}$$

Next, we compute the total derivative of u, evaluated at the high type allocation. We have

$$du(b_H, k_H, s_H; \theta_i) = (1 - s_H)\Omega_k^{iH} * dk + (1 - s_H)\Omega_b^{iH} * db - \Omega^{iH} * ds. \tag{35}$$

Setting the derivative to zero, one obtains the slope of indifference curves evaluated at the high type allocation. The manager's willingness to exchange equity ownership for additional capital is determined by

$$\frac{ds}{dk}(b_H, k_H, s_H; \theta_i) = \frac{(1 - s_H)\Omega_k^{iH}}{\Omega^{iH}}. \tag{36}$$

This indifference curve notation allows us to rewrite the optimality condition for the high-type capital stock (33) as

$$\begin{aligned}
& 1 - \beta \left[\int_{\varepsilon_{HH}^d}^{\infty} [v_H'((1 - \delta)k_H + \theta_H \varepsilon k_H^{\alpha} - b_H)][1 - \delta + \alpha\theta_H \varepsilon k_H^{\alpha - 1}]f(\varepsilon)d\varepsilon \right] \\
& -\beta \left[\int_{\underline{\varepsilon}}^{\varepsilon_{HH}^d} [1 - \delta + \alpha\theta_H \varepsilon k_H^{\alpha-1}]f(\varepsilon)d\varepsilon + \frac{\partial \varepsilon_{HH}^d}{\partial k_H} f(\varepsilon_{HH}^d)\phi w_H^d \right] \\
= & \left(\frac{\mu\Omega^{LH}}{\lambda_H} \right) \left[\frac{ds}{dk}(b_H, k_H, s_H; \theta_H) - \frac{ds}{dk}(b_H, k_H, s_H; \theta_L) \right].
\end{aligned} \tag{37}$$

Due to diminishing marginal product of capital, the left-side of equation (37) is increasing in k_H. Therefore, the optimality condition tells us that the high-type's capital stock varies positively with the signal content of capital investment, as measured by the difference in the slope of the two type's indifference curves in k-s space. For example, in Figure 1 the indifference curves are drawn under the assumption that the high type has a greater willingness to exchange equity for capital. In this case, higher capital investment provides a positive signal which encourages overinvestment relative to first-best.

Intuition suggests that four factors determine the relative slopes of the types' indifference curves in k-s space. First, the high type generates more future cash than a low type for a given level of capital. Second, persistence in θ implies that a high type has a high probability of being a high type in the subsequent period. For such a firm, the future internal cash generated by installed capital may be more valuable since internal funds reduce exposure to adverse selection costs. These first two effects serve to increase the slope of the high-type indifference curve. However, the low type knows his equity is less valuable than that of the high type, which increases his willingness to exchange equity for capital. In addition, the low type *may* place a higher shadow value on a marginal dollar at the end of the period, since it necessarily realizes lower net worth when it receives the high-type allocation. The next subsection presents additional analysis of the signal content of capital investment under uniformly distributed idiosyncratic (ε) shocks.

The manager's willingness to exchange equity for debt reductions is determined by

$$\frac{ds}{db}(b_H, k_H, s_H; \theta_i) = \frac{(1-s_H)\Omega_b^{iH}}{\Omega^{iH}}. \tag{38}$$

Using this indifference curve relationship allows us to rewrite the debt optimality condition for the high type (32) as

$$\begin{aligned} &\beta\left[\int_{\varepsilon_{HH}^d}^{\infty}[v_H'((1-\delta)k_H+\theta_H\varepsilon k_H^{\alpha}-b_H)-1]f(\varepsilon)d\varepsilon-\frac{\partial\varepsilon_{HH}^d}{\partial b_H}f(\varepsilon_{HH}^d)\phi w_H^d\right]+\chi\gamma b_H \\ =\;& \left(\frac{\mu\Omega^{LH}}{\lambda_H}\right)\left[\left|\frac{ds}{db}(b_H,k_H,s_H;\theta_L)\right|-\left|\frac{ds}{db}(b_H,k_H,s_H;\theta_H)\right|\right]. \end{aligned} \tag{39}$$

Equation (39) tells us that the high-type's borrowing depends upon the signal content of debt, as measured by the difference in the slope of the indifference curves in b-s space. Recall that when the constraint NM_{LH} is slack, the high type will choose to save an amount $b_H^P < 0$ such that the left-side of (39) is equal to zero. Starting at this point, if the firm were to decrease its saving, the left-side of (39) would increase. With this in mind, consider the indifference curves in Figure 2. In Figure 2, the low type is assumed to be more willing to exchange equity for a reduction in debt. Therefore, debt provides a positive signal and the high type borrows more/saves less in order to separate from the low type. An exact treatment of the signal content of debt is provided in the following two subsections, which makes specific distributional assumptions regarding the ε shocks. Anticipating, we obtain unambiguous analytical predictions that debt provides a positive signal for both uniformly and exponentially distributed ε shocks.

Finally, we note that an efficient mix of real and financial signals equates the ratio of "distortion" to signal content at the margin. In particular, equations (37) and (39) imply that

$$\begin{aligned} &\frac{1-\beta\left[\int_{\varepsilon_{HH}^d}^{\infty}(v_H')*[1-\delta+\alpha\theta_H\varepsilon k_H^{\alpha-1}]f(d\varepsilon)+\int_{\underline{\varepsilon}}^{\varepsilon_{HH}^d}[1-\delta+\alpha\theta_H\varepsilon k_H^{\alpha-1}]f(d\varepsilon)+\frac{\partial\varepsilon_{HH}^d}{\partial k}f(\varepsilon_{HH}^d)\phi w_H^d\right]}{\frac{ds}{dk}(b_H,k_H,s_H;\theta_H)-\frac{ds}{dk}(b_H,k_H,s_H;\theta_L)} \\ =\;& \frac{\beta\left[\int_{\varepsilon_{HH}^d}^{\infty}[(v_H')-1]f(d\varepsilon)-\frac{\partial\varepsilon_{HH}^d}{\partial b_H}f(\varepsilon_{HH}^d)\phi w_H^d\right]+\chi\gamma b_H}{\left|\frac{ds}{db}(b_H,k_H,s_H;\theta_L)\right|-\left|\frac{ds}{db}(b_H,k_H,s_H;\theta_H)\right|}. \end{aligned}$$

3.5 Example: Uniformly Distributed Idiosyncratic Shocks

This subsection assumes ε is distributed uniformly on the interval $[\underline{\varepsilon}, \overline{\varepsilon}]$, with θ being i.i.d. We define

$$\begin{aligned} \underline{w}^{ij} &\equiv (1-\delta)k_j+\theta_i\underline{\varepsilon}k_j^{\alpha}-b_j \\ \overline{w}^{ij} &\equiv (1-\delta)k_j+\theta_i\overline{\varepsilon}k_j^{\alpha}-b_j. \end{aligned} \tag{40}$$

It will also be useful to note that for type-i taking the type-j allocation

$$dw = \theta_i k_j^{\alpha} d\varepsilon \Rightarrow d\varepsilon = \frac{dw}{\theta_i k_j^{\alpha}}. \tag{41}$$

