Decomposing Swap Spreads\textsuperscript{1}

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\textsuperscript{1}This paper - including earlier versions entitled "A model for corporate bonds, swaps and Treasury securities" and "A model of swap spreads and corporate bond yields" - was presented at the BIS workshop on "the pricing of credit risk", the inaugural WBS fixed income conference in Prague, a meeting of the Moody’s Academic Advisory Research Committee in New York, Moody’s Second Risk Conference 2005, the Credit Risk Workshop at Aarhus School of Business, Copenhagen Business School, the Quantitative Finance conference at the Isaac Newton Institute, Cambridge and Cornell University. We would like to acknowledge helpful discussions with Richard Cantor, Pierre Collin-Dufresne, Joost Driessen, Darrell Duffie, Dwight Jaffee, Bob Jarrow, Jesper Lund, Lasse Pedersen, Wesley Phoa, Tony Rodrigues, Ken Singleton, Etienne Varloot, and Alan White. Both authors are at the Department of Finance at the Copenhagen Business School, Solbjerg Plads 3, DK-2000 Frederiksberg, Denmark (e-mail: Feldhütter: pf.fi@cbs.dk, Lando: dl.fi@cbs.dk).
Abstract

We analyze a six-factor model for Treasury bonds, corporate bonds, and swap rates and decompose swap spreads into three components: A convenience yield from holding Treasuries, a credit risk element from the underlying LIBOR rate, and a factor specific to the swap market. In the later part of our sample, the swap-specific factor is strongly correlated with hedging activity in the MBS market. The model further sheds light on the relationship between AA hazard rates and the spread between LIBOR rates and GC repo rates and on the level of the riskless rate compared to swap and Treasury rates.
1 Introduction

Interest rate swaps and Treasury securities are the primary instruments for hedging interest rate risk in the mortgage-backed security (MBS) and corporate bond markets but the large widening of swap spreads - the difference between swap rates and comparable Treasury yields - in the fall of 1998 clearly revealed that there are important differences between the two markets. The ability to accurately hedge interest rate risk critically depends on understanding these differences. This paper decomposes the term structure of swap spreads into three components: A convenience yield for holding Treasury securities, a credit spread arising from the credit risk element in the LIBOR rates, which define the floating rate payments of interest rate swaps, and a residual component which, starting with the onset of an MBS refinancing period towards the end of 2000, is heavily correlated with MBS hedging activity. The dynamic decomposition of the evolution for the 5-yr swap spread is depicted in Table 1.

As we will explain below, the size of the convenience yield is what separates the Treasury yield from the riskless rate and the two other components separate the swap yield from the riskless rate. Hence we also obtain a clear picture of the differences between these three key interest rates.

We obtain these decompositions through a joint pricing model for Treasury securities, corporate bonds and swap rates using six latent factors. Two factors are used in the model of the government yield curve, one factor is used in modeling the convenience yield in Treasuries, two factors are used in the credit risk component in corporate bonds, and one is a factor unique to the swap market. The components that go into the swap spread are all part of our pricing model. This is in contrast to the approach in Duffie and Singleton (1997) who fit a model for the swap spreads and then regress the fitted spreads onto different proxies for liquidity and credit to obtain a decomposition. By building the relevant components into our pricing model we avoid the problem that the proxies may be inaccurate. In fact, instead of using proxies as regressors, our model allows us to assess whether certain proxies are appropriate. He (2001), Liu, Longstaff, and Mandell (2004), and Li (2004) use the spread between LIBOR and general collateral (GC) repo rates as a proxy for the loss-adjusted AA hazard rates, but our estimates
indicate that the LIBOR-GC repo spread is too volatile to use as a proxy. This and other comparisons are only possible because we include corporate bonds in our pricing model.

A key finding of our model is that to fit the markets simultaneously, we need a factor which is specific to the swap market, similar to the idiosyncratic swap factor used in Reinhart and Sack (2002). The presence of this factor implies that the assumption of homogeneous credit quality (defined below) in the LIBOR and AA corporate markets cannot be maintained. We show that after 2000, this factor has a strong correlation with hedging activity in the MBS market. In addition, we use our full pricing model to estimate this effect across maturities.

Our model builds upon and extends a number of previous models and empirical studies. In Duffie and Singleton (1997) the 6-month LIBOR rate is based on an adjusted short rate process $R$ which includes the Treasury rate, an adjustment for liquidity differences in Treasury and swap markets, and a loss adjusted default rate. By simultaneously using $R$ to discount the cash-flows of the swap and for determining the floating rate payments of the swap, the fair swap rates depend only on $R$ and not on the contributions from the individual components to $R$. In their subsequent analysis, the swap rates are therefore regressed on proxies for liquidity and credit risk, but the components are not included separately in the pricing model$^1$.

We follow Collin-Dufresne and Solnik (2001) and find the fair swap rate by pricing the cash flows of the swap separately using an (estimated) riskless rate (instead of using the refreshed LIBOR rate as in Duffie and Singleton (1997)). This is reasonable given the fact that counterparty risk on plain vanilla interest rate swap is typically eliminated through posting collateral and netting agreements. As noted in Collin-Dufresne and Solnik (2001), future paths of the LIBOR-rate are critical in determining the swap rates. Large future LIBOR rates will imply higher swap rates. The viewpoint in our paper is that only by including corporate bond rates can we reasonably hope to separate out from LIBOR that part which is due to credit risk. The credit risk is reflected in part in the corporate AA-curve, but this in turn is affected by adjacent curves since a bond currently rated AA is affected by default risk in adjacent rating-categories. Our joint modeling of corporate

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$^1$Duffie and Singleton do discuss a specification which separates out the riskless short rate and a combined liquidity and credit risk adjustment, but this specification is not estimated in their paper.
curves and the swap curve therefore gives a much more detailed view on the future path of LIBOR rates and on the AA corporate curve than the one used in Collin-Dufresne and Solnik (2001). Also, we focus on explaining factors influencing swap spreads while Collin-Dufresne and Solnik (2001) focus on the difference between the swap curve and the AA corporate curve.

Both Duffie and Singleton (1997) and Collin-Dufresne and Solnik (2001) assume that the 6-month AA corporate rate and 6-month LIBOR are the same - an assumption Duffie and Singleton (1997) refer to as homogeneous credit quality. We cannot maintain this assumption and obtain a simultaneous fit of swap and corporate bond rates.

Our approach is similar to that of Liu, Longstaff, and Mandell (2004), who use a five-factor model using three factors to model Treasury yields, one factor to model the 'liquidity' (i.e. what we refer to as the convenience yield) of Treasury securities and one factor for default risk. Their identification of the credit risk and the liquidity component in swap spreads relies critically on the use of 3-month GC repo rates as a short term riskless rate and 3-month LIBOR as a credit-risky rate. Their default factor is in fact equal by definition to the difference between 3-month LIBOR and 3-month GC repo rates, an assumption used also (for 1-month rates) by He (2001). By including the information of corporate bonds in our study we do not need to rely on short-term interest rate spreads as proxies for credit risk and Treasury components and we show that this strongly alters conclusions about the size and time series behavior of these components.²

Most of the various proxies that we discuss in this paper, can be found in the model of Reinhart and Sack (2002) who specify a multivariate time series model for 10 year swap rates, off-and on-the run Treasury rates, Refcorp rates (to be defined below) and AA corporate rates. However, their model does not contain any pricing model or full term structure modeling of the relevant rates.

Grinblatt (2001) takes a different approach and views swap rates as riskless rates and the spread between government and swap rates as a liquidity spread. The argument presented in Grinblatt (2001) relies on AA refreshed credit as being virtually riskless. While it is true that historical default experience for AA issuers over a three month or six month period is extremely low, we do find a credit risk component in swap spreads.

The rating-based approach explicitly incorporates different dynamics for

²This also distinguishes our approach from that of Li (2004).
bonds of different rating categories. This is consistent with empirical evidence in Duffee (1999) who finds that the dynamics (and not just the levels) of the hazard rate process depends on the rating category. We incorporate this finding into our model by letting the default intensity process evolve as a diffusion with ‘regime-shifts’, i.e. a diffusion with rating-dependent parameters. An alternative approach would be to model the default-intensities for different rating categories by adding positive valued processes for lower categories, but unless we include a migration component as well we cannot price bonds consistently.

Our approach also allows for a stochastic variation in spreads which can be due either to a time varying risk premium on default event risk or to changing hazard rates. Driessen (2005) finds evidence in support of the existence of an event risk premium of default, but the evidence is not conclusive. He assumes, however, in his empirical specification that a possible risk premium on default event risk is constant. If there is a time varying risk premium on default event risk, the risk premium on variations of default risk will not be estimated properly. Our specification allows for time varying event risk premium and therefore also for a more reliable estimation of the risk premium on variations in default risk.

The outline of the paper is as follows: In Section 2 we describe the structure of our model. The explicit pricing formulas are relegated to an appendix. Section 3 describes the US market data, that we use, and Section 4 explains our estimation methodology. In Section 5 we report our parameter estimates along with residuals from the estimation, and we elaborate on our main findings, as outlined above. Section 6 concludes.

2 The Model

Our model of Treasury bonds is an affine short rate model with a liquidity component, and we use an intensity-based, affine framework for corporate bonds and swaps as introduced in Duffie and Singleton (1997,1999) and Lando (1994, 1998). Since our pricing of corporate bonds includes rating information we also use the affine, rating based setting introduced in Lando (1994, 1998).

