Industry Evolution with Endogenous Entry and Exit*

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Abstract

The timing of entry is a critical decision for a firm that is interested in a new industry. The decision not only depends on exogenous market conditions but also depends endogenously on the decision of other potential entrants. This paper presents a multi-period dynamic model that investigates firms' entry and exit decisions and industry evolution. It focuses on the formative period of a new industry that is characterized by rapid growth in both firms and sales, or firms and sales "take-off". Firms' entry and exit decisions are considered as the equilibrium outcome of a dynamic discrete game, and sales and prices are the equilibrium of incumbents' competition. To capture the dynamic nature of a new industry, I allow the potential demand to expand exogenously instead of assuming it to be stable over time. The model also includes random shocks of demand and cost each period that persist over time. The model captures the essence of firms' strategic interactions and is effective in explaining some of the key industry evolution features in terms of change of the number of firms, prices and total outputs in the introductory periods of an industry. Simulation of the model also replicates the empirical finding that firms take off before sales take off. The model is estimated using data from the first twenty years of the US clothes washer industry. The dynamic model provides explanation for observed industry evolution patterns and also has important policy implications.

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1 Introduction

Whether and when to enter the market is a critical decision for a firm that is interested in a new industry. While early entry often means facing less competition and enjoying more market power, the small demand of a new industry during the infancy period and the large uncertainty as to when the sales will finally "take-off" may discourage early moves.[1] However, if a firm waits until the industry reaches maturity to enter, it can be suboptimal because competitors may have already occupied the market. Therefore, a firm's choice of entry not only depends on exogenous conditions but also depends endogenously on the choice of other potential entrants. The evolution of industry structure is the aggregate result of each firm's individual choice.

In this paper, I address firms' endogenous entry and exit decisions by presenting and estimating a multi-period dynamic model of industry evolution. I am particularly interested in the firms' decisions of entry (and exit) during the formative periods of a new industry. This period of time starts with the commercial introduction of a new product, and is characterized by a period of rapid growth in the number of firms as well as new product sales, or "take-off" in firms and sales. Numerous studies across different industries have found some stylized facts for this period of industry evolution: the number of active firms increases dramatically, the total industry output expands quickly and the price generally decreases over time (e.g., Klepper 1996). Moreover, Agarwal and Bayus (2002) found that firms take-off systematically occurs before sales take-off when they examine a sample of thirty different consumer and industrial product innovations commercialized in US over the past 150 years. One question is, why would we observe these empirical patterns that are not unique to one industry but common to many different industries? How does the industry structure unfold itself with the firms' behavior?

The model in this paper attempts to explore the answers to these questions. The time-series of observed firms' entry and exit decisions are considered as the equilibrium outcome of a dynamic discrete game. Firms make strategic decisions each period as whether to enter the industry immediately by paying an entry cost or to wait additional periods as potential entrants, or whether to exit the industry and receive a sell-off value or to continue operation

[1] Sales take-off corresponds to the first large increase in sales. As noted by Agarwal and Bayus (2002), "early market evolution of successful consumer and industrial product innovations is generally characterized by an initial period of slow growth immediately after commercialization that is eventually followed by a sharp increase".

as incumbents, given the actions taken by other firms and expectations of future firms' behavior and market conditions. In addition, industry sales and prices are the equilibrium outcome of incumbents' competition. The equilibrium dynamic model aims to explain and predict the evolution path of the number of firms, price and sales during the early development of an industry.

To capture the dynamic changing nature of a new industry, I allow the potential demand to expand exogenously instead of assuming it to be stable over time. Thus, price reduction is not the sole reason for sales increases. That exogenous demand expansion is a cause of sales increases echoes the finding of Agarwal and Bayus (2002), which shows that a large portion of sales increase in the early stage of an industry is not explained by price decreases alone. Analogously, firms' production cost is allowed to change over time. Meanwhile, there are random shocks of demand and cost each period that persist over time. For example, a negative demand shock in a certain period not only directly hurts sales for that period, but also has an indirect negative impact on later periods.

Simulation results show that the model successfully replicates important industry evolutionary features in terms of changes in the number of firms, total industry output, and price. Moreover, the model simulations show that firm take-off occurs before sales take-off, as documented in Agarwal and Bayus (2002). The underlying rationale is that firms are forward-looking instead of myopic profit-maximizers. As long as the entry cost is not prohibitively high, firms enter an industry despite minimal demand and even zero profit in early periods, anticipating growing demand in the future (though with uncertainty),. The concern with late entry is that the market may be occupied by other competitors as the market can only sustain a limited number of firms. When the number of active firms is large enough, the profit for each participants becomes very thin and not attractive for new entry. In this sense, the number of existing incumbents plays a role as a natural deterrent of entry as the market becomes crowded.[2] The model is estimated using data from the first twenty years (1921-1940) of the US clothes washer industry. The model's predictions of the general patterns of the evolution of number of active firms and the evolution of price and output are close to the observed data.

The strategic interactions between firms have been explicitly explored in the market structure literature, in which firms' entry decisions are modelled as equilibrium outcomes of a discrete game played between potential entrants. Bresnahan and Reiss (1990) and

[2]For discussion of entry deterrent, see, for example, Gilbert (1986).

Berry (1992) are among the first ones that estimate an equilibrium entry model of this type. Mazzeo (2002) extends the entry model by endogenizing firms' product choices together with location choice and estimates the model with oligopoly motel market data. Seim (2004) studies the market structure of the video rental industry as the equilibrium outcome of retailers' simultaneous entry and location choices, in which location serves as product differentiation between video retailers.[3] However, most previous studies of firm entry consider mature industries and focus on "market entry", i.e., a firm's choice of entering into one or multiple markets of the many markets that are geographically separated with different demand and competition conditions, and most of these studies use a static setup.[4] Little attempt has been made to directly analyze the inherently dynamic process of firms entering and exiting a developing industry for a time-series of market structure observations.

The remainder of the paper is organized in the following way. Section 2 describes the dynamic model in detail. Section 3 presents the simulation results. Section 4 discusses the data, estimation strategy and results. Section 5 provides further discussion of the model specification and results. The paper is concluded in Section 6.

2 The Model

2.1 The basic setup

Consider a dynamic game with discrete time, $t = 1, 2, ..., \infty$. A total number of $\overline{E}$ firms, including both potential entrants and incumbents indexed by $i \in \{1, 2, ..., \overline{E}\}$, are making entry/exit decisions each period in a new industry. The total number of firms is fixed and does not change over time, although not all firms are in the industry.

At the beginning of each period t, potential entrants and incumbents simultaneously make entry and exit decisions.[5] All potential entrants receive i.i.d. draws of an entry cost k_i from an exponential distribution G^κ. The realizations of the entry costs are private information while the distribution is common knowledge. A potential entrant decides

[3]Orhun (2006) and Zhu, Singh and Manuszak (2006) use a similar model to investigate retailer's location choice. Holmes (2007) estimates a monopoly dynamic location choice model to assess the economies of density to Wal-Mart.

[4]An exception is Aguirregabiria, Mira and Roman (2007).

[5]See Pakes, Ostrovsky and Berry (2005) for a similar model setup.

whether to pay the entry cost it draws and enter the industry. If it decides to enter, it pays the entry cost in t and becomes active in period $t+1$.

At the same time, incumbents simultaneously receive i.i.d. draws of sell-off value ϕ_i from an exponential distribution G^{ϕ} and decide whether to continue operating in the industry or accept the sell-off value and exit at the beginning of the next period. Again, the realizations of sell-off value are private information while the distribution is common knowledge. When an incumbent exits, it may re-enter in later periods by paying an entry cost again as a new entrant.

Incumbents are assumed to compete in quantities with homogenous products.[6] In each period, each incumbent decides how much to produce given the information available at the beginning of the period. Profit in period t depends on the the number of incumbents as well as a vector of exogenous factors $X_{t.}$ In this model, X_t includes demand and cost characteristics at period t, which will be defined in detail in Section 2.3. Price and sales in each period are the equilibrium outcome from competition among incumbents in that period.

Given the structure of the discrete game, all the possible changes of the number of incumbents are realized in the beginning of the following period before the production decisions of that period. The action space regarding entry/exit for each player can be defined as $A = \{out, in\}$, where '*out*' means staying outside of the industry for potential entrants or exit the industry for incumbents, and '*in*' means staying in the industry for incumbents or entry for potential entrants. The actions are taken after firms observe the state and their private information of the period.

We are interested in the Markov Perfect Equilibrium (Maskin and Tirole, 1988) in this model: firms' dynamic entry and exit decisions and incumbents' quantity decisions only depend on pay-off relevant state variables. The payoff-relevant state variables in the model are the number of incumbents, the exogenous demand and cost shifters and firm's private information of entry cost or sell-off value.

2.2 Firms' Dynamic Decisions

To capture the heterogeneity of firms while keep the dynamic model trackable, I allow two types of incumbents with one type enjoying a lower production cost than the other. The

[6] Discussion of price competition and product differentiation is presented in Section 5.

low cost incumbents are called 'low type' and high cost incumbents 'high type'. Considering that the average sell-off value of low cost incumbents may be different from that of high cost incumbents, I assume incumbents draw sell-off value from different exponential distributions, G_L^ϕ or G_H^ϕ, depending on their type.[7]

At the beginning of each period, given public and private information, firms simultaneously decide whether to stay in or exit (for incumbents) or to enter or not (for potential entrants). Since I assume firms play Markov strategies, then firm $i's$ decision for period t and period τ is the same if the information sets of period t and τ are identical. Therefore, I drop time subscript and use the superscript '$'$' to denote the next period state variables, e.g. X', N'.

2.2.1 Potential entrants' problem

Each potential entrant faces a dynamic optimization problem as to enter now or later: next period's draw of entry cost and market conditions are uncertain.[8] When making decisions, potential entrants observe the current industry structure and market conditions, but have to form expectations about the number of active competitors of each type as well as demand and cost conditions for future periods. I assume that the potential entrant becomes a low type incumbent with probability s and becomes a high type with probability $1-s$.[9] The type is realized at the beginning of next period (after entry cost is paid) and becomes public information. Therefore, when a potential entrant becomes active, its type is revealed to everyone else. A potential entrant only observes its type after entry, and the firm type does not change until exit.

Consider a potential entrant i. The Bellman equation for the value of firm i is the maximum of the values associated with the two options 'in' or 'out':

$$V_i^{out}(N,X,k_i) = \max\left\{-k_i + \beta \cdot E_e[V_i^e(N',\ X',\phi_i')|N,X],\ \beta \cdot E_o[V_i^{out}(N',X,'k_i')|N,X]\right\}$$

[7]The estimation of the model (using the clothes washer industry data) shows that the two distributions are not statistically different.

[8]Decision akin to this is optimal product launch. Hitsch (2006) examines a firm's optimal dynamic product launch and exit under demand uncertainty.