Consider now the signal content of the high-type issuing an amount of debt such that $\underline{\varepsilon}< \varepsilon_{HH}^d < \varepsilon_{LH}^d < \overline{\varepsilon}$.[9] Using the change of variable in (41) allows us to rewrite the marginal effect of debt on equity value, presented in equation (18), as follows

$$\begin{aligned}
\Omega_b^{ij} &= -\beta \int_{\varepsilon_{ij}^d}^{\overline{\varepsilon}} v'[(1-\delta)k_j + \theta_i \varepsilon k_j^{\alpha} - b_j] \left(\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right) d\varepsilon \\
&= \frac{-\beta}{\overline{w}^{ij} - \underline{w}^{ij}} \int_{w^d}^{\overline{w}^{ij}} v'(w) dw \\
&= \frac{-\beta v(\overline{w}^{ij})}{\overline{w}^{ij} - \underline{w}^{ij}}.
\end{aligned} \tag{42}$$

Applying the same change of variable, it is readily verified that

$$\Omega^{ij} = \frac{\beta}{\overline{w}^{ij} - \underline{w}^{ij}} \int_{w^d}^{\overline{w}^{ij}} v(w) dw. \tag{43}$$

It follows that the debt signaling term in (32) can be expressed as

$$\frac{\Omega_b^{HH}}{\Omega^{HH}} - \frac{\Omega_b^{LH}}{\Omega^{LH}} = \frac{[\overline{w}^{LH} - w^d]^{-1} v(\overline{w}^{LH})}{[\overline{w}^{LH} - w^d]^{-1} \int_{w^d}^{\overline{w}^{LH}} v(w) dw} - \frac{[\overline{w}^{HH} - w^d]^{-1} v(\overline{w}^{HH})}{[\overline{w}^{HH} - w^d]^{-1} \int_{w^d}^{\overline{w}^{HH}} v(w) dw}. \tag{44}$$

From Lemma 2 and Proposition 4 we know v exhibits concavity as the shadow value of internal funds falls to unity for net worth sufficiently high. When the value function is concave, the numerator of the first term in (44) exceeds that of the second term. Next, note that the respective denominators are simply the average value of v, which is strictly larger for the high type. It follows that the difference in (44) is positive, which implies that marginal increases in debt provide a positive signal under this subsection's assumptions that the persistent θ shocks are i.i.d. and the idiosyncratic ε shocks are uniformly distributed.

The mathematical arguments in the preceding paragraph are closely related to the economic intuition regarding why debt issuance is a positive signal in this setting. First note that we invoked concavity of the value function (v). Concavity of the value function implies that the low type attaches a higher shadow cost to a dollar of future debt service, since the low type realizes lower net worth if it mimics the high-type. Second, the mathematical argument invoked the fact that the low type has lower equity value. Low equity value increases the low type's willingness to part with equity in return for debt reductions. Both effects cause the low type's b-s indifference curve to have a steeper slope.

[9] Similar results are obtained if one considers the issuance of safe debt by the high type. Here we confine the analysis to defaultable debt in the interest of brevity.

Consider next the signal content of capital accumulation. Using the change of variable in (41), we obtain

$$\frac{\Omega_k^{HH}}{\Omega^{HH}} - \frac{\Omega_k^{LH}}{\Omega^{LH}} = \frac{\partial \overline{w}^{HH}}{\partial k_H} * \left|\frac{\Omega_b^{HH}}{\Omega^{LH}}\right| - \frac{\partial \overline{w}^{LH}}{\partial k_H} * \left|\frac{\Omega_b^{LH}}{\Omega^{HH}}\right|. \tag{45}$$

Recall that the signal content of capital accumulation depends on the manager's willingness to trade equity for capital. In the preceding subsection we argued that capital accumulation tends to have positive signal content since the high type has a higher marginal product of capital. This effect is captured in equation (45), with $\partial \overline{w}^{HH}/\partial k_H > \partial \overline{w}^{LH}/\partial k_H$ increasing the difference between the high-type and low-type indifference curve slopes. However, it follows from (44) that $\left|\Omega_b^{HH}/\Omega^{HH}\right| < \left|\Omega_b^{LH}/\Omega^{LH}\right|$, so the sign of the capital signaling expression (45) is ambiguous. Again, the mathematical argument runs parallel to the economic intuition. The fact that $\left|\Omega_b^{HH}/\Omega^{HH}\right| < \left|\Omega_b^{LH}/\Omega^{LH}\right|$ was attributed to concavity of v and the fact that the low type has a lower average value of v. Economically, concavity of v implies that the low type imputes a higher shadow value to a marginal dollar of future cash, such as that stemming from installed capital. In addition, the low expectation of v for the low type causes it to exhibit a greater willingness to exchange equity for capital.

3.6 Example: Exponentially Distributed Idiosyncratic Shocks

This subsection assumes ε is exponentially distributed with $f(\varepsilon) \equiv \xi e^{-\varepsilon\xi}$. Under this distributional assumption, we are able to obtain unambiguous results regarding the signal content of debt even allowing for correlated θ shocks. This contrasts with the previous subsection, where we needed to assume that θ was i.i.d. in order to obtain clear results. In the interest of brevity, we confine attention to the signal content of debt, as the signal content of capital is impossible to sign analytically. We confine attention to the signal content of defaultable debt, although similar results are obtained when we consider the issuance of safe debt.

We begin by rewriting the expression for the marginal cost of debt (18) as follows

$$\Omega_b^{ij} = \frac{-\beta}{\theta_i k_j^\alpha} \int_{\varepsilon_{ij}^d}^{\infty} f(\varepsilon) \left(\frac{\partial}{\partial \varepsilon} v_i((1-\delta)k_j + \theta_i \varepsilon k_j^\alpha - b_j) \right) d\varepsilon. \tag{46}$$

Using integration by parts it follows that

$$\Omega_b^{ij} = \frac{-\beta\xi\Omega^{ij}}{\theta_i k_j^\alpha}. \tag{47}$$

It follows directly that debt issuance is a positive signal, with

$$\frac{\Omega_b^{HH}}{\Omega^{HH}} - \frac{\Omega_b^{LH}}{\Omega^{LH}} = \xi k_H^{-\alpha}(\theta_L^{-1} - \theta_H^{-1}) > 0. \tag{48}$$

4 Numerical Simulation

The procedure used to solve the model numerically is presented in Appendix B. The persistent profit shocks are assumed to be i.i.d. and the idiosyncratic shocks are exponentially distributed. The assumed values of exogenous parameters are presented in Table 1. Once the model is solved, we use the wealth-contingent equilibrium policy functions (a_L^*, a_H^*) to generate a panel data set for simulated firms. In particular, we draw 3000 samples consisting of 300 draws of the two profit shocks. We then use the policy functions generated by the model to determine shock-contingent policy paths. We drop the first 2970 periods, leaving us with a panel of 3000 firms with 30 years of data for each firm. The simulated regressions only use 20 years of data, since the construction of some variables requires the use of lagged data. The simulated panel data set is similar in size to those commonly used in empirical testing.

Figure 3 plots the equity value function v. Since the θ shocks in the simulation are assumed to be i.i.d., there is only one equity value function ($v_H = v_L = v$). A couple of points are worth noting. First, v is concave, which implies that there are precautionary motives for savings and capital accumulation. Second, note that the firm continues even when realized net worth is negative. This stems from the fact that there is option value inherent in equity ownership.