We use a six-factor models based on independent translated CIR processes. More precisely, we assume that the latent state vector $X$ consists of 6
independent diffusion processes with an affine drift and volatility structure,

\[ X_t = (X_{1t}, \ldots, X_{6t})' \]

\[ dX_{it} = k_i(X_{it} - \theta_i)dt + \sqrt{\alpha_i + \beta_iX_{it}}dW_{P_i}^{P_i}, \quad i = 1, \ldots, 6, \]

where the Brownian motions \( W_{P_1}^{P}, \ldots, W_{P_6}^{P} \) are independent. This specification nests the Vasicek (\( \beta = 0 \)) and CIR (\( \alpha = 0 \)) processes as special cases. We assume that the market price of risk for factor \( i \) is proportional to its standard deviation and normalize the mean of \( X_i \) under \( Q \) to zero for identification purposes, so the processes under \( Q \) are given by

\[ dX_{it} = k_i^*X_{it}dt + \sqrt{\alpha_i + \beta_iX_{it}}dW_{Q_i}, \]

where

\[ k_i^* = k_i - \lambda_i \beta_i \]

\[ \lambda_i = \frac{k_i \theta_i}{\alpha_i}. \]

From the state vector we now define the short rate processes, intensities and liquidity adjustments needed to jointly price the Treasury and corporate bonds and the swap contracts.

We work in an arbitrage-free model with a riskless short rate and this rate \( r \) is given as a three-factor process,

\[ r(X) = a + X_1 + X_2 + (e + X_5), \quad (1) \]

where the first two factors \( X_1 \) and \( X_2 \) are the factors governing the Treasury short rate while the last factor \( X_5 \) is a Treasury premium which distinguishes the Treasury rate from the riskless rate. The premium is a convenience yield on holding Treasury securities arising from among other things a) repo specialness due to the ability to borrow money at less than the General Collateral rates\(^3\), b) that Treasuries are a desired mechanism for hedging interest rate risk, c) that Treasury securities must be purchased by financial institutions to fulfill regulatory requirements, d) that the amount of capital required to be held by a bank is significantly smaller to support an investment in Treasury securities relative to other near default-free securities, and to a lesser extent

e) the ability to absorb a larger number of transactions without dramatically affecting the price. The constant $a$ is the $Q$-mean of the government short rate while $e$ is the $Q$-mean of the convenience yield. Consequently, the Treasury short rate process is given as

$$r^g(X) = a + X_1 + X_2.$$  

(2)

The Treasury premium is positive and in the empirical work we restrict the parameters such that $e + X_5$ is positive. From this affine specification, prices of government bonds are given as

$$P^g(t, T) = \exp(A^g(T - t) + B^g(T - t)X_t),$$

where $A^g$ and $B^g$ can be found in appendix B.2.

Our model for corporate bonds prices a ‘generic’ bond with initial rating $i$ by taking into account both the intensity of default for that rating category and the risk of migration to lower categories with higher default intensities. Spread levels within each rating category are stochastic, but for all rating categories they are modeled jointly by a stochastic credit spread factor.

We use the reduced form representation with fractional recovery of market value to price a corporate bond which at time $t$ is in rating category $\eta_t = i$:

$$v^i(t, T) = E_t^Q \exp \left(- \int_t^T (r(X_u) + \lambda(X_u, \eta_u)du) \right).$$  

(3)

where $\lambda(X, \eta)$ is the loss-adjusted default intensity when the rating-class is $\eta$. The default intensities for the different categories are assumed to have a joint factor structure

$$\lambda(X, i) = \nu_i \mu(X)$$

where $\nu_i$’s are constants, and $\mu(X)$ is a strictly positive process which ensures stochastic default intensities for each rating category and plays the role as a common factor for the different default intensities. We specify $\mu$ as

$$\mu(X) = b + X_3 + X_4 + c(X_1 + X_2).$$

Note that the process $\mu$ is allowed to depend on government rates through the constant $c$ while the two processes $X_3$ and $X_4$ are used only in the definition of $\mu$. Hence we have in essence a two factor model for the credit spreads across different rating categories. We now have the definition of the loss
adjusted default intensity as a function of the state variable process and the rating category, so all that is left to specify before (3) can be evaluated, is the stochastic process for the rating migrations. We work with a 'conditional' Markov assumption as in Lando (1994, 1998) which means that the transition intensity from category \( i \) to category \( j \) is given as

\[
a_{ij}(X_t) = \lambda_{ij} \mu(X_t)
\]

where \( \lambda_{ij} \) is a constant for each pair \( i \neq j \) and \( \mu(X_t) \) is the same factor that governs credit spreads. We can collect all the conditional transition intensities and the loss adjusted default intensities into one common matrix given as

\[
A_X(s) = \Lambda \nu \mu(X_s).
\]

Note that the scalar function \( \mu(X_s) \) is multiplied onto every element of the (loss-adjusted) generator matrix \( \Lambda \nu \). This means essentially that the intensities of rating and default activity is modulated by the process \( \mu(X_s) \). The multiplicative effect has the effect of modifying the default intensity by a scalar function which captures both stochastic variation in this and possibly a compensation for event risk.\(^4\) As shown in Lando (1994, 1998), this specification generates pricing formulas for corporate bonds in all rating categories which are sums of affine functions. Hence the price of a zero coupon corporate bond in rating class \( i \) at time \( t \) is of the form

\[
v^i(t, T) = \sum_{j=1}^{K-1} c_{ij} E_t(\exp(\int_t^T d_j \mu(X_u) - r(X_u) du))
\]  

(4)

where the constants \( c_{ij} \) and \( d_j \) are given in appendix B.3. Our model does not take into account the difference in the tax treatment between corporate bonds and Treasury securities. Grinblatt (2001) argues that 'the tax equilibrium argument is implausible because the state tax advantage does not apply to broker-dealers, tax-exempt investors like pension funds, or international investors\(^5\) who would then arbitrage away these differences'. Elton et al.

\(^4\)Since we do not observe empirical default intensities, we cannot decompose this multiplicative factor into event risk premium and variation in default risk under the empirical measure.

\(^5\)Tony Crescenzi of BondTalk.com reports in a market Commentary on November 18, 2004, that foreign corporate bond purchases reached a record of 44 billion dollars in September 2004
(2001) employ a ‘marginal investor’ tax rate argument and estimate the tax premium on corporate bonds to be significant. They do however measure the bond spreads using Treasury bonds as a benchmark. The convenience yield that we estimate for Treasury bonds easily explains a spread of similar magnitude. Longstaff, Mithal, and Neis (2005) in their analysis of the non-default component of credit spreads for corporate bonds find only weak support for a tax effect.

With the specification of the Treasury and corporate bond prices in place, we can now find swap rates. First, we need to define the 3-month LIBOR rate used to determine the floating-rate payment on the swap:

\[
L(t, t + 0.25) = \frac{1}{v^{LIB}(t, t + 0.25)} - 1
\]

where \( v^{LIB}(t, t + 0.25) \) is the present value of a 3-month loan in the interbank market:

\[
v^{LIB}(t, t + 0.25) = E^Q_t \exp(- \int_t^{t+0.25} \lambda^{LIB}(X_s) ds). \tag{5}
\]

The adjusted short rate to value this loan is given as

\[
\lambda^{LIB}(X_s) = r(X_s) + \nu_{AA} \mu(X_s) + S(X_s). \tag{6}
\]

There are three stochastic components in the determination of LIBOR rates. The first component is the riskless rate \( r(X_s) \). The second component, \( \nu_{AA} \mu(X_s) \), is the loss adjusted AA-intensity of default. If these were the only two components defining LIBOR, we would be working under the assumption that the three-month LIBOR rate and the yield on a 3-month AA corporate bond are equal. This is an assumption typically used in the literature (Duffie and Singleton (1997), Collin-Dufresne and Solnik (2001), Liu, Longstaff, and Mandell (2004), and He (2001)). However, Duffie and Singleton (1997) note that the assumption - which they call homogeneous LIBOR-swap market credit quality - is nontrivial since the default scenarios, recovery rates, and liquidities of the corporate bond and swap markets may differ. The additional component \( S(X_s) \), which we use, accounts for such differences and as we will discuss later this component has important consequences for the model’s ability to fit swap rates. We assume that the component \( S(X_s) \) that allows for differences in swap and corporate bond markets is defined by

\[
S(X) = d + X_0.
\]
In contrast to the other 5 factors, $S(X)$ only comes into play when pricing swaps. With the floating-rate payments on the swap in place, we proceed to value the swap, i.e. to find the fixed rate payments needed to give the contract an initial value of zero. We compute the value of the swap by taking present values separately of the fixed-and floating payments, and by discounting both sides of the swap using the riskless rate. This amount to ignoring counterparty risk in the swap contract - a standard assumption in recent papers. From a theoretical perspective this assumption is justified in light of the small impact that counterparty default risk has on swap rates when default risk of the parties to the swap are comparable as shown in Duffie and Huang (1996) and Huge and Lando (1999). From a practical perspective posting of collateral and netting agreements reduce - if not eliminate - counterparty risk. Bomfim (2002) shows that even under times of market distress there is no significant role for counterparty risk in the determination of swap rates.

With these assumptions we can value the swap rates in closed form. The swap data in the empirical section are interest rate swaps where fixed is paid semi-annually while floating is paid quarterly. We consider an interest rate swap contract with maturity $T - t$, where $T - t$ is an integer number of years. Defining $n = 4(T - t)$ as the number of floating rate payments at dates $t_1, ..., t_n$ and $F(t, T)$ as the $T - t$-year swap rate, the three-month LIBOR, $L(t_{i-1}, t_i)$, is paid at time $t_i, i = 1, ..., n$ while the fixed-rate payments $\frac{F(t,T)}{2}$ are paid semi-annually, i.e. at times $t_2, t_4, \ldots, T$. The resulting formula for the swap rate is

$$F(t, T) = \frac{2 \sum_{i=1}^{n} (e^{A^s(t_{i-1}-t)}+B^s(t_{i-1}-t)'X_t - P(t, t_i))}{\sum_{i=1}^{n} P(t, t_{2i})},$$

where the functions $A^s$ and $B^s$ are found in the appendix.