[9]Alternatively, one could allow a potential entrant to choose its own type by choosing a certain investment level in addition to entry cost, for example. This adds another layer of decision beyond the entry decision, which brings more complexity to the model and is outside the research scope of this paper. See Pakes and Ericson (1995) for a dynamic model in which firms invest to enhance capability to earn profits.

where $\beta \in (0,1)$ is the discount factor. The variable $N = [\ n^l,\ n^h\ ]$ denotes the number of low type (n^l) and high type incumbents (n^h), and X is the vector of exogenous state variables including demand and cost conditions. In the '*max*' operator , the first term is the value of entry now, while the second term is the value of not entering now. The variable k_i is the entry cost that the potential entrant i draws from the exponential distribution $G^{\kappa} = 1 - e^{-(1/\upsilon)\cdot k}$. Note that each period the potential entrant would draw a new entry cost from the same distribution if it has not entered the industry. If the potential entrant i decides to enter the industry, it becomes one of the incumbent firms from next period on and would receive a private information of sell-off value ϕ_i' as other incumbents. The term $E_e[V_i^e(N',\ X', \phi_i')|N, X]$ is the expected value of entering the industry where $E_e \equiv E_{N,X,\phi}$. Since a new entrant does not observe its type (low or high cost firm) when making the entry decision, the value of entering reflects the expectation over types:

$$V_i^e(N, X, \phi_i) = s \cdot V_i^l(N, X, \phi_i^l) + (1-s) \cdot V_i^h(N, X, \phi_i^h)$$

where $V^l(N, X, \phi^l)$ and $V^h(N, X, \phi^h)$ are value of being in the industry for a low and a high type firm respectively. When necessary, I use a superscript to denote the status of an agent, '*out*' for being out of the industry (potential entrants), '*l*' for low type incumbents and '*h*' for high type incumbents.

If the potential entrant i decides not to enter, then it is still in the pool of potential entrants in the next period, and would receive a new private information of entry cost k_i' and undergo the same decision process. The term $E_o[V_i^{out}(N', X,' k_i')|N, X]$ is the expected value of staying out of the industry and $E_o \equiv E_{N,X,k}$. Therefore, a potential entrant would choose to enter the industry if the discounted expected value of entry net of entry cost is bigger than the discounted expected value of staying out, or

$$k_i < \beta \cdot \{E_e[V_i^e(N',\ X', \phi_i')] - E_o[V_i^{out}(N', X,' k_i')]|N, X]\} \tag{2.1}$$

Therefore, if k_i is sufficiently large, the potential entrant may choose to delay entry even if the discounted expected value of entry outweighs the entry cost. This is because, for example, the potential entrant may expect to draw a much lower entry cost in the next period. As a result, there is effectively a delay option for potential entrants.

Note that $E_e(\cdot)$ and $E_o(\cdot)$ depend on the firm's expectation about the number of active competitors, demand and cost characteristics as well as relevant private information of the next period. I assume firms' expectations are rational in the sense that they are consistent with observed behavior, which I describe in more detail in Section 2.4.

2.2.2 Incumbents' problem

At the beginning of each period, each incumbent decides whether to continue operating in the industry. Upon receiving its private information of sell-off value, the incumbent decides whether to accept it and exit in the next period or continue staying in the industry. The Bellman equation for the value of a incumbent firm i can be expressed as:

$$
\begin{aligned}
V_i^m(N, X, \phi_i^m) = \max \Big\{ & \pi^m(N, X) + \beta \cdot E_e[V_i^m(N', X', \phi_i^{m\prime})|N, X], \\
& \pi^m(N, X) + \beta \cdot (\phi_i^m + E_o[V_i^{out}(N', X', k_i')|N, X]) \Big\} \\
& m \in \{l,\ h\}
\end{aligned}
$$

where m denotes an incumbent's type as low or high, and ϕ_i^m is the realization of sell-off value for a type m incumbent. For low-type incumbents, sell-off values are drawn from the distribution $G_L^\phi = 1 - e^{-(1/\eta)\cdot\phi^l}$ and for high-type firms, from the distribution $G_H^\phi = 1 - e^{-(1/\mu)\cdot\phi^h}$. Note that parameters η and μ are the means of the two distributions. If an incumbent decides to exit, it can choose to re-enter the industry in later period as a new entrant and therefore would be in the pool of potential entrants from next period on. An incumbent forms expectations about the next period's number of competitors, demand and cost characteristics as well as private information to evaluate the expected value of being in the industry $E_e[V_i^m(N', X', \phi_i^{m\prime})|N, X]$ and the expected value of exiting the industry $E_o[V_i^{out}(N', X', k_i')|N, X])$. A type m incumbent would choose to exit the industry if it draws a sell-off value greater than the threshold value defined as

$$
\phi_i^m > E_e[V_i^m(N', X', \phi_i^{m\prime})|N, X] - E_o[V_i^{out}(N', X', k_i')|N, X] \tag{2.2}
$$

To complete the model, we need to: (1) describe the static payoff of incumbents $\pi^m(N, X)$; (2) describe how the expectations are formed. The next two subsections are devoted to these issues.

2.3 Static Payoffs and Supply-Demand Equilibrium

This subsection characterizes the competition among incumbents each period and the static payoffs. I assume that firms produce homogenous products and compete in quantities. To detail incumbents' production decision and period payoffs, I start with the specification of demand and cost functions.

2.3.1 Demand function

An important feature characterizing the early stage of a new industry is the outward shifting of the demand curve. To capture this dynamic nature of demand, I model period demand as a function of the exogenous demand shifter F_t and price P_t:

$$D_t = F_t - bP_t, \qquad b > 0$$

where

$$F_t = F_{t-1} + (M - F_{t-1}) \cdot (p + q \cdot F_{t-1}) + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \sigma^2) \tag{2.3}$$

This specification of the exogenous demand shift function F_t permits an 'S' shaped diffusion process (Bass 1969). The idea is that the growth of the consumer base is fast in early periods but becomes negligible when the new product is known to most of the potential consumers. Therefore, the sales increase in the early stages of the industry could be attributed to both price decreases and exogenous demand expansion, while in later stages sales variation is mainly due to price variation and demand shocks. The exogenous expansion process slows down when the market potential is reached. The random shock ε_t is assumed to be *i.i.d.* normal. Note that the random shock ε_t not only has a direct effect on F_t but also an indirect effect on F_τ for $\tau > t$. The parameters to be estimated are b, M, p, q and σ.

Given the Markov process of F_t as depicted in (2.3), the transition density of F_t can be calculated as:

$$g^F(F_t | F_{t-1}) = h((F_t - F_{t-1} - (M - F_{t-1}) \cdot (p + q \cdot F_{t-1}))/\sigma)$$

where $h(\cdot)$ stands for the standard normal distribution.

2.3.2 Cost function

I assume that marginal costs evolve according to some exogenous stochastic process, and that firms of the same type share the same cost structure. Ideally, one could have separate cost evolution functions for different types of firms and estimate the set of parameters associated with each function. To reduce the dimensionality of the state space, I assume that the marginal cost for the low-type firms is proportional to the high-type's cost:

$$mc_t^l = \gamma \cdot mc_t^h, \qquad 0 < \gamma \leq 1$$

where mc_t^l denotes the marginal cost of low-type firms at period t, and γ is the parameter capturing the cost difference between low type and high type firms. As γ approaches 1, the cost difference becomes smaller. With this specification, only mc^h enters into the state space.

I use a first-order autoregressive (AR(1)) process to capture the evolution of cost:

$$\log(mc_t^h) = c_0 + c_1 \log(mc_{t-1}^h) + u_t, \qquad u_t \sim N(0, \omega^2) \tag{2.4}$$

where mc_t^h is the marginal cost of high type incumbents at period t, u_t is the random shock, and $c_{0,}$ c_1 and ω are parameters to be estimated. The transition density of $\log(mc_t^h)$ can be calculated as:

$$g^C(\log mc_t^h | \log mc_{t-1}^h) = h((\log(mc_t^h) - c_0 - c_1 \log(mc_{t-1}^h))/\omega).$$

2.3.3 Equilibrium payoffs per period

At the beginning of period t, each active firm chooses the quantity to produce in this period to maximize its current period profit. The demand and cost random shocks in period t are observable to all the firms before the production decision. Therefore, given the assumption of constant marginal cost within each period and the specification of demand and cost function, the equilibrium price and industry output can be derived as:

$$P_t^* = \frac{F_t + b(n_t^l \cdot mc_t^l + n_t^h \cdot mc_t^h)}{b(n_t^l + n_t^h + 1)} \tag{2.5}$$

$$Q_t^* = \frac{(n_t^l + n_t^h)F_t - b(n_t^l \cdot mc_t^l + n_t^h \cdot mc_t^h)}{n_t^l + n_t^h + 1} \tag{2.6}$$

The static supply-demand equilibrium shows that with the number of active firms of each type in each period, one can predict the market price and sales quantity of that period, given any parameters values.

Given the endogenously evolved state variables $N_t = [\ n_t^l,\ n_t^h\ ]$ which depends on firms' entry and exit decisions, the exogenously evolved state variables $X_t = [F_t, mc_t^h]$, and the parameter values $\theta^p = [b, M, p, q, c_0, c_1, \gamma, \sigma, \omega]$, one can calculate an incumbent's profit at period t as[10]

$$\pi_t^l(N_t, X_t, \theta^p) = \frac{[F_t + bn_t^h(mc_t^h - mc_t^l) - bmc_t^l]^2}{b(n_t^l + n_t^h + 1)^2} \tag{2.7}$$

$$\pi_t^h(N_t, X_t, \theta^p) = \frac{[F_t - bn_t^l(mc_t^h - mc_t^l) - bmc_t^h]^2}{b(n_t^l + n_t^h + 1)^2} \tag{2.8}$$

It is straightforward to obtain that a low-type incumbent earns a higher profit than that of a high type incumbent.

2.4 Equilibrium Expectations and Transition Probabilities

As noted above, state variables include both endogenous variables, the number of active firms of each type N, and exogenous variables, demand and cost characteristics X. The number of active firms of each type in each period evolves as the aggregate result of the individual firm's entry and exit decisions, while demand and cost conditions evolve exogenously according to the Markov processes described in Section 2.3. The conditional transition density $f(N', X'|\ N, X)$ can be written as

$$f(N', X'|N, X) \equiv \Pr(N'|N, X) \cdot g(X'|X) \tag{2.9}$$

where

$$g(X'|X) = g^F(F'|F) \cdot g^C(\log mc^{h\prime}|\log mc^h) \tag{2.10}$$

While it is straightforward to derive $g(X'|X)$, calculating $\Pr(N'|N, X)$ requires knowledge of each firm's expectations of all the other firms actions. I assume that firms in the same set, potential entrants, low incumbents or high incumbents, are identical up to their draw of private information. Since all the firms draw their private information from the same distributions, $G^\kappa(\cdot)$, $G_L^\phi(\cdot)$ or $G_H^\phi(\cdot)$, then the value functions for each firm in the same set become identical when private information is integrated out. Let $\overline{V}^{out}(N, X) \equiv \int V_i^{out}(N, X, k_i) dG^\kappa, \overline{V}^l(N, X) \equiv \int V_i^l(N, X, \phi_i) dG_L^\phi$, and $\overline{V}^h(N, X) \equiv \int V_i^h(N, X, \phi_i) dG_H^\phi$ be

[10] Note that mc_t^l is a fixed function of mc_t^h, thus it is not a separate state variable.

the integrated value functions, or *ex ante value functions* before private information is observed.