Figure 4 plots the capital allocations of each type relative to first-best. *This figure casts doubt on the conventional wisdom that asymmetric information leads to underinvestment.* In the least-cost separating equilibrium, the low type invests roughly 9% more than the first-best level, regardless of realized net worth. This overinvestment pattern reflects the fact that informational asymmetries create a precautionary motive for capital accumulation. The high type also invests more than first-best in all states, with the extent of the distortion decreasing in net worth. The overinvestment of the high type reflects both signaling and precautionary motives. When net worth is low, the no-mimic constraint binds and the high type overinvests in order to signal private information. When net worth is sufficiently high, the no-mimic constraint becomes slack, but the high type still overinvests a bit due to precautionary motives. The high type has a weaker precautionary motive than the low type, since, ceteris paribus, he will realize higher net worth for any given realized ε.

Figures 5 and 6 plot the wealth-contingent financing policies for each firm type. Consistent with Proposition 3, the low type uses dividends and equity issuance as the sole means of achieving budget-balance, while retaining a wealth-invariant level of savings. When net worth is low, the dividend is set to zero and the firm issues a large amount of equity. Equity issuance for the low type then declines monotonically in net worth. The debt of the high type declines monotonically in net worth. Effectively, higher net worth allows the high type to reduce its need for costly external funds. By way of contrast, the dollar value of equity issuance is non-monotonic in net worth. The rising part of the equity issuance curve is mechanical. As shown in Figure 7, the equity stake (s_H) sold by the high type stays roughly constant for low-intermediate net worth. Since the firm's debt commitment falls with net worth, Ω^{HH} increases, leading to an increase in $s * \Omega$. Eventually, net worth becomes sufficiently high such that the high type begins cutting s_H which leads to a reduction in the value of equity flotations. Both types of firms only pay dividends if net worth is sufficiently high. This prediction is consistent with the empirically observed positive relation between dividends and firm size.

Figure 8 depicts the evolution of the leverage ratio for an arbitrary simulated firm. Note that the leverage ratio always hits the same level, roughly -0.22, when the firm realizes a negative productivity shock (θ_L). The leverage ratio rises when the firm experiences a positive shock. The leverage ratio in good states varies, depending upon the firm's endogenous net worth. For example, the leverage ratio is approximately zero in year 15 despite the fact that the firm has drawn θ_H. The leverage ratio exceeds 0.20 in year 25, which reflects the fact that the firm has drawn θ_H and has low net worth. Upon seeing a data series like the one depicted in Figure 8, an advocate of trade-off theory might be tempted to view the firm as being governed by trade-off theory cum transactions costs, given that the leverage ratio looks to be mean-reverting. However, this conclusion would clearly be incorrect. In our model, the leverage ratio is mean-reverting. However, this is caused by endogenous fluctuations in net worth.

Table 2 presents summary statistics for the simulated firm. The leverage ratio is negative on average, although this property of the model is sensitive to the assumed probability of realizing θ_H and the cost of cash retentions (γ). The average leverage ratio increases when we use higher probabilities of θ_H and higher values of γ. In this set of simulations, the firm issues defaultable debt infrequently, but issues equity frequently. Consequently, there are frequent violations of the traditional pecking-order. In particular, roughly 26% of the simulated firms issue equity despite having access to default-free debt.

Table 3 reports the results of regressions that mimic those commonly found in the literature. In each of the four regressions, the dependent variable is the book leverage ratio. The first row mimics the specification in equation number 2 in Shyam-Sunder and Myers (1999). Essentially, this specification tests the pecking-order prediction that debt is used to fill any financing gaps. In the notation of our model, the financing gap is equal to desired capital plus dividends minus internal resources ($k + d - w$). According to the pecking-order as traditionally specified, the predicted coefficient on the financing gap is *one*. Inspecting Table 2 we see that the simulated firms fill only 44% of the financing gap with debt. Although this point estimate is sensitive to the assumed probability of θ_H, the point that we want to stress is that *there is no theoretical basis for expecting the coefficient on the financing gap to equal one in an economy with asymmetric information between managers and investors.*

The second row of Table 3 mimics leverage regressions in Rajan and Zingales (1995). Consistent with empirical observation, and with the prediction of Myers (1984), in our model leverage ratio does indeed decline in lagged profits. The causal mechanism in our model is as follows. Leverage is invariant to wealth if the firm draws θ_L, since the low type is given a second-best cash buffer stock in the least-cost separating equilibrium. However, conditional upon θ_H being drawn, the leverage of the firm will decline with net worth.

The third row mimics the specification in equation number 3 in Shyam-Sunder and Myers (1999), regressing leverage on the difference between lagged leverage and the firm's "target" where the target is computed as the sample average leverage ratio. Consistent with empirical observation, the mean-reversion coefficient is small but positive. In the simulated data, the mean-reversion coefficient is 0.27. By way of contrast, Shyam-Sunder and Myers report a mean-reversion coefficient of 0.33.

The last row mimics the market-timing regression of Baker and Wurgler (2002). In simulated data, the coefficient on their market-timing variable (external finance weighted average q) is actually insignificant. In a rational economy with Bayesian updating by investors, managers with market-timing preferences do not generate significant market-timing coefficients. Given the behavioral spin Baker and Wurgler place on their results, it is likely that they would take exception with the notion that investors do engage in Bayesian updating. We here note that the well-documented existence of announcement effects is clearly inconsistent with the notion that investors stubbornly cling to their priors. Therefore, their story must be predicated upon some notion of misreaction.

5 Conclusions

There is little doubt that the informational asymmetries stressed by Myers and Majluf (1984) play an important role in corporate finance. As evidence, one may point to the large fees paid to underwriters in return for performing due diligence. As additional direct evidence, one may note the existence of announcement effects surrounding changes in corporate financial structure. However, this paper shows that the types of reduced-form regressions commonly found in the literature are uninformative about the validity of corporate financing theories predicated on asymmetric information. In particular, it has been shown that firms operating in an environment with asymmetric information do not necessarily generate the types of regression coefficients that advocates of the pecking-order predict.

In addition to this critique of econometric practice, the model generates theoretical predictions of independent interest. We show that concern over *future* adverse selection costs effectively converts a risk-neutral manager into a pseudo-risk-averse manager. This risk-aversion weakens the attractiveness of debt relative to what one obtains in a single-period model where (by construction) the firm only faces adverse selection once. This same risk-aversion adds to the signal content of debt. The optimal mix of debt and equity can be viewed as balancing efficient risk-sharing against information revelation. Thus, the results of our dynamic model have clear linkages with static contracting theory. This insight may prove useful in subsequent analysis, as recursive techniques can be used to convert dynamic contracting problems into simpler static problems.

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Appendix A: Proofs

Proof of Lemma 1.

Let a_L^* denote the solution to Program L. The allocation a_L^* was in the feasible set when the type-H program was solved. To verify, the NM_{LH} constraint would be trivial as the allocation would be type-independent. Since BC_L is satisfied at a_L^* we know BC_H would be slack. Optimality then demands that the payoff to the high type must be at least as high as what would he would obtain under a_L^*. Therefore:

$$d_H^* + (1 - s_H^*)\Omega^{HH} \geq d_L^* + (1 - s_L^*)\Omega^{HL}. \blacksquare$$

Proof of Lemma 2.