3 Data Description

Data consist of Treasury yields, swap rates, and corporate yields for the rating categories AAA, AA, A, and BBB on a weekly basis from 1996 to 2003. The rates obtained from Bloomberg are from the US market and covering the period from December 20, 1996, to February 14, 2003. In total 322 observations for each time series. The rates reported are closing rates on Fridays.

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Treasury rates are zero coupon yields and covers the maturities 0.5, 1, 2, 3, 4, 5, 6, and 7 years\textsuperscript{7}. Swap rates are for swaps with a semi-annually fixed rate versus 3-month LIBOR and are means of the bid and ask rates from major swap dealers’ quoted rates. Data covers the maturities 2, 3, 4, 5, and 7 years. In addition to the swap data, 3-month LIBOR is used in estimation. Corporate rates are zero coupon yields obtained from Bloomberg’s Fair Market Yield Curves (FMYC) for banks/financial institutions\textsuperscript{8} for the investment grade categories AAA, AA, A, and BBB. Corporate bond data cover the maturities 1, 2, 3, 4, 5, 6, and 7 years. Yield curves for the rating category BBB are missing in the period May 5, 2000, to January 11, 2002. The 5-year BBB yield exhibits a peculiar ”jump” not present in the other time series just before the missing period starts. This ”jump” is present in the BBB time series for other maturities as well and indicates mispricing. The four observations for the BBB curve for the ”jump” dates, from April 7, 2000 to April 28, 2000, are therefore removed. This expands the period where no BBB curves are available to the period from April 7, 2000, to January 11, 2002.

Figure 2 shows the average yield curves for the period December 20, 1996, to February 14, 2003. Not surprisingly, the swap curve is well above the Treasury curve. However, the swap curve is below the AA curve and the average spread between AA yields and swap rates increases with maturity. The average 2-year spread is 14.8 basis points increasing to 34.8 at a maturity of 7 years.

[Figure 2 about here.]

4 Estimation Methodology

Similar to Duffee (1999) and Driessen (2005) we estimate the model using both the cross-sectional and time-series properties of the observed yields by use of the extended Kalman filter.

Each week we observe 42 yields (we return to missing observations later):

- 8 government yields

\textsuperscript{7}For a review of Bloomberg’s estimation methodology see OTS (2002).
\textsuperscript{8}For more information see Doolin and Vogel (1998)
• 7 AAA corporate yields
• 7 AA corporate yields
• 7 A corporate yields
• 7 BBB corporate yields
• 1 LIBOR rate
• 5 swap rates

We recall that $X_t = (X_{t1}, ..., X_{t6})'$ where $X_{t1}, ..., X_{t6}$ are 6 independent affine processes. Suppressing the dependence on the parameters the measurement and transition equation in the Kalman filter recursions are\(^9\)

\[
y_t = A_t + B_t X_t + \epsilon_t, \quad \epsilon_t \sim N(0, H_t) \tag{7}
\]

\[
X_t = C_t + D_t X_{t-1} + \eta_t, \quad \eta_t \sim N(0, Q_t) \tag{8}
\]

where $N(0, \Sigma)$ denotes a normal distribution with mean 0 and covariance matrix $\Sigma$.

We first set up the transition equation (8). The conditional mean and variance of $X_t$ are linear functions of $X_{t-1}$\(^10\),

\[
E(X_t|X_{t-1}) = C + DX_{t-1}, \quad \text{Var}(X_t|X_{t-1}) = Q^1 + Q^2 X_{t-1}
\]

where the matrices $D$ and $Q^2$ are diagonal since the processes are independent. We do not observe $X_{t-1}$ and therefore we use the Kalman filter estimate $\hat{X}_{t-1}$ in the calculation of the conditional variance, $Q_t = Q^1 + Q^2 \hat{X}_{t-1}$.

When pricing corporate bonds, it is more convenient to work with the upper-left $K - 1 \times K - 1$ submatrix of $\Lambda$ which we denote $\tilde{\Lambda}$. Theoretically, we could treat all the entries in the matrix $\tilde{\Lambda}$ as parameters. However, this adds $(K - 1)^2$ parameters to the parameter vector, and this will make the maximizing of the likelihood function over the parameter vector infeasible. Instead we use a generator matrix empirically estimated using Moody's corporate bond default database for the period 1987-2002 The matrix is shown in Table I.

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\(^9\)See Harvey (1990) for a treatment of the Kalman filter.  
We assume that rating transitions are conditionally Markov and ignore downward drift effects. That is, for given level of $\mu(X_s)$ the intensity of downgrade is a function only of the current state and not of the previous rating history. Results in Lando and Skødeberg (2002) indicate that this may be a reasonable approximation for financial firms.

As mentioned, the data include corporate yields for the rating categories AAA, AA, A, and BBB, i.e. investment grade ratings. The remaining rating categories BB, B, and C, which are all speculative grade rating categories, are treated as one rating category denoted $SG$. The generator matrix in Table I is therefore reduced in the following way:

- for the investment grade rating categories the transition intensities for changing rating to BB, B, and C are added and used as the transition for changing rating to $SG$,

- the intensities for going from BB to investment grade ratings are used as the intensities for going from $SG$ to investment grade. The intensity for a jump to a different rating from $SG$ ($\lambda_{SG,SG}$) is changed such that the last row in the new generator matrix still sums to zero.
Table II: This table shows the transition intensity matrix intensities from Table I, where speculative grade states are gathered in one state, SG.

The resulting generator matrix is given in Table II. The new category SG can be regarded as a “downward adjusted” BB category, because the transitions intensities from BB are kept, while the transition intensities to SG are slightly higher than the original transition intensities to BB. The problems of reducing the generator matrix are concentrated in the SG rating category. If we were to price speculative grade bonds the way of reducing the generator matrix would be problematic, but since we only price AAA, AA, A, and BBB rated bonds the adjusted transition intensities do not cause problems for the modelling. However, care has to be taken when interpreting the SG rating category.

Corporate bonds and swap rates are nonlinear functions of the state variables and we write the observed yields as \( y_t = f(X_t) \). A first-order Taylor approximation of \( f(X_t) \) around the forecast \( \hat{X}_{t|t-h} \),

\[
f(X_t) \simeq f(\hat{X}_{t|t-h}) + \hat{B}_t (X_t - \hat{X}_{t|t-h})
\]

\[
= f(\hat{X}_{t|t-h}) - \hat{B}_t \hat{X}_{t|t-h} + \hat{B}_t X_t,
\]

where

\[
\hat{B}_t = \left. \frac{\partial f(x)}{\partial x} \right|_{x=\hat{X}_{t|t-h}}
\]

yields the matrix \( B_t \) in the measurement equation\(^{11}\).

We assume that all 42 yields and rates are measured with independent errors with identical variance, so \( \text{var}(\epsilon_t) = \sigma^2 I_{42} \). Furthermore, we assume that the processes are stationary under \( P \) (implying \( k_i < 0 \)) and use the unconditional distribution as initial distribution in the Kalman filter recursions.

\(^{11}\)It is not necessary to calculate \( A_t \) in the linearization since it is not used in the extended Kalman filter.
BBB yields are missing for a period but the Kalman filter can easily handle missing observations and we refer to Harvey (1990) p. 143-144. The reason for restricting the $Q$-mean of all the processes $X_1, ..., X_6$ to be zero is that empirically, not all of the parameters can be estimated. For example in $r = X_1 + X_2$ the mean and $\alpha_i$ of each factor are not separately identified\textsuperscript{12}. With this normalization $\alpha$ can be interpreted as the average volatility of each factor. In addition, we added a constant mean to the processes describing the government rate, the convenience yield, the default and rating adjustment process $\mu$, and the swap factor $S$ processes. In summary, we therefore have the following model:

\begin{align*}
    r^g(X) &= a + X_1 + X_2, \\
    r(X) &= a + X_1 + X_2 + (e + X_5), \\
    \mu(X) &= b + X_3 + X_4 + c(X_1 + X_2), \\
    S(X) &= d + X_6.
\end{align*}

Restricting $D$ to be a positive CIR process implies the restriction $\alpha_5 = e/\beta_5$\textsuperscript{13}.

The outlined extended Kalman filter does not yield consistent parameter estimates for two reasons. First, in the estimate of $\text{Var}(X_t | X_{t-1})$ we use $\hat{X}_{t-1}$ instead of $X_{t-1}$ and set $\hat{X}_t = -\frac{\alpha_i}{\beta_i}$ if $\hat{X}_t < -\frac{\alpha_i}{\beta_i}$. Nevertheless, Monte Carlo studies in Lund (1997), Duan and Simonato (1999), and de Jong (2000) indicate that the bias of this approximation is negligible. Second, the pricing function $f$ in the measurement equation is linearized around $\hat{X}_{t|t-1}$. In order to assess the possible bias in parameter estimates we conducted a small Monte Carlo experiment suggesting that the approximate Kalman filter works well in estimating our model. Appendix C gives the details of the Monte Carlo experiment along with further estimation details.

\section*{5 Empirical Results}

Before we turn to the main results of the paper we examine our model along several dimensions to check whether the implications of the model are consistent with key features of the data. In this section we look at the average pricing errors of the model and compare our estimated parameters with findings in the previous literature. We also interpret the latent variables, and to

\textsuperscript{12}See de Jong (2000).

\textsuperscript{13}The process $Y = e + X$ has dynamics $dY = k(e - Y)dt + \sqrt{\alpha} - e\beta + \beta Y dW$. 

