Nominal Shocks and Long-Term Contracts

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Abstract

The paper analyzes how nominal shocks affect the terms of long-term contracts. If the price level is measured with delay, renegotiation-proof contracts do not yield full insurance against nominal risk. The focus is on the impulse response of wages and investment to nominal shocks.

PRELIMINARY

1 Introduction

Few contracts are fully indexed to the price level, and so inflation produces winners and losers. This is surprising if agents are risk averse because then they would look to remove all nominal risk to their consumption and labor-supply decisions. This paper explains imperfect indexation without resort to menu costs or to bounded rationality.

In the model a risk-neutral firm employs a risk-averse worker whose effort it cannot see. The firm also chooses a capital stock to combine with the worker’s effort. Delayed observation of the price level also allows a parallel analysis of investment which also depends on price-level shocks because a time-to-build delay requires that the firm invest before the nominal uncertainty has been resolved. The aggregate shock will throughout be a price-level shock. The real effects of such shocks are roughly proportional to the amount of price-level uncertainty. In light of the easy-money policies of the past few months, however, U.S. price volatility is probably on the rise again after a fairly steady decline since the early 1980s.

The model assumes that the price-level is observed with delay\(^1\) and that contract renegotiation is possible and derives imperfect indexation from these assumptions.

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\(^1\)In the U.S. it takes up to a year for the consumer price index to be gathered reliably (Bullard 1994).
Jovanovic and Ueda (1997) and (with caveats) Monnet and Martin (2006) do this for one-period contracts between firms and workers, and Meh, Quadrini and Terajima (2008) do it for long-term contracting between a lender and borrower. These papers show that the optimal contract insures the agent only imperfectly against nominal shocks, not because principal is risk averse but because contracts are renegotiation proof and shocks are observed with delay so that some contingencies are negotiated into the contract \textit{ex post}.

On the effects of monetary policy the model has time-series and cross-section implications.

\textit{Time series}.—Times when prices are when price-level shocks do not often occur. On the other hand, a nominal shock of a given size will exactly then have a larger impact on activity because it will more likely be mis-perceived as real. This implication is as in Lucas (1972), but he modeled signal confusion between a nominal shock and an aggregate real shock. Here, however, the contract emphasizes a local agency problem, to which idiosyncratic risk is central. Any evidence that idiosyncratic risk changes over time is relevant for the time-series implications of the model.\textsuperscript{2}

\textit{Cross section}.—the model implies that full indexation of contracts should occur in sectors that have no agency problems, sectors where the price of the output is immediately verifiable, and sectors where contracts can set terms that conflict with ex-post pressures to renegotiate.

\textit{Real shocks}.—The renegotiation argument applies to any aggregate shock be it nominal or real, that (a) affects the agent’s output and (b) is observed with delay. So for instance, it could explain why firms do not fully insure workers against sectoral demand shocks, as Guiso \textit{et al.} (2006) and Katay (2008) found.

2 Real model

This section briefly shows the real side of the model. A risk-averse worker is in a contract with a risk-neutral firm. We look at the contract in isolation, i.e., in partial equilibrium.

\textit{Production function}.—Output depends on capital, \( k \), and effort \( n \):

\[ y = zk^\alpha n. \]

The shock \( z \) is Markov: \( z \sim G(z, z_{-1}) \). Capital \( k \) depreciates 100\% but is subject to a one-period time-to-build, and effort takes on two values.

\textsuperscript{2}Publicly traded firms seem to have become more subject to idiosyncratic risk, whereas private firms less – Davis \textit{et al.} (2007). Lorenzoni (2008) has discussed misperception as it relates to local risk.
Preferences.—The common discount factor is \( \beta \). The worker’s preferences are 
\[
E_0 \sum \beta^t U(c_t, n_t) \quad \text{where} \quad c_t \text{ is date-}t \text{ consumption and } n_t \text{ is effort. }
\]
The firm can borrow and lend at the rate \( \beta^{-1} - 1 \), the worker cannot. The parties are committed to the contract for ever. We shall assume that 
\[
U(c, n) = U(c) - \kappa \frac{n_t - n_L}{1 - n_L} \quad \text{where} \quad n \in \{n_L, 1\}.
\]

Decision problem.—Assume that wages must be paid up front, before effort is exerted. Assume that the firm always induces effort \( n = 1 \). The firm’s state is the worker’s promised utility \( v \), and the pair \((k, z_{-1})\). The firm’s value is 
\[
F(v, k, z_{-1}) = \max_{c, k', c'} \left\{ -c + \int (zk^\alpha - k' + \beta F(v'_z, k'_z, z)) \, dG(z, z_{-1}) \right\}.
\]

s.t. 
\[
v \leq U(c) - \kappa + \beta \int v'_z \, dG(z, z_{-1}) \tag{2}
\]
\[
\kappa \leq \beta \int v'_z \, d(G - G^L) \tag{3}
\]

where 
\[
G^L(z, z_{-1}) = G\left(\frac{z}{n_L}, z_{-1}\right)
\]
is the distribution of \( z \) when low effort is exerted.\(^3\)

We assume throughout that the worker is committed to the contract for ever in which case there is no reason for the firm to pay him more than it promised. The Appendix shows both (2) and (3) hold with equality.\(^4\) The Lagrangean is 
\[
L = F(v, k, z_{-1}) - \lambda_{PK} \left(v - U(w) + \kappa - \beta \int v'_z \, dG\right) - \lambda_{IC} \left(\kappa - \beta \int v'_z \, d(G - G^L)\right)
\]
The FOCs are 
\[
c : \quad 1 = \lambda_{PK} U'(c), \tag{4}
\]
and for every \( z \), 
\[
v'_z : \quad F_v(v'_z, k'_z, z) \, dG + \lambda_{PK} dG + \lambda_{IC} d\left(G - G^L\right) = 0 \tag{5}
\]

