

Arbitrage and Implied Price Relationships Among the
S&P500 Cash Index, SPDRs, and Futures

by

Ronald Tarantino

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Professor Marti G. Subrahmanyam

Faculty Adviser

Professor Stephen Figlewski

Thesis Advisor

Introduction

The very idea of arbitrage, the allure of risk free profit, should be the strong motivational factor that causes market participants to monitor related prices and ensure they do not deviate from their no arbitrage prices. If prices do deviate, then arbitrage trades should occur to drive prices back to their no arbitrage levels. However, the difficulties that arise in executing arbitrage trades, beyond transaction costs, can cause deviations to persist for longer than a moment.

One way to magnify the problems that can occur with arbitrage trades is to look at price relationships among an index and its derivatives. The S&P500 Index is suited for this purpose because its collection of 500 stocks, all of varying market capitalization, makes arbitrage trades difficult. Also, its importance as a leading market indicator keeps the attention and attracts the capital of traders and investors.

For both of these reasons, numerous securities have been developed to facilitate investor participation in the index. The S&P500 Index futures contract, a derivative, and the Standard & Poor's Depository Receipt (SPDR), an ETF, are widely known and traded securities that serve such a purpose. ETFs are equities that trade like a single share of stock, with some exceptions, whose values depend in part on the values of their underlying portfolio of securities.

Interestingly, although one may think that price movements of the cash index are fully translated into movements of the future and SPDR, closer inspection shows that price movements of the future and SPDR are more related to each other than either is to the cash index. My hypothesis is that one reason these relationships exist is because of differences in the degree of difficulty in exploiting arbitrage opportunities. Furthermore, these relationships change through time, and knowledge of their existence can shed light on a trading strategy that produces statistically significant profits.

Explanation of Securities

This study used three securities: S&P500 cash index, SPDRs, and S&P500 index futures. To understand the relationships among the securities, it is first necessary to know the details of each one individually.

S&P500 Cash Index

The S&P500 Index is a market value weighted collection of 500 stocks designed to represent the stock market as a whole. As of December 2001, the index constituted about 80% of the value of all stocks traded in the US¹, so it is an important and highly visible index. In addition, the index is subject to rebalancing, where price weights are adjusted, and reconstitution, where some stocks are completely replaced by others. These represent risks of holding a position in the index. Finally, if one wanted to take a position on the index at a certain level, he would have to buy or sell 500 stocks before the price of any individual stock changed. This is a difficult trade to do² because there may simply not be enough size in the market at the desired levels. Furthermore, even though the index is at a level, the trade still has to be executed and during that process the index may move away. Even program trading cannot guarantee the index portfolio will trade at an observed level.

Futures

An S&P500 index futures contract is an agreement to exchange cash equal to \$250 times the difference between the current futures price and the price of the index at maturity. Futures are marked to market each day, so cash does change hands before expiry. Nevertheless, if a futures contract were entered into at a level of 1000, for example, and on expiry the level of the index was 1050, the holder of the futures contract would be entitled to \$12,500 ($50 * \250).

¹ Siegel, Jeremy. "Stocks for the Long Run."

² It is because of this reason that futures and SPDRs are desirable securities. They conveniently summarize the price action of 500 stocks into the price action of one security. While it is true that the relationship is not one to one, in the long run, index appreciation will translate to futures and SPDR appreciation as well.

There is no up front cost for the futures contract since the trader is only taking a position on the future. To determine the futures price, the following equation for Futures Fair Value may be used:

$$(1) \quad F_0 = I_0 (1 + (r-d) * t / 365)$$

where:

F_0 = Futures Fair Value

I_0 = Index Level at Time 0

r = Annualized Risk Free Rate

d = Dividend Yield

t = Time to Futures Expiry

In words, Futures Fair Value is the cost of borrowing funds at the risk free rate to purchase the S&P500 cash index portfolio, holding the index portfolio, and collecting dividends until maturity. For example, if the S&P500 index is trading at a level of 1000 and there remain 90 days until the next futures expiry, an investor can borrow \$1000 at the risk free rate of 5% to purchase the index portfolio. Over the next 90 days, he can collect dividends at an annual rate of 2.5% on the index portfolio. Therefore, at the end of 90 days, the investor will have paid \$12.33 ($\$1000 * 5% * 90/365$) in interest costs, but will have received \$6.16 ($\$1000 * 2.5% * 90/365$) from dividends to help offset that cost. The total cost of this strategy at the end of 90 days, then, is \$1006.17 ($\$1000 + \$12.33 - \6.16). This is futures fair value. Note that this trade is described per unit of index, and that the dollar size is much greater because of the multiplication by \$250.

To calculate the index level implied in a futures contract, the following rearranged Fair Value Equation may be used:

$$(2) \quad I_0 = F_0 / (1 + (r - d) * t / 365)$$

where:

I_0 = Implied Index Level at Time 0

F_0 = Futures Market Price

r = Annualized Risk Free Rate

d = Dividend Yield

t = Time to Futures Expiry

A futures contract is a derivative, and it can be replicated through a strategy of borrowing, investing, and collecting. Thus, if the futures price deviates from the cost of the replication strategy, then an arbitrage opportunity exists. If, for example, the futures contract discussed above were trading at \$1010 then an arbitrage opportunity would exist. To exploit this price disparity, an arbitrageur would sell the over priced future at \$1010 and replicate the future as discussed above for the predetermined cost of \$1006.17 to earn a risk free profit of \$3.83 (\$1010 – \$1006.17). If, on the other hand, the futures contract were only trading at \$1005, then an astute arbitrageur would buy the underpriced future for \$1005. He would then short the index and loan \$1000 at the risk free rate. Over the next 90 days, he would pay the dividends of \$6.16 to the lenders of the shares sold short while collecting interest in the amount of \$12.33 on the loan he made. The total flows to the arbitrageur after 90 days equals \$1006.17 (\$1000 + \$12.33 – \$6.16). Since the futures contract he purchased was only at a level of \$1005, his risk free profit is \$1.16 (\$1006.17 - \$1005). Note that to trade the index in efficient size, an arbitrageur typically trades \$10 million worth of the index portfolio or more.