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justify our interpretation of the Treasury convenience yield, we compare the estimated Treasury premium at different maturities with the proxy spread between Refcorp bonds and Treasury bonds.

To assess the model’s simultaneous fit to all the curves, the mean, standard error, and first-order autocorrelation of the residuals are shown in Table III. The average pricing error for all yields is less than half a basis point while the average standard error is less than 8 basis points. The BBB yield curve has the worst fit, which is seen by the largest average standard errors. This suggests that the our specification of the generator matrix enables us to price highly rated corporate bonds well while the pricing of lower rated bonds might be more problematic. However, for our purpose the fit of the corporate bonds is satisfactory.

The yield curve with the smallest average standard error is the swap curve and the average error of 5.6 basis points is comparable to other papers estimating the swap curve\textsuperscript{14}. We note that a sign of misspecification of the model is that the first-order correlations of the residuals are strongly positive which is also found in other papers such as Duffie and Singleton (1997) and Collin-Dufresne and Solnik (2001).

\textsuperscript{14}The average swap pricing standard error is 6.1 in Duffie and Singleton (1997), 4.5 in Collin-Dufresne and Solnik (2001), and 7.1 in Liu, Longstaff, and Mandell (2004). He (2001) and Grinblatt (2001) do not report pricing errors.
Table III: This table shows statistics for the residuals of the government, corporate, LIBOR, and swap rates measured in basis points. The residual is $\epsilon_t = y_t - \hat{y}_t$ where $\hat{y}_t$ the model-implied yield. The means, standard deviations, and first-order autocorrelations $\rho$ are shown.

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Table IV: This table shows parameter estimates resulting from the Kalman filter estimation described in section 4. The model is

(10) $r^{govt} = a + X_1 + X_2$
(11) $r^{riskless} = r^{govt} + e + X_5$
(12) $\mu = b + c(X_1 + X_2) + X_3 + X_4$
(13) $\lambda^{LIBOR} = \lambda^{AA} + d + X_6$

where the numbers are the corresponding equation numbers in the text. The first set of standard errors are calculated as $\hat{\Sigma}_1 = \frac{1}{T} (\hat{A} \hat{B}^{-1} \hat{A})^{-1}$ where $\hat{A} = -\frac{1}{T} \sum_{t=1}^{T} \frac{\partial \log l_t(\hat{\theta})}{\partial \theta}$ and $\hat{B} = \frac{1}{T} \sum_{t=1}^{T} \frac{\partial \log l_t(\hat{\theta})}{\partial \theta \theta'} \frac{\partial \log l_t(\hat{\theta})}{\partial \theta'}$, while the second set of standard errors are calculated as $\hat{\Sigma}_2 = [\hat{B}]^{-1}$. Note that there are no standard errors on $\alpha_5$ because of the restriction $\alpha_5 = d\beta_5$ ensuring that $D(X) = e + X_5$ remains positive.
Next, we report the estimated parameters in Table IV. Two sets of standard errors are reported: White (1982) heteroscedasticity-corrected standard errors and standard errors without the correction. The former is theoretically more robust while the latter is numerically more stable and details are given in appendix C.

The means of the variables are difficult to estimate reliably and therefore are hard to interpret which is a common problem encountered in for example Duffee (1999) and Duffee and Stanton (2001). From the table we see that credit risk is inversely related to government rates because the parameter $c$ is significantly negative. This is consistent with research using only Treasury and corporate bond data (Duffee (1999) and Driessen (2005)) but in contrast to research using only swap and Treasury data (Liu, Longstaff, and Mandell (2004)). Using both Treasury, swap and corporate bond data we find a negative relationship as in Collin-Dufresne and Solnik (2001) and we suspect that the result in Liu, Longstaff, and Mandell (2004) might be due to the use of the LIBOR - GC repo spread as a proxy for credit risk - a point we will discuss more thoroughly in section 5.1.

Turning to the filtered state variables, the two variables $X_1$ and $X_2$ have the usual interpretation as the level and slope of the government curve as seen in Figure 3. More interestingly, the variables $X_3$ and $X_4$ governing the credit risk process $\mu$ has a similar interpretation. As we see in Figure 4 $X_3$ can be interpreted as the mean credit level defined as $\frac{y^{AAA} + y^{AA} + y^{A}}{3} - y_{g}^{\mu}$ and $X_4$ as the mean credit slope as $\frac{y_{f}^{AAA} + y^{AA} + y^{A}}{3} - y_{f}^{\mu} - \left[\frac{y^{AAA} + y^{AA} + y^{A}}{3} - y_{f}^{\mu}\right]$ even though the picture is not as convincing as for the government factors. This is to be expected since the a) the credit risk factor also depends on the government rate, and b) the spread between corporate bonds and government rates consists of both a liquidity and credit risk factor and $\mu$ accounts only for the credit risk factor.

[Figure 3 about here.]

[Figure 4 about here.]

Finally, we check whether implications of the model is consistent with other sources of data. Longstaff (2004) and Longstaff et al. (2005) suggest using the spread between Refcorp bonds/ strips and the government curve as a proxy for the Treasury premium. Refcorp bonds are implicitly backed by the U.S. Treasury and therefore the credit risk premium is essentially zero.
Six Refcorp bonds are issued maturing in 2019, 2020, 2021, and 2030 and the principal amounts outstanding range from 4.5 to 5.5 billion dollars\textsuperscript{15}. As with Treasury bonds, Refcorp bonds can be held in stripped form so a whole range of Refcorp zero coupon bonds exists. From these a yield curve is estimated daily and readily available at Bloomberg. We can therefore compare the average model-implied term structure of Treasury premia with the average term structure of Refcorp - government spreads to see whether our model and the proxy imply the same shape of term structure. In our model the process \( L = e + X_5 \) is a measure in basis points of the Treasury premium at the very short end of the yield curve. For longer-term maturities we can estimate the effect in basis points of the Treasury factor since the price of a riskless bond is given as

\[
P(t, T) = E_t(\exp(-\int_t^T r^g(s) + L(s)ds)) = P^a(t, T) E_t(\exp(-\int_t^T L(s)ds)),
\]

and therefore

\[
y(t, T) = y^a(t, T) - \frac{1}{T-t} \log(E_t(\exp(-\int_t^T L(s)ds))). \tag{14}
\]

Figure 5 shows the average Refcorp - government curve along with the average estimated Treasury premium curve. Both yield curves are downward sloping consistent with Driessen (2005) who also finds a downward-sloping term structure of liquidity using a different methodology. However, the average model-implied Treasury premium is more downward-sloping than the average Refcorp - government spread. A possible explanation for this could be that the Refcorp - government spread is a noisy proxy for Treasury convenience yield. Out of the 322 weekly observations in the estimation period 17 observations of the 1-year Refcorp - government spread are negative while one observation of the 7-year spread is negative. A negative Treasury premium is hard to interpret and this suggests that there are other factors than a Treasury premium influencing the Refcorp - Treasury spread.

[Figure 5 about here.]

In summary, we have found that our model has small average pricing errors. We have interpreted the first two latent factors as the level and slope

\textsuperscript{15}For more information see Longstaff (2004).
of the Treasury curve, and the two factors governing credit risk to be strongly related to the level and slope of the average credit spread curves. We have also found a downward sloping term structure of Treasury premia consistent with the commonly used proxy Refcorp minus Treasury, and therefore our factor proxying the convenience yield for Treasuries seems well specified. We will return to the interpretation of the sixth factor – the swap factor – below. In passing, we noted that the relation between credit risk and the government short rate is strongly negative.

5.1 Credit risk

The market’s perception of credit risk has large effects on swap spreads as documented in Duffie and Singleton (1997) and other papers, although swap rates carry less default risk than AA corporate rates as showed in Collin-Dufresne and Solnik (2001). However, papers separating out the credit risk component in swap spreads have to our knowledge all relied on using proxies for credit risk. In their time series analysis of the 10-year swap spread, Reinhardt and Sack (2002) proxy the size of the credit risk component in the swap spread as a constant fraction of the 10-year AA corporate- Refcorp spread. In a full pricing framework He (2001) and Liu, Longstaff, and Mandell (2004) separate out the credit risk component in swap spreads by proxying short-term AA credit risk with the LIBOR - GC repo spread - a spread which we label the LGC spread. In this section we assess the quality of this proxy by comparing the implied short-term AA credit risk in our model with the observed LGC spread.

LIBOR rates are rates on unsecured loans between counterparties rated AA on average while GC Repo rates are rates on secured loans and therefore the difference is thought to be due to a credit risk premium. In figure 6 we compare the 3-month LGC spread with the estimated 3-month AA credit risk premium. The 3-month AA credit risk premium on date $t$ is calculated as the difference in basis points between the yield on a 3-month AA corporate bond and a 3-month riskless bond (with no liquidity), while the 3-month LIBOR and GC repo rates are from Bloomberg. The average estimated premium is 10.0 basis points while the average observed LGC spread is 14.7 basis points. The difference in averages is to a large extent due to large difference before the year-end in 1999 and 2000 and if we exclude the last three months before 1999 and 2000 the averages are 10.0 and 12.2. Even though the averages are similar, the LGC spread is very volatile while the estimated AA default
premium is much more persistent. A possible explanation for the different behavior of the two time series is given in Duffie and Singleton (1997). In their model the LIBOR rate is poorly fitted and they suggest that there might be noncredit factors determining LIBOR rates. Support for this view is given in Griffiths and Winters (2004) who examine one-month LIBOR and find a turn-of-the-year effect. The rate increases dramatically at the beginning of December, remains high during December, and decreases back to normal at the turn-of-the-year, with the decline in rates beginning a few days before year-end. This effect is a liquidity effect unrelated to credit risk. If the GC repo rate does not have a turn-off-the-year effect, the LGC spread will mirror this liquidity effect.