\(^3\)This is because
\[
\Pr(n_L \leq z \mid z_{-1}) = \Pr\left(\tilde{z} \leq \frac{z}{n_L} \mid z_{-1}\right) = G\left(\frac{z}{n_L} \mid z_{-1}\right)
\]

\(^4\)The analogues of (2) and (3) in the nominal model will also hold with equality.
and
\[ k'_z : \quad \beta F_k (v'_z, k'_z, z) = 1. \] (6)

Integrating both sides of (5) over \( x \), we get
\[ \int F_v (v'_s, k'_s, z) dG (z, z_{-1}) + \lambda_{PK} = 0. \] (7)

Envelope theorem says that \( F_v (v, k, z_{-1}) = \lambda_{PK} \), and from (4), \( \lambda_{PK} = 1/U'' (w) \) from which we get the inverse Euler equation for the agent’s consumption
\[ E \left( \frac{1}{U''(c')} \mid z \right) = \frac{1}{U''(c)}. \]

Envelope theorem also says that \( F_k (v, k, z_{-1}) = \alpha_k^{\alpha - 1} E (z \mid z_{-1}) \) which, when substituted into (6) yields \( \beta \alpha k^{\alpha - 1} E (z \mid z_{-1}) = 1 \), i.e.,
\[ k'_z = [\alpha \beta E (z \mid z_{-1})]^{1/(1-\alpha)}. \]

Thus \( k'_z \) does not depend on \( v \), i.e., investment is independent of the contracting problem, conditional at least on \( n = 1 \) always being elicited.

### 3 Nominal model

Formally, the only effect of renegotiation is turn the real signal on the agent’s output into a noisier, nominal signal. Thus the agent’s pay at each date will depend on the revenues that he has generated in the past, and not on the outputs themselves.

Denote the policy functions of the real problem by
\[
\begin{align*}
v' &= \alpha_v (v, z, z_{-1}) \\
c &= \alpha_c (v, z_{-1}) \\
k' &= \alpha_k (z)
\end{align*}
\] (8)

The production function is still (1). Now we introduce the the price level, \( p \), which also is the price of the only consumption good in this economy, and the only good that the principal and agent consume. Nominal revenue is
\[ s = pz. \] (9)

**Full indexation.**—A fully-indexed contract can achieve the same outcome as the real contract by replacing \( z \) by \( s/p \) in the three policy functions defined in (8). Letting \( W \) denote the nominal wage, the fully-indexed contract rewards the worker as follows:
\[
\begin{align*}
W &= p\alpha_c (v, z_{-1}) \\
v' &= \alpha_v \left( v, \frac{s}{p}, z_{-1} \right).
\end{align*}
\] (10) (11)
Thus (10) is simply the nominal payment that provides real consumption equal to \( \alpha_c (v, z_{-1}) \).\(^5\) If the firm could hold off on its choice of \( k' \) until it could see both \( s \) and \( p \), then its nominal spending on capital would be

\[
p\alpha_k \left( \frac{s}{p} \right)
\]

(12)

In this case, real decisions would be as in (8), i.e., the principal and the agent would be fully insulated from nominal-price variability and the outcome would be the same as in the ‘real contract’ described in section 2.

In what follows we shall make assumptions that render the outcome implied by (10) and (11) non-renegotiation proof. We also shall assume that the firm cannot implement (12) because time-to-build is so long that it must choose \( k' \) after it sees \( s \) but before it learns \( p \). All this requires that we be explicit about the timing of events within the period.

**Timing.**—The timing of events is:

0. start period with \((z_{-1}, p_{-1}, k)\) common knowledge,
1. choose effort \( n \in \{n_L, 1\} \),
2. both observe the product \( s = pz \), but not \( p \) and \( z \) separately,\(^6\)
3. firm must choose \( k \) for the following period,
4. get a chance to renegotiate,
5. observe \( p \),
6. employer pays worker nominal wage \( W \)

Let \( m = p/p_{-1} \) be the gross inflation rate and let it follow the Markov process

\[
m \sim \Phi (m, m_{-1}).
\]

Since \( z \sim G (z, z_{-1}) \) is also Markovian, the vector \( x = (z, m) \) will itself follow the Markov process with the conditional C.D.F. \( H \) defined by

\[
x \sim H (x, x_{-1}) = G (z, z_{-1}) \Phi (m, m_{-1}).
\]

5 We chose not to replace \( z_{-1} \) by \( s_{-1}/p_{-1} \) in (10) and (11) because \( z_{-1} \) is observed by both parties by the time that any period-\( t \) decisions are taken and does not offer any renegotiation incentives.