In practice, such an arbitrage trade is difficult to do because, while buying or selling a futures contract while the index is at a certain level is relatively straightforward, buying and selling all 500 securities at that same level poses several problems. First, there is a chance that

the index may move away from the level at which the arbitrageur bought or sold the future before he can trade in the index. Second, if trading volume is light for one of the 500 securities, there is a chance that the arbitrageur can move the market. Third, while holding the position in the index, either long or short, the arbitrageur may face index rebalancing. This would require him to adjust his holdings and may result in an over or under performance, the latter of which could quickly consume his arbitrage profits. Finally, selling the index short is harder than buying it because an arbitrageur would have to borrow the stocks from a lender and would need to short all 500 stocks on an uptick.

SPDRs

SPDRs are an exchange traded fund (ETF) that allows investors to participate in the S&P500 cash index by trading a single security that, unlike the future, does not mature. SPDRs are approximately equal to 1/10 of the cash index and trade like traditional equity with some differences. One difference is that SPDRs can be shorted on a downtick. Another is that SPDRs can be created and redeemed at NAV. This second distinction creates a whole host of interesting issues, which are explored below. Some important concepts related to understanding SPDRs are as follows:

- Net Asset Value (NAV): NAV is the value at which SPDRs can be created and redeemed. According to the Specifications page included with SPDR prospectus from the AMEX, "The net asset value (NAV) per SPDR is computed each business day at the close of trading in the underlying stocks...NAV represents the aggregate closing market value of the underlying portfolio of S&P500 securities in the SPDR Trust, plus any accrued dividends, minus accumulated Trust expenses on a per-SPDR basis." In the form of a mathematical equation:

(3) NAV = (S&P500 Index)/10 + Accumulated Dividends – Management Fees

The S&P500 index level is self explanatory. To create or redeem SPDRs, two rules apply. First, a minimum number of 50,000 SPDRs must be created or deleted, with no upper limit. Second, for the service of creation and deletion, the Trust charges a fixed fee of \$3000, regardless of whether SPDRs are being created or deleted and the quantity of SPDRs in the transaction.

- Accumulated dividends: Represents the dividends that the Trust holds for and delivers to SPDR holders on the last business day of the month following calendar quarter end (March quarter end paid in April, June quarter end paid in July, September quarter end paid in October, and December quarter end paid in January).
- Management Fees: Represents deductions from accumulated dividends that management takes as compensation for administering the Trust.

As mentioned before, SPDR prices in the market deviate from NAV. In principle, any such deviation represents an opportunity for ETF arbitrage. If the SPDR price is greater than NAV, then an arbitrageur should short the SPDR and deliver the cash index portfolio, accumulated dividends, and \$3000 minus management fees to obtain SPDRs which he can then use to cover his short position. If the SPDR price is trading below NAV, the arbitrageur can simultaneously short the index portfolio, buy the SPDR, and for \$3000 deliver it to the Trust to receive the cash index portfolio plus accumulated dividends minus management fees. He can then use the index portfolio to cover his short position. For example, if the index level is at 1000 and the accumulations minus fees is \$0.12 per SPDR, then the SPDR should be trading at \$100.12 ($\$1000/10 + \0.12). If the SPDR is trading at \$101, then an arbitrageur would short (at least) 50,000 shares of the SPDR for an inflow of \$5,050,000. He would simultaneously deliver

\$5,000,000 worth of the index ($50,000 * \$100$) plus \$6000 ($50,000 * \$0.12$) plus \$3000 for a total cost of \$5,009,000 to receive 50,000 SPDRs, which he would use to cover his short position. The arbitrage profit would be \$41,000. On the other hand, if the SPDR were trading at \$99, for example, then the arbitrageur would purchase at least 50,000 units of the SPDR for an out flow of \$4,950,000 while simultaneously shorting \$5,000,000 worth of the index. He would then pay \$3000 to trade in those SPDRs for \$5,000,000 worth of the index, plus \$6000 of dividends less management fees, which he would keep, for a total inflow of \$5,003,000. Total arbitrage profit would be \$53,000.

Note that the Trust only calculates NAV once per day. However, arbitrage is still possible because SPDR NAV is reported every fifteen seconds. And the fact that the transaction to create or delete SPDRs takes at most three days to clear does not affect the arbitrage profits. Clearing the trade can occur as quickly as two days, and if the shorted shares of the index portfolio were called in, that transaction would also take up to three days to clear so that the SPDR trade would clear first, and the arbitrageur would have the index portfolio to deliver.

Literature Review

“Spiders: Where are the Bugs?” by Professors Elton et. al. is the impetus for this study.