[Figure 6 about here.]

We see the largest difference between the LGC spread and estimated AA credit premium in the last three months before the Millenium Date Change (Y2K). Three months before Y2K the LGC spread jumps from 11 to 79 basis points. If the jump was caused by general credit risk concerns all LGC spreads for various maturities would jump simultaneously. If the jump was caused by credit risk concerns right after Y2K we would see LGC spreads remaining high until Y2K. Neither is happening as we see in Figure 7. As argued in Sundaresan and Wang (2004), due to concern around Y2K, lenders in the interbank market wanted a premium to lend cash due shortly after Y2K and the jump is therefore due to a liquidity premium on short-term lending. Because the 3-month credit risk premium in our model is estimated on basis of a range of yields and maturities, we see in Figure 7 that the premium is practically unaffected by the Y2K.

[Figure 7 about here.]

Liu, Longstaff, and Mandell (2004) and Li (2004) proxy credit risk with this spread. This implies that the credit risk component inherits the properties of the LGC spread in being volatile and rapidly mean-reverting. This in turn implies that long-term swap spreads are only weakly affected by fluctuations in the credit risk component. The credit spread fluctuations in our model are not as mean-reverting and therefore cause larger fluctuations in long-term swap spreads.

The evidence in this section suggests that the LGC spread is an inappropriate proxy for credit risk. Although the floating rate leg in the swap...
contract is directly tied to 3-month LIBOR, short term liquidity effects in the LGC spread imply that this spread is not suitable for catching the credit risk premium in longer term swaps.

5.2 Swap Rates and MBS Hedging

In contrast to earlier literature modeling the term structure of swap spreads, we allow for a unique liquidity factor for the swap market. We now show that this factor has a strong impact on swap spreads both cross-sectionally and in the time series dimension, and we find that from the beginning of late 2000, the factor correlates strongly with variables influencing the demand to hedge MBS securities.

We calculate the contribution to the swap spread from the swap factor cross-sectionally by computing a ’zero-coupon spread’

\[-\frac{1}{T} \log(E_t(\exp(-\int_t^{t+T} S(u)du)))\]

and transforming this into par rates.\(^\text{16}\) We will return to the term structure of these par rates in the next section, and focus now on the time-variation of the swap factor and MBS related hedging activity.

Before turning to the evidence, we briefly recall why there is likely to be a connection between the two markets. Due to the prepayment risk embedded in MBSs - borrowers are allowed to prepay the mortgage which creates uncertainty regarding the timing of cash flows of MBSs - movements in interest rates often result in significant changes in the option-adjusted duration of an MBS. When interest rates drop, borrowers can refinance their mortgages by exercising their option to call the mortgages at par value. This causes a fall in the duration of mortgage backed securities. Hedging activity in connection with this duration change has the potential for creating large flows in the fixed-income markets. The US mortgage market has more than doubled in size since 1995 and in 2000 the size surpassed the Treasury market as noted in BIS (2003).

Furthermore, as Wooldridge (2001) notes, non-government securities were routinely hedged with government securities until the financial market crisis

\(^{16}\)This is actually an approximation. We can find the exact effect of the swap factor by calculating swap rates with and without the swap factor and find the effect as the difference between the two calculated swap rates. However, the approximation differs less than one basis point from the exact calculation so for simplicity we use the approximation.
in 1998. However, periodic breakdowns in the normally stable relationship between government and non-government securities lead many market participants to switch hedging instruments from government to non-government securities such as interest rate swaps. Today, interest-rate swaps and swap-tions are the primary vehicle for duration hedging of MBS portfolios, a point also confirmed in studies by Perli and Sack (2003), Duarte (2005) and Chang, McManus, and Ramagopal (2005) all of which are primarily concerned with volatility effects of this hedging. In BIS (2003), it is argued, that the concentration of OTC hedging activity in a small number of dealers in the swap market seems to make the market more vulnerable to a loss of liquidity. If a few dealers breach their risk limits and cut back on the market-making activity the whole market loses liquidity. Therefore falling (rising) option-adjusted duration can cause swap rates to fall below (rise above) their long run level.

[Figure 8 about here.]

In Figure 8 we show the swap factor in our model along with the Lehman modified duration index for mortgage backed securities obtained from Datas-stream. The factor varies greatly during the estimation period with a difference of around 50 basis points from the high in the middle of 2000 to the low in the end of 2002. We see that the factor spans a much wider interval after the turn of the Millenium than before. Before 2000 there is no relation (correlation -0.01), while after 2000 there is a strong relation (correlation 0.86) and we see that the factor influences swap rates by as much as 40 basis points after 2000. In the pre-2000 period, the year of 1998 saw a considerable amount of mortgage refinancing but we see that it did not strongly affect swap rates. In contrast the large refinancing wave beginning in the end of 2000 and lasting until the end of the sample resulted in swap rates falling more than Treasury rates. The fact that the swap market reacted differently in the two refinancing waves before and after 2000 is consistent with the change in preferred hedging instruments mentioned in Wooldridge (2001).

In November 2002, the effect of the swap factor on 2-yr swap rates was as much as 32 basis points and from August 2002 until the end of the estimation period in February 2003 the swap component was -20 basis points on average suggesting a strong pressure to enter as the fixed-receiver in an interest rate swap. IMF (2003, p. 17) reports that "In August 2002, the duration gap between Fannie Mae’s assets and liabilities widened to minus 14
months, as falling interest rates increased likely prepayment rates and thus shortened the expected duration of its mortgages. This gap prompted Office of Federal Housing Enterprise Oversight to require an action plan to correct this imbalance and to monitor Fannie Mae’s maintenance of its duration gap for the following six months before it declared itself satisfied in April 2003”. Since Fannie Mae is a large player in this market, this is certainly consistent with a pressure to enter as fixed receiver being formed in the market.

Reinhart and Sack (2002) analyze the properties of the 10-year swap spread and estimate an idiosyncratic swap factor similar to our estimated swap factor. In their data set, which runs until the second half of 2001, they find a larger influence of the swap factor in 2001 and conjecture that it is due to the use of swaps as hedging instruments in the MBS market. We confirm this conjecture using a swap factor that is part of a full pricing model controlling for credit risk and convenience yields in Treasury securities.

As a final indication of the relationship between MBS hedging and our swap factor, we consider statistics on the holdings of swaps and swaptions from one of the two largest mortgage issuers, Fannie Mae. As documented for example in Jaffee (2003) and Jaffee (2005), the mortgage agencies Fannie Mae and Freddie Mac have strongly increased the amount of mortgage bonds retained on their own books. Both institutions have guidelines available to the public regarding their hedging of duration. In order to hedge interest rate risk they seek to keep the net total duration of their balance sheet within a range around zero by hedging their holdings mainly with interest rate swaps or related derivatives such as swaptions and caps. Other market participants follow a similar trading strategy although to a lesser extent. If interest rates drop, as outlined above, duration on MBSs drop and Fannie Mae/Freddie Mac may offset the duration drop on their holdings by entering as the fixed-receiver in an interest rate swap or in options to enter as a fixed receiver. Figure 9 graphs our estimated swap factor with the net quarterly changes in Fannie Mae’s holdings of the fixed leg (or options to enter to the fixed leg) of interest rate swaps.

[Figure 9 about here.]

When the change is negative, the receive-fixed demand has gone up, pressing the swap factor down, consistent with what we saw above. A detailed study of these holdings is in itself a large research project. The exact composition of the swaption portfolio would have to be examined as the ‘moneyness’
of the swaptions has implications for the amount by which they contribute to swap hedging.\footnote{If we consider swap positions only from the beginning of 2001, we get a similar result, but before 2001 where we have swap data only, we do not get the same picture.} Despite the roughness of the measure, the graph lends further support to the interpretation of the swap factor being related to MBS hedging after 2000.

### 5.3 Decomposing swap spreads

We have analyzed three components in swap spreads - a Treasury, a credit risk, and a swap component - separately and now turn to the joint effect of the components on swap spreads.

In Figure 10 the estimated 5-year swap spread is decomposed into the three components. We see that while the Treasury factor accounted for the largest part of the swap spread in first half of the estimation period, the size of the factor diminished in the second half of the period. This suggests that the importance of the Treasury factor has lessened during the estimation period. The swap factor accounted for a relatively small part of the swap spread before 2000, while the factor has contributed to a contraction of the swap spread in 2000-2003.

While the credit risk factor had a relatively small impact on the 5-year swap spread prior to 2000 we see that the factor has grown in importance after 2000. The larger impact of credit risk is due to steeper credit spread slopes in the corporate bond market as seen on the right graph in Figure 4. This highlights the importance of incorporating the term structure of corporate bonds in modeling swap spreads, since the slope of the corporate curve has information on the credit component in long-term swap rates which cannot be deduced from a 'short' credit spread alone.

Our model not only estimates the impact of the swap factor through time but also allows us to see its term structure implications. As shown in Table V, the swap factor gives rise to a significant maturity premium in interest rate swap relative to government bonds. While this premium on average is approximately zero at a maturity of two years the average premium is 15.4
basis points at seven years\textsuperscript{18}. The difference between the 7-year premium and the 2-year premium fluctuates between 15.2 and 18.0 basis points so the maturity premium is not very time-varying. This evidence supports the existence of a risk premium for holding swaps relative to government bonds - a point also discussed by He (2001).