6 I assume the parties cannot reverse engineer \( p \) from \( s \) at this stage. They cannot observe \( y \) directly, perhaps because \( y \) denotes the quality of a differentiated service such as sales effort. The assumed delay between stages 2 and 5 exists in fact since it takes time to assemble aggregate price data and even money-supply figures are usually revised months later. Jovanovic and Ueda (1998) deal with this in their equilibrium OLG model in which agents work only when young and consume only in old age.
We assume the interest rate is fully indexed to inflation so that everything except for the wage contract that we seek to determine is in real terms independent of $p$. Normalize $p_{-1} = 1$ so that\footnote{Forecastable changes in $p$ do not affect real variables in the RPC and so we shall re-initialize the prices to unity in each period so as to save on notation.}

$$s = \frac{my}{k^\alpha} = mz \equiv s(x). \quad (13)$$

**Renegotiation-proof contracts** (RPCs).—In contrast to full indexation expressed by (10) and (11), RPCs insure the worker’s consumption fully ex post, conditional on $s$. That is,

**Proposition 1** RPCs are of the form

$$W = mw(v, z_{-1}) \quad (14)$$

$$v' = V(v, s, z_{-1}) \quad (15)$$

In particular, $(w, V)$ do not depend directly on $m$

**Proof.** RPCs do not restrict real consumption since $w(\cdot)$ in (14) depends on the same list of variables as $\alpha_c(\cdot)$ in (10). The difference is that $V$ excludes $m$ from the list of its arguments. Suppose then that, on the contrary, $V(\cdot)$ depended on $m$, as in $v' = V(v, s, m, z_{-1})$ But then his next-period consumption would be

$$w [V(v, s, m, z_{-1}), z_{-1}].$$

But $w$ is strictly increasing in $v$ (this follows from $F$ being strictly decreasing in $v$ and from (26) and (30) below) this introduces risk into the next-period wage, risk that the principal can costlessly absorb. This leads $m$ to be renegotiated away after $s$ is observed, i.e., it removes $m$ from the list of arguments in $V(\cdot)$. Once it is in the form (14) and (15), no unresolved uncertainty remains: Once $(v, s, z_{-1})$ are specified, the outcome is ex-post Pareto optimal and, hence, not vulnerable to renegotiation.

**Timing of investment.**—We assume above that the decision on $k'$ has to be made after $s$ is observed, but before $p$ can be observed. Therefore in parallel to (14) and (15), next period’s capital decision cannot depend directly on $p$:

$$k_s = k(s). \quad (16)$$

In sum, the RPC replaces (10), (11) and (12) by (14), (15) and (16).
3.1 Recursive formulation of the RPC

The recursive contract maximizes the firm’s value subject to being of the form (14), (15) and (16), and subject to \( n = 1 \) always being elicited. We can pose the problem as follows:

\[
F(v, k, x_{-1}) = \max_{(w, k'_s, v'_s) \in \Omega} \left\{ -w + \int \left( zk^\alpha - k'_s(x) + \beta F(v'_s(x), k'_s(x), x) \right) dH(x, x_{-1}) \right\}
\]

(17)

where

\[
\Omega = \{(w, k'_s, v'_s) \geq 0 \mid (18) \text{ and } (19) \text{ hold}\}.
\]

where (18), and (19) are the promise-keeping constraint

\[
v \leq U(w) - \kappa + \beta \int v'_s(x) dH(x, x_{-1}),
\]

and the incentive-compatibility constraint

\[
\kappa \leq \beta \int v'_s(x) d(H - H_L),
\]

in which

\[
H_L(x, x_{-1}) \equiv G\left(\frac{z}{n_L}, z_{-1}\right) \Phi(m, m_{-1})
\]

(20)
is the C.D.F. of \( x \) conditional on \( x_{-1} \) and on \( n = n_L \).

3.1.1 Decomposing the problem

The Appendix shows that (17) has at most one solution for \( F \) and that this solution separates into the following three terms:

\[
F(v, k, x_{-1}) = E(z \mid x_{-1}) k^\alpha + R(x_{-1}) - C(v, x_{-1}).
\]

(21)
The individual components have the following economic interpretation:

1. 

\[
E(z \mid x_{-1}) k^\alpha = \int z k^\alpha dH(x, x_{-1})
\]

(22)
is the return to the capital in place. The firm’s nominal profit is \( m \left( zk^\alpha - w - k'_s(x) \right) \).

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8 A larger \( k \) confers no informational advantage to the firm in observing the worker’s output. That is the case when \( z \) is multiplicative. The additive separability in (21) would fail if \( z \) was an additive shock, i.e., if, say, \( y = k^\alpha n + z \). In that case, a larger \( k \) would raise the signal-to-noise ratio of \( y \).

9 The firm’s nominal profit is

\[
\pi = m \left( zk^\alpha - w - k'_s(x) \right)
\]
2. $R(x_{-1})$ is the rent (not counting the cost of the unit of effort) associated with the firm’s lifetime right or option to exploit the decreasing-returns technology; $R$ solves

$$R(x_{-1}) = \max_{k'_s \geq 0} \left\{ \int (-k'_s(x) + \beta \left[ z \left( k'_s(x) \right)^\alpha + R(x) \right]) \, dH(x, x_{-1}) \right\}$$

(23)

with the FOC $\beta \alpha k^{\alpha - 1} E(z' \mid s) = 1$. The resulting relation between nominal shocks and investment is

$$k_s = [\alpha \beta E(z' \mid s)]^{1/(1-\alpha)}$$

(24)

where $E(z' \mid s) = \int E(z' \mid x) \, dP(x \mid s)$.

3. $C(v, x_{-1})$ is the lifetime cost to the firm of eliciting $n = 1$ while providing the worker with utility $v$, and it satisfies

$$C(v, x_{-1}) = \min_{(w, v'_s) \in \Omega_1} \left\{ w + \beta \int C(v'_s(x), x) \, dH(x, x_{-1}) \right\}.$$ 

(25)

where $\Omega_1 = \{(w, v'_s) \geq 0 \mid (18) \text{ and } (19) \text{ hold}\}$. I.e., $C$ is the minimized expected present value of the wage payments needed to elicit effort at each date.