Some of the interesting findings and concepts in that paper were the following:

- Futures and Implied Index Levels: The market price of an S&P 500 index future is determined by supply and demand. If dividends on the index can be accurately forecasted, then the dividend yield, risk free rate, time to futures expiry, and the market price of the future can be combined to calculate the index level implied in that futures price. Theoretically, this index level should be the same as that of the cash index. However, that is not always the case. One explanation may be that because futures prices can adjust more quickly to new information entering the market than all 500 stocks

comprising the index, there is an element of price staleness in the index. Another explanation, one that is more consistent with the results of my study, is that an arbitrage trade between the SPDR and the future is much easier to execute than one involving the cash index. Therefore, SPDR and futures prices can adjust to their proper levels relative to each other, while both the implied index level from the future and the SPDR NAV differ from the level at which the cash index is then trading.

- Correlations: SPDR net asset value (NAV) is explicitly calculated using the level of the S&P 500 index, according to the SPDR Trust. Ostensibly, changes in the index level should translate one for one to changes in NAV and SPDR price, and therefore price changes in the index and the SPDR should be perfectly positively correlated. However, SPDR prices deviate from NAV in varying magnitude through time, so such a high correlation between SPDRs and the cash index is not necessarily fundamental. In search of a better relationship, once Elton et. al. had determined the implied index levels in futures prices, they regressed SPDR returns on implied index returns and found an adjusted R^2 of 0.98 with a slope coefficient of 0.99 and concluded that SPDRs and futures prices move closely together.
- Price Deviations from NAV and ETF Arbitrage: Since SPDRs can be created and redeemed at NAV, it is a wonder that they should ever deviate from that value. Yet, because SPDRs, and not NAV, are traded, supply and demand determines their prices. Just as traders can use a replication strategy in an arbitrage trade to profit from price differences between derivatives and the cost of replicating those derivatives, they can also execute ETF arbitrage wherein they make trades in the cash index and the SPDR to guarantee profit from deviations between SPDR price and NAV.

Purpose of Study

The purpose of this study is to explore the hypothesis that price pressures on the SPDR and S&P500 futures contracts that drive the securities away from their no arbitrage prices, and the arbitrage trades that bring them back in line, cause the securities to be more correlated with each other than either is with the underlying S&P500 cash index.

The results of this study are achieved by exploring some of the results that “Spiders: Where are the Bugs?” presented. Namely, the authors found that deviations from NAV were not autocorrelated. That is, there is no relationship between tomorrow’s deviation and today’s deviation. This study asks whether that relationship holds with more contemporary data. The authors also determined that a strong price relationship between SPDRs and the index level implied in futures exists. Elton et. al. covered the years 1993 to 1998, while this study explores 1996 through 2002. This study asks whether the results for this later set of data differ from the first.

The second purpose of this study is to extend the exploratory results. The study asks how well the cash index explains movements in SPDR prices versus the futures, whether the explanatory power of the index and futures changes through time, and whether the results can be used to implement a trading strategy that produces a statistically significant profit.

Data Set

The data in this study run from 1996 to 2002, inclusive, because the OptionMetrics database was only able to furnish risk free zero-coupon interest rates and S&P 500 dividend yields for those years. CRSP provided S&P 500 index and SPDR prices and returns. Bloomberg provided SPDR NAV and Accumulations Net of Expenses data. Finally, the Commodity Research Bureau’s InfoTech Database provided futures prices. Daily closing prices were used for level analysis and to compute daily percentage changes for change analysis.

This study used standard S&P 500 total returns, meaning the percentage increase in the level of the index, plus dividends. The dividend yield on the index is calculated as the difference between CRSP Total Return minus the price index return. The dividend yield is then added to the actual price index return on the S&P 500. Calculating actual total return is necessary because S&P 500 total returns, which would be best suited for the purposes of study, are not publicly available from S&P. Elton et. al. developed this methodology because they found that the S&P 500 total returns calculated by CRSP did not fit the SPDR returns very well. This discovery was confirmed when CRSP admitted to using a different methodology for calculating the total return than S&P does³.

CRSP provides SPDR prices and returns. Both were needed for level and change analysis. To be consistent, SPDR returns were calculated from the SPDR prices that were given. Plotting the price and return data to search for outliers proved to be very fruitful. The graphs exhibited tremendous peaks and troughs, in addition to large outliers. Closer inspection revealed SPDR prices that were only a fraction of a dollar. Adding 100 to numbers like these was the obvious solution to correcting the error, since doing so set the closing price between the daily high and low. Furthermore, the difference between CRSP calculated returns and the returns calculated using these modified prices was virtually zero.

Dividend Yields on the S&P 500 index provided by OptionMetrics were highly volatile. This is because of the way OptionMetrics computes the dividend yield. In short, the database uses a model based on put-call parity (appropriate for index options because they are European) that assumes the risk free rate is known to calculate dividend yields. By assuming the risk free rate is stable, when in actuality there is uncertainty and variability surrounding it, the model

³ Email correspondence with George Comer, a former doctoral student at NYU Stern and coauthor of “Spiders; Where are the Bugs?”

passes the variability to the dividend yield. To smooth these yields, I used a twenty-two day moving average of the dividends yields. I used twenty-two days because that was the number of trading days for which dividend yields were available in January 1996 and I wanted to use full months of data in my study for my pre and post expiry analysis. Therefore, by averaging the data over the month of January, I was able to have risk free rates beginning the first trading day of February 1996. Unfortunately, because I encountered bad data later on, I did not always end up with full months.

Bloomberg's NAV and accumulated dividend data posed the most problems by far. To determine the level of the S&P 500 index used to compute NAV, it is necessary to subtract accumulations (net management fees) from NAV [refer to equation (3)]. Fortunately, Bloomberg provided the NAV and accumulation (net management fees) data. After subtracting out accumulations from NAV and comparing the resulting index levels to that of the S&P 500, it was clear that NAV was off by one day. Therefore, I shifted NAV and dividend data up by one day. The result was that the standard deviation between the calculated index level and reported index level went from 15.77 before the shift to 6.18 after the shift. That represents a 61% decrease in variability.