<table>
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<tr>
<th>maturity</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>average swap effect</td>
<td>-0.8</td>
<td>2.6</td>
<td>5.9</td>
<td>9.1</td>
<td>15.4</td>
</tr>
<tr>
<td>average Treasury effect</td>
<td>34.9</td>
<td>29.3</td>
<td>25.1</td>
<td>21.8</td>
<td>17.3</td>
</tr>
<tr>
<td>average credit risk effect</td>
<td>17.2</td>
<td>20.4</td>
<td>23.1</td>
<td>25.4</td>
<td>28.9</td>
</tr>
</tbody>
</table>

\textbf{Table V}: This table shows the average effect in basis points of the swap-specific factor, Treasury convenience yield, and LIBOR credit risk on swap rates across maturities. See caption on Figure 10 for details of the calculations.

The swap maturity premium is evident also from Figure 11 which shows the average effect of the three components on swap spreads across maturity. The Treasury component accounts for the largest part of the swap spread in the short end of the swap curve while the size of the Treasury factor decreases with maturity. Finally, the average effect of the credit risk component is 28.9 basis points at maturity 7 years and 17.2 basis points at maturity 2 years. Thus, the size of the average credit risk factor increases with maturity - an effect mainly caused by the period after 2000.

Finally, our decomposition of swap spreads allows us to measure the distance from the riskless rate to the Treasury rate and to the swap rate. Figure 12 shows the 5-year spreads between the riskless par rate and the Treasury par rate and between the riskless par rate and the swap rate. While the risk free rate was roughly halfway between the Treasury and swap rate before 2000 it has been closer to the Treasury par rate after 2000, especially during 2000-2002. As explained earlier in the section the increased distance between the riskless rate and the swap rate in the latter part of the sample is mainly due to an increased credit risk component in swap rates. In addition there is on average a swap maturity premium of nine basis points adding to the separation of the risk free rate and the swap rate. Therefore our results support

\textsuperscript{18}In an earlier version of the paper we estimated the model without the swap factor and the swap rates were systematically overfitted at short maturities and underfitted at long maturities.
the conclusion in Wooldridge (2001) that the attractiveness of the swap curve as a benchmark yield curve is diminished by the over-the-counter structure of the swap market and the credit risk component inherent in swap spreads. Our model also allows us to decompose swap spreads at different maturities. Since convenience yield separates the riskless rate from the Treasury rate, we see in Figure 11 that for short maturities the swap rate on average is a better proxy for the riskless rate while for longer maturities the Treasury yield is closer to the riskless rate.

6 Conclusion

We have analyzed a six-factor model for Treasury bonds, corporate bonds, and swap rates and decomposed the swap spreads, for each time and maturity, into three components: One arising from a convenience yield to holding Treasuries, one explained by the credit risk element of the underlying LIBOR rate, and one factor specific to the swap market. We have seen that in the later part of our sample, the swap-specific factor has a strong correlation with hedging activity in the MBS market.

Our model shows that the credit risk factor is important for understanding swap spreads, but it is better captured by the information in the corporate bond market than by the commonly used proxy LIBOR - GC repo spread.

The credit risk component is important in swap spreads and it is on average increasing with the maturity of the swap. We find that the Treasury factor accounts for most of the swap spread in the short end but that it is decreasing with maturity. The average effect of the swap factor is increasing with maturity but is virtually zero at two years. As a consequence of these observations, the riskless rate is better proxied by the Treasury rate for long maturities whereas the swap rate is a better proxy for the riskless rate in the short end.
A Result on Univariate Affine Processes

For the univariate affine process
\[ dX_t = k(X_t - \theta)dt + \sqrt{\alpha + \beta X_t}dW, \] (15)
we know from Duffie, Pan, and Singleton (2000) that there exists \( A \) and \( B \) such that
\[ E_t(e^{-\int_t^T c_1 X_u du} e^{c_2 X_T}) = e^{A(t,T)+B(t,T)X_t}. \] (16)

Christensen (2002) has derived explicit expressions for \( A \) and \( B \) and they are given as
\[
B(t, T) = \frac{-2c_1(e^{\gamma(T-t)} - 1) + c_2e^{\gamma(T-t)}(\gamma + k) + c_2(\gamma - k)}{2\gamma + (\gamma - k - c_2\beta)(e^{\gamma(T-t)} - 1)} \\
A(t, T) = \frac{-2k\theta}{\beta} \ln\left(\frac{2\gamma e^{\frac{1}{2}(\gamma-k)(T-t)}}{2\gamma + (\gamma - k - c_2\beta)(e^{\gamma(T-t)} - 1)}\right) \\
\gamma = \sqrt{k^2 + 2\beta c_1} \quad \text{for} \quad c_1 > -\frac{k^2}{2\beta}.
\]

B Pricing Formulas

B.1 Riskless Bonds

The riskless rate is given in equation (11) as
\[ r = X_1 + X_2 + X_5. \]
As noted in section 4, in order to have identification of the parameters we set the means of the processes to zero and add a constant,
\[ r = (a + X_1 + X_2) + (e + X_5). \]
Prices of riskless bonds are

\[ P(t, T) = E_t(\exp(-\int_t^T r(u)du)) = \]

\[ = e^{-(a+e)(T-t)} e^{-\int_t^T X_1u du} e^{-\int_t^T X_2u du} e^{-\int_t^T X_3u du} \]

\[ = e^{-(a+e)(T-t)} e^{A^1(T-t)+B^1(T-t)X_1} e^{A^2(T-t)+B^2(T-t)X_2} e^{A^5(T-t)+B^5(T-t)X_3} \]

\[ = e^{A(T-t)+B(T-t)'X_t}, \]

where we have used the independence of the processes and a special case of
the result in appendix A, and \( A \) and \( B \) are given as

\[ A(T-t) = -(a+e)(T-t) + A^1(T-t) + A^2(T-t) + A^5(T-t), \]
\[ B(T-t)' = (B^1(T-t), B^2(T-t), 0, 0, B^5(T-t), 0). \]

### B.2 Government Bonds

In equation (10) the government rate is given as

\[ r^g = X_1 + X_2. \]

For identification this becomes

\[ r^g = a + X_1 + X_2, \]

and the prices of government bonds are

\[ P^g(t, T) = e^{A^g(T-t)+B^g(T-t)'X_t}, \]

where \( A(T-t)^g = -a(T-t) + A^1(T-t) + A^2(T-t) \) and \( B^g(T-t)' = (B^1(T-t), B^2(T-t), 0, 0, 0, 0) \) are derived exactly as in the riskless bond case.

### B.3 Corporate Bonds

In the pricing of corporate bonds we choose to work with a generator matrix
excluding default states,

\[ \tilde{A}_X(s) = \tilde{A}' \mu(X_s) = \begin{pmatrix} -\lambda_1 - \nu_1 & \lambda_1 & \cdots & \lambda_1 & \cdots & \lambda_{1,K-1} \\ \lambda_2 & -\lambda_2 - \nu_2 & \cdots & \lambda_2 & \cdots & \lambda_{2,K-1} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \lambda_{K-1} & \cdots & \cdots & -\lambda_{K-1} & \cdots & -\lambda_{K-1,K-1} - \nu_{K-1} \end{pmatrix} \mu(X_s), \]

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for notational reasons. We can decompose $\tilde{\Lambda}^\nu$ into $\tilde{\Lambda}^\nu = \tilde{B} \tilde{D} \tilde{B}^{-1}$, where $\tilde{D}$ is a diagonal matrix with the eigenvalues of $\tilde{\Lambda}^\nu$ in the diagonal and $\tilde{B}$ is a $K - 1 \times K - 1$ matrix with columns given by the $K - 1$ eigenvectors of $\tilde{\Lambda}^\nu$. The price of a corporate bond with rating $i$ can be written as

$$v^i(t, T) = \sum_{j=1}^{K-1} -[B]_{ij}[B^{-1}]_{j,K} E_t (e^{\int_t^T \tilde{D}_{jj} \mu(X_u) - r(X_u) du})$$

according to Lando (1998), so we have $c_{ij} = -[B]_{ij}[B^{-1}]_{j,K}$ and $d_j = \tilde{D}_{jj}$ in (4). From the specification in (11) and (12) we have that $r = a + X_1 + X_2 + (e + X_5)$ and $\mu = b + X_3 + X_4 + c(X_1 + X_2)$ so we can solve the conditional expectation

$$E_t (e^{\int_t^T \tilde{D}_{jj} \mu(X_u) - r(X_u) du}) = e^{-(T-t)(a+e-\tilde{D}_{jj}b)} E_t (e^{-\int_t^T ((1-c\tilde{D}_{jj})X_{1u}+(1-c\tilde{D}_{jj})X_{2u}+(-\tilde{D}_{jj})X_{3u}+(-\tilde{D}_{jj})X_{4u}+X_{5u} du)}) =$$

$$e^{-(T-t)(a+e-\tilde{D}_{jj}b)} \prod_{i=1}^{2} [E_t (e^{-\int_t^T (1-c\tilde{D}_{jj})X_{iu} du})] \times$$

$$\prod_{i=3}^{5} [E_t (e^{-\int_t^T (-\tilde{D}_{jj})X_{iu} du})] E_t (e^{-\int_t^T (X_{5u} du)}) =$$

$$e^{-(T-t)(a+e-\tilde{D}_{jj}b)} \prod_{i=1}^{5} [e^{A^i(t-T)+B^i(T-t)X_{iu}}] = e^{A^i(T-t)+B^i(T-t)X_{it}}$$

where

$$A^i(T-t) = -(T-t)(a+e-\tilde{D}_{jj}b) + \sum_{i=1}^{5} [A^i(t)]$$

$$B^i(T-t) = (B^1_1(T-t), ..., B^5_5(T-t), 0).$$

### B.4 Swap Rates

To price interest rate swaps we value the floating rate payments and fixed rate payments separately. In the following we assume that the floating rate is paid quarterly while the fixed rate is paid semi-annually. With $n$ being the maturity of the swap in quarters of a year the present value of the floating
rate payments in the swap is
\[
E_t^Q \left[ \sum_{i=1}^{n} e^{-\int_{t_i}^{t} r_u \, du} \left( \frac{1}{u^{\alpha/2}(t_{i-1}, t_i)} - 1 \right) \right],
\]
while the present value for the fixed rate payments are
\[
\frac{F(t, T)}{2} \sum_{i=1}^{n} P(t, t_i).
\]