*Analysis of the cost-minimization problem (25).*— The Lagrangean is

$$L = C(v, x_{-1}) + \lambda_{PK} \left( v - U(w) - \beta \int v'_s(x) \, dH \right) + \lambda_{IC} \left( \kappa - \beta \int v'_s(x) \, d(H - H^L) \right)$$

The Lagrangean has the FOCs

$$1 = \lambda_{PK} U'(w)$$

(26)

and, recalling that $x = (z, m) = \left( \frac{s}{m}, m \right)$, we have for every $s$,

$$\int C(v'_s \left( \frac{s}{m}, m \right), x_{-1}) \, dG(s - m, z_{-1}) \, d\Phi(m, m_{-1}) - \lambda_{PK} h - \lambda_{IC} (h - h^L) = 0$$

(27)

where $h$ and $h^L$ are the densities of $s$ under effort and shirking respectively:

$$h = h(s, x_{-1}) \equiv \int g(s - m, z_{-1}) \, \phi(m, m_{-1}) \, dm$$

(28)

and

$$h^L = h^L(s, x_{-1}) \equiv \frac{1}{n_L} \int g \left( \frac{s}{n_L} - m, z_{-1} \right) \, \phi(m, m_{-1}) \, dm$$

(29)
The envelope theorem says that
\[ C_v (v, x_{-1}) = \lambda_{PK}. \] (30)

Then (26) implies \( C_v = 1/U' \). Substituting into (27) yields for each \( s \)
\[
\frac{1}{U'(w)} + \lambda_{IC} \left( 1 - \frac{h^L(s)}{h(s)} \right) = \int \frac{1}{U'(w'[v'_s, \frac{w'}{m}, m])} dG(s - m, z_{-1}) \, d\Phi(m, m_{-1}) \]
\[
= E \left( \frac{1}{U'(w_{t+1})} \mid s \right)
\]
where \( w \) and \( \lambda \) depend on \((v, x_{-1})\). With the ‘single crossing property’ on \( h \) and \( h^L \) we have \(-\frac{h^L(s)}{h(s)}\) and hence \( E \left( \frac{1}{U'(w_{t+1})} \mid s \right) \) is increasing in \( s \). To find the slope of \( E \left( \frac{1}{U'(w_{t+1})} \mid s \right) \) w.r.t. \( s \), we need to solve for \( \lambda_{IC} \), which one can do as follows: Since \( \lambda_{PK} = C_v (v'_s) = \frac{1}{U'(w_s)} \),
\[
v'_s = C_v^{-1} \left( \frac{1}{U'(w_s)} \right) = C_v^{-1} \left( \frac{1}{U'(w)} + \lambda_{IC} \left( 1 - \frac{h^L(s)}{h(s)} \right) \right).
\]
Then if (19) holds with equality
\[
\kappa = \beta \int C_v^{-1} \left( \frac{1}{U'(w(v))} \right) + \lambda_{IC} \left( 1 - \frac{h^L(s)}{h(s)} \right) \, d \left( H - H^L \right)
\]
(32)

3.1.2 The i.i.d. case

Let us consider the special case where \( x \) is i.i.d., which requires that both \( z \) and \( m \) be i.i.d.. In that case \( x \) drops out as a state from \( C \) and (31) becomes
\[
\frac{1}{U'(w_s)} = \frac{1}{U'(w)} + \lambda_{IC} \left( 1 - \frac{h^L(s)}{h(s)} \right).
\]
The familiar utility functions yield the following processes:

- \( U(c) = \ln c \), wages follow the random walk
  \[
w'_s = w + \lambda_{IC} \left( 1 - \frac{h^L(s)}{h(s)} \right)
  \]
  (33)

- If \( U(c) = \frac{c^{1-\sigma}}{1-\sigma}, \)
  \[
w'_s = \left( w + \lambda_{IC} \left( 1 - \frac{h^L(s)}{h(s)} \right) \right)^{1/\sigma}
  \]
  (34)
Proposition 2  When $x$ is i.i.d. and $U(c) = \ln c$,

$$w(v) = C (1 - \beta) e^{(1-\beta)v}$$  \hspace{1cm} (35)$$

$$w' = w(v) \left[ 1 + \beta (1 - C) \left( 1 - \frac{h^L(s)}{h(s)} \right) \right]$$  \hspace{1cm} (36)$$

$$\lambda IC = \beta C (1 - \beta) (1 - C) e^{(1-\beta)v}$$  \hspace{1cm} (37)$$

$$F(v) = Ce^{(1-\beta)v}$$  \hspace{1cm} (38)$$

and where $C \in (0,1)$ solves

$$\frac{(1 - \beta)}{\beta} \kappa = \int \ln \left[ 1 + \beta (1 - C) \left( 1 - \frac{h^L(s)}{h(s)} \right) \right] (h(s) - h^L(s)) \, ds$$  \hspace{1cm} (39)$$

and where we must have

$$h^L(s) \leq \left[ 1 + \frac{1}{\beta (1 - C)} \right] h(s) \quad \text{for all } s.$$  \hspace{1cm} (40)$$

The following solution indicates that the response of wages to $s$ must grow with $\kappa$ and decline with $\beta$. Wage-growth dispersion via (36) is

$$\frac{w'}{w} = 1 + \beta (1 - C) \left( 1 - \frac{h^L(s)}{h(s)} \right)$$

and is governed by the coefficient $\beta (1 - C)$. We find that this coefficient is increasing in $\kappa$ and decreasing in $\beta$, meaning that it is easier to provide incentives when $\kappa$ is low and when $\beta$ is high.