After the SPDR data was aligned, I had to fill in missing values. Since the return on the cash index should be the same as the return on the calculated index, I used the following equation to fill in missing NAV values:

$$(4) \quad NAV_t = NAV_{t-1} * S\&P_t / S\&P_{t-1}$$

where:

NAV_t = net asset value at time t

NAV_{t-1} = net asset value at time t-1

$S\&P_t$ = level of the S&P500 index at time t

$S\&P_{t-1}$ = level of the S&P500 index at time t-1

The dividend data offered its share of problems as well. To fill in missing dividend data, I used the following methodology:

1. Assumed the accrued dividend to be zero if the dividend before the missing point was greater than the dividend after the missing point, and the dividend after the missing point was zero.
2. Assumed the accrued dividend to be the same as on the previous day if the dividend before the missing point was the same as the dividend after the missing point.
3. Assumed the accrued dividend to be equal to the prior dividend if the dividend before the missing point was less than the dividend after the missing point.

Finally, because SPDRs trade ex-dividend on the third Friday of the month, it is natural to see dividends accumulate and then drop to zero. However, in many cases the dividend dropped to zero on the third Thursday of the month, resulting in a calculated index level that was several points away from the reported index. In these cases, I assumed the dividend on the Thursday before expiry was the same as on the day before. Making such an assumption reduced deviations from five points to fractions of a point in some cases. Note that not all Thursdays before expiry were missing dividend data, so this procedure was not always necessary.

Once the NAV and accumulations data was shifted and dividends corrected, there were still NAV values which were obviously wrong. For example, on August 13, 1996 the calculated index level was 450.50, while the reported cash index level was 660.2. Fortunately, there were only a few such blatant errors, and they were discarded.

Futures mature every three months. Therefore, it is impossible to use one futures contract for all seven years of this study, although I required returns on futures and their implied index levels for all seven years. The solution to this problem was to assume trading in one contract until some point in time before it matured, and then roll over to begin trading the contract that was next to mature. This process is called rolling. I rolled to the next contract on the first

trading day of March, June, September, and December because trading volume in the contracts maturing in those months falls as traders begin to roll their positions to the more distant contracts.

The set of risk free zero-coupon bond rates provided by OptionMetrics was relatively complete. Days with missing rates were deleted from all data in the set (cash index, SPDRs, NAV, futures, and implied).

Results

Now that the valuation of the ETFs and futures, as well as the arbitrage trades surrounding them and the issues that make arbitrage difficult have been discussed, it is appropriate to turn to the discussion of the results of this study.

Exploratory

As mentioned above, the first purpose of this study is exploratory. Elton et. al. examined the autocorrelation between SPDR price deviations from NAV at time t and time $t+1$. That is, they tested whether a deviation observed today will disappear by tomorrow as expected because of arbitrage. The general equation for such a regression is:

$$(5) \quad D_{t+1} = B_0 + B_1 D_t + u_t$$

where:

D_{t+1} = SPDR – NAV at time $t+1$

D_t = SPDR – NAV at time t

u_t = error term

Table 1: Regression of NAV Deviation at time t+1 on NAV Deviation at time t

Study	Constant	Coefficient	R ²	# Obs.
Tarantino (T-stat)	0.112 (13.46)	0.108 (4.48)	1.2%	1709
Elton/Gruber	-1.34	0.0620	0.4%	-

The results of Elton et. al. show an R² of only 0.4%, which means deviations at time t explain very little of the deviation in time t+1, and therefore NAV deviations do not exhibit autocorrelation. My results are similar. Both the constant and slope coefficient are significant, and the R² is 0.8% higher. Nevertheless, the lack of autocorrelation in NAV deviations is apparently persistent through time.

The second exploratory test is to determine whether futures are still highly correlated with SPDRs. Elton et. al. regressed the percentage changes in SPDR prices on changes in index levels implied in futures prices. The general equation for such a regression is:

$$(6) \quad \text{SPDR Return}_t = B_0 + B_1 \text{Future Return}_t + u_t$$

where:

SPDR Return_t = return on SPDR from time t-1 to time t

Future Return_t = return on Future from time t-1 to time t

u_t = error term

The expectation here is that the constant will be close to zero and the slope coefficient will be close to one because purchases of SPDRs can be financed in similar ways, with small differences coming from the payment of dividends. In addition, futures and SPDRs can adjust quite quickly to news entering the markets.

Table 2: Regression of SPDR return on Implied Index returns

Study	Coefficient	Adjusted R ²	# Obs.
Tarantino (T-stat)	0.99 (240.88)	97.1%	1711
Elton/Gruber	0.99	98.0%	-

The constant found by Elton et. al. was not reported, although I suspect that it was not statistically significant because mine was not either. However, both studies produced a slope coefficient close to one and adjusted R²'s of almost 100%. Clearly, price movements in SPDRs and futures are closely related. Furthermore, since SPDRs and futures are still highly correlated, I present a trading strategy below to show these results in dollar terms.

Extension

The extension analysis of this paper consists of level analysis, change analysis, and error correction analysis.