Consider first a firm with lagged type θ_L. Since $w > 0$ at the start of the next period (on the equilibrium path), equity will have positive value even if the next period's type is also θ_L. Applying the Envelope Theorem, the value of internal funds if the low type is realized is $\lambda_L = 1$. The value of a dollar of internal funds to the high type is given by $\lambda_H + \mu$. To see this, it must be noted that the low type's equilibrium payoff (which enters the NM_{LH} constraint) can be expressed as $w + \kappa^*$. This explains why the shadow value of internal funds to the high type is not simply λ_H. Rather, one must account for the fact that higher wealth adds slack to the NM_{LH} constraint. Taking a probability weighted average of these values yields (20). The derivation of expression (21) is identical, except that one must account for the fact that a dollar of internal funds is worth zero if the low type is drawn following a high type and the equity becomes worthless.

Proof of Proposition 4.

We begin by demonstrating

$$\mu(\widehat{w}) = 0 \Rightarrow \mu(w) = 0 \ \forall \ w > \widehat{w}.$$

From the Envelope Theorem

$$\frac{\partial}{\partial \widetilde{w}} \left[d_L^* + (1 - s_L^*)\beta \int_{\underline{\varepsilon}}^{\infty} v_L[(1-\delta)k_L^* + \theta_L \varepsilon (k_L^*)^\alpha - b_L^*] f(\varepsilon) d\varepsilon \right] = 1.$$

Next, consider that NM_{LH} demands

$$d_L + (1 - s_L)\beta \int_{\varepsilon_{LL}^d}^{\infty} v_L[(1-\delta)k_L + \theta_L \varepsilon k_L^\alpha - b_L] f(\varepsilon) d\varepsilon \geq d_H + (1 - s_H)\Omega^{LH}. \quad (49)$$

The left-side has a slope of one in wealth. Suppose now that $\mu(\widehat{w}) = 0$ which implies that the high type implements (b_H^P, k_H^P) at $\widehat{w}$. Now consider the slope of the right-side of NM_{LH} under the conjecture that the constraint remains nonbinding as $\widetilde{w}$ increases. Under the hypothesis that NM_{LH} remains nonbinding, the high type continues to implement (b_H^P, k_H^P) which are invariant to $\widetilde{w}$. The slope of the right-side of (49) may then be computed as

$$\frac{d}{d\widetilde{w}}[d_H + (1 - s_H)\Omega^{LH}] = \frac{\partial d_H}{\partial \widetilde{w}} - \Omega^{LH}\frac{d}{d\widetilde{w}}s_H. \tag{50}$$

If $\partial d_H/\partial\widetilde{w} > 0$, it follows from (31) that $\mu = 0$. Suppose instead that $\partial d_H/\partial\widetilde{w} \leq 0$. From BC_H it follows that

$$\frac{d}{\partial\widetilde{w}}s_H = \frac{-1 + \partial d_H/\partial\widetilde{w}}{\Omega^{HH}}. \tag{51}$$

Substituting (51) into (50) one obtains

$$\frac{\partial d_H}{\partial\widetilde{w}} - \frac{ds_H}{d\widetilde{w}} * \Omega^{LH} = \frac{\Omega^{LH}}{\Omega^{HH}} + \frac{\partial d_H}{\partial\widetilde{w}}\left[1 - \frac{\Omega^{LH}}{\Omega^{HH}}\right] < 1.$$

Thus, the left-side of (49) has a steeper slope than the right and the NM_{LH} constraint remains nonbinding as conjectured. The high-type allocation for $\mu = 0$ follows directly from (32) and (33).■

Appendix B: Details of Computational Algorithm

The computational procedure is based on value function iteration. The individual steps are as follows. The idiosyncratic shock ε is implemented by discretizing its domain using N possible values. Each maximization is implemented by discretizing the domain of the decision variables.

1. Guess "going-concern" values w_j^d of the firm.

2. Guess value functions v_j of the firm.

3. Solve for the low type allocation a_L that maximizes the value of the low type firm subject to its budget. Pick the policy in the optimal set that minimizes the dividend payout.

4. Solve for the high type allocation a_H that maximizes the value of the high type firm subject to its budget and the incentive constraint that guarantees the low type prefers a_L to a_H.

5. Compute new value functions $v_j^{'}$ from

$$\begin{aligned} v_j^{'} = \pi_{Hj}&\left[d_H + \beta(1 - s_H)\sum_{n=1}^{N} f(\varepsilon_n)v_H[(1-\delta)k_H + \theta_H\varepsilon_n k_H^{\alpha} - b_H]\right] \\ &+ \pi_{Lj}\left[d_L + \beta(1 - s_L)\sum_{n=1}^{N} f(\varepsilon_n)v_L[(1-\delta)k_L + \theta_L\varepsilon_n k_L^{\alpha} - b_L]\right]. \end{aligned}$$

6. The functions $v_j^{'}$ are the new guess for v_j. The procedure is then restarted from step 2 until convergence.

7. Check the option value inherent in the firm by verifying $v_j(w_j^d) = 0$. If these conditions are not satisfied, update the initial guesses w_j^d and restart the procedure from step 1 until convergence.

Figure 1: **Higher capital investment as positive signal**

This figure shows the indifference curves drawn under the assumption that the high type has a greater willingness to exchange equity for capital. In this case, higher capital investment provides a positive signal which encourages overinvestment relative to first-best.

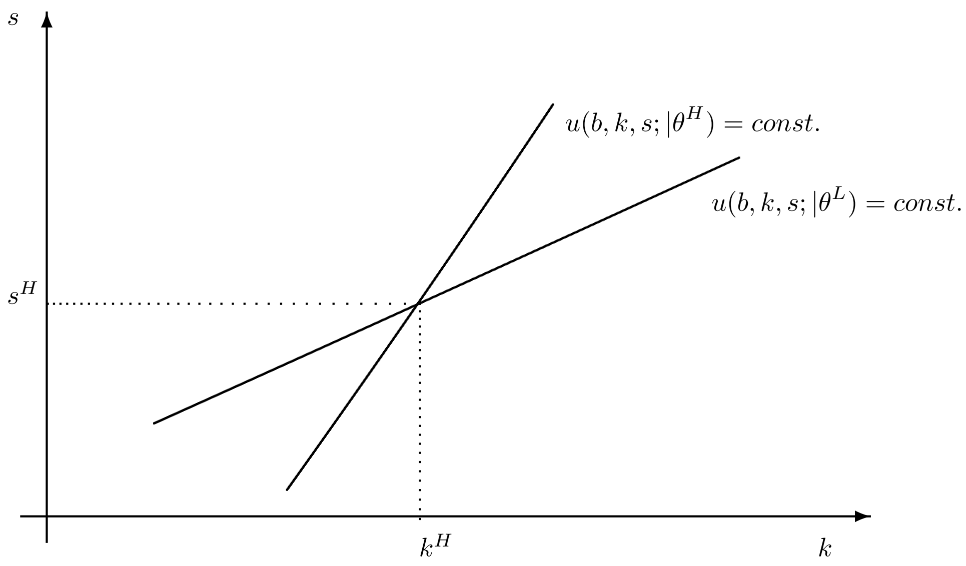

Figure 2: **Higher debt as positive signal**

This figure shows the indifference curves drawn under the assumption that the low type is more willing to exchange equity for a reduction in debt. Therefore, debt provides a positive signal and the high type borrows more/saves less in order to separate from the low type.