In evaluating the present value of the floating rate payments we follow the idea outlined in Duffie and Liu (2001). The \( i \)’th floating rate payment can be rewritten as
\[
E_t^Q \left[ e^{-\int_{t_i}^{t} r_u \, du} \left( \frac{1}{u^{\alpha/2}(t_{i-1}, t_i)} - 1 \right) \right] = E_t^Q \left[ e^{-\int_{t_i}^{t} r_u \, du} \left( \frac{1}{u^{\alpha/2}(t_{i-1}, t_i)} \right) \right] - P(t, t_i).
\]
Assumption (5) and (6) gives
\[
v^{LIB}(t_{i-1}, t_i) = E_{t_{i-1}}^Q \left( e^{-\int_{t_{i-1}}^{t} r_u \, du} \left( \frac{1}{u^{\alpha/2}(t_{i-1}, t_{i-1})} + r_u + d + X_u \right) \right)
= e^{-0.25(a + d + e + \nu_2 b)} \prod_{j \in \{1, 2, 5, 6\}} E_{t_{i-1}}^Q \left( e^{-\int_{t_{i-1}}^{t} r_u \, du} \left( \frac{1}{u^{\alpha/2}(t_{i-1}, t_{i-1})} + X_u \right) \right) \prod_{j \in \{3, 4\}} E_{t_{i-1}}^Q \left( e^{-\int_{t_{i-1}}^{t} r_u \, du} \left( \frac{1}{u^{\alpha/2}(t_{i-1}, t_{i-1})} \nu \right) \right) = e^{-0.25(a + d + e + \nu_2 b)} \prod_{j=1}^{6} e^{A_j(0.25) + B_j(0.25) X_{t_{i-1}}} = e^{\mathcal{A}(0.25) + \mathcal{B}(0.25) X_{t_{i-1}}},
\]
where
\[
\mathcal{A}(0.25) = -0.25(a + d + e + \nu_2 b) + \sum_{j=1}^{6} A_j(0.25),
\]
\[
\mathcal{B}(0.25)' = (B_1(0.25), ..., B_6(0.25)).
\]
Now, using the law of iterated expectations the expectation in the \( i \)’th floating rate payment given in equation (20) is
\[
E_t^Q \left[ e^{-\int_{t_i}^{t} r_u \, du} \left( \frac{1}{u^{\alpha/2}(t_{i-1}, t_i)} \right) \right] = E_t^Q \left[ e^{-\int_{t_i}^{t} r_u \, du} e^{-\mathcal{A}(0.25) - \mathcal{B}(0.25) X_{t_{i-1}}} \right] = e^{-\mathcal{A}(0.25)} E_t^Q \left[ e^{-\int_{t_i}^{t} r_u \, du} e^{-\mathcal{B}(0.25) X_{t_{i-1}}} E_{t_{i-1}}^Q \left[ e^{-\int_{t_{i-1}}^{t} r_u \, du} \right] \right],
\]
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and since
\[
E^Q_{t_{i-1}} \left[ e^{-\int_{t_{i-1}}^{t_{i}} r_u du} \right] = e^{-0.25(a+e)} \prod_{j=1}^{4} E^Q_{t_{i-1}} \left[ e^{-\int_{t_{i-1}}^{t_{i}} X_j u du} \right] = e^{-\bar{A}(0.25) - \bar{B}(0.25) X_{t_{i-1}}},
\]
where
\[
\bar{A}(0.25) = -0.25(a + e) + \sum_{j=1}^{4} \bar{A}_j(0.25),
\]
\[
\bar{B}(0.25) = (\bar{B}_1(0.25), \bar{B}_2(0.25), 0, 0, \bar{B}_5(0.25), 0),
\]
we have
\[
E^Q_t \left[ e^{-\int_{t_{i-1}}^{t_{i}} r_u du} \frac{1}{vLIB(t_{i-1}, t_i)} \right] = e^{-\bar{A}(0.25) - \bar{B}(0.25)} E^Q_t \left[ e^{-\int_{t_{i-1}}^{t_{i}} X_j u du} \right] E^Q_t \left[ e^{-\int_{t_{i-1}}^{t_{i}} X_j u du} \right]
\]
\[
= e^{-\bar{A}(0.25) - \bar{B}(0.25) - (t_{i-1} - t)(a + e)} \prod_{j=1}^{4} E^Q_t \left[ e^{-\int_{t_{i-1}}^{t_{i}} X_j u du} \right] \prod_{j=1}^{6} E^Q_t \left[ e^{-\int_{t_{i-1}}^{t_{i}} X_j u du} \right]
\]
\[
= e^{A^*(t_{i-1} - t) + B^*(t_{i-1} - t) X_t},
\]
where
\[
A^*(t_{i-1} - t) = 0.25 \nu_2 b - \sum_{j=3}^{4} \bar{A}_j(0.25) - (t_{i-1} - t)(a + e) + \sum_{j=1}^{6} \bar{A}_j(t_{i-1} - t),
\]
\[
B^*(t_{i-1} - t) = (\bar{B}_1(t_{i-1} - t), ..., \bar{B}_6(t_{i-1} - t)).
\]
Inserting this in formula (18) for the floating rate payments,
\[
\sum_{i=1}^{n} (e^{A^*(t_{i-1} - t) + B^*(t_{i-1} - t) X_t} - P(t, t_i)),
\]
and equating the present value of the fixed and floating rate payments we get the swap rate
\[
F(t, T) = \frac{2 \sum_{i=1}^{n} (e^{A^*(t_{i-1} - t) + B^*(t_{i-1} - t) X_t} - P(t, t_i))}{\sum_{i=1}^{n} P(t, t_{2i})}. 
\]
When calculating the current three-months LIBOR rate we allow for downgrades, so it is calculated as

\[ L(t, t + 0.25) = \frac{1}{v^{LIB}(t, t + 0.25)} - 1 \]

where

\[
v^{LIB}(t, t + 0.25) = \sum_{j=1}^{K-1} -[B]_{ij}[B^{-1}]_{ij} E_t(e^{\int_t^T \hat{b}_{j}\mu(X_u) - (r(X_u) + d + X_{6u})du})
\]

\[ = \left( \sum_{j=1}^{K-1} -[B]_{ij}[B^{-1}]_{ij} E_t(e^{\int_t^T \hat{b}_{j}\mu(X_u) - (r(X_u) + d + X_{6u})du}) \right) E_t(e^{-\int_t^T d + X_{6u}du}) \]

\[ = v^{AA}(t, t + 0.25) E_t(e^{-\int_t^T d + X_{6u}du}). \]

C Estimation Details

C.1 Monte Carlo Experiment

In the main text we noted that the extended Kalman Filter does not yield consistent parameter estimates partly due to the linearization in the measurement equation in the Kalman filter. In this appendix we set up a small Monte Carlo experiment in order to assess the possible bias due to the linearization. The experiment is set up as follows. We assume that the government short interest rate \( r \) is constant and that convenience yield is assumed to be identical zero while the factor \( \mu(X) \) is assumed to be one-dimensional. Since we focus on the possible bias arising from the linearization in the measurement equation we assume that \( X \) is Gaussian, i.e.

\[ dX_t = k^*(X_t - \theta^*)dt + \sigma dW_t^Q. \]

The risk premium \( \lambda \) does not enter the pricing functions and therefore we regard it as fixed in the simulations. Besides the three parameters \( k^*, \theta^*, \) and \( \sigma \) there is the magnitude of the measurement error \( \sigma_e \) and the five default intensity parameters \( \nu_1, ..., \nu_5 \) to estimate. In each simulation run the latent Gaussian variable \( X \) is simulated using that \( X_t|X_{t-1} \) is normal distributed. At time \( t \) the observed yields are calculated using the pricing formulas for swap rates and corporate yields and adding independent normal distributed
errors with variance $\sigma^2$ to each yield. Finally, the parameters are estimated maximizing the likelihood function in the Kalman filter with the pricing function linearized around $\hat{X}_{t|t-1}$. This procedure is repeated 500 times. The parameters are chosen at reasonable values and in Table VI we see the result of the parameter estimation. It is seen that there does not appear to be any significant bias in the parameter estimates and the standard errors on the parameter estimates are quite small, so for parameter estimation the extended Kalman filter can be considered to work very well. In an earlier version of the paper we also showed that the bias on the filtered estimate of $X$ is negligible.