The goal of level analysis was to determine how well deviations of the implied index from the actual cash index can explain deviations of SPDR price from NAV. The general equation for such a regression is:

$$(7) \quad (\text{SPDR} - \text{NAV})_t = \mathbf{B}_0 + \mathbf{B}_1(\text{Implied Index} - \text{Index})_t + \mathbf{u}_t$$

where:

$(\text{SPDR} - \text{NAV})_t$ = SPDR deviation from NAV from at time t
 $(\text{Implied Index} - \text{Index})_t$ = difference between implied index and S&P500 cash index at time t
 \mathbf{u}_t = error term

The expectation here is that the slope coefficient should be positive because the SPDR and future are highly positively correlated. The R² and slope coefficient should also be relatively small

because as the future nears maturity it becomes the index and uncertainty regarding dividends is resolved. Therefore, the deviations that occur between the implied index and the index should reduce in magnitude over time, while the magnitude and variation of the deviations between the SPDR and NAV could persist because the SPDR does not mature into NAV.

Table 3: Regression of NAV Deviations on Implied Index Deviations

Sample	Constant	Coefficient	R²	# Obs.
Full	0.130	0.030	17.9%	1711
(T-stat)	(18.49)	(19.28)		

The constant and coefficient are statistically significant with a small R² as expected. Note that the slope coefficient of 0.030 is more important than it seems because, while the prices used to calculate NAV deviations are in the tens and hundreds, the prices used to calculate the implied index deviations are in the hundreds and thousands. In other words, if the NAV deviations and implied index deviations were on the same scale, the slope coefficient would be around 0.30, which is ten times larger than it currently is.

To further explore the results of this regression and the idea that as the future moves closer to expiry deviations in the implied index level from the actual index explain progressively fewer of the deviations of the SPDR from NAV, I divided the full sample into three months to expiry (3MTE), two months to expiry (2MTE), and one month to expiry (1MTE). In other words, for 3MTE, I only used futures contracts that had three months until the month of expiry. The general regression equation I fit is the same as before [see equation (7)]. For these regressions, the slope coefficient and R² should fall as time to expiry decreases. I hypothesize

that as the futures mature into the index, uncertainty regarding the dividend yield, macroeconomic factors, and microeconomic factors that would affect the index are resolved so that implied index levels become clearer and induce arbitrage, thereby reducing the deviations through time.

Table 4: Regression of NAV Deviations on Implied Index Deviations as a Function of MTE

Sample	Constant	Coefficient	R²	# Obs.
3MTE (T-stat)	0.176 (14.88)	0.0373 (12.85)	22.4%	574
2MTE (T-stat)	0.050 (3.72)	0.040 (12.26)	20.9%	570
1MTE (T-stat)	0.167 (17.01)	0.019 (9.76)	14.5%	565

The constants and slope coefficients are statistically significant. As expected, the explanatory power of implied index deviations decreases with time. Notice that the coefficient increased slightly from 3MTE to 2MTE, contrary to what was expected. But the much lower constant of the 2MTE regression means that the 3MTE and 2MTE lines do not intersect until the implied deviation reaches about 47, which is a very large deviation. In 1MTE, the results converge with expectations since the slope coefficient is much smaller than in 3MTE.

The next step in level analysis is to determine whether the difference between the implied index and the cash index can explain NAV deviations, and whether the number of months to futures expiry is important in this explanation. The general equation for such a regression is:

$$(8) \quad (\text{SPDR-NAV})_t = B_0 + B_1 (\text{Implied} - \text{Cash})_t + B_2 Z_2^*(\text{Future D2})_t + B_3 Z_3^*(\text{Future D3})_t + u_t$$

where:

$(\text{SPDR} - \text{NAV})_t$ = NAV deviation at time t

$(\text{Implied} - \text{Cash})_t$ = implied index deviation at time t

Z_2 = 1 if MTE is 2, 0 otherwise

$(\text{Future D2})_t$ = $(\text{Implied} - \text{Cash})$ for 2MTE at time t

Z_3 = 1 if MTE is 3, 0 otherwise

$(\text{Future D3})_t$ = $(\text{Implied} - \text{Cash})$ for 3MTE

u_t = error term

The expectation here is that the slope coefficients should be positive because the SPDR and implieds should be moving together, as should the NAV and cash index. Furthermore, if month to expiry is important, then the B_2 and B_3 should be statistically significant.

Table 5: Regression of NAV Deviations on Implied Index Deviations as a Function of MTE

Importance of Futures MTE in Explaining NAV Deviations					
Constant	$(\text{Implied} - \text{Cash})_t$	$Z_2^*(\text{Future D2})_t$	$Z_3^*(\text{Future D3})_t$	R^2	# Obs.
0.131	0.0186	0.0229	0.0185	20.0%	1711
(18.89)	(7.93)	(6.13)	(4.92)		

$$(9) \quad (\text{SPDR-NAV})_t = 0.131 + 0.0186 (\text{Implied} - \text{Cash})_t + 0.0229 Z_2^*(\text{Future D2})_t + .0185 Z_3^*(\text{Future D3})_t$$

The results clearly show that all coefficients are significant. Furthermore, the slope coefficient actually increases from 0.0186 for one month to expiry to 0.0415 ($0.0186 + 0.0229$) for two months to expiry month because then, $Z_2 = 1$ and $Z_3 = 0$, and $(\text{Implied} - \text{Cash})_t$ is equal to $(\text{Future D2})_t$. The slope coefficient for three months to expiry is about double what it is for one month to expiry, and is calculated by summing 0.0186 and 0.0185 to obtain 0.0371. This occurs because when there are three months to expiry month, $Z_2 = 0$ and $Z_3 = 1$ and like before,

(Implied – Cash)_t is equal to (Future D3)_t. Therefore, time to expiry affects the relationship between futures deviations and NAV deviations and is statistically significant. Finally, note that as in the first exploratory analysis, the coefficients are ten times as important as they seem because implied index deviations represent the difference between numbers which are in the hundreds and thousands, while NAV deviations occur from values that are only in the tens and hundreds.