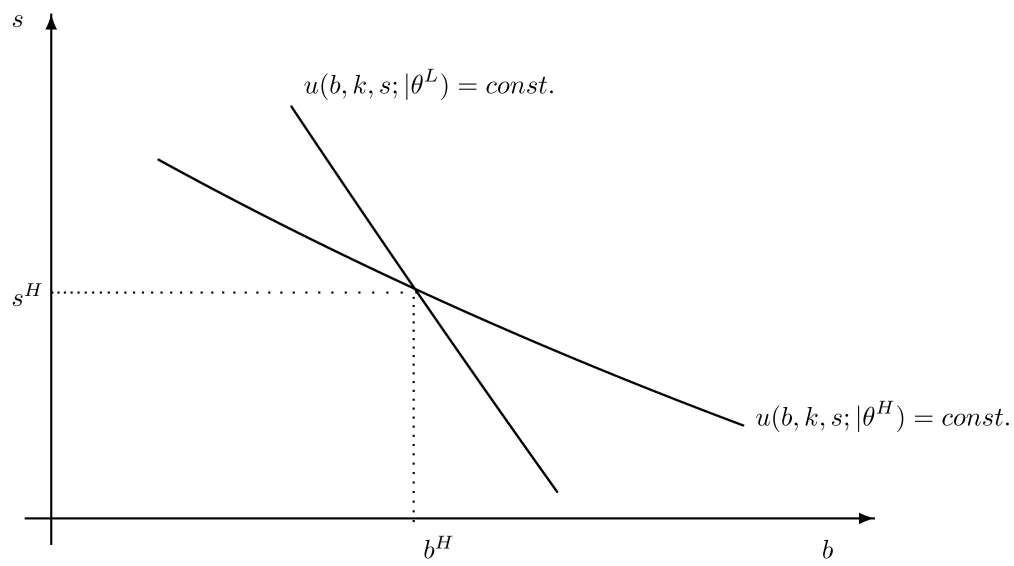

Figure 3: **Equity value function**

The equity value function v is plotted as a function of the realized net worth, w. The productivity shock ε is exponentially distributed and both productivity shocks are *i.i.d.* The parameter choices are reported in Table 1.

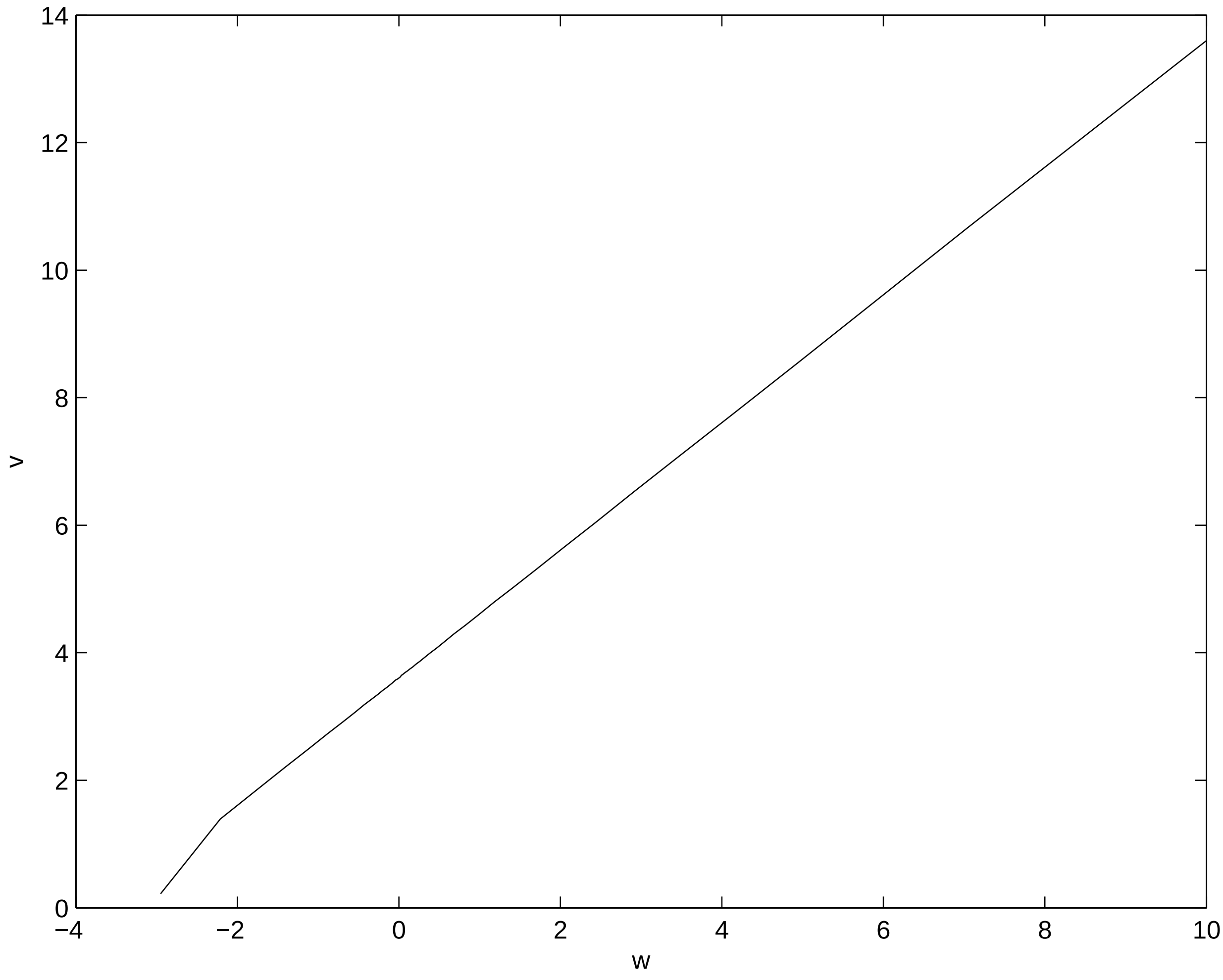

Figure 4: **Optimal capital allocations**

Optimal capital allocations, k_i^*, scaled by the first-best allocations, k_i^{FB}, are plotted as a function of the realized net worth, w, for both high and low values of θ. The productivity shock ε is exponentially distributed and both productivity shocks are *i.i.d.* The parameter choices are reported in Table 1.