Corollary 1

$$\frac{\partial (1 - C)}{\partial \kappa} > 0 \quad \text{and} \quad \frac{\partial \beta (1 - C)}{\partial \beta} < 0.$$  \hspace{1cm} \text{(40)}$$

Proof. By the implicit function theorem, a solution $C(\kappa)$ exists because the derivative of the RHS of (39) w.r.t. $C$ is

$$- \int \left[ 1 + \beta (1 - C) \left( 1 - \frac{h^L(s)}{h(s)} \right) \right]^{-1} \beta \frac{(h(s) - h^L(s))^2}{h(s)} \, ds < 0$$

the sign follows because every term under the integral sign is positive as long as (40) holds. Similarly the derivative of the RHS of (39) w.r.t. $\beta$ is positive whereas the derivative of the LHS w.r.t. $\beta$ is negative which means that $C$ must rise.  \hspace{1cm} \blacksquare
### 3.1.3 When $m$ and $z$ are both log normal

The user-friendly log-normal case is

\[
\begin{align*}
\ln m &\sim N(\mu_m, \sigma^2_m) \\
\ln z &\sim N(\mu_z, \sigma^2_z)
\end{align*}
\]

Then if $\text{Cov}(m, z) = 0$,

\[
\ln s \sim N(\mu_s, \sigma^2_s)
\]

where $\mu_s = \mu_m + \mu_z$ and $\sigma^2_s = \sigma^2_m + \sigma^2_z$. Then

\[
h(s) = \frac{1}{s \sqrt{2\pi \sigma^2_s}} \exp \left\{ -\frac{(\ln s - \mu_m - \mu_z)^2}{2\sigma^2_s} \right\}
\]

and

\[
h^L(s) = \frac{1}{sn_L \sqrt{2\pi \sigma^2_s}} \exp \left\{ -\frac{(\ln s - n_L \mu_s)^2}{2n^2 \sigma^2_s} \right\}
\]

Then

\[
\frac{h^L(s)}{h(s)} = \frac{1}{n_L} \exp \left\{ \frac{(\ln s - \mu_s)^2}{2\sigma^2_s} - \frac{(\ln s - n_L \mu_s)^2}{2n^2 \sigma^2_s} \right\}
\]  

(41)

According to (33) and (34), the wage is an increasing function of $s$ in the range where $h^L/h$ is decreasing. It turns out that in the log-normal case this will not happen for all $s$, although we can produce examples where it will be declining most of the time. The likelihood ratio is not monotone even though $H(x)$ first-order dominates $H^L(x)$.

The ratio $h^L/h$ peaks at the value where the derivative of the RHS of (41) w.r.t. $s$ is zero. This happens at the point $s^*$ satisfying $\frac{\ln s - \mu_s}{\sigma^2_s} = -\frac{\ln s - n_L \mu_s}{n^2 \sigma^2_s}$ which implies that the likelihood ratio peaks when $s = s^*$ given by\(^\text{10}\)

\[
\ln s^* = \frac{n_L + n^2 \mu_s}{1 + n^2 \sigma^2_s} \mu_s
\]

In this case the likelihood ratio is monotone in the right way for about two thirds of the observations. We plot $\frac{h^L(s)}{h(s)}$ for $n_L = 1/2$, and $\mu_s = \sigma^2_s = 1$, alongside $5h(s)$

\[
\ln s^* = \left( \frac{1}{\sigma^2_s} \left[ 1 + \frac{1}{n^2_L} \right] \right)^{-1} \left( 1 + \frac{1}{n_L} \right) \frac{\mu_s}{\sigma^2_s} \\
= \frac{n^2_L}{1 + n^2_L} \left( 1 + n_L \right) \frac{\mu_s}{\sigma^2_s} = \frac{n_L + n^2_L \mu_s}{1 + n^2_L \sigma^2_s}
\]

\(10\)
which has a mean of $e^{1.5} = 4.48$, and \( \frac{h^L(s)}{h(s)} \) peaks at the value \( s^* = e^{\frac{75}{125}} = 1.82 \).

![Graph showing \( \frac{h^L(s)}{h(s)} \) and \( 5h(s) \)]

We conclude that for reasonable processes for \( m \) and \( z \) it will be hard to produce a monotone likelihood ratio \( h^L / h \) and, hence, the wage will not be monotone in \( s \), though we may hope that it will be increasing ‘most’ of the time.

### 3.1.4 Properties of the contract

The results rely on inflation uncertainty and on asymmetry of risk attitudes. The more risk averse the agent is, the larger the monetary incentive he needs to meet the incentive constraint, i.e., the greater the response of income to performance must be. Since the RPC can provide this incentive as a function of the nominal signal alone, the distortions that such signals induce will be proportional to the degree of asymmetry in risk attitudes.

**Nominal shocks and real wages.**—Nominal shocks raise real wages, which corresponds to the impulse responses that Christiano, Eichenbaum and Evans (2000, ‘CEE’) report in their Figure 1. The effect on the wage is predicted to be persistent: Integrating both sides of (27) over \( x \), it becomes

\[
\int C_v (v'_s(x), x) \, dH (x, x-1) = \lambda_{PK}. \tag{42}
\]

From (26), \( \lambda_{PK} = 1/U''(w) \) which, when combined with (30) and updated yields the inverse Euler equation

\[
E \left( \frac{1}{U''(w')} \mid s(x) \right) = \frac{1}{U''(w)}. \tag{43}
\]
Therefore money shocks will have persistent effects because $U^{t-1}$ is a Martingale. \footnote{If there is saving, the persistence of consumption will be governed not by $(43)$ but by the usual Euler equation}

Therefore nominal shocks will have persistent effects because $U^{t-1}$ is a Martingale. If there is saving, the persistence will be transformed, at least in part, to something like the persistence of the effects of a windfall shock to income.\footnote{These would work through the Euler equation} he model delivers persistence without insisting that prices and wages are much more sticky than they are in the data. Empirically there seems room for a new source of monetary propagation because sticky-price models under-provide it. Chari, Kehoe, and McGrattan (2000, 2002) show that stickiness of prices (in the sense of infrequent price adjustment calibrated to fit the micro facts) cannot generate anywhere near the level of persistence in output seen in the data.