Equilibrium expectations require a firm's perception about other firms' entry and exit choices to be consistent with behavior. Firm i's rational expectation about the probability of a potential entrant j choosing action a_j to be entry should be the probability that j receives a signal of entry cost that is smaller than the threshold as shown in (2.1), which implies:

$$\begin{aligned}\Pr(a_j = entry|N, X) &= \Pr\{k_j < \beta \cdot \{E_e[V_j^e(N', X', \phi_j')] - E_o[V_j^{out}(N', X', k_j')]\}|N, X\} \\ &= G^{\kappa}\{\beta \cdot [E[\overline{V}^e(N', X')] - E[\overline{V}^{out}(N', X')]|N, X\} \\ &\equiv P^{entry}(N, X)\end{aligned} \tag{2.11}$$

where $E \equiv E_{N,X}$, and $G^{\kappa}(\cdot)$ is the cumulative distribution function of the exponential distribution of entry cost. Obviously, the equilibrium expectation involves each potential entrant having the same probability of entry $P^{entry}(N, X)$. Therefore, the probability of having a number of e new entrants given state $[N, X]$ is simply:

$$p(e|N, X) = \binom{\bar{E} - n^l - n^h}{e} \cdot P^{entry}(N, X)^e \cdot [1 - P^{entry}(N, X)]^{\bar{E} - n^l - n^h - e} \tag{2.12}$$

which is the probability that among all the potential entrants, e of them draw a favorable entry cost while the rest draw a big entry cost that makes immediate entry a less attractive option than waiting for a future opportunity. Note that with probability s, a new entrant is a low-type firm and with probability $(1-s)$ it is a high type firm. Let e^l be the number of new entrants that draw a low type and $e^h = e - e^l$ the number of new entrants that draw a high type. The probability of having e^l low-type entrants or e^h high type entrants is,

$$p(e^l|N, X) = \binom{e}{e^l} s^{e^l} \cdot (1-s)^{e-e^l} \cdot p(e|N, X) \tag{2.13}$$

Similarly, firm i's expectation of a low-type or high type incumbent j choosing exit should be that j receives a signal of sell-off value that is greater than the threshold as defined in (2.2). This yields:

$$\begin{aligned}\Pr(a_{j,l} = exit|N,X) &= \Pr\{\phi_j^l > E_e[V_i^l(N',X',\phi_i^{l\prime})] - E_o[V_i^{out}(N',X',k_i')]|N,X\} \\ &= 1 - G_L^\phi\{E[\overline{V}^l(N',X')] - E[\overline{V}^{out}(N',X')]|N,X\} \\ &\equiv P^{l.exit}(N,X)\end{aligned} \tag{2.14}$$

$$\begin{aligned}\Pr(a_{j,h} = exit|N,X) &= \Pr\{\phi_j^h > E_e[V_i^h(N',X',\phi_i^{h\prime})] - E_o[V_i^{out}(N',X',k_i')]|N,X\} \\ &= 1 - G_H^\phi\{E[\overline{V}^h(N',X')] - E[\overline{V}^{out}(N',X')]|N,X\} \\ &\equiv P^{h.exit}(N,X)\end{aligned} \tag{2.15}$$

where $G_L^\phi(\cdot)$ and $G_H^\phi(\cdot)$ are the cumulative distribution functions of exponential distributions of sell-off value for low-type and high type incumbents respectively. The equilibrium expectation of firm i is that each low-type (high type) firm has the same probability of exit as described above. Therefore, the probability of having x^m number of type m incumbents to exit is:

$$p(x^m|N,X) = \binom{n^m}{x^m} \cdot [1 - P^{m.exit}(N,X)\ ]^{n^m - x^m} \cdot P^{m.exit}(N,X)^{x^m},\ m = \{l,h\} \tag{2.16}$$

Combining the expectation of entry from potential entrants and exit from incumbents, one can form expectations over the the number of active firms N' in the next period. The number of low-type (high type) firms competing in the next period is the number of low-type (high type) incumbents in this period that decide to stay plus the number of new entrants of this type: $n^{l\prime} = n^l - x^l + e^l$ and $n^{h\prime} = n^h - x^h + e^h$.

Therefore, the probability of having $n^{m\prime}$ type m incumbents in next period given the state $[N,X]$ is:

$$\Pr(n^{m\prime}|N,X) = \sum_{\substack{x^m,e^m, \\ s.t.n^{m\prime}=n^m-x^m+e^m}} p(x^m|N,X) \cdot p(e^m|N,X) \tag{2.17}$$

Since there are different combination of number of exitors and new entrants that lead to the same net result, $\Pr(n^{m\prime}|N,X)$ is the probability over all the possible combinations such that $n^{m\prime} = n^m - x^m + e^m$.

Now we can obtain the transition probability of state N as a function of $\Pr(n^{l\prime}|N,X)$ and $\Pr(n^{h\prime}|N,X)$. Note that $\Pr(n^{l\prime}|N,X)$ and $\Pr(n^{h\prime}|N,X)$ are not independent, but are related through e^l and e^h which satisfy $e^l + e^h = e$. Therefore,

$$\Pr(N'|N,X) = \sum_{B} \binom{n^l}{x^l} \cdot [1 - P^{l.exit}(N,X)\ ]^{n^l - x^l} \cdot P^{l.exit}(N,X)^{x^l} \cdot \binom{n^h}{x^h} \cdot [1 - P^{h.exit}(N,X)\ ]^{n^h - x^h} \cdot P^{h.exit}(N,X)^{x^h} \cdot \binom{e}{e^l} s^{e^l} \cdot (1-s)^{e-e^l} \cdot \binom{\bar{E} - n^l - n^h}{e} \cdot P^{entry}(N,X)^e \cdot [1 - P^{entry}(N,X)]^{\bar{E} - n^l - n^h - e} \tag{2.18}$$

$$B \in \{x^l, e^l, x^h, e^h | n^{l\prime} = n^l - x^l + e^l, n^{h\prime} = n^h - x^h + e^h, e^l + e^h = e, x^l \leq n^l, x^h \leq n^h\}$$

which is the probability over all the possible combination of number of entrants and exitors of each type that leads to the same result $[n^{l\prime}, n^{h\prime}]$. Note that the expectations are held toward the choices of *other* firms, therefore $\Pr(N'|N,X)$ is slightly different depending on whether a firm is a potential entrant, a low-type incumbent or a high-type incumbent. More detailed discussion can be found in the Appendix.

With the transition probabilities as calculated above, I complete the description of the (integrated) value functions for being in the industry and being out of the industry. Since scrap value is exponentially distributed, one can express the integrated value functions for low-type and high-type incumbents as:[11]

$$\overline{V}^l(N,X) = \pi^l(N,X) + \beta \int_{X'} \sum_{N'} \overline{V}^l(N',X') \Pr(N'|N,X) \cdot g(X'|X) dX' + \beta \cdot P^{l.exit}(N,X) \cdot \eta \tag{2.19}$$

$$\overline{V}^h(N,X) = \pi^h(N,X) + \beta \int_{X'} \sum_{N'} \overline{V}^h(N',X') \Pr(N'|N,X) \cdot g(X'|X) dX' + \beta \cdot P^{h.exit}(N,X) \cdot \mu \tag{2.20}$$

Similarly, with exponential distribution of entry costs, the integrated value function for potential entrants is:

$$\overline{V}^{out}(N,X) = \beta \int_{X'} \sum_{N'} \overline{V}^e(N',X') \Pr(N'|N,X) \cdot g(X'|X) dX' - p^{entry}(N,X) \cdot \upsilon \tag{2.21}$$

where

$$\overline{V}^e(N',X') = s\overline{V}^l(N',X') + (1-s) \cdot \overline{V}^h(N',X')$$

As noted above, $P^{l.exit}(N,X), P^{h.exit}(N,X)$ and $P^{entry}(N,X)$ are the probability of exit of low-type incumbents, the probability of exit of high-type incumbents and the probability of entry of potential entrants under state $[N,X]$. Also as noted above, η and μ are the mean of sell-off value distributions for low-type and high-type incumbents, and υ is the mean of the entry costs distribution. Therefore, the firms' equilibrium choice

[11] See appendix for the derivation of the integrated value functions.

probability of entry and exit depends on the expected value of being in the industry and being out of the industry, which in turn depends on the the equilibrium choice probabilities of all the firms.

3 Model Simulation

To explore the market forces at work in the model, we can use simulations to see how the model predicts industry evolution. Set the parameters as follows: $b = 0.2, M = 5, p = 0.005, q = 0.1, c_0 = 0.2, c_1 = 0.85, \gamma = 0.9, \sigma = 0.8, \omega = 0.05, \upsilon = 50, \eta = 1.25, \mu = 1, s = 0.4$ and $\beta = 0.95$; the total number of players $\bar{E}$ is 25 and the initial state $N_0 = [0, 0]$ and $X_0 = [0.01, 9.5]$. The total number of periods used in the simulation is 50. The following results are based on 100 simulations with draws of random shocks. The parameter values are chosen to ensure that the potential demand and marginal cost stabilize within 50 periods.

Figure 1 depicts the evolution of the number of active firms. The two lines with dots are examples of a simulation, while the line with circles represents the average of 100 simulations. We can see that the number of active numbers increases dramatically in early periods, and becomes relatively stable quickly. The intuition is the following: with constant expected entry cost, potential entrants want to enter the industry early. However, with entry cost serving as a filter, only those potential firms drawing a sufficiently small sunk cost are able to enter the industry. When more firms are competing in the market, the profit earned by each firm shrinks. This prospect makes even smaller the fraction of firms to enter the market. In later periods, only the few firms that happen to draw an extremely small entry fee enter, and incumbents that draw a good sell-off value would exit. In general, the variation in later periods is small.

Figure 2 shows the share of low-cost incumbents over all the incumbents over time. The percentage of low-cost firms increases steadily from 40% to 65%. This is because low-cost firms are more competitive than high-cost firms and have higher expected value of staying in the industry. As competition becomes more intense with an increased number of incumbents, high-cost firms are the first ones to exit.

The evolution paths of price and industry output are consistent with observed industry reality. Generally, price declines over time as a result of decreasing production cost and increasing competition. As evident with many new products, price declines fast in early periods and this process slows down in later periods. Figure 3 shows two examples from a simulation (dotted lines) and the average of 100 simulations (line with stars). Figure 4 shows the evolution of sales or industry output. Sales are virtually zero in the first few periods when price is high. Declining price, together with the out-shifting demand curve, leads to fast industry sales growth.

To compare the timing of firms take-off with the timing of sales take-off, the evolution paths of firms and sales are depicted in one graph with a common scale in Figure 5: the index represents the percentage of the number of firms or sales as compared with their respective maximums: $IndexN_t = N_t / \max_{t=1,\ldots,50}\{N_t\}$, and $IndexQ_t = Q_t / \max_{t=1,\ldots,50}\{Q_t\}$. The comparison of the two paths shows clearly that take-off of firms occurs ahead of the sales take-off, an important empirical finding first documented in Agarwal and Bayus (2002). In that paper, Agarwal and Bayus studied sales take-off of a sample of thirty consumer and industrial product innovations commercialized in US over the past 150 years. One of their findings is that take-off of firms occurs systematically before take-off of sales. According to our model, this can be a result of rational expectations and strategic interaction. Firms enter anticipating growing demand of new products even though the demand is negligible at the time of entry. It is not the current-period profit but the prospect to earn profit in the future by occupying a space in the market that drives early entry. Flows of new entrants crowd the market early, which drives down the expected future earnings and makes entry less attractive in later periods even though the market is still growing. As an example that shows the take-off pattern, Figure 6 depicts the evolution of number of firms and total industry output of the early US black-and-white TV industry, which actually is similar to the pattern we show in Figure 5.[12]

The above simulation results show that the dynamic model developed in section 2 is able to replicate several important industry regularities. Now I turn to the estimation of the model in the US clothes washer industry.

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[12]The data of US B&W TV industry is generously provided by Zhu Wang. The data are drawn from periodic editions of *Television Factbook*.

4 Data and Estimation

I use data from the US clothes washer industry from 1921 to 1940 to estimate the dynamic model detailed in Section 2.[13] This is the introductory period of the clothes washer industry, which witnessed a dramatic increase in the number of firms from as few as 5 in the early 1920s to over 70 in the beginning of 1940s. During the same period, clothes washer sales started to take off and price fell over time.