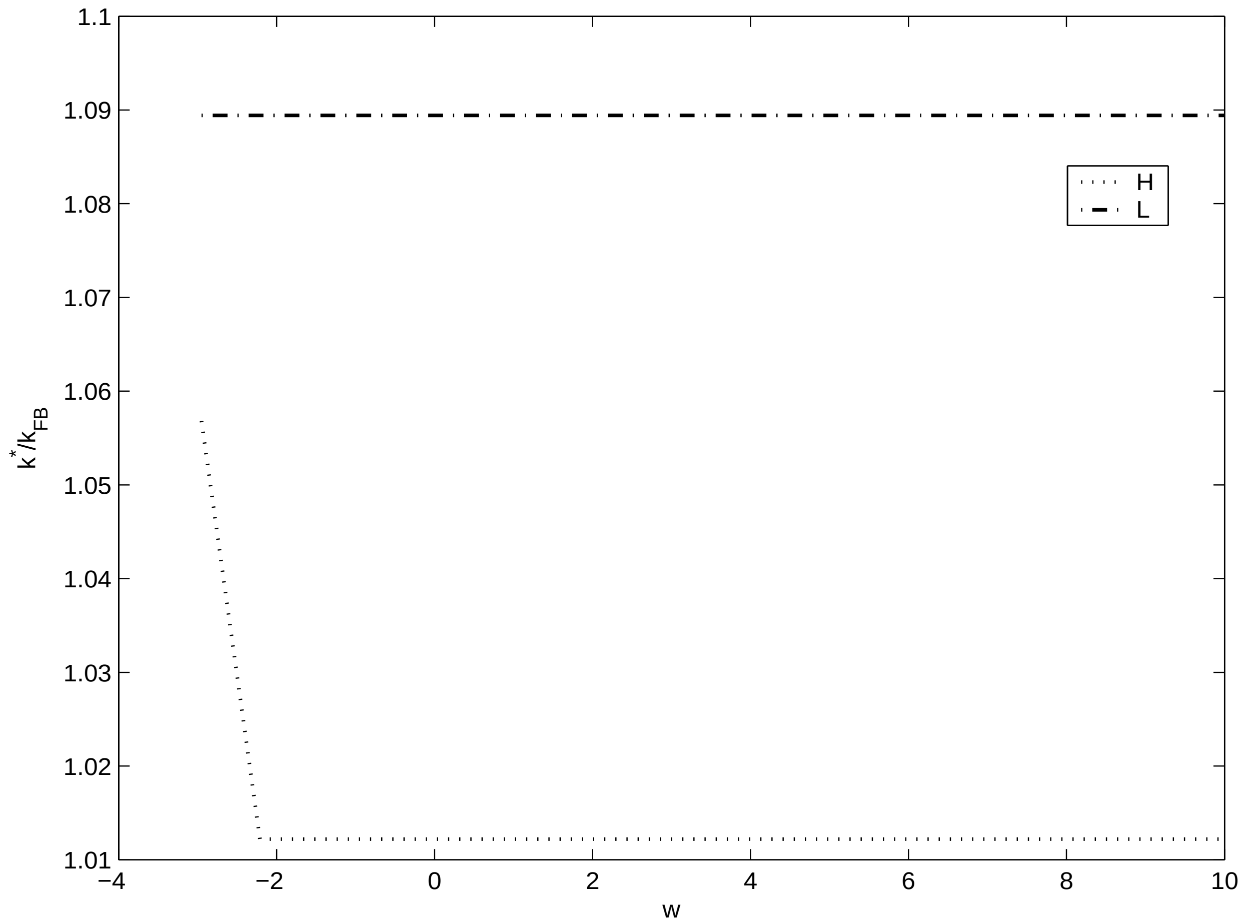

Figure 5: **Optimal financing policies - high value of θ**

Optimal financing policies: debt, ρ_H, and equity, $s_H^*\Omega^{HH}$, as well as optimal dividend policy, d_H^*, are plotted as functions of the realized net worth, w, for the case of high value of θ. The productivity shock ε is exponentially distributed and both productivity shocks are *i.i.d.* The parameter choices are reported in Table 1.

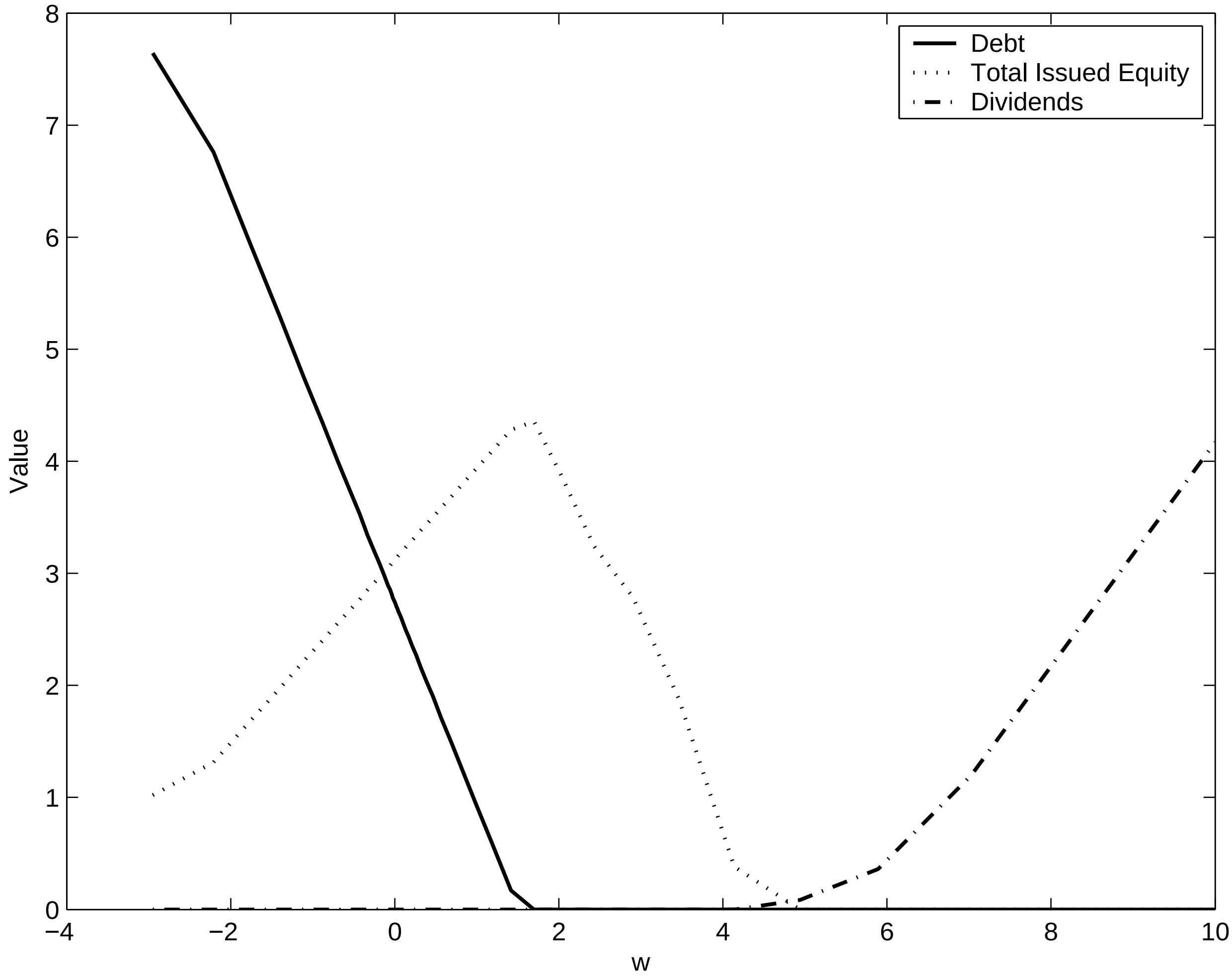

Figure 6: **Optimal financing policies - low value of θ**

Optimal financing policies: debt, ρ_L, and equity, $s_L^* \Omega^{LL}$, as well as optimal dividend policy, d_L^*, are plotted as functions of the realized net worth, w, for the case of low value of θ. The productivity shock ε is exponentially distributed and both productivity shocks are *i.i.d.* The parameter choices are reported in Table 1.