**Nominal shocks and investment.**—Nominal shocks raise investment, but the effects are predicted to be purely transitory; once $z$ and $m$ are observed separately, only the value $z$ guides expectations about $z'$. A caveat is the possible effect of inflation, $m$ on future uncertainty of inflation via the law of motion expressed by $\Phi (m', m)$, but even if it exists, this is probably a second-order effect.\footnote{To check if dispersion of $m$ depends on $m_{-1}$ one would look for differential persistence in the transition-probability matrix a first-order Markov chain for $m$ (reference?) reveal....Intuitively one would expect volatility to be increasing in $m$ – low-inflation states are probably more stable than high-inflation states.} But the CEE show that real wages and investment respond with roughly equal persistence to shocks. Returning to the model: Investment rises with a positive nominal shock because a high signal is perceived as good news for next period’s MPK. Contrast this with the Phillips curve in Meh et al. (2008) where $z$ is i.i.d., and where, instead, high investment is a reward for loan repayments.

**Nominal shocks and firm value.**—Let us measure stock returns from the start of a period when $x_{-1}$ is known, until the start of the following period when $x$ becomes known. Positive inflation surprises have a qualitatively different impact on stock returns from negative inflation surprises:

- Positive inflation surprises (i.e., above-average realizations of $m$) lead to lower stock returns for two reasons. First, they redistribute income to the agent

\[
U'(w) = \beta (1 + r) U'(w') = U'(w')
\]

because $\beta (1 + r) = 1$.

\[
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\]

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because $v'_s$ is increasing in $s$. Second they induce overoptimism about next-period $z$ and thereby lead to overinvestment at the average $z$.

- Negative inflation surprises have an ambiguous effect. On the one hand they redistribute to the firm which is good for returns but, on the other hand, they distort investment in the negative direction thereby lowering returns.

The possible non-monotonicity arises because the effect of surprises in $m$ on the efficiency of the capital decision is non monotonic: A departure in either direction of $m$ from its expected value implies (at the average realization of $z$ conditional on $z_{-1}$) a distortion in the capital decision, and imposes losses on the firm ex post.

4 Aggregate vs. local shocks

The model has assumed that $p$ is the price of the agent’s output and the price of the good that the agent consumes. Let us return to the notation of equations (9), (10) and (11). Let $p$ be the price of the output that the agent has produced, and let $P$ be the price level, the numeraire being a bundle of goods and $P$ is the price of that bundle. The signal is still

$$s = pz$$

but the agent wants, ex-post insurance against $P$, whereas the principal is risk-neutral with respect to it.

**Full indexation.**—Instead of (10) and (11), the first-best, fully-indexed contract is

$$W = P\alpha_c(v, z_{-1})$$

$$v' = \alpha_v(v, s, z_{-1})$$

Thus comparing this to (10) and (11), we find that

1. (11) and (45) are identical: Promised utility should reflect only the agent’s output

2. Full insurance against the outside bundle that the agent is interested in consuming requires that nominal wages be proportional to its price. Thus $p$ in (10) is replaced by (44)

**The RPC.**—A result almost identical to Proposition 1 holds here:

**Proposition 3** The contract (44), (45) is not renegotiation proof. Rather, the RPC has the form

$$W = P\alpha_c(v, z_{-1})$$

$$v' = \alpha_v(v, s, z_{-1})$$
Proof. Suppose, on the contrary, that the contract is RPC. The agent’s lifetime utility then would not depend on $p$. But his next-period consumption would, conditional on $p$, then equal

$$C = \alpha_v [\alpha_v (v, s, p, z_{-1})]$$

In other words, the RPC changes (45) to (47), just as it changed (11) to (15). Optimal investment is still (16). What changes, then is not the general form of the dependence of wages and investment to $p$ but, rather, the effect of aggregate shocks on these decisions and, in particular, the correlation of these decisions with $P$. Since incentives are provided through $p$, what matters is the correlation between $p$ and $P$.

Other shocks.—The model extends to allow for an aggregate productivity shock $\eta$ to output so that

$$y \sim F(y \mid e, \eta).$$

As long as $\eta$ too is observed with delay, and as long as the firm is risk neutral with respect to $\eta$ or at least less risk averse to it, it too will not affect the agent’s real consumption in the RPC. The model can also help us understand why firms do not fully insure their workers.14

5 Extensions & discussion

1. Nominal price rigidity.—Although the model is not one of nominal rigidity, it is possible for it to generate a prediction very similar to that of a sticky price model. Consider a firm that has a unit of output to sell. Suppose that it must keep its price fixed. Now let there be a price inflation by the factor equalling $m$. The firm’s real income will then be lower by a factor of $m$, i.e., lower by the same percentage as percentage inflation. In other words, the buyer’s real surplus from the transaction then has an $m$-elasticity of unity. Inspired by this analogy, we ask: “When is the agent’s reward unit elastic with respect to $s$?” (In the multiplicative case, unit elasticity w.r.t. $p$ or $m$ is the same as that w.r.t. $s$). Since $w$ is paid before $s$ is observed, we need the elasticity of the real wage with respect to $s_{-1}$:

$$\frac{s_{-1} \partial w}{w} \frac{\partial v}{s_{-1}} \approx 1.$$ (49)

New evidence suggests that wages rise with firm-level shocks — temporary as found by Guiso, Pistaferri and Schivardi (2005) and both temporary and permanent as found by Kátay (2007). Delay is essential, however. For example, the model does not apply to shocks such as changes in the exchange rate because these are instantaneously observable by all parties. Rather, the model predicts that indexation to foreign-exchange shocks should be full.
The model cannot explain zero indexation implicit in nominal price or wage rigidity, except in knife-edge situations, i.e., on a zero-measure parameter subset.