The goal of change analysis was to determine which securities explained the greatest amount of SPDR price deviation. I began by taking the full sample and regressing SPDR price returns on returns on the cash index, implied index, and both. The general equations for such regressions are:

$$(10) \quad \text{SPDR Return}_t = B_0 + B_1 \text{Cash Index Return}_t$$

$$(11) \quad \text{SPDR Return}_t = B_0 + B_1 \text{Implied Index Return}_t$$

$$(12) \quad \text{PDR Return}_t = B_0 + B_1 \text{Cash Index Return}_t + B_2 \text{Implied Index Return}_t$$

where:

SPDR Return_t = return on SPDR from time t-1 to time t

Cash Index Return_t = return on cash index from time t-1 to time t

Implied Index Return_t = return on implied index from time t-1 to time t

The expectation here is that the constant should be very close to zero because if there is no change in the index from time t-1 to time t, then there should be almost no price change in the SPDR. The slope coefficient should be positive and close to one because SPDR returns depend in part on cash index returns, as do futures returns. Furthermore, I expected that the slope coefficient of the cash index would be greater than that of the future, perhaps greater than one, because SPDR prices, and therefore returns, deviate from NAV in addition to moving with the cash index. So I would expect cash index returns to be amplified slightly. As for the future,

volatility declines as the contract matures into the index since, so I would expect the slope coefficient to be less than one to represent this declining volatility.

Table 6: Regression of SPDR Returns on the Cash Index, Implied Index, and Both

Sample	Constant	Cash Index	Implied Index	R²	# Obs.
Full (T-stat)	0.000 (-0.56)	1.014 (149.87)	-	92.9%	1711
Full (T-stat)	0.000 (0.19)	-	0.991 (240.88)	97.1%	1711
Full (T-stat)	0.000 (0.06)	0.100 (5.42)	0.898 (50.86)	97.2%	1711

The results satisfy what I am looking for. The constants are not statistically significant from zero. The cash index has a statistically significant slope coefficient that is positive and slightly greater than one, while its R^2 is 92.9%. As stated before, futures returns are closely related to SPDR returns, and a significant slope coefficient of 0.991 along with an R^2 of 97.1% show this. Finally, the cash and implied index together explain 97.2% of the variation, only 0.1% more than the future alone. The futures returns receive a much higher weighting than the cash index returns do. This is consistent with the fact that futures are more closely related to SPDRs than either one is with the cash index.

To explore how the explanatory ability of the above securities changes as a function of time to futures maturity, I regressed SPDR returns on the cash index, implied index, and both. As the futures mature into the cash index, the implied index level is more transparent, meaning

that as the futures price becomes the index level, there are fewer dividends to forecast and less uncertainty about macroeconomic and microeconomic factors that could impact the index. As a result, there is increasingly greater certainty in the inputs used to calculate the implied index from the futures fair value equation. So arbitrage trades in the SPDR and future are clearer to identify and thus should cause the futures to explain more of the SPDR price returns through time. The general equations for such regressions are equations (10), (11), and (12). The coefficient expectations are similar here. In addition, the futures coefficient and R^2 should increase from 3MTE to 1MTE.

Table 7: SPDR Returns versus the Cash Index, Implied Index, and Both As A Function of MTE

Sample	Constant	Cash Index	Implied Index	R²	# Obs.
3MTE (T-stat)	0.000 (-0.99)	1.014 (87.02)	-	93.0%	574
3MTE (T-stat)	0.000 (-1.47)	-	0.986 (153.75)	97.6%	574
3MTE (T-stat)	0.000 (-1.56)	0.143 (5.54)	0.854 (34.92)	97.8%	574
2MTE (T-stat)	0.000 (-0.08)	1.018 (81.06)	-	92.0%	570
2MTE (T-stat)	0.000 (0.31)	-	0.994 (123.07)	96.4%	570
2MTE (T-stat)	0.000 (0.29)	0.029 (0.76)	0.967 (26.14)	96.4%	570

Table 7: SPDR Returns versus the Cash Index, Implied Index, and Both As A Function of MTE (continued)

Sample	Constant	Cash Index	Implied Index	R²	# Obs.
1MTE (T-stat)	0.000 (0.09)	1.010 (96.08)	-	94.2%	565
1MTE (T-stat)	0.000 (1.50)	-	0.991 (156.69)	97.8%	565
1MTE (T-stat)	0.000 (1.33)	0.136 (4.65)	0.863 (30.59)	97.8%	565

The results show that the explanatory power of futures increases over time from 97.60% to 97.80%. The anomaly is that the explanatory power falls 1.2% from 3MTE to 2MTE. This contradicts what was expected. One explanation is that the explanatory power is so great in the period before expiry that it more than compensates for the drop in explanatory after expiry, thus resulting in a large R^2 for the total 3MTE period. To test this, I divided the 3MTE expiry into two sets: one containing pre-expiry returns and one containing post expiry returns. The general forms of these equations are given by equations (10), (11), and (12). The expectation is that R^2 will be highest before expiry and lowest after expiry for the 3MTE data.