The data set consists of two parts. The first part is firm-level information, which is compiled from the *Thomas Register of American Manufacturers*, a source that has been widely used to study industry evolution. The information includes the year a firm entered the industry and the year it exited if exit occurred during the observation years, as well as an indicator variable showing the asset size type of a firm. Firms are characterized as large or small based on the asset size class reported in the *Thomas Register* after adjusting the boundaries of the classes to account for inflation.[14] Large firms correspond to the low-type firms, and small firms correspond to the high-type firms in the model. This is consistent with the idea that larger firms have more resources to obtain state-of-the-art technology and therefore enjoy lower production costs. The remaining of the data consists of the aggregate information of industry average price and total output each year.[15] Total industry output per period is also treated as total sales per period in the model estimation.

The firm-level information is then summarized into aggregate variables according to its type: the total number of low-type (large) and high-type (small) incumbents each year, the total number of new entrants and the number of low-type and high-type exitors each year.

The parameters to be estimated are summarized in $\theta = [b, M, p, q, c_0, c_1, \gamma, \sigma, \omega, \upsilon, \eta, \mu]$, including the parameters in the demand and cost functions as well as the parameters that determine the distribution of entry cost and sell-off value. Recall that there is one more parameter in the model, s, which is the probability of becoming a low type for a new incumbent. This parameter is treated as an "observable parameter" with $s = 0.5$. In the data period, the average ratio of low type entrants over all the entrants is 0.51.

[13]This data set is generously provided by Barry Bayus.

[14]See Agarwal and Bayus (2003) for more detailed description of the data collection and categorization process.

[15]Data for average price and sales were compiled from *Dealerscope Merchandising* (previously named *Merchandising*, *Merchandising Week*), by Agarwal and Bayus.

The parameters in the distribution of entry cost and sell-off value are essentially identified through the observed turn over of number of active firms of each type, and the parameters in the profit function are identified through the evolution of price, total output as well as number of firms over time.

Notice that the expected number of entrants in each period depends on the total number of potential entrants of the period $\overline{E} - n_t^l - n_t^h$, which in turn depends on the total number of firms (both potential entrants and incumbents) $\overline{E}$. Since only the firms that ever make entry attempt were observed, while the firms that were interested in the industry but never entered are not observable, it is difficult to determine the number of potential entrants. To estimate the model, I set $\overline{E}$ fixed such that:

$$\overline{E} = \max_{t \in (1,2,\ldots T)} \{nn_t\} + \max_{t \in (1,2,\ldots T)} \{en_t\},$$

where nn_t is the observed total number of incumbents at period t, and en_t is the total number of new entrants at period t in the data set.[16]

4.1 Estimation Procedure

The estimation procedure is close to the sequential estimation method proposed by Aguirregabiria and Mira (2007), except that I use a simulated method of moments (SMM) estimator instead of a maximum likelihood estimator. Markov perfect equilibrium is represented as a fixed point of a best response mapping in the space of the firms' choice probability. The objective of the algorithm is to find a vector of firms' choice probabilities and a vector of parameters such that given the vector of choice probabilities, this vector of parameters minimizes the SMM objective function, and, given the vector of parameters, this vector of firms' choice probabilities is a fixed point.

4.1.1 Discretization of state space

One part of the algorithm is to find a fixed point in firms' choice probabilities under each state. Notice that all the potential entrants and all the incumbents of each type are *ex ante* identical. That is, every potential entrant has the same probability of entry and

[16] The total number of potential players is commonly treated as fixed instead of estimated in entry models. For example, Aguirregabiria and Mira (2007) fixed the number of potential entrants at $\max\{\max_{t \in (1,2,\ldots T)} \{nn_{t-1} + en_t\}, 2\}$.

every low-type (high-type) incumbent has the same probability of exit. A firm's expected profit depends on the number of incumbents of each type, its own type (potential entrant, low-type incumbent or high-type incumbent) and the exogenously evolved state of demand and cost characteristics. Therefore, let FS represent the number of states associated with the number of incumbents of each type, and let MS the number of states associated with demand and cost characteristics, the size of the full state space becomes $3 \times FS \times MS$. Although the symmetry in the Markov perfect equilibrium simplifies the solution, two computational difficulties need to be solved to implement the algorithm. These are the continuous state space of demand and cost characteristics and the large number of possible states for the number of firms.

By observing (2.19)-(2.21), we notice that evaluating value functions and computing a fixed point requires integrating over the two-dimensional continuous state space X_t which includes demand and cost characteristics. Since accurate integration is not feasible, I use a randomization technique to approximate the value functions (Rust, 1996). Let $\mathbf{X} \subset R_+^2$ be the compact set of the state vector of demand and cost conditions. I discretize the continuous state vector with a $R-$point support by taking R i.i.d. uniform random draws from the compact set $\mathbf{X}$. Denote the random draws as $\left\{\tilde{X}_1, \tilde{X}_2, ..., \tilde{X}_R\right\}$. Replace the original continuous transition function of X with the normalized discrete probability densities $\Pi(\tilde{X}'|\tilde{X})$ in the value functions:

$$\Pi(\tilde{X}'|\tilde{X}) = \frac{g(\tilde{X}'|\tilde{X})}{\sum_{i=1}^{R} g(\tilde{X}_i'|\tilde{X})}$$

Another issue is the large number of active firms. Many industries, including the clothes washer industry, have observed dramatic increase in the number of firms.[17] Thus, although the state space of the number of active firms is discrete in nature, it could be prohibitively large for estimation purposes. Let $\overline{N}$ be the maximum possible number of active firms within a period; the possible states of $N_t = [n_t^l,\, n_t^h]$ as a combination of low-type and high-type incumbents is $\overline{N} \times \overline{N}/2$, which gets very large with a big $\overline{N}$. To keep the state space manageable, I re-discretize the state space of the number of incumbents by selecting a small number of points with an even interval. Let the interval be λ (an

[17]Most industries witness large net entry in the early stage of evolution, varying from dozens to hundreds. For example, the number of firms that produce electric shavers grows from 2 in year 1937 to 33 in 1941. In 1974, the beginning of the personal computer industry, there were only two PC producers. In ten years, the number of producers grew to over 140. (Please see Agarwal and Bayus, 2003.)

integer); the states of low-type incumbents are: $\{1, \lambda+1, 2\lambda+1, ...[\overline{N}/\lambda]\}$, where $[\cdot]$ is the operator of rounding the element inside to the nearest integer. The same discretization technique applies to the states of high-type incumbents. Denote the re-discretized states of number of firms as $\left\{\tilde{N}_1, \tilde{N}_{2,}...\tilde{N}_W\right\}$ with $\tilde{N}_i = [\tilde{n}_i^l, \tilde{n}_i^h]$, $i = 1, 2, ..., W$. To ensure that the discretized probabilities sum to one, the transition probability is normalized so that

$$\Pi(\tilde{N}'|\tilde{N}) = \frac{\Pr(\tilde{N}'|\tilde{N})}{\sum_{i=1}^{W} \Pr(\tilde{N}_i'|\tilde{N})}$$

With the state space discretization described above, the approximate value function for a low-type incumbent after integrating out private information becomes:

$$\hat{V}^l(\tilde{N}, \tilde{X}) = \pi^l(\tilde{N}, \tilde{X}) + \beta \sum_{\tilde{N}'} \sum_{\tilde{X}'} \hat{V}^l(\tilde{N}', \tilde{X}')]\Pi(\tilde{N}'|\tilde{N})\Pi(\tilde{X}'|\tilde{X}) + \beta \cdot P^{l.exit}(\tilde{N}, \tilde{X}) \cdot \eta \quad (4.1)$$

where $\pi^l(\tilde{N}, \tilde{X})$ is the current period profit of the low type incumbents under state $[\tilde{N}, \tilde{X}]$, $P^{l.exit}(\tilde{N}, \tilde{X})$ is the probability to exit and η is the mean of the sell-off value distribution for low type firms. Similarly, the discretized version of value function for high-type incumbents (after integrating out private information) is:

$$\hat{V}^h(\tilde{N}, \tilde{X}) = \pi^h(\tilde{N}, \tilde{X}) + \beta \sum_{\tilde{N}'} \sum_{\tilde{X}'} \hat{V}^h(\tilde{N}', \tilde{X}')\Pi(\tilde{N}'|\tilde{N})\Pi(\tilde{X}'|\tilde{X}) + \beta \cdot P^{h.exit}(\tilde{N}, \tilde{X}) \cdot \mu \quad (4.2)$$

The discretized version of value functions for being out of the industry (after integrating out private information) is:

$$\hat{V}^{out}(\tilde{N}, \tilde{X}) = \beta \sum_{\tilde{N}'} \sum_{\tilde{X}'} [s \cdot \hat{V}^l(\tilde{N}', \tilde{X}') + (1-s) \cdot \hat{V}^h(\tilde{N}', \tilde{X}')]\Pi(\tilde{N}'|\tilde{N})\Pi(\tilde{X}'|\tilde{X}) - P^{entry}(\tilde{N}, \tilde{X}) \cdot \upsilon \quad (4.3)$$

where $P^{entry}(\tilde{N}, \tilde{X})$ is the probability of entry for an potential entrant under state $[\tilde{N}, \tilde{X}]$ and υ is the mean of entry costs distribution.

To find the Markov Perfect equilibrium of this dynamic game is to find the fixed point in the space of firms' choice probabilities. Starting with any vector of an initial guess of firms' choice probabilities $\hat{P}^0$ under each state (the probability to enter for potential entrants and the probability to exit for incumbents of each type) and some initial guess of parameter value $\hat{\theta}^0$, one can calculate the integrated value functions $\hat{V}^l(\tilde{N}, \tilde{X})$, $\hat{V}^h(\tilde{N}, \tilde{X})$ and $\hat{V}^{out}(\tilde{N}, \tilde{X})$, from which an iterated vector of choice probabilities $\hat{P}^1 = \Psi(\hat{P}^0, \hat{\theta}^0)$ is obtained, where Ψ represents the mapping in the choice probability space. The K-stage

$(K > 1)$ iteration uses the iterated choice probability vector $\hat{P}^{K-1}$ from stage $K-1$ as a starting value to obtain a new probability vector $\hat{P}^K(\hat{P}^{K-1}, \hat{\theta}^K)$.[18] The iteration continues until the vector of choice probabilities converges. Then, the firms' equilibrium strategies are obtained under these states. I use interpolation to obtain firms' equilibrium choice probabilities under states that are not among the selected grid points.

4.1.2 Implementation of the Estimation Algorithm

The parameters are estimated using the simulated method of moments.[19] At the start of the estimation process, i.i.d. random draws are obtained from the standard normal distribution for each period $t = 1, .., T$, to imply realizations of (standardized) random shocks of demand $\{\tilde{\varepsilon}_t\}$ and random shocks of cost $\{\tilde{u}_t\}$. The number of simulations, S, is set to 100, which means S i.i.d. $T \times 1$ sequences of random vectors are generated. These random draws are held fixed throughout the estimation.