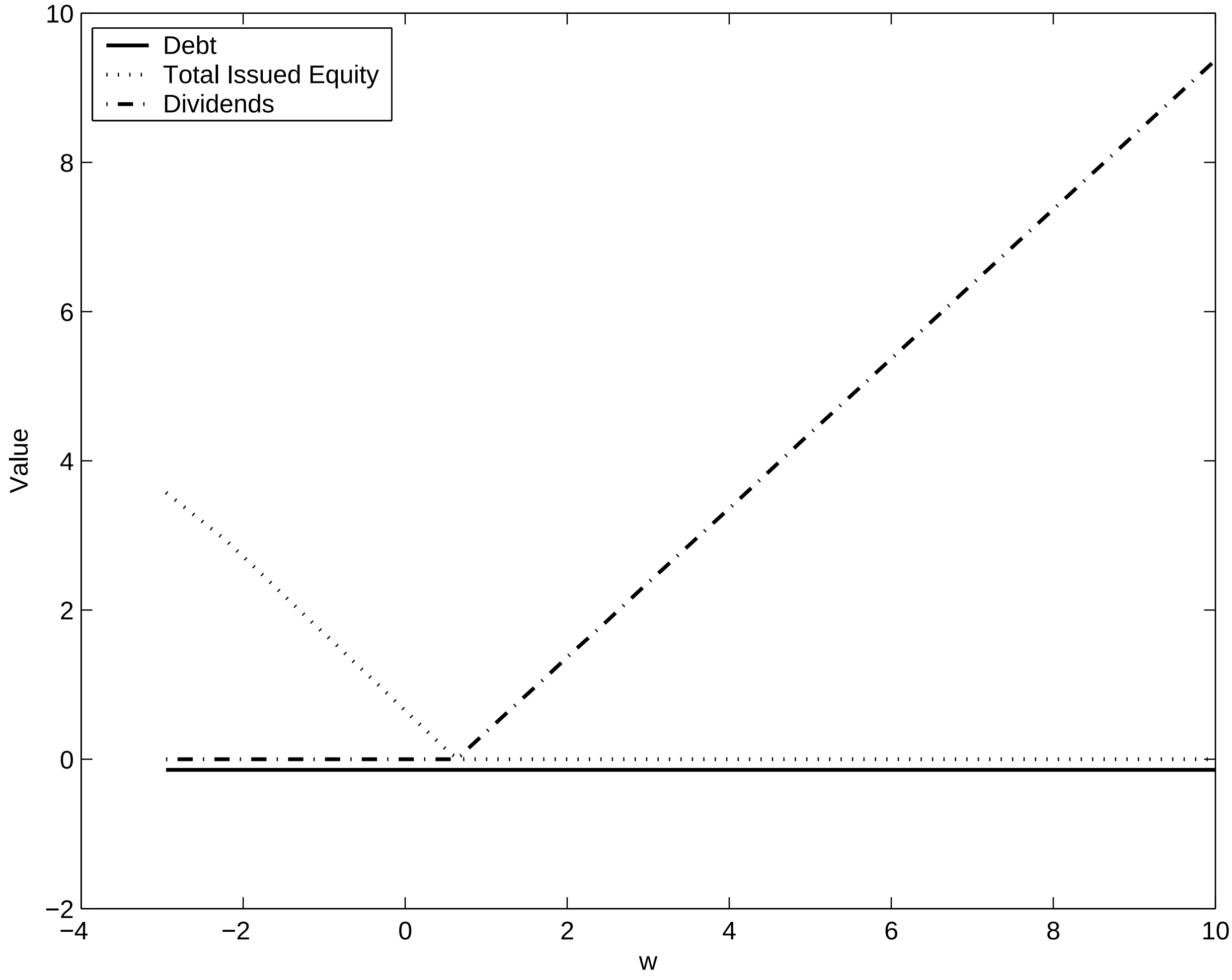

Figure 7: **Percentage of equity sold to new shareholders**

Optimal percentage of equity sold to new shareholders, s_i^*, is plotted as a function of the realized net worth, w, for cases of both high and low values of θ. The productivity shock ε is exponentially distributed and both productivity shocks are *i.i.d.* The parameter choices are reported in Table 1.

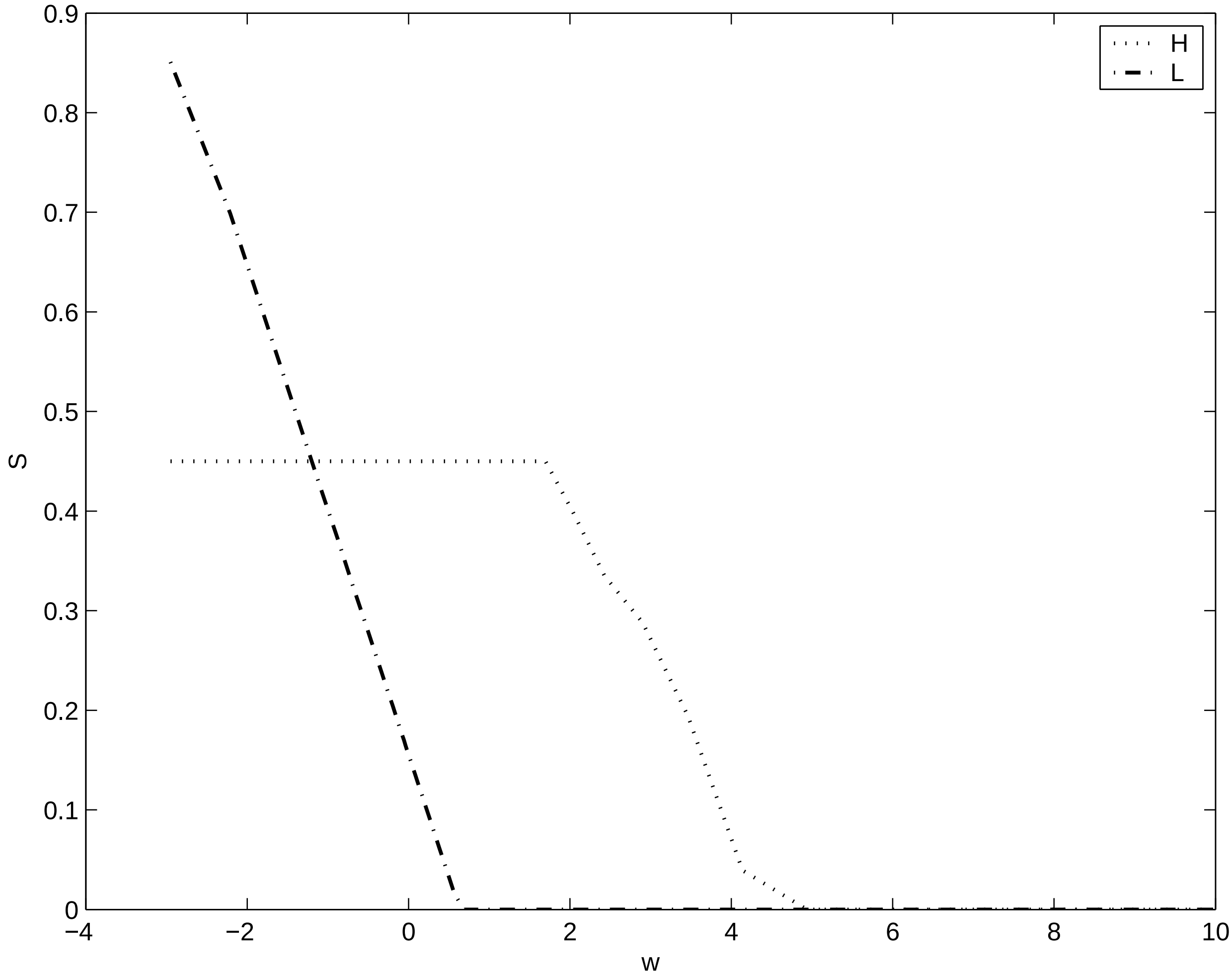

Figure 8: **Evolution of leverage ratio**

Simulated time series of the firm-level leverage ratio defined as a ratio of debt, $\rho(t)$, to total assets, $k(t)$, is shown. This is a randomly chosen sample from the simulated panel of firms that contains 3,000 firms over 300 time periods, where only the last thirty time periods are kept for each firm. The productivity shock ε is exponentially distributed and both productivity shocks are *i.i.d.* The parameter choices are reported in Table 1.