2. **Long-term contracts between borrowers and lenders and between firms.**—Borrowers are arguably more risk averse than lenders, and the model should therefore apply naturally to loan contracts. It also should extend to the long-term contracts that Stigler, Kindahl and Carlton studied. If a small firm is more risk-averse than a large firm, e.g., a supplier of parts, then if there also is an agency problem, the model implies that price surprises should redistribute to the small firm. Arguably, small firms do better in times of unforeseen inflation. E.g., trade between manufacturers and retailers. Meh, Quadrini and Terajima (2008) analyze lending contracts in a modification of the reporting problem of Clementi and Hopenhayn (2006) to include delayed observation and renegotiation. My model applies to employment contracts and accompanying investment financed internally, which arguably constitute the bulk of economic activity but which, at any rate, is complementary to Meh et al. (2008) since it has nothing to say about lending contracts. Doepke and Schneider (2006) study the effects of inflation on borrowers and lenders and find real effects.

3. **The original Phillips curve.**—The original relation was between unemployment and wage inflation. We can generate unemployment in this model by introducing outside opportunities, perhaps in a GE setting such as Rudanko (2008). With outside opportunities for both parties and resulting participation constraints, nominal shocks will generally induce quits and layoffs as in Spear and Wang (2005). Thus we may hope to have the Phillips curve relating inflation and unemployment.

4. **Borrowing and lending.**—When we allow workers to borrow and lend, the dynamic contract will generally lead to a smaller (and perhaps even a zero) improvement over the series of static contracts. Allen (1985) and Cole and Kocherlakota (2001) deal with these questions. We shall ask if the Allen-Cole-Kocherlakota results survive the introduction of nominal shocks and RPC.

## 6 Conclusion

The standard view of non-neutrality and imperfect indexation is that contracts being infrequently revised so that nominal rigidities creep into them. Here, full indexation is prevented from the possibility of renegotiation – too frequent renegotiation, in fact, from the viewpoint of first best.

## References


[9] Fama


7 Appendix

Proposition 4 If there is no outside option for the worker and if $U (\cdot)$ is unbounded below then both (2) and (3) hold with equality.

Proof. First, (2). Suppose (2) was slack, so that at the optimal policy $v'_s$,$$U (w) - \kappa + \beta \int v'_s dG = v + \varepsilon$$for $\varepsilon > 0$. Then instead of $v'_s$ you can choose, for each $s$,$$\hat{v}'_s = v'_s - \frac{1}{\beta} \varepsilon$$and still satisfy (3). This is possible because the principal can reduce utility by as much as he wants to because it is unbounded from below and because the worker has no alternatives. The same argument applies to (3).

Proposition 5 Any solution to (17) have the form (21)

Proof. The function $v'_s(x)$ depends on $s = s(x)$ and does not involve $k$ at all. Nor does $k$ enter (18) and (19). Therefore $\Omega$ is the product of the set of non-negative $k$ values and of $\Omega_1$. Therefore maximizing the RHSides of (23) and (25) over $k'_s \geq 0$ and $(w, v'_s) \in \Omega_1$ respectively and then summing the results yields the same outcome.
as would first summing the RHSides of (23) and (25) and then maximizing them over \( \Omega \). We substitute an \( F \) of the form given in the RHS of (23) yields the same form on the LHS of (17). Since the RHS of (17) is a contraction on functions \( F \), it has at most one solution which must be of the form given in (21).

\[ \text{Proof of proposition 2} \]

**Proof.** Let \( A \equiv 1 - \beta \). Then

\[ C(v) = Ce^{Av} \]

(50)

for some \( C > 0 \).

\[ C_v(v_s') = ACe^{Av_s'} = \frac{1}{U''(w)} = w. \]

This is the policy function for the real wage

\[ w = ACe^{Av} \]

(51)

and its inverse relation

\[ v = \frac{1}{A} (-\ln AC + \ln w). \]

(52)

Now (33) says that for each \( s \),

\[ w + \lambda I_C \left( 1 - \frac{h^L(s)}{h(s)} \right) = (w [v'_s]) \]

(53)

\[ = ACe^{Av'_s} \quad \text{(using (51))} \]

Solving the above equation for \( v'_s \) yields

\[ v'_s = \frac{1}{A} \left( -\ln AC + \ln \left[ w + \lambda I_C \left( 1 - \frac{h^L(s)}{h(s)} \right) \right] \right) \]

(54)

where the last equality follows from (51). Now substitute for \( v'_s \) into (19), assume that it holds with equality, and divide both sides by \( \beta \) to get\(^{15}\)

\[ \frac{A \kappa}{\beta} = \int \ln \left[ w + \lambda I_C \left( 1 - \frac{h^L(s)}{h(s)} \right) \right] d (H - H^L). \]

(55)

because \( \frac{1}{A} \int (\ln AC) d (H - H^L) = 0 \). Now we go back to (25). Substituting the functional form from (50) for \( C(v) \), from (51) for \( w \), and from (54) for \( v'_s \), (25) (i.e.,