The estimation algorithm is a recursive procedure. Let $\hat{P}^{K-1}$ be a K-1 stage iterated vector of firms' choice probabilities. Then in stage K, find a vector of parameters $\hat{\theta}^K$ that minimizes the SMM objective function:

$$\hat{\theta}^K = \arg\min_{\theta \in \Theta} \; Q_s(\theta, \hat{P}^{K-1})$$

Given a vector of parameters θ and the iterated firms' choice probabilities $\hat{P}^{K-1}$, the evolution path of the number of firms of each type is simulated given initializing conditions.[20] Then with the simulated evolution of the number of firms and the pre-drawn random shocks of each period, one can calculate the equilibrium of price and quantity of each period and therefore obtain the evolution path for price and industry output as described in (2.5) and (2.6). The SMM estimator is simply the parameter values that minimizes the distance between a set of simulated moments and the sample moments for the actual data, and generates consistent estimates.

Formally, let M_T be a $J \times 1$ (J is larger than the number of parameters to be estimated)

[18]The derivation of $\hat{\theta}^K$ is discussed in the next subsection.

[19]The simulated method of moments was proposed by McFadden (1989) and Pakes and Pollard (1989) in discrete choice models; Lee and Ingram (1991) extend it to time series settings where the errors can have serial correlation. See also Duffie and Singleton (1993), and Hall and Rust (2003).

[20]I use the first observation in the data as the initializing condition. Explicitly, F_0 and mc_0^h can be expressed as a function of the first observation of price, sales and number of firms, i.e., P_0, Q_0, n_0^l, n_0^h, and parameters to be estimated. Therefore, F_0 and mc_0^h vary with the parameter vectors.

vector of observed data moments and $m_s(\theta)$ be a $J \times 1$ vector of model moments from a single simulation:

$$\hat{\theta}^K = \arg\min_{\theta \in \Theta} [M_T - M_S(\theta, \hat{P}^{K-1})]' W_T [M_T - M_S(\theta, \hat{P}^{K-1})]$$

where $M_S(\theta, \hat{P}_k) = \frac{1}{S} \sum_{s=1}^{S} m_s(\theta, \hat{P}_k)$ and W_T is a $J \times J$ positive definite weighting matrix. The moments used in the estimation are the means and histograms (four of the five quintile bins)[21] of the evolution processes of price $\{P_t\}$, total industry output $\{Q_t\}$, the number of low type incumbents $\{n_t^l\}$, the number of high-type exitors $\{x_t^h\}$ and the number of entrants $\{e_t\}$ for a total of 25 moment conditions. Efficient estimates are obtained by setting W_T as the empirical variance-covariance matrix.

With the stage-K estimates $\hat{\theta}^K$, the choice probability vector in stage K+1 is iterated as $\hat{P}^K = \Psi(\hat{P}^{K-1}, \hat{\theta}^K)$. Iteration continues until a fixed point is found, where a fixed point is a pair $\left(\hat{P}, \hat{\theta}\right)$ such that $\hat{P}$ is a fixed point in the probability mapping $\hat{P} = \Psi(\hat{P}, \hat{\theta})$ and $\hat{\theta}$ minimizes the the objective function $Q_s(\theta, \hat{P})$. More details of the algorithm are provided in the Appendix.

4.2 Estimation Results

Table 2 provides the values and standard errors of the estimated parameters. The standard errors are computed by bootstrapping. The parameter estimates are all with reasonable sign and magnitude and are statistically significant.

The parameter estimates in Table 2 show that low-type incumbents' marginal cost is about 87% of that of high-type incumbents, which is the source of the low-type's competitive advantage. The estimates of entry cost and sell-off value parameters need to be interpreted with caution. The estimated mean of the exponential distribution of entry cost is $\hat{v}$. However, it could not be interpreted as the average entry cost that observed new entrants pay to enter the industry. The latter should be calculated as the mean of entry cost conditional on firms' decision to enter. Similarly, $\hat{\eta}$ and $\hat{\mu}$ are the estimated means of the exponential distributions of sell-off value for low-type and high-type incumbents. However, to estimate the average sell-off value that exitors receive, one needs to calculate the conditional mean of the sell-off value.

[21]See Hall and Rust (2003) for a similar treatment. Following Hall and Rust (2003), I use logistic transforms of the indicator functions to smooth the criterion function. The indicator functions arise when computing histogram bins.

The estimated model fits the data quite well in terms of predicting the evolution pattern of the number of active firms of each type, price and industry output. Figures 7-10 provide estimates and the actual data of key variables for comparison. In all figures, the solid lines represent data while the dashed lines represent what is predicted by the model. Predictions for period t are based on the parameter estimates and the realized state in $t-1$. In other words, the predictions for period t are based on the actual observed state in $t-1$ and simulated random shocks in all the future periods. The results show that as the industry evolves, the share of low-type firms outweighs the share of high-type firms. According to the model, this could be explained by the cost advantage of low-type firms. When the number of firms competing in the industry increases dramatically, it drives down profit, and high-type firms are more likely to exit.

While the model predicts well the general evolution pattern of the number of firms, it overestimates the number of entrants in the early periods and underestimates in the later periods, which could be due to unobserved factors that are not captured by the model.

5 Discussion

5.1 Policy Experiment of Entry Cost

The results from simulation and estimation of the model indicates that the proposed model is able to explain and generate some of the general industry evolution patterns commonly observed across industries. The model may also have policy implications. One variable that affects the industry structure is entry cost. By changing the distribution of entry costs, the policy maker may effectively change how fast the number of firms grows in a certain industry as well as the maximum number of firms.

To demonstrate how entry costs may change the evolution of the number of firms over time, I simulate the model with different parameters of the entry cost distribution while keeping the rest of the parameters the same across all the simulations.[22] I consider the following value of $\upsilon = \{5, 25, 50\}$, where υ is the mean of the entry cost distribution. The total number of players is $\overline{E} = 40$. Figure 11 presents the simulation results with the three

[22]Since here we are interested in the changes of total number of firms over time with respect to the change of entry cost distribution, I simulate a simplified version of the model without firm heterogeneity (all the incumbents are of the same type). The parameters used in the simulations are: $b = 0.5, M = 5, p = 0.005, q = 0.1, c_0 = 0.2, c_1 = 0.85, \sigma = 0.8, \omega = 0.05, \eta = \mu = 1$.

entry cost levels; each path is the average across 100 simulations with random demand and cost shocks. It seems that when the mean entry cost is low ($\upsilon = 5$), the number of firms increases dramatically in a relatively short period. However, when the mean entry cost is high ($\upsilon = 50$), the number of firms grows much slower than in the low entry cost scenario. Moreover, the simulation shows that the maximum number of incumbents in equilibrium is higher when the entry cost is lower, as expected.

5.2 Product Differentiation and Price Competition

The model presented assumes homogenous products in the market. The basic intuition behind the dynamic entry/exit decisions, however, does not change if the model is extended to allow for product differentiation. The key driver of the behavior pattern is the expected profits, which essentially depends on the number of competitors N_t, the potential demand F_t and the marginal cost mc_t. The profit functions (2.7)-(2.8) indicate that:

(1) If more firms of the same type are competing in the industry, the current period profit is lower. The decrease in profit caused by entry of a similar competitor is decreasing with the number of competitors: $\frac{\partial \pi^m}{\partial n^m} < 0$, $\frac{\partial^2 \pi^m}{\partial n^{m2}} > 0$, for $m = l, h$.

(2) The entry of firms of the other type also lowers the current period profit of one's own type. Moreover, for both low-type and high-type incumbents, the entry of an low-type firm hurts profits more than the entry of a high-type firm: $\frac{\partial \pi^l}{\partial n^h} < 0, \frac{\partial \pi^h}{\partial n^l} < 0$, and $\left|\frac{\partial \pi^m}{\partial n^h}\right| < \left|\frac{\partial \pi^m}{\partial n^l}\right|$, for $m = l, h$.

(3) Higher potential demand leads to higher the current period profit. Moreover, the increase in profit caused by expansion in potential demand is increasing with the potential demand:$\frac{\partial \pi^m}{\partial F} > 0$, $\frac{\partial^2 \pi^m}{\partial F^2} > 0$, for $m = l, h$.

(4) Profits increase with a decrease in marginal costs:$\frac{\partial \pi^m}{\partial mc^m} < 0$, for $m = l, h$

Result (2) comes from the fact that low-type firms enjoy a lower cost structure and therefore are more competitive. This explains why entry of low-type firms threatens the profits of incumbents more than that of high-type firms.

Now consider an industry with differentiated products. Then, products are not perfect substitutes as in the homogenous market the model inspected. It is straightforward to see that results (3) and (4) may still hold in most product differentiation setups. The entry of new firms has an asymmetric effect on incumbents' profit. Intuitively, an incumbent's profit hurts more if the new entrant's product positioning is close to its own product than farther away in the product space. Nevertheless, the basic argument remains the same that, with the entry of new competitors, the profit prospect becomes less attractive and therefore, the increase in the total number of firms slows down and finally levels off.[23]

The current model assumes that incumbents are involve in quantity competition. Alternatively, we could investigate a model with price competition and product differentiation. The objective function for firm i is to choose the optimal price to maximize its current period profit[24]

$$\max_{P_i}\ (P_i - c)Q_i(\bar{P})$$

in which $\bar{P}$ denotes the vector of prices which including competitors prices. The first order condition yields:

$$Q_i(\bar{P}) + (P_i - c)\frac{\partial Q_i(\bar{P})}{\partial P_i} = 0$$

On the demand side, we use a nested logit model to describe consumers' choice behavior in which the first layer of buyer choice is 'to buy' or 'not to buy', and within the 'nest' of 'to buy', the alternatives are different products. The nested logit model allows for flexible correlations over alternatives in the market.

Assume that the net utility a consumer obtains from the purchase of product i is:

$$U_i = F - bP_i + \varepsilon_i, \quad i = 1, 2, ..., N$$

where F represents the fixed utility from consuming the product.[25] We normalize the deterministic utility of not buying to zero, therefore $U_0 = \varepsilon_0$. The term ε_i $(i > 0)$ is the demand shock for product i that may correlate with the demand shocks of other products while it does not correlate with $\varepsilon_{0,}$ and is assumed to follow a generalized extreme value distribution. Let $1 - \rho^2$ be the correlation between ε_i for $i > 0$. Then the probability of purchasing product i given that the consumer decides to buy some product is

[23] See Mazzeo (2002) and Seim (2004) for a static entry model that explicitly deals with product differentiation by endogenizing the firms' product choice together with entry decision.

[24] We assume that consumers are not forward-looking and the price of period t does not affect the future profit flow. But we acknowledge that allowing for consumers' forward-looking behavior and the strategic interactions between firms and consumers could be important in the dynamic model. See, for example, Desai and Purohit (1999) and Villas-Boas (2004). Erdem, Imai and Keane (2003), Gordon (2006) and Nair (2007) examine demand models with forward-looking consumers. Che, Sudhir and Seetharaman (2007) look at dynamic competition with consumer state dependence.

[25] One could also allow F to vary across products, but we are not able to identify them with aggregate data.

$$\Pr(y = i | y \neq 0) = \frac{\exp[(F - bP_i)/\rho]}{\sum_{j=1}^{N} \exp[(F - bP_j)/\rho]} = \frac{\exp(-bP_i/\rho)}{\sum_{j=1}^{N} \exp(-bP_j/\rho)}$$

The probability of buying one of the products is

$$\Pr(y \neq 0) = \frac{\exp(F + \rho \cdot \ln \sum_{j=1}^{N} e^{-bP_j/\rho})}{1 + \exp(F + \rho \cdot \ln \sum_{j=1}^{N} e^{-bP_j/\rho})}$$

The unconditional probability of purchasing i is simply the product of the above two probabilities. Now we can derive the sales for product i as:

$$Q_i(\bar{P}) = \Pr(y = i | y \neq 0) \cdot \Pr(y \neq 0) \cdot M$$

where M is the total market potential. With the expression for $Q_i(\bar{P})$ and the assumption that firms are symmetric, the optimal price P_i^* can be can be calculated. Then the equilibrium sales or total output could be obtained as $Q^* = \sum_{i=1}^{N} Q_i(\bar{P}^*)$.