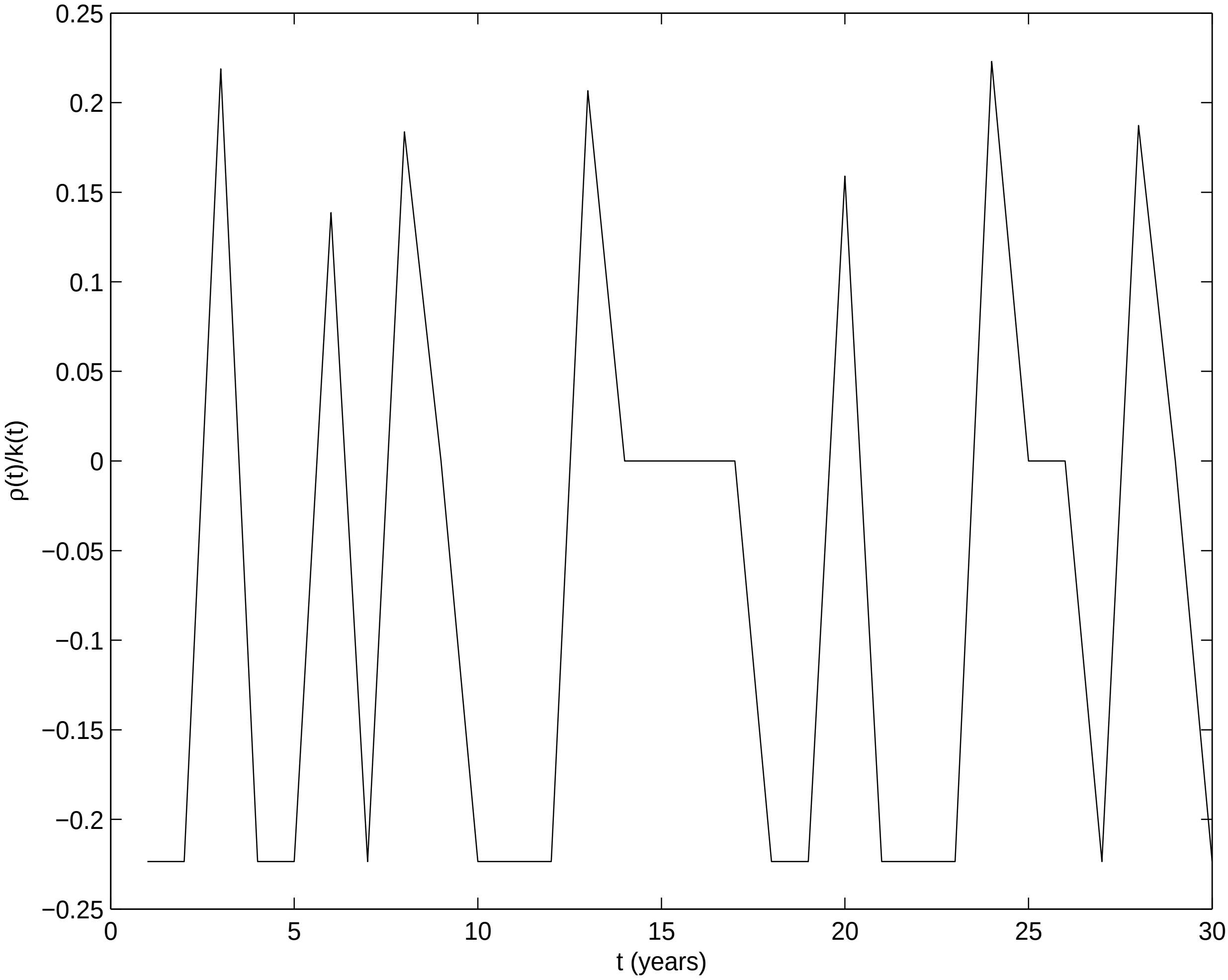

Table 1: **Parameter Choices**

This table reports the values of parameters used in simulation. The profit shock ε is distributed exponentially.

β	1/1.065
α	0.7
δ	0.1
γ	0.05
ϕ	0.1
ξ	2.0
θ_H	0.8
θ_L	0.4
π_H	0.5

Table 2: **Summary Statistics from Simulated Firms**

This table presents summary statistics from simulated panel of firms. The simulated panel of firms is generated from the model and contains 3,000 firms over 300 time periods, where only the last thirty time periods are kept for each firm. The productivity shock ε is exponentially distributed and both productivity shocks are *i.i.d.* The parameter choices are reported in Table 1. The first column reports statistics for all firms, while the next two columns report the results for firms with "high" and "low" values of θ separately. The frequency of the event $A > 0$, $f(A)$, is defined as

$$f(A) = \frac{\sum_{i=1}^{N} \chi(A_i > 0)}{N},$$

where $\chi(A_i > 0)$ is an indicator function.

Variable	All	High	Low
Leverage			
mean	-0.0618	0.1032	-0.2236
median	-0.1126	0.0840	-0.2236
std	0.1789	0.1042	0.0000
Investment			
mean	0.3258	3.0433	-2.3946
median	0.3247	3.1386	-2.4682
std	3.5922	2.3649	2.3617
	Frequency		
Equity Issuance			
all debt	0.2633	0.2633	0.0000
default-free debt	0.2633	0.2633	0.0000
Dividend Payment	0.7367	0.2368	0.4999
Repurchasing	0.0000	0.0000	0.0000

Table 3: **Leverage Regressions**

This table reports results of several regressions on the simulated data with leverage ratio, $\frac{\rho(t)}{k(t)}$, as the dependent variable. Here $q(t)$ is the Tobin's Q defined as $q(t) = \frac{v(t)+b(t-1)}{k(t-1)}$. The financing gap is defined as $\frac{d(t)+k(t)-w(t)}{k(t)}$. Operating profits are defined as $\theta(t)\varepsilon(t)k^{\alpha-1}(t)$. EFWAQ$(t)$ is a weighted average of each of the prior ten year's q ratios, with the weight on any given lagged q equal to the market value of external finance in that year, $\rho_i(t)(1-\chi)+s_i(t)\Omega^{ii}(t)$ divided by the total market value of external finance obtained over the entire ten year lag period. The simulated panel of firms contains 3,000 firms over 300 time periods, where only the last thirty time periods are kept for each firm. The productivity shock ε is exponentially distributed and both productivity shocks are *i.i.d.* The parameter choices are reported in Table 1.

Financing Gap	$q(t)$	Lagged Operating Profits	$\mathrm{E}[\frac{\rho}{k}] - \frac{\rho(t-1)}{k(t-1)}$	EFWAQ(t)
0.4416	-	-	-	-
(0.0315)	-	-	-	-
-	0.0299	-0.0071	-	-
-	(0.0131)	(0.0028)	-	-
-	-	-	0.2683	-
-	-	-	(0.0612)	-
-	0.0267	-0.0816	-	-0.0138
-	(0.0254)	(0.0118)	-	(0.0256)