Table 8: 3MTE SPDR Returns Regressed on the Cash Index, Implied Index, and Both

Sample	Constant	Cash Index	Implied Index	R²	# Obs.
Pre 3MTE (T-stat)	0.000 (0.48)	1.011 (68.37)	-	93.2%	340
Pre 3MTE (T-stat)	0.000 (0.64)	-	0.974 (128.32)	98.0%	340
Pre 3MTE (T-stat)	0.000 (0.66)	0.134 (4.31)	0.852 (29.28)	98.1%	340
Post 3MTE (T-stat)	-0.0005 (-2.19)	1.018 (53.91)	-	92.6%	233
Post 3MTE (T-stat)	-0.0004 (-2.85)	-	1.006 (89.36)	97.2%	233
Post 3MTE (T-stat)	-0.0004 (-3.02)	0.154 (3.48)	0.862 (20.14)	97.3%	233

For the most part, the regressions confirm the expected relationship between time and R². The 3MTE post expiry implied index returns give an R² of 97.2%, which then falls to 96.40% for 2MTE, then rises again to 97.80% for 1MTE, and finally rises again to 98% before expiry in the months of expiry (3MTE). Still dissatisfied with these results, I drilled down into the 2MTE return data to see if there was a reason for the explanatory drop. As it turns out, there were one or two large positive and negative returns in the data. But since the data had already been cleaned for anomalies, there was no reason to throw these points out.

The error correction model developed in this study is an attempt to search for some relationship between NAV deviations in time t+1 and time t. Both Elton et. al. and this study found that deviations at time t+1 are not explained by deviations at time t, suggesting that NAV deviations are not autocorrelated. However, this study explores the relationships in deviations further by asking whether the NAV deviation at time t will disappear at time t+1. That is, will the deviation at time t fully correct by time t+1. If so, then the correction should be equal to the opposite of the time t deviation. The regression equation is:

$$(13) \quad (D_{t+1} - D_t) = B_0 + B_1(D_t)$$

where:

$(D_{t+1} - D_t)$ = difference between NAV deviation at time t+1 minus NAV deviation at time t
 D_t = NAV deviation at time t

The expectation here is that B_0 will be zero and B_1 will be negative one. B_0 should be zero or very close to it because if there is no deviation at time t, then there should be no correction between t+1 and t. B_1 should definitely be negative and one, or close to one, because a positive deviation at time t should result in a negative correction to bring the deviation to zero at time t+1, and a negative deviation at time t should result in a positive correction to bring the deviation to zero at time t+1. For example, if D_t is 2 at time t, then the deviation should be corrected by the close at t+1 so that $D_{t+1} = 0$. The difference between D_{t+1} and D_t is -2, which can be achieved in the regression equation by multiplying D_t by -1. Similarly, if D_t is -2 at time t, then the deviation should be corrected by the close at t+1 so that $D_{t+1} = 0$. The difference between D_{t+1} and D_t is 2, which can be achieved in the regression equation by multiplying D_t by -1.

Table 9: NAV Deviation Differences from time t to t+1 Regressed on NAV Deviation at time t

Constant	NAV Deviation	R ²	# Obs.
0.112 (13.45)	-0.892 (-37.09)	44.6%	1709

$$(14) \quad (D_{t+1} - D_t) = 0.112 - 0.892 D_t$$

with $(D_{t+1} - D_t)$ and D_t defined as in (13). Both the constant and slope coefficient are statistically significant, and the R^2 indicates that time t NAV deviations explain 44.6% of the next period change in NAV deviation. As expected, the slope coefficient is negative and fairly close to one.

To better understand what a typical correction is, I calculated the average positive deviation and average negative deviation. I then entered each average number to calculate the predicted correction. The results are reported below.

Table 10: Average Positive and Negative Deviation with their Respective Predicted Corrections

SPDR - NAV	Average NAV Deviation(t)	Correction
Positive	0.260	-0.120
Negative	-0.220	0.308

These results show that when NAV deviation is positive, the deviation at time t will fall on average by -0.120 at t+1 so that the deviation at time t+1 will not be 0 but 0.14 (which is 0.260 + -0.120), slightly greater than 0. Thus, the adjustment is not complete. On the other hand, when the deviation is negative, the deviation at time t will tend to rise by 0.308 at t+1 so that the deviation at time t+1 will not be 0 but 0.088 (which is -0.200 + 0.308). Therefore, when the

deviation is negative the adjustment is not only complete, it also overshoots the 0 deviation mark to a deviation that is actually positive.

Trading Strategy

Thus far, this study has confirmed that SPDRs are more closely related to futures than either is to the cash index, and that there is a fairly strong relationship between SPDR price deviations from NAV and the amount of that deviation that disappears by the next day. Using this information, two trading strategies can be implemented to demonstrate in dollar terms what these results mean.

The first strategy is the following:

- Buy SPDR, short the S&P index portfolio, when $(\text{SPDR} - \text{NAV}) < 0$
- Short SPDR, buy the S&P index portfolio, when $(\text{SPDR} - \text{NAV}) > 0$

The idea here is that when the SPDR is over NAV, for example, the deviation will be corrected within a day. There are several ways this can happen:

- SPDR moves to NAV
- SPDR moves to NAV and NAV moves to SPDR
- NAV moves up and so does SPDR, but by a smaller amount
- NAV moves down and so does SPDR, but by greater amount

These movements represent risks to the trade. For example, if $(\text{SPDR} - \text{NAV}) > 0$ then the strategy says to short the SPDR and buy the S&P index portfolio. If it is the case that the deviation is reduced by NAV increasing and the SPDR increasing, but by a smaller amount, then the short position will lose money as the long one gains.