Under the assumption that firms compete in price, we have new expressions for equilibrium price, quantity and current period profit as functions of the number of firms N, demand condition F, cost c and demand parameters, which are ready to be used to implement the estimation procedure.

6 Conclusion

In this paper, I construct and estimate a multi-period dynamic entry and exit model. It contributes to the small body of empirical literature of dynamic models that investigate industry behavior. Although several restrictive assumptions have been imposed to simplify the model, it is effective in explaining important industry evolution patterns. I further use the data of the US clothes washer industry (1921-1941) to estimate the parameters of the model. Model predictions about the evolution paths of the number of firms, price and output nicely match the data pattern observed.

The model is built to capture the essence of the dynamic interaction while being parsimonious. There are several ways to extend the current model. First, further research could enrich the modelling of the demand-side dynamics. For example, Agarwal and Bayus (2002) suggest that new firms entry in the formative stage of a new market not only affects supply but also shifts the demand curve directly through mechanisms such as increased consumer awareness, expanded distribution, perceived and actual product improvements and market infrastructure development.[26] Therefore, one can model the potential demand at period t as a function of the number of firms ever entered up to t. The conjecture is that in this case, everything else being equal, the model would predict an even faster increase in the number of firms in the early stage of an industry, which would strengthen the effect that the take-off of the number of firms occurs ahead of sales take-off. However, this element is not necessary to reproduce the empirical pattern that firms take-off first. The current model shows that by simply assuming strategic interactions among firms, this empirical pattern can be replicated as clearly shown in Figure 5.

Further steps on integrating more sophisticated demand-side dynamics into the current dynamic entry game are subject to computational difficulties. In the above example, the total number of firms that ever entered each period would have a permanent effect on payoff in later periods. Therefore, this effect should be kept track of as an additional state variable in the dynamic optimization problem. Pakes (2000) discusses how to extend such models to incorporate the complicated feature that future demand depends on current quantity choice.

Another avenue of future research is to allow for more flexible firm heterogeneity. The current model only allows for limited firm heterogeneity by having two types of firms. A more realistic model is to have multiple types to fully capture the heterogeneity among firms.[27] Furthermore, the current model assumes that firms only differ in cost structure, while there could be other source of heterogeneity, such as product differentiation or idiosyncratic profit shocks.

Moreover, future work is required to explain the “shake-out” phenomenon during in-

[26]This implies that strategic variables, such as the level of advertising over time, would affect demand and may also be considered in the model. For studies of dynamic advertising policies, see, for example, Villas-Boas (1993) and Dubé, Hitsch and Manchanda (2005).

[27]See Ericson and Pakes (1995) for a dynamic framework that allows for full firm heterogeneity; see Pakes and McGuire (1994, 2001) for numerical solutions. Due to the complexity of such dynamic models, the number of firms is limited to very few. For example, as an empirical application of the proposed framework, Gowrisankaran and Town (1997) study the hospital industry which allows for a maximum of three firms in the model estimation. Pakes and McGuire (2001) and Pakes, Ostrovsky and Berry (2005) propose estimation methods to break the ‘curse’ of dimensionality, which are applicable to cross-sectional data. For application of Pakes and McGuire (2001)’s method, see, for example, Goettler, Parlour and Rajan (2005). Weintraub, Benkard and Van Roy (2007) propose an approximation method to analyze and estimate Ericson and Pakes (1995) model by using a new equilibrium concept to approximate Markov Perfect equilibrium, which has computational advantage and could allow for many firms.

dustry evolution that is commonly observed in new industries. A "shake-out" phase is characterized by a noticeable decrease in the number of firms after a period of heavy entry. While the current model accounts for stochastic exit, it does not predict a period of "shake-out". Future research is needed to incorporate both "take-off" and "shake-out" of firms in the model.

Tables and Figures

Table 1: Descriptive Statistics

	Minimum	Maximum	Mean	Std
Number of Firms[a]	4	71	41.10	23.64
—Type L Firms[b]	1	45	25.15	16.34
—Type H Firms[c]	3	29	15.95	7.72
Number of New Entrants	0	19	7.20	5.29
—Type L Entrants	0	13	3.75	3.29
—Type H Entrants	0	9	3.45	2.68
Number of Firms Exit	0	13	5.15	3.27
—Type L Exitors	0	8	1.70	2.11
—Type H Exitors	0	9	3.45	1.95
Price (in 100 dollars)	4.30	9.35	6.31	1.61
Output(in 100,000 units)	4.35	20.14	10.36	4.39

[a]Descriptive statistics are based on yearly data from 1921-1940.

[b]Type L firms correspond to firms with large asset size in the data.

[c]Type H firms correspond to firms with small asset size in the data.

Table 2: Parameter Estimates

Parameters	Estimates	Standard Errors
Demand function:		
b	1.9521	0.0992
M	70.8872	6.4999
p	0.0011	0.0000
q	0.0001	0.0000
σ	2.2542	0.0263
Cost function:		
c_0	0.1672	0.0038
c_1	0.9064	0.0138
ϖ	0.0909	0.0021
γ	0.8699	0.0229
Entry cost:		
υ	76.5992	10.1540
Sell-off Value:		
η	3.8143	0.7755
μ	3.1082	0.6056

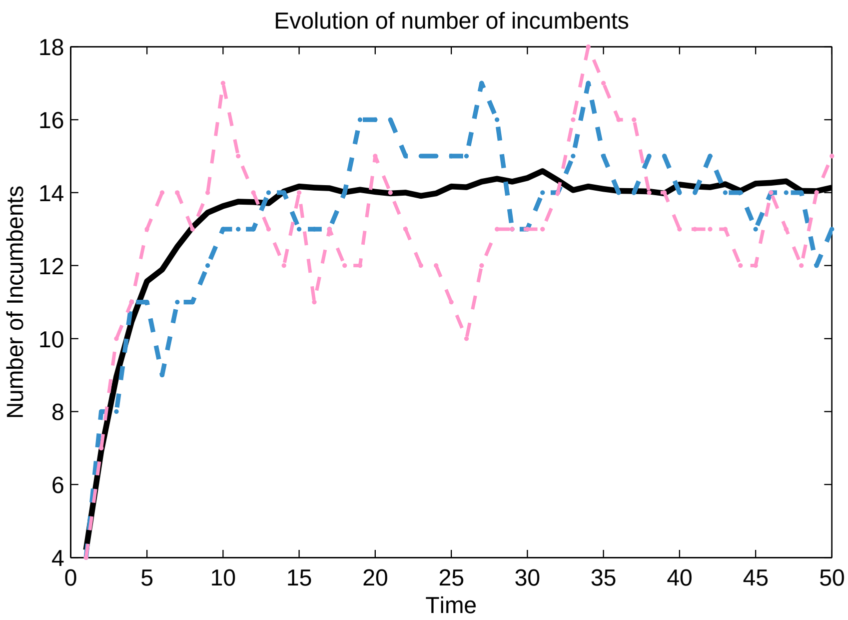

Figure 1: Simulated evolution path of number of firms

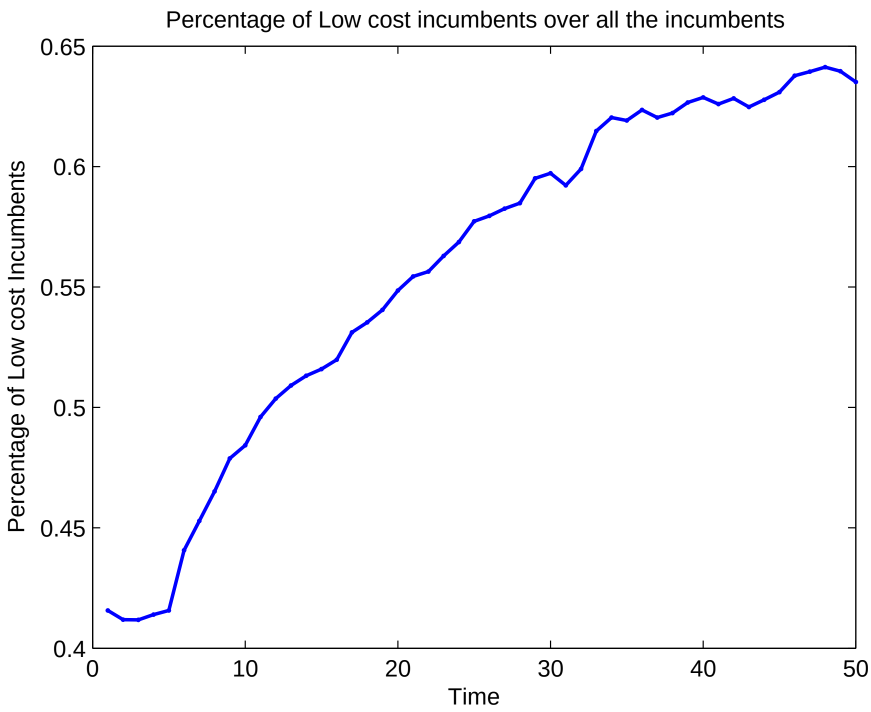

Figure 2: The change of percentage of low-cost incumbents over time (simulation)