\(^{15}\)In (55) this equation, \( H^L \) is defined in (20), \( h \) in (28) and \( h^L \) in (29).
\( C(v) = \min_{(w,v') \in \Omega} \left\{ w + \beta \int C \left( v'_{s(x)} \right) dH \right\} \) reads

\[
C e^{Av} = AC e^{Av} + \beta C \int e^{Av'} dH = \int \left[ w + \lambda_{lc} \left( 1 - \frac{h^L(s)}{h(s)} \right) \right] dH
\]

so that

\[
e^{Av} = Ae^{Av} + \frac{\beta}{A} \int \left[ AC e^{Av} + \lambda_{lc} \left( 1 - \frac{h^L(s)}{h(s)} \right) \right] dH
\]

(56)

(57)

(these second equality follows from (53) by which \( Ce^{Av} = \frac{1}{A} w' \) which must be true for all \( v \) which, in turn requires that

\[
\lambda_{lc} = \Lambda e^{Av}
\]

in which case \( v \) cancels and (57) becomes

\[
1 = A + \frac{\beta}{A} \int \left[ AC + \Lambda \left( 1 - \frac{h^L(s)}{h(s)} \right) \right] dH
\]

\[
= A + \beta C + \frac{\Lambda}{AC}
\]

because \( \int \left( 1 - \frac{h^L(s)}{h(s)} \right) dH = 0 \). Substituting for \( \lambda_{lc} \) into (55) yields

\[
\frac{\kappa}{\beta} = \int v'_{s(x)} d\left( H - H^L \right)
\]

\[
= \frac{1}{A} \left( -\ln AC + \ln \left[ w + \lambda_{lc} \left( 1 - \frac{h^L(s)}{h(s)} \right) \right] \right) d\left( H - H^L \right)
\]

\[
= \frac{1}{A} \left( -\ln AC + \ln \left[ AC e^{Av} + \Lambda e^{Av} \left( 1 - \frac{h^L(s)}{h(s)} \right) \right] \right) d\left( H - H^L \right)
\]

\[
= \frac{1}{A} \int \ln \left[ AC + \Lambda \left( 1 - \frac{h^L(s)}{h(s)} \right) \right] d\left( H - H^L \right)
\]

(58)

because

\[
\int \frac{1}{A} \left( -\ln AC + Av \right) d\left( H - H^L \right) = 0 \text{ for each } v
\]

Therefore \( (C, \Lambda) \) solve (59), (60):

\[
1 = A + \beta C + \frac{\Lambda}{AC}
\]

(59)

\[
1 = \frac{1}{A} \int \ln \left[ AC + \Lambda \left( 1 - \frac{h^L(s)}{h(s)} \right) \right] d\left( H - H^L \right)
\]

(60)
and from (59)

$$\beta = \frac{\Lambda}{AC} \implies 1 = C + \frac{\Lambda}{A \beta C}$$

and therefore

$$\Lambda = \beta CA \,(1 - C)$$

as claimed. Substituting for $\Lambda$ in (58),

$$\frac{A \kappa}{\beta} = \int \ln \left[ AC^+ \beta CA \,(1 - C) \left( 1 - \frac{h_L(s)}{h(s)} \right) \right] d \,(H - H^L)$$

$$= \int \ln \left[ 1 + \beta \,(1 - C) \left( 1 - \frac{h_L(s)}{h(s)} \right) \right] d \,(H - H^L)$$

because $\int \ln \,[AC] \,d \,(H - H^L) = 0$.

Finally we need to check that the solution satisfies promise keeping:

$$v = \ln w + \beta \int v'dH$$

$$= \ln AC + Av + \beta \int \left[ \frac{1}{A} \left( -\ln AC + \ln \left[ w + \lambda_C \left( 1 - \frac{h_L(s)}{h(s)} \right) \right] \right) \right] dH$$

$$= \ln AC + Av + \beta \int \left[ \frac{1}{A} \left( -\ln AC + \ln \left[ e^{Av} \left[ AC + \Lambda \left( 1 - \frac{h_L(s)}{h(s)} \right) \right] \right) \right) \right] dH$$

$$= \ln AC + v - \frac{\beta}{A} \ln AC + \frac{\beta}{A} \int \ln \left[ AC^+ \beta CA \,(1 - C) \left( 1 - \frac{h_L(s)}{h(s)} \right) \right] dH$$

(61)

so indeed if promise-keeping holds holds for any $v$, it holds for all $v$.

Finally in (61) after we cancel $v$ from both sides of the equation the resulting constants must sum to zero:

$$0 = \ln AC - \frac{\beta}{A} \ln AC + \frac{\beta}{A} \int \ln \left[ AC^+ \beta CA \,(1 - C) \left( 1 - \frac{h_L(s)}{h(s)} \right) \right] dH$$

$$= \ln AC + \beta \left( \int \ln \left[ 1 + \beta \,(1 - C) \left( 1 - \frac{h_L(s)}{h(s)} \right) \right] dH \right)$$

This places the stated restrictions on $h^L$ and $h$ so that wages remain positive. $C$ also must solve this equation. REMAINS TO CHECK THAT

$$-\ln AC = \beta \left( \int \ln \left[ 1 + \beta \,(1 - C) \left( 1 - \frac{h_L(s)}{h(s)} \right) \right] dH \right) = 0.$$
If this number is not zero, the firm consistently provides either more or less than it promised. It must be zero, but that is yet to be shown. All other conditions of optimality are met, and so we strongly suspect that this condition is met as well. ■