Before presenting the results, it would be useful to explain how the strategy was implemented. I first calculated the NAV deviations, and then used Excel to follow the strategy

by longing/shorting the SPDR and shorting/longing the index when appropriate. I then calculated the mean, standard deviation, max, min, and total profits, along with the statistical significance of the profits and the number of trades made using all trades. I then set screens that allowed the trade only if the absolute value of the deviation was greater than a certain amount. I varied the required minimum deviations from 0.05 to 1.25 and calculated the summary measures for the screened trades just as I had for all trades. The profitability of the strategy is as follows:

Table 11: SPDR/Cash Index Trading Strategy (Traded Only) in Dollars

	All	>.05	>.1	>.15	>.2	>.5	>1	>1.25
Mean	1.60	1.79	2.09	2.30	2.66	5.90	17.11	25.95
Standard Deviation	3.99	4.13	4.18	4.38	4.59	7.32	12.54	13.64
Maximum	43.76	43.76	43.76	43.76	43.76	43.76	41.98	41.98
Minimum	-25.20	-25.20	-23.84	-23.84	-23.84	-23.21	5.24	11.13
Total Profit	2719.90	2636.55	2606.72	2425.26	2330.45	967.42	256.58	181.62
Significance [$\mu/(\sigma/\text{sqrt}(n))$]	16.52	16.61	17.67	17.06	17.16	10.31	5.28	5.03
Number of Trades	1701	1475	1248	1055	875	164	15	7

Clearly trading on all 1701 deviations yields the highest profit of \$2719.90 per unit of index portfolio traded over seven years, although trading on deviations greater than 1 yields the highest mean profit. Furthermore, trading on deviations larger than 1.25 provided a maximum and minimum profit that are both positive. To better understand the results, first notice the top row shows the screen. The term “>.2” means that all deviations whose absolute value is greater than \$0.2 are traded on. The mean profit of this strategy is \$2.66, with a standard deviation of \$4.59.

The maximum profit made on any single trade was \$43.76, while the minimum profit/maximum loss was \$23.84. The total profit, computing by simply adding the profits and losses of each trade over seven years, was \$2330.45, which is substantially different from zero (highly significant, with a significance of 17.16). To produce these results, 875 trades were executed.

If the profits to this strategy seem high, they should not. Executing such trades is difficult to do and therefore there is a considerable amount of profit to be enjoyed by one who can do it. Since only one unit of the index portfolio and 10 units of the SPDRs were used, the profit potential of this strategy may be greater than it seems. Note however, that there are diminishing returns to scale as the size is increased.

The next strategy is designed to demonstrate that because the trading strategy used when the SPDR is overpriced relative to the implied index in futures is relatively easier to do, profits are much lower. This strategy also demonstrates why SPDRs and implied futures move so closely. Strategy two is as follows:

- Buy SPDR, short future, when $(\text{SPDR} - \text{Implied Index}) < 0$
- Short SPDR, buy future, when $(\text{SPDR} - \text{Implied Index}) > 0$

As in Strategy 1, price convergence is the idea behind this strategy. The results are as follows:

Table 12: SPDR/Futures Trading Strategy (Traded Only) in Dollars

	All	>.05	>.1	>.15	>.2	>.5	>1	>1.25
Mean	0.30	0.28	0.30	0.36	0.46	1.59	4.63	4.30
Standard Deviation	2.89	2.85	2.68	2.77	2.88	5.05	11.90	13.13
Maximum	48.38	48.38	48.38	48.38	48.38	48.38	48.38	48.38
Minimum	-15.84	-15.84	-15.84	-15.84	-15.84	-15.84	-13.12	-13.12
Total Profit	504.80	450.17	424.82	458.25	508.57	366.88	134.41	90.31
Significance [$\mu/(\sigma/\text{sqrt}(n))$]	4.21	3.97	4.18	4.66	5.32	4.78	2.10	1.50
Number of Trades	1701	1589	1439	1264	1106	231	29	21

In contrast to the first strategy outcomes, the mean and total profit numbers are much smaller. The screen that produces the highest total profit is >.2, with trading all deviations as a very close second. Again, the profits are statistically significantly different from zero except for >1.25 at the 95% level. These results are more consistent with the fact that SPDRs are more closely related to Futures by arbitrage in the market than either is with the cash index.

One caveat is warranted for the second trading strategy. When buying and shorting futures, no cash changes hands. So to buy the future and short the SPDR would mean a net cash inflow. Some traders, like investment banks, may have the opportunity to invest the proceeds from the SPDR sale to earn a rate of interest. On the other hand, traders who short the futures to buy the SPDR will have to borrow money (or invest capital) to finance the SPDR purchase and therefore bear an interest cost. Although the interest earned on invested short sale cash or paid for a loan used to buy SPDRs is small, not including it gives biased results.

Conclusion

Price behavior among the S&P 500 Index, SPDRs, and futures described for the period 1993 to 1998 by “Spiders: Where are the Bugs?” holds true in recent times. Futures are the best predictors of SPDR price returns, and deviations of implied index levels in future prices can help explain NAV deviations. Furthermore, month to expiry in explaining the variation in NAV deviations is significant, since futures mature into the index at expiry.

SPDR deviations are not autocorrelated, yet deviations at time t can be used to determine how much of the deviation will be corrected by $t+1$, since not all of the deviation disappears as expected.

Finally, trading in the SPDR and cash index when the SPDR deviates from NAV can produce significant profits to someone who can surmount the difficulties of such a trade. And because arbitrage keeps the SPDR and futures prices closely related, trades involving these securities when the SPDR is overpriced relative to the implied index produces only a small, though statistically significant, profit.

It would be interesting to extend this study to other ETFs, particularly the QQQs and Diamonds, to determine whether the size of the index affects the relationships among the cash index, ETFs, and futures.

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