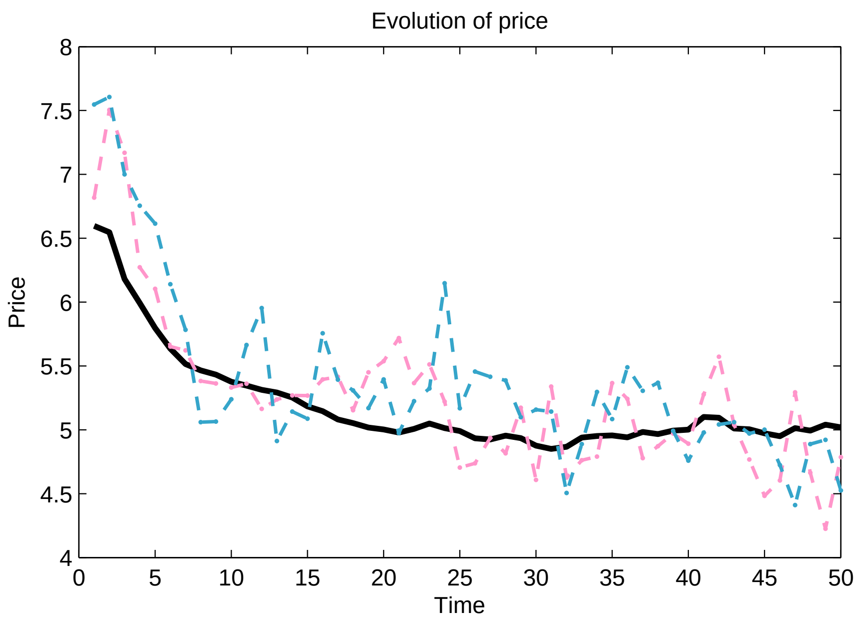

Figure 3: Simulated evolution path of price

Figure 4: Simulated evolution path of industry sales

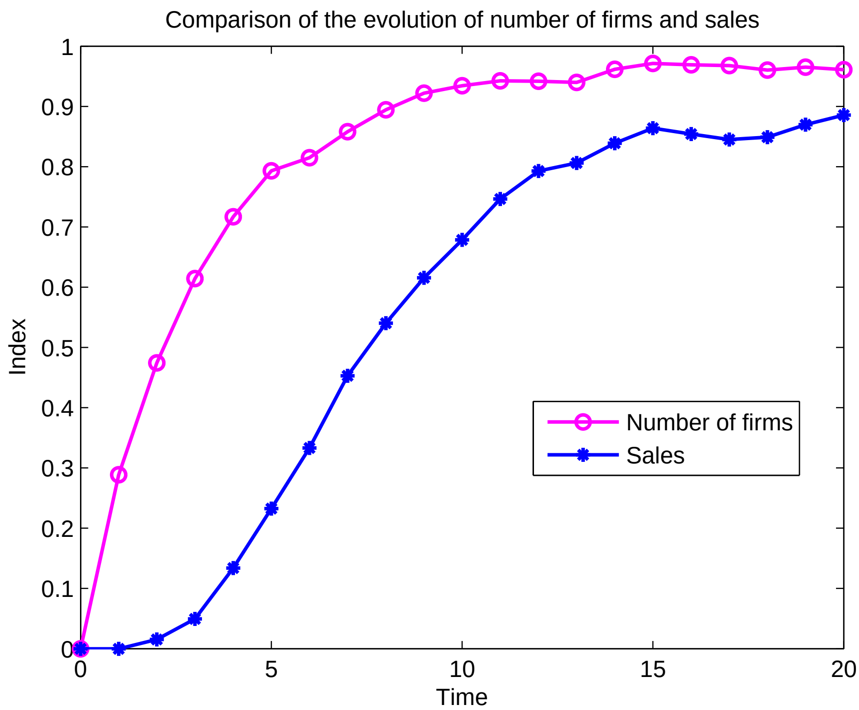

Figure 5: Simulated Take-off of the number of firms and sales

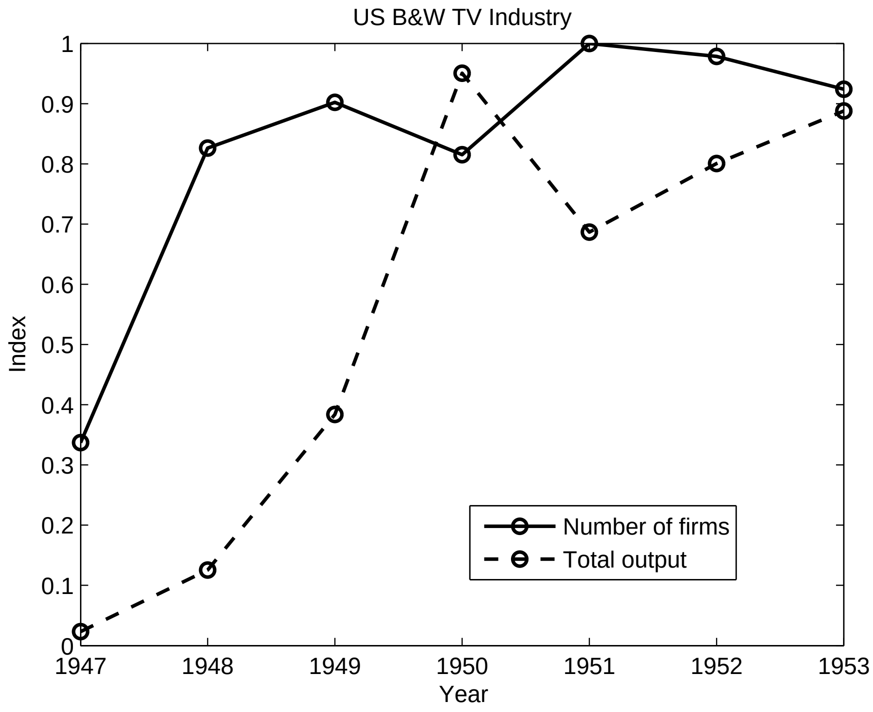

Figure 6: Take-off of the number of firms and sales of TV industry

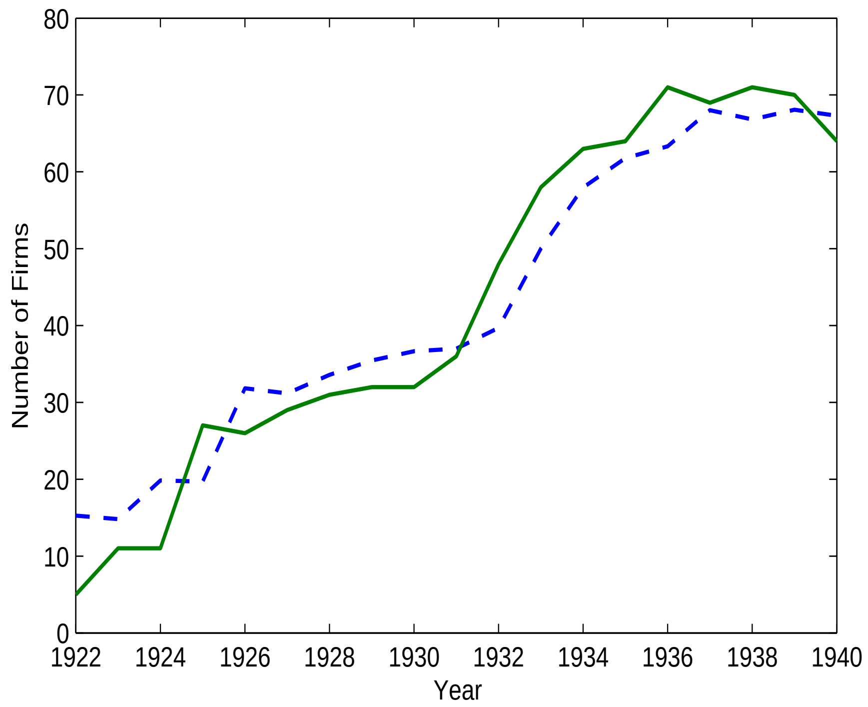

Figure 7: Actual vs. predicted evolution path of the total number of firms

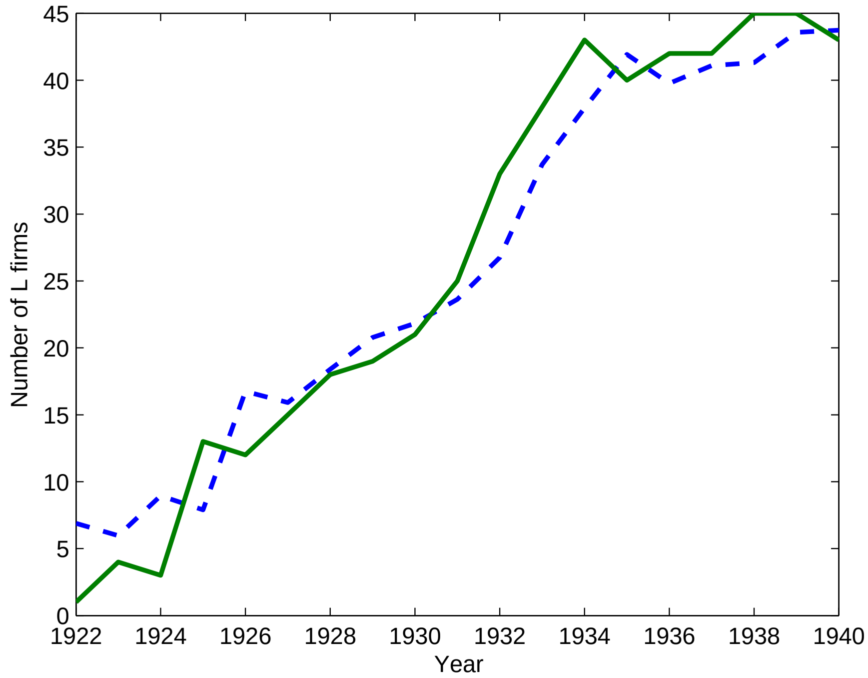

Figure 8: Actual vs. predicted evolution path of the number of low cost (L) firms