<table>
<thead>
<tr>
<th>Par.</th>
<th>True val.</th>
<th>Mean (Std.Err)</th>
<th>Par.</th>
<th>True val.</th>
<th>Mean (Std.Err)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k^*$</td>
<td>-0.1</td>
<td>-0.1006 (0.0062)</td>
<td>$\nu_1$</td>
<td>0.002</td>
<td>0.001998 (0.000097)</td>
</tr>
<tr>
<td>$\theta^*$</td>
<td>1</td>
<td>1.002 (0.028)</td>
<td>$\nu_2$</td>
<td>0.005</td>
<td>0.004994 (0.000195)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.3</td>
<td>0.3003 (0.0153)</td>
<td>$\nu_3$</td>
<td>0.01</td>
<td>0.009985 (0.000407)</td>
</tr>
<tr>
<td>$\sigma_\epsilon$</td>
<td>0.0009</td>
<td>0.0008995 (0.000068)</td>
<td>$\nu_4$</td>
<td>0.02</td>
<td>0.01997 (0.00083)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\nu_5$</td>
<td>0.03</td>
<td>0.02995 (0.00112)</td>
</tr>
</tbody>
</table>

Table VI: Results of a Monte Carlo experiment for the QML estimator of a one-factor Vasicek model $\mu_t = k^*(\mu_t - \theta^*)dt + \sigma dW_t^Q$. $\nu_i\mu_t$ is the recovery-adjusted hazard rate for rating class $i$. The number of simulation runs is 500. In each simulation, 322 weekly observations on 2-, 3-, 4-, 5-, 7-year swap rates, 1-, 2-, 3-, 4-, 5-, 6-, 7-year AAA-, AA-, A-, BBB-yields and 3-month AA yield are observed. Measurement error $\sigma_\epsilon I_{34}$ is added to the observations.

### C.2 Optimization

The optimization of the likelihood function is complicated due to two reasons. First, in the Kalman filter recursions it might happen that $\hat{X}_t < -\frac{a}{\beta}$ in which case we set $\hat{X}_t = -\frac{a}{\beta}$ to ensure that $Var(X_t|X_{t-1})$ remains positive definite. In this case we follow Duffee and Stanton (2001) and set the log-likelihood function to a large negative number, so it will not be optimal hit the boundary. Second, the likelihood function has a large number of
local maxima as is common in these types of models. We employ the usual procedure of repeatedly generating a random vector of starting values and maximize the log-likelihood function. This was done 100 times using the Nelder-Mead maximization algorithm and the largest of the 100 resulting values was chosen. For the 10 largest values we compared the parameter values and they were generally in the same range. More importantly in regard to the conclusions in our paper, the filtered processes were very close to each other across the 10 values.

C.3 Standard Errors

Because the log-likelihood function is misspecified for non-Gaussian models, a robust estimate of the variance-covariance matrix can be found using White (1982) as

\[
\hat{\Sigma}_1 = \frac{1}{T}[\hat{A}\hat{B}^{-1}\hat{A}]^{-1},
\]

where

\[
\hat{A} = -\frac{1}{T} \sum_{i=1}^{T} \partial^2 \log l_t(\hat{\theta}) \frac{\partial}{\partial \theta \partial \theta'},
\]

\[
\hat{B} = \frac{1}{T} \sum_{i=1}^{T} \frac{\partial \log l_t(\hat{\theta})}{\partial \theta} \frac{\partial \log l_t(\hat{\theta})'}{\partial \theta}.
\]

In order to minimize the concern of numerical instability in the calculation of second derivatives, we estimate "smoothed" versions of \( \hat{A} \) and \( \hat{B} \) which are calculated as follows. The \( \Delta \theta_1 \) and \( \Delta \theta_2 \) vectors leading to the most stable calculation of first and second derivatives are found. \( \hat{A} \) is found by calculating \( \hat{A}_i \) using \((0.8 + 0.02i)\Delta \theta_1\), \( i = 1, ..., 20 \), and letting \( \hat{A} = E(\hat{A}_i) \). \( \hat{B} \) is found by calculating \( \hat{B}_i \) using \((0.8 + 0.02i)\Delta \theta_1 \) and \((0.8 + 0.02i)\Delta \theta_2\), \( i = 1, ..., 20 \), and letting \( \hat{B} = E(\hat{B}_i) \). Standard errors using the smoothed estimates and the formula (22) are reported in the first row after parameter estimates. In the second row after parameter estimates we report standard errors using the less theoretically but numerically more robust estimator of the variance-covariance matrix,

\[
\hat{\Sigma}_2 = \frac{1}{T}[\frac{1}{T} \sum_{i=1}^{T} \frac{\partial \log l_t(\hat{\theta})}{\partial \theta} \frac{\partial \log l_t(\hat{\theta})'}{\partial \theta}]^{-1} = [T\hat{B}]^{-1},
\]
where the smoothed estimate of $\hat{B}$ is used.
References


Figures

Figure 1: This figure shows a decomposition in basis points of the estimated 5-year swap spread into the effect of the swap specific factor, LIBOR credit risk, and Treasury convenience yield. For details of the calculations, see caption on Figure 10.
Figure 2: This figure shows the average term structure of swap rates, Treasury yields, and A-, AA-, AAA-corporate yields. Data is from Bloomberg and covers the period December 20, 1996, to February 14, 2003. The Treasury and corporate yields are semi-annual par yields.
Figure 3: The government rate is given as $r^g(X) = a + X_1 + X_2$ in the model. This figure shows $X_1$ and $X_2$ plotted with a function of the level and slope of the government yield curve, and we see that $X_1$ and $X_2$ can be interpreted as the level and slope of the government yield curve.
Interpretation of credit risk factors

Figure 4: The factor associated with credit risk is given as $\mu(X) = b + X_3 + X_4 + c(X_1 + X_2)$ in the model. This figure shows a) $-X_3$ plotted with the level of the corporate spread curves calculated as $\frac{2y_{AAA} + y_{AA} + y_A}{3} - y_g$ and measured in basis points, b) $X_4$ plotted with the steepness of the corporate spread curves calculated as $\frac{y_{AAA} + y_{AA} + y_A}{3} - y_g - \left(\frac{y_{AAA} + y_{AA} + y_A}{3} - y_1\right)$ and measures in basis points. The two factors can be interpreted as the level and slope of the credit spread curves although the relationship is not precise due to reasons explained in the text.
Figure 5: This figure shows the average Refcorp - government spread and average estimated Treasury premium for maturities 0.5 to 7 years. The average is taken over the estimation period December 20, 1996 to February 14, 2003. Both Refcorp and government yields are zero coupon yields from Bloomberg and Refcorp yields are estimated by Bloomberg using stripped zero coupon yields from Refcorp bonds.
Figure 6: This figure shows the estimated 3-month AA credit risk premium and 3-month LIBOR-GC Repo spread (called LGC spread). The 3-month AA credit risk premium on date $t$ is calculated as the difference in basis points between the yield on a 3-month AA corporate bond and a 3-month riskless bond (with no liquidity). The 3-month LIBOR and GC repo rates are from Bloomberg. The average estimated 3-month AA credit risk premium is 10.0 basis points, while the average LGC spread is 14.7 basis points. Excluding the last three months of 1999 and 2000, the average LGC spread is 12.2 basis points.
Figure 7: Before Y2K: This figure shows the 1-, 2-, and 3-month LIBOR - GC repo spreads along with estimated 3-month Treasury and credit risk premia. The 3-month AA credit risk premium on date $t$ is calculated as the difference in basis points between the yield on a 3-month AA corporate bond and a 3-month riskless bond (with no liquidity). The 3-month Treasury premium on date $t$ is calculated as the difference in basis points between the yield on a 3-month riskless bond (with no liquidity) and a 3-month government bond according to formula (14). LIBOR and GC repo rates are from Bloomberg.
Figure 8: We relax the assumption of homogeneous LIBOR-swap market credit quality and the resulting swap factor accounts for differences in the two markets such as default scenarios, recovery rates, and liquidity. This figure shows the estimated swap specific factor in the swap market and the Lehman option-adjusted duration index for mortgage backed securities downloaded from Datastream. Before 2000 there is a correlation of -0.01 while after 2000 the correlation is 0.86.
Figure 9: This figure shows the effect in basis points of the idiosyncratic swap factor on 5-year swap rates and the quarterly change of Fannie Mae’s holdings of receive-floating minus receive-fixed swaps and swaptions which are calculated as follows. At the end of each quarter the notional amount of Fannie Mae’s receive-fixed swaps and options to enter receive-fixed swaps are subtracted from the notional amount of receive-floating swaps and options to enter receive-floating swaps. The effect of the swap specific factor at time $t$ is calculated as $-\frac{1}{5} \log E_t(\exp(- \int_t^{t+5} S(u)du))$. Source: Fannie Mae (2002, p.34) and Fannie Mae (2002b, p.37).
Figure 10: This figure shows a decomposition in basis points of the estimated 5-year swap spread into the effect of the swap-specific factor, LIBOR credit risk, and Treasury convenience yield through the estimation period. The effect of the swap specific factor at time $t$ is calculated as $-\frac{1}{5} \log(E_t(\exp(-\int_t^{t+5} S(u)du)))$. The size of Treasury convenience yield at time $t$ is calculated as $-\frac{1}{5} \log(E_t(\exp(-\int_t^{t+5} e + X_3(u)du)))$. The size of the LIBOR credit risk factor at time $t$ is calculated as the difference between the estimated 5-year swap spread and the sum of the Treasury and swap factor at time $t$. The effects are consequently transformed to basis points in par rates.
Figure 11: This figure shows the average effect across maturity of the swap maturity premium, LIBOR credit risk, and Treasury convenience yield on swap spreads in the estimation period. The size of the swap maturity premium at maturity $T$ at time $t$ is calculated as $-\frac{1}{T} \log(E_t(\exp(-\int_t^{t+T} S(u)du)))$ and transformed to basis points in par rates. The average is then calculated over the estimation period December 20, 1996 to February 14, 2003. The average effects of Treasury convenience yield and LIBOR credit risk are calculated similarly (see caption on Figure 10).
Figure 12: This figure shows the distance from the 5-year riskless par rate to the government par rate and the swap rate. The riskless par rates are calculated from maturity $T$ continuously compounded riskless rates at time $t$ which are $-\frac{1}{T}\log(E_t(\exp(-\int_t^T r^{govt}(u) + c + X_5(u)du)))$. The calculation of swap rates and government par rates are explained in the caption on Figure 10.