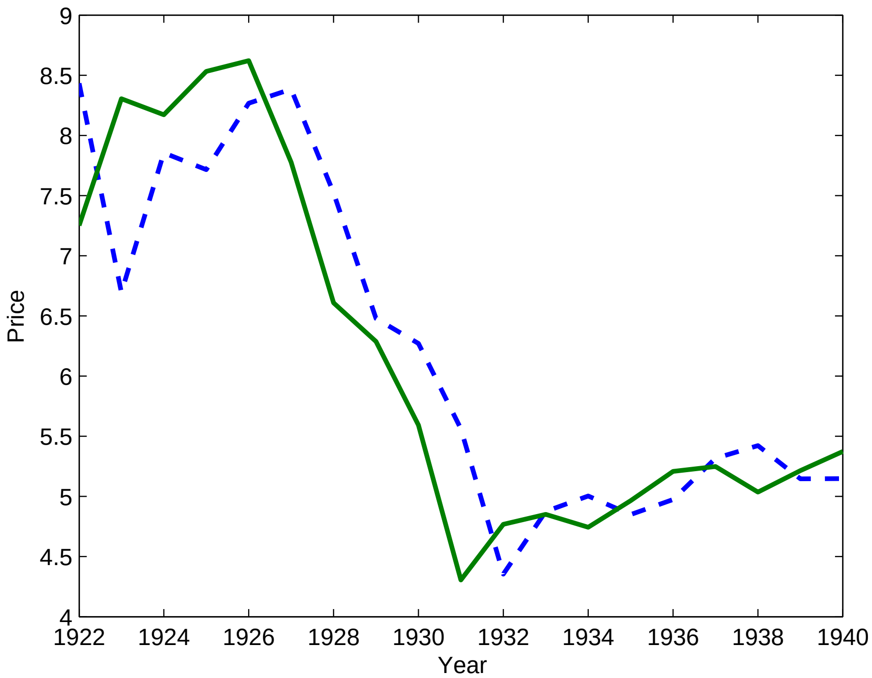

Figure 9: Actual vs. predicted evolution of price

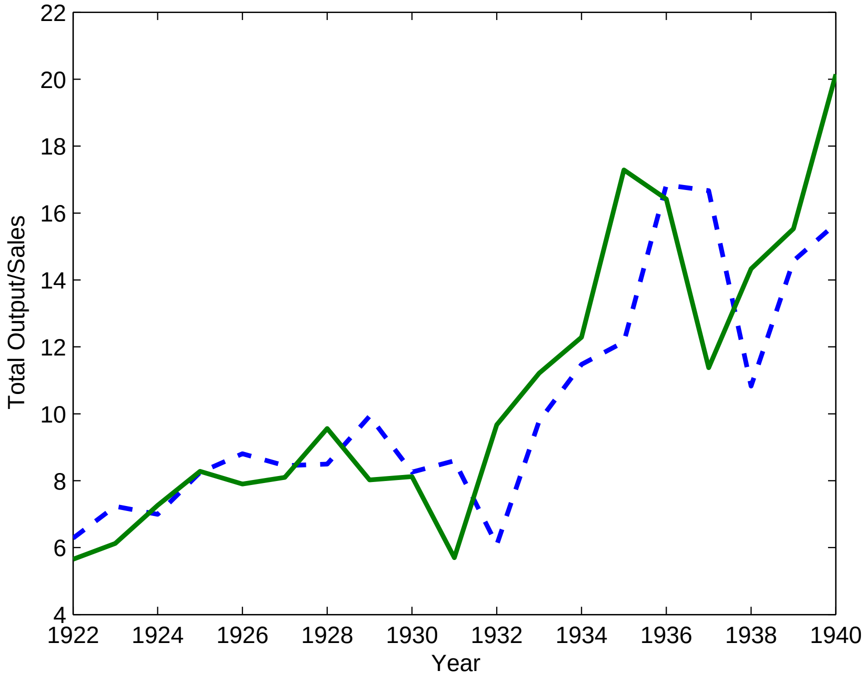

Figure 10: Actual vs. predicted evolution of industry output

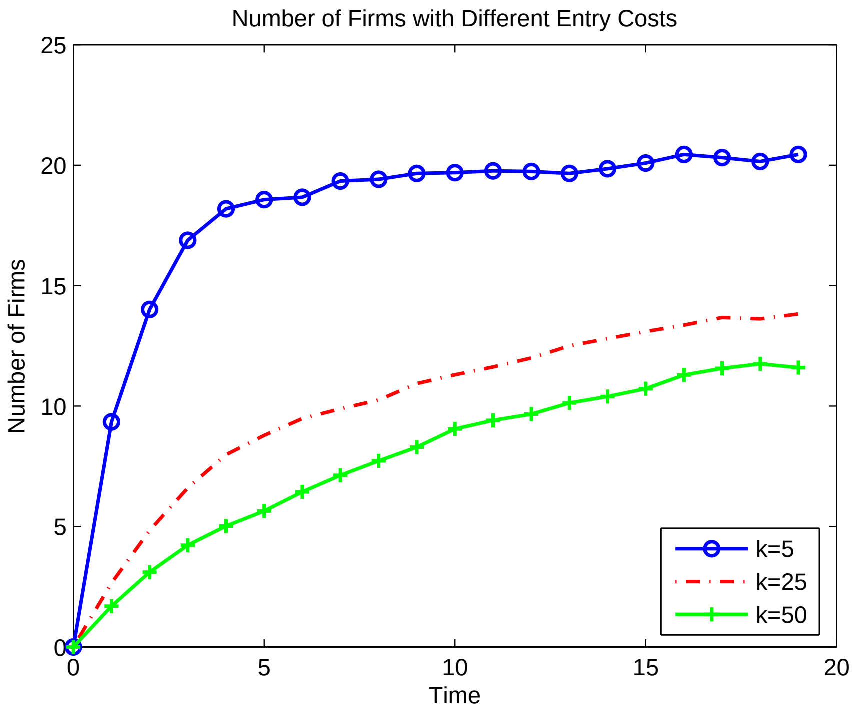

Figure 11: Number of firms with different entry costs ($\overline{E}$=40)

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Appendix

Incumbents vs. potential entrants' expectation

Section 2.4 describes the firms' equilibrium expectation of the number of active competitors of each type in the next period. An important point to make is that firms form expectation about the action of *other* active or potential competitors. As a result, the probabilities calculated from the potential entrants, low-type or high-type incumbents are slightly different.

The probability of having a number of e new entrants given state $[N, X]$ from the incumbents' perspective is

$$p(e|N,X) = \binom{\bar{E} - n^l - n^h}{e} \cdot P^{entry}(N,X)^e \cdot [1 - P^{entry}(N,X)]^{\bar{E}-n^l-n^h-e}$$

while from a potential entrant's perspective,

$$p(e|N,X) = \binom{\bar{E} - 1 - n^l - n^h}{e} \cdot P^{entry}(N,X)^e \cdot [1 - P^{entry}(N,X)]^{\bar{E}-1-n^l-n^h-e}$$

The difference is that the number of other potential entrants is $\bar{E} - n^l - n^h$ for incumbents while it is $\bar{E} - 1 - n^l - n^h$ for a potential entrant. Similarly, the probability of having x^l low-type firms to exit is

$$p(x^l|N,X) = \binom{n^l}{x^l} \cdot [1 - P^{lexit}(N,X)\ ]^{n^l-x^l} \cdot P^{lexit}(N,X)^{x^l}$$

from the perspective of potential entrants and high-type incumbents, while it is

$$p(x^l|N,X) = \binom{n^l - 1}{x^l} \cdot [1 - P^{lexit}(N,X)\ ]^{n^l-1-x^l} \cdot P^{lexit}(N,X)^{x^l}$$

from the perspective of low-type incumbents.

Integrated Bellman Equation

The integrated value functions $\overline{V}^l(N,X)$ can be calculated as the following:

$$\begin{aligned}\overline{V}^l(N,X) = &\Pr\{\phi^l < E[\overline{V}^l(N',X')] - E[\overline{V}^{out}(N',X')]|N,X\} \cdot [\pi^l(N,X) + \beta E[\overline{V}^l(N',X')|N,X]] \\ &+ \Pr\{\phi^l > E[\overline{V}^l(N',X')] - E[\overline{V}^{out}(N',X')]|N,X\} \cdot \{\pi^l(N,X) + \beta E[\overline{V}^{out}(N',X')|N,X] \\ &+ \beta \cdot E[\phi^l|\phi^l > E[\overline{V}^l(N',X')|N,X] - E[\overline{V}^{out}(N',X')|N,X]]\}\end{aligned}$$

Let $P^{lexit}(N,X) \equiv \Pr\{\phi^l > E[\overline{V}^l(N',X')] - E[\overline{V}^{out}(N',X')]|N,X\}$ be the probability of exit of low-type firm. Together with the assumption that ϕ^l is distributed exponentially ($G_L^{\phi} = 1 - e^{-(1/\eta)\cdot\phi^l}$), the above equation can be simplified as:

$$\overline{V}^l(N,X) = \pi^l(N,X) + \beta E[\overline{V}^l(N',X')|N,X] + \beta \cdot P^{lexit}(N,X)\cdot\eta$$

if $E[\overline{V}^l(N',X')|N,X] > E[\overline{V}^{out}(N',X')|N,X]$ or $P^{lexit}(N,X) < 1$. A property of exponential distribution is used to obtain the above result:[28]

$$E[\phi|\phi > V] = V + \eta, \;\; V \in R^+$$

where η is the mean of the exponential distribution. Since $\phi^l >= 0$, if $E[\overline{V}^l(N',X')|N,X] < E[\overline{V}^{out}(N',X')|N,X]$, then $P^{lexit}(N,X) = 1$, and $\overline{V}^l(N,X) = \pi^l(N,X) + \beta E[\overline{V}^{out}(N',X')|N,X] + \beta\cdot\eta$.

Similarly, let $P^{hexit}(N,X) \equiv \Pr\{\phi^h > E[\overline{V}^h(N',X')] - E[\overline{V}^{out}(N',X')|N,X]$ be the probability of exit of high-type firm. Then with the assumption that ϕ^h distributed exponentially ($G_H^{\phi} = 1 - e^{-(1/\mu)\cdot\phi^h}$),

$$\overline{V}^h(N',X') = \pi^h(N,X) + \beta E[\overline{V}^h(N',X')] + \beta\cdot P^{hexit}(N,X)\cdot\mu$$

if $E[\overline{V}^h(N',X')|N,X] > E[\overline{V}^{out}(N',X')|N,X]$ or $P^{hexit}(N,X) < 1$. When $E[\overline{V}^h(N',X')|N,X] < E[\overline{V}^{out}(N',X')|N,X]$, then high-type incumbents exit with probability 1 and $\overline{V}^h(N',X') = \pi^h(N,X) + \beta E[\overline{V}^{out}(N',X')] + \beta\cdot\mu$.

To obtain $\overline{V}^{out}(N',X')$, first note that:

$$\begin{aligned}\overline{V}^{out}(N,X) = {} & \Pr\{k < E[\overline{V}^e(N',X')] - E[\overline{V}^l(N',X')|N,X\} \\ & \cdot\{-E[k|k < E[\overline{V}^e(N',X')|N,X] - E[\overline{V}^l(N',X')|N,X]] + \beta E[\overline{V}^e(N',X')|N,X]\} \\ & + \Pr\{k > E[\overline{V}^e(N',X')] - E[\overline{V}^l(N',X')|N,X\}\cdot\beta E[\overline{V}^{out}(N',X')|N,X]\end{aligned}$$

Let $P^{entry}(N,X) = \Pr\{k < E[\overline{V}^e(N',X')] - E[\overline{V}^l(N',X')|N,X\}$ be the probability of entry of potential entrants. Together with the assumption that k follows exponential distribution ($G^{\kappa} = 1 - e^{-(1/\sigma)k}$), the above equation can be expressed as:

$$\overline{V}^{out}(N,X) = \beta E[\overline{V}^e(N',X')] - p^{entry}(N,X)\cdot\upsilon$$

[28] Pakes, Ostrovsky and Berry (2005) also used exponential distribution for entry cost and used this property to derive the expressions for Bellman equations.

if $E[\overline{V}^e(N',X')|N,X] > E[\overline{V}^l(N',X')|N,X]$. Again, there is discontinuity when $E[\overline{V}^e(N',X')|N,X] < E[\overline{V}^l(N',X')|N,X]$, which results in no-entry and $\overline{V}^{out}(N,X) = \beta E[\overline{V}^{out}(N',X')]$.

Estimation details

Let $S = \{S_1, S_2, ..., S_R\}$ be the set of discretized states, where $S_i = [\tilde{n}_i^l, \tilde{n}_i^h, \tilde{F}_i, mc_i^h] \equiv [\tilde{N}_i, \tilde{X}_i]$ for $i = 1, 2, ..., R$, is a 4×1 vector containing information about the number of low-type and high-type incumbents as well as demand and cost characteristics of state i. Let P^0 be a vector of initial guess of firms' choice probabilities (the probability to enter of potential entrants and the probability to exit of incumbents) under each state S_i. The iteration process proceeds as follows:

Step 1 : Given the choice probabilities, compute the $R \times R$ transition matrix for number of firms $\Pr(\tilde{N}'|S) = [f_{ij}^N]$,where each element f_{ij}^N in the matrix is the probability of having the outcome $[\tilde{n}_i^l, \tilde{n}_i^h]$ incumbents in the next period from current state S_j, $i, j = 1, ..., R$.f_{ij}^N is obtained by applying equation (2.18).

Step 2: Find the vector of parameters $\hat{\theta}^1$ that minimize the SMM objective function given $\Pr(N'|S)$ from step 1. Within this step, for each vector of parameters, compute transition probability $\Pi(\tilde{X}'|S)$. Together with $\Pr(\tilde{N}'|S)$, one can obtain $\hat{V}^l(\tilde{N},\tilde{X}), \hat{V}^h(\tilde{N},\tilde{X})$ and $\hat{V}^{out}(\tilde{N},\tilde{X})$ by solving a system of linear equations as shown in equations (4.1) to (4.3). The results are used to predict the evolution paths of number of incumbents, price and output, etc. Evaluate the moments and calculate the value of objective function.

Step 3: Iterate the choice probability by applying formula (2.14), (2.15) and (2.11). For example,

$$\hat{P}^{entry} = \hat{G}^{\kappa}\{\beta \cdot [\hat{E}[\overline{V}^e(\tilde{N}',\tilde{X}',\hat{\theta}^1)] - \hat{E}[\overline{V}^{out}(\tilde{N}',\tilde{X}',\hat{\theta}^1)]|\tilde{N},\tilde{X},\hat{\theta}^1\}$$

with $\hat{E}(x) = \sum_{\tilde{N}'}\sum_{\tilde{X}'} x \cdot \Pi(\tilde{N}'|\tilde{N})\Pi(\tilde{X}'|\tilde{X})$.

Step 4: Repeat from Step 1 until both convergence in $\hat{\theta}$ and $\hat{P}$ is